RICE

Course: PHYS 516
Term: Fall 2022
Room: Herman Brown 423
Class: MW, 1:40-2:55pm

Instructor Contact Information

Instructor: Matthew G Baring
Office: Herman Brown 366
Email: baring@rice.edu

Office Hours: Mon 12:00-1:30pm (HBH 310); Zoom: Fri 4:00-5.30pm.

Course Objectives and Learning Outcomes

Mathematical Methods is taught at the graduate level. It gathers together many of the concepts and techniques that are discovered in specialized undergraduate mathematics courses, highlighting their application to physics pedagogy so as to facilitate learning throughout Rice's Physics and Astronomy graduate curriculum. It is therefore to a large extent a survey course, and complete mathematical rigor and depth is not possible in such a condensed format. Yet the material presented provides an excellent starting point for those interested in delving deeper into particular subfields. For graduates, it serves as a grounding for problems they might tackle in their research work. For any undergraduates taking this course, it serves a fast-track in providing the mathematical foundation for future graduate studies in physics or astronomy.

Objectives: The goal is to provide students with a basic understanding of many of the important elements of mathematics, developing a working knowledge of standard mathematical techniques that are useful to physicists and astronomers, especially for problems involving differential equations (ordinary and partial), integrals and their transforms, special functions, complex analysis, series, matrices and tensors. While much of the course is analytic in character by necessity, to maintain contact with contemporaneous computational methods as research tools, there is also a modest numerical component to the course. This will address algorithm development and also provide limited practical applications of numerics for contained problems in applied mathematics. Topics covered include interpolation and curve fitting, iterative solution of ordinary differential equations and finite difference techniques, numerical integration and quadrature, and matrix decomposition methods.

Students will discern how particular techniques can be applied to different physics disciplines. They will learn how to assemble various of elements of mathematics from disparate areas, and integrate them to cohesively to attack a particular problem. Students will be taught to think logically and critically about what are reasonable assumptions in mathematical developments and what are not, and how to evaluate and check derived results. They will also be taught how to think "laterally" in the sense of connecting seemingly unrelated techniques in broaching a given task. This will apply both to analysis methods and also in developing computational skils.

Learning Outcomes: By completing the course, students will be much better equipped to use mathematical tools in their research, thereby enhancing the Rice graduate study experience. They will be in a position to assess posed problems, suggest algorithms for approaching analytic and computational tasks, test their ideas, and develop efficient strategies for reducing a problem to a tractable form, or an endpoint that will facilitate numerical evaluation. They will receive training in communicating their ideas, hypotheses and understanding of select issues to an ensemble of their peers. This is the essence of the research process: think critically, probe, discover, revise one's perspective and tell the broader community of the results and the path taken to get there, and where to go next. This is invaluable training for an array of professions and potential careers down the line, including industry, business, government laboratories and academia.

COVID-19 INFORMATION

This course will be taught simultaneously in person and on-line. Lecture slides will be available to students before each class, and will be posted on the course web pages (see below). Office Hours will be a mix of in person and virtual (via Zoom). Homeworks will be submitted electronically, and directly to the Teaching Assistant, CC'd to Prof. Baring, unless otherwise negotiated. Details of various logistics such as masking elements will be communicated via email throughout the course.

REQUIRED TEXTS AND MATERIALS

Mathematical Methods for Physicists, by George Arfken and Hans Weber (Elsevier, Amsterdam)

We will structure the course around the Sixth Edition (2005) of this book: it is an essential supplement to the lectures. The 7th edition of this book, *Mathematical Methods for Physicists*, by Arfken, Weber & Harris (Elsevier, 2012), can also be used. Notes for course material not contained therein will be provided, for example from Mathews and Walker, listed just below.

SUPPLEMENTARY SUPPORTING TEXT

Mathematical Methods of Physics, by Jon Mathews & Robert L. Walker. (Addison-Wesley, New York)
[Fragments of a more formal nature will come from this text throughout the course.]

EXAMS AND PAPERS

The course assessment will consist of approximately seven to eight to problem sheets, cumulatively constituting 65% of the total grade, one ***open-book*** **mid-term take-home exam** during the semester that constitutes 15% of the grade, and an ***open-book, open notes*** **take-home final exam** at the end of the semester, constituting the remaining 20% of the assessment.

GRADE POLICIES

All parts of the assessment will be graded on a curve, determined commensurately with the overall performance of past students who have taken this course at Rice. This means that present students will not only be measured relative to their peers, but also relative to the long-term body of high-caliber Rice students who have enjoyed the experience of this course.

Late homeworks will automatically receive a **5% reduction in credit**, unless an extension has been negotiated with Prof. Baring. Homeworks that are **4-7 days or more overdue** will be **reduced by 30%** in total credit. Beyond that timeframe, late homeworks will not be graded and score zero. This policy is implemented because it is (i) not fair to other students to have the return of their homeworks on a timeframe that is delayed due to tardiness by any student, and (ii) it is not fair to impose logistical constraints on Prof. Baring in terms of grading elements.

Extensions of homework deadlines must be negotiated with Prof. Baring prior to the original deadline, with the student defining good cause for the extension. The negotiated deadline will substitute for the original one in terms of the aforementioned late penalties.

The final exam must be submitted (electronically) prior to the University-mandated deadline of 5pm on Tuesday, 13^{th} December, the end of Fall Semester, 2022.

Exceptions to these late policies can occur for extenuating circumstances such as student illness, family illness or emergency. In such cases, it is the **student's responsibility to let Prof. Baring know** (ahead of time, if possible) what is going on so that he is not "in the dark." The student will need to (retroactively) document the circumstances.

