• The impact of pressure buoyancy in disrupting localized collapse in the radiation era is potentially crimped by the influence of photon diffusion beyond the Jeans lengthscale. This effect is known as **Silk damping** of radiation pressure in overdense regions, and is due to electron Thomson scattering. If $d_s = 1/(n_{\rm b}\sigma_{\rm T})$ is the effective scattering mean free path, assuming that $n_e = n_{\rm b}$, then a simple random walk analysis indicates that the photon scattering/diffusion [`sketch`] radial scale is

$$\lambda_s \sim \sqrt{\frac{d_s ct}{3}} = \sqrt{\frac{ct}{3n_{\rm b}\sigma_{\rm T}}} \sim \frac{7.6\,{\rm Gpc}}{\Omega_{\rm b}^{1/2}\Omega_{\rm rad}^{1/4}}\,\frac{1}{(1+z)^{5/2}} \quad . \tag{30}$$

Into this, we have inserted $t = (2H_0\sqrt{\Omega_{\rm rad}})^{-1}(1+z)^{-2}$ for a radiation-dominated universe, and $n_{\rm b} = \Omega_b\rho_c(1+z)^3/m_p$ for the baryonic matter content. If $\lambda_s > \lambda_{\rm J} \propto (1+z)^{-2}$, then obviously photon leakage inhibits pressure support and gravitational collapse is enhanced. We find

$$\frac{\lambda_s}{\lambda_{\rm J}} \sim \frac{0.4}{\sqrt{1+z}} \quad , \tag{31}$$

with $(\lambda_s/\lambda_{\rm J})^2 \propto Gm_p/[H_0 c\sigma_{\rm T}]$. Clearly, at $z > z_{\rm eq}$, the Silk damping radius is much smaller than the ideal radiation-era Jeans length, both being computed in terms of averages from the Universe.

∗ $\Rightarrow$ perturbation growth can proceed only on the Silk radius λ_s.

• The growth mass scale is therefore defined by the **Silk mass**

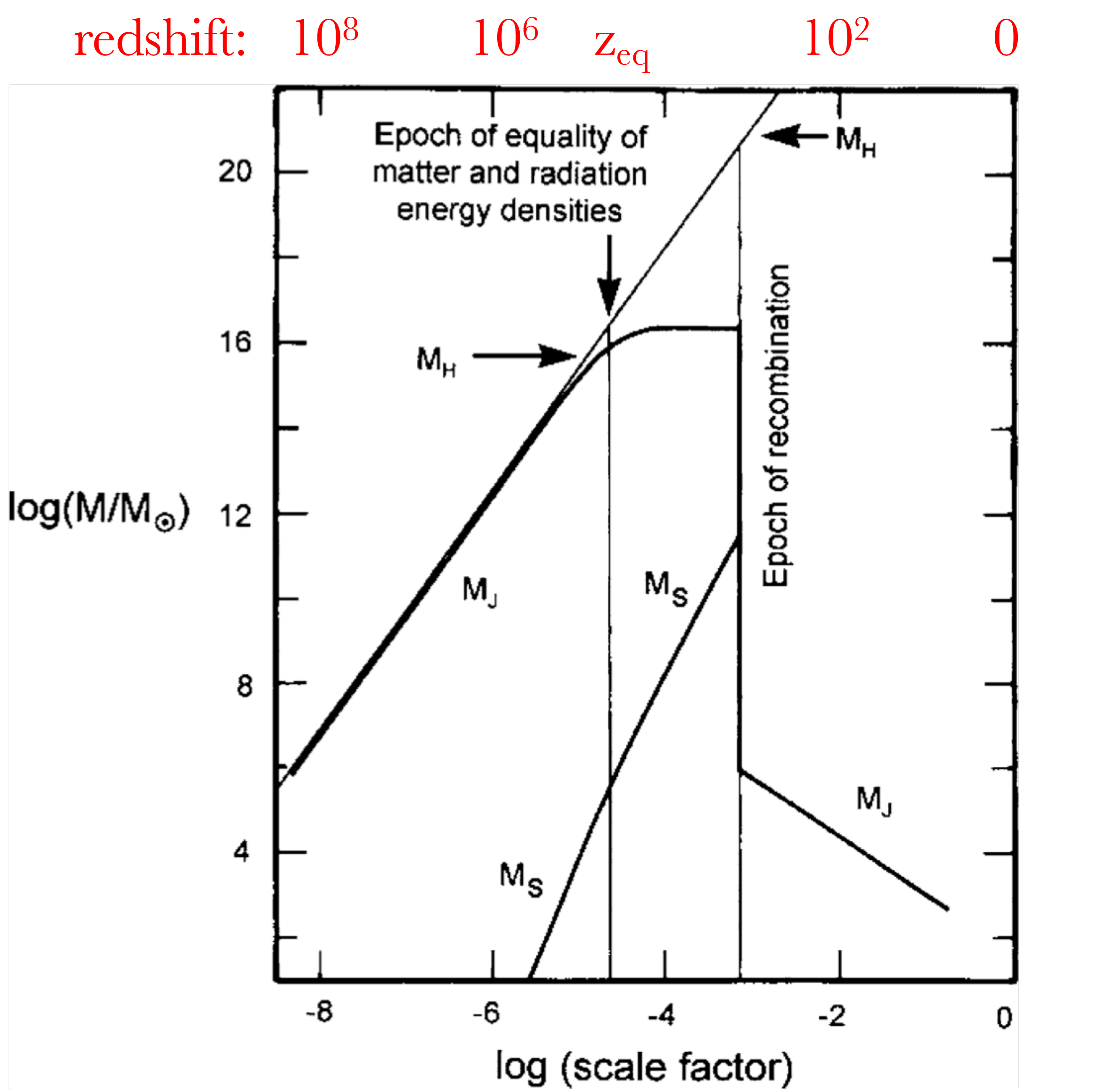
$$M_s = \frac{4\pi}{3}\lambda_s^3\rho_{\rm b} \sim \frac{2.3\times 10^{27}\,M_\odot}{(1+z)^{9/2}} \quad . \tag{32}$$

For $z \sim 10^4$, this gives perturbations on the scale of $\sim 10^9 M_\odot$, not vastly different from the globular cluster Jeans scale for the matter-dominated era.

∗ Accordingly, Silk damping helps bring structure evolution prior to equipartition more closely into line with structure evolution after recombination.

Jeans Mass Evolution

- Evolution of Jeans mass M_J with scale factor in radiation and matter-dominated epochs.
  - Observe the abrupt drop in M_J at the z_{eq} boundary.
- Also shown are the horizon scale mass M_H and a mass M_S associated with electron-photon scattering scales (Silk damping).
- Fig. 12.1 of Longair book.
- Note the high value of z_{eq} in this older presentation.



11. THE VERY EARLY UNIVERSE

Matthew Baring – Lecture Notes for ASTR 360, Spring 2025

1 The Planck Era

C & O, pp. 1233-36