CLASS ATTENDANCE

The purpose of the lectures is to impart knowledge distilled to its essentials on the subject matter of the course and in a manner more efficient than is afforded by merely reading textbooks and browsing Web sites; these important out-of-class learning paths are intended to supplement, not replace lectures. A central ingredient of this class forum is leveraging the extensive research experience and science connections of the Lecturer, *and this is best done by attending lectures*. The smaller average class size underpins an exceptional learning experience that sets Rice apart from many of its peer institutions. **Students should take advantage of this opportunity by habitually attending classes**; their learning curve will be enhanced by such dedication.

Classes will be given in dual delivery format. The live classes in Herman Brown 423 will be presented via Zoom so that students not able to be in the classroom can attend in real time as suits. The presentations will be recorded so that students may view them asynchronously, providing the ability to meet all the needs of students as circumstances vary during the semester.

ABSENCE POLICIES

Infrequent absences are not a problem. If a student is noted to be absent for an extended period of time, or frequently, the student must communicate with Prof. Baring the reasons of the absence(s). Such cases normally will degrade the efficiency of learning for the student. Again, if there are extenuating circumstances such as student illness or family illness or emergency, accommodations will be made, and Prof. Baring should be informed. Otherwise, concerning any absence, it is the student's responsibility to read through the pertinent notes/materials provided to guide their study accordingly. Prof. Baring is under no obligation to "re-lecture" such material during office hours.

RICE HONOR CODE

In this course, all students will be held to the standards of the Rice Honor Code, a code that you pledged to honor when you matriculated at this institution. If you are unfamiliar with the details of this code and how it is administered, you should consult the Honor System Handbook at http://honor.rice.edu/honor-system-handbook/. This handbook outlines the University's expectations for the integrity of your academic work, the procedures for resolving alleged violations of those expectations, and the rights and responsibilities of students and faculty members throughout the process.

The mid-term and final (take-home) exam questions are not to be discussed at all with other students, faculty or graders, and are subject to the provisions of the Rice Honor Code. Please verify this by **writing the word pledge and your signature on each exam**. Questions specifically about exams should be directed only to Prof. Baring.

DISABILITY SUPPORT SERVICES

If you have a documented disability or other condition that may affect academic performance you should: 1) make sure this documentation is on file with Disability Resource Center (Allen Center, Room 111 / adarice@rice.edu / x5841) to determine the accommodations you need; and 2) contact Prof. Baring to discuss your accommodation needs *during the first two weeks of class*.

Any letter from DRC to the instructor requesting accommodations for the student should be delivered in the first three weeks of semester (either via email or hardcopy by campus mail), so that Prof. Baring can plan accordingly.

TITLE IX RESPONSIBLE EMPLOYEE NOTIFICATION

Rice University cares about your wellbeing and safety. Rice encourages any student who has experienced an incident of harassment, pregnancy discrimination, gender discrimination or relationship, sexual, or other forms interpersonal violence to seek support through The SAFE Office. Students should be aware when seeking support on campus that most employees, including myself, as the Instructor, are required by Title IX to disclose all incidents of non-consensual interpersonal behaviors to Title IX professionals on campus who can act to support that student and meet their needs. For more information, please visit the http://safe.rice.edu/ webpage or email titleixsupport@rice.edu .

SYLLABUS

The detailed syllabus below gives the layout of the course material. For further information, such as scheduling, pointers to related chapters in the Required Text, etc., see the PHYS 516 course web pages at https://www.ruf.rice.edu/~baring/phys516/phys516_syllabus.html .

Complex Analysis

- Uses of Complex Variables
- Functions of a Complex Variable
  - Complex Functions of a Complex Variable
- Differentiation
  - The Cauchy-Riemann Relations
- Integration
  - The Cauchy Integral Theorem
  - The Cauchy Integral Formula
- Taylor and Laurent Series
- The Theorem of Residues

Interpolation, Fitting and Root Solving

- Lagrange Interpolation
- Cubic Splines
- Least Squares Fitting
- Padé Approximations
- Root Solving
  - Numerical Bisection

Ordinary Differential Equations

- Linear, First Order Ordinary Differential Equations
- Linear ODEs with Constant Coefficients
- Linear ODEs with Non-Constant Coefficients
- Power Series Solutions: Frobenius' Method
- WKB Approximation
- Numerical Solutions: Euler's Method
- Runge-Kutta and Finite Difference Techniques

Special Functions I: Gamma, Beta and Zeta Functions

- The Gamma Function
  - Definitions and Simple Properties
  - Polygamma Functions
  - Stirling's Series
- Functions Related to the Gamma Function
  - Beta and Incomplete Gamma Functions
  - Riemann Zeta Function

Special Functions II: Bessel Functions

- Bessel Functions J_v and N_v
  - Generating Function and Recurrence Relations
  - An Integral Representation
  - Neumann Functions and Wronskians
- Modified Bessel Functions I_v and K_v
  - Integral Representations and Asymptotic Series

Special Functions III: Orthogonal Polynomials

- The Legendre Polynomials P_n
  - Legendre Functions of the 2nd Kind Q_n
  - Associated Legendre Functions
- Hermite Polynomials H_n
- Laguerre Functions
- Hypergeometric Functions

Partial Differential Equations

- Global Characterization of PDEs
- Solution of Linear Second Order PDEs
  - Separation of Variables
  - Transform Techniques

Eigenfunctions

- Eigenfunctions and Eigenvalues for Linear PDEs
- Homogeneous Problems: Green's Functions
  - Green's Functions for Initial Conditions

Integral Equations

- Fredholm and Volterra Equations
- Transform Techniques
- Neumann Series Solutions
- Hilbert-Schmidt Theory

Syllabus Change Policy

This syllabus is a general guide for the course and is subject to change without advanced notice.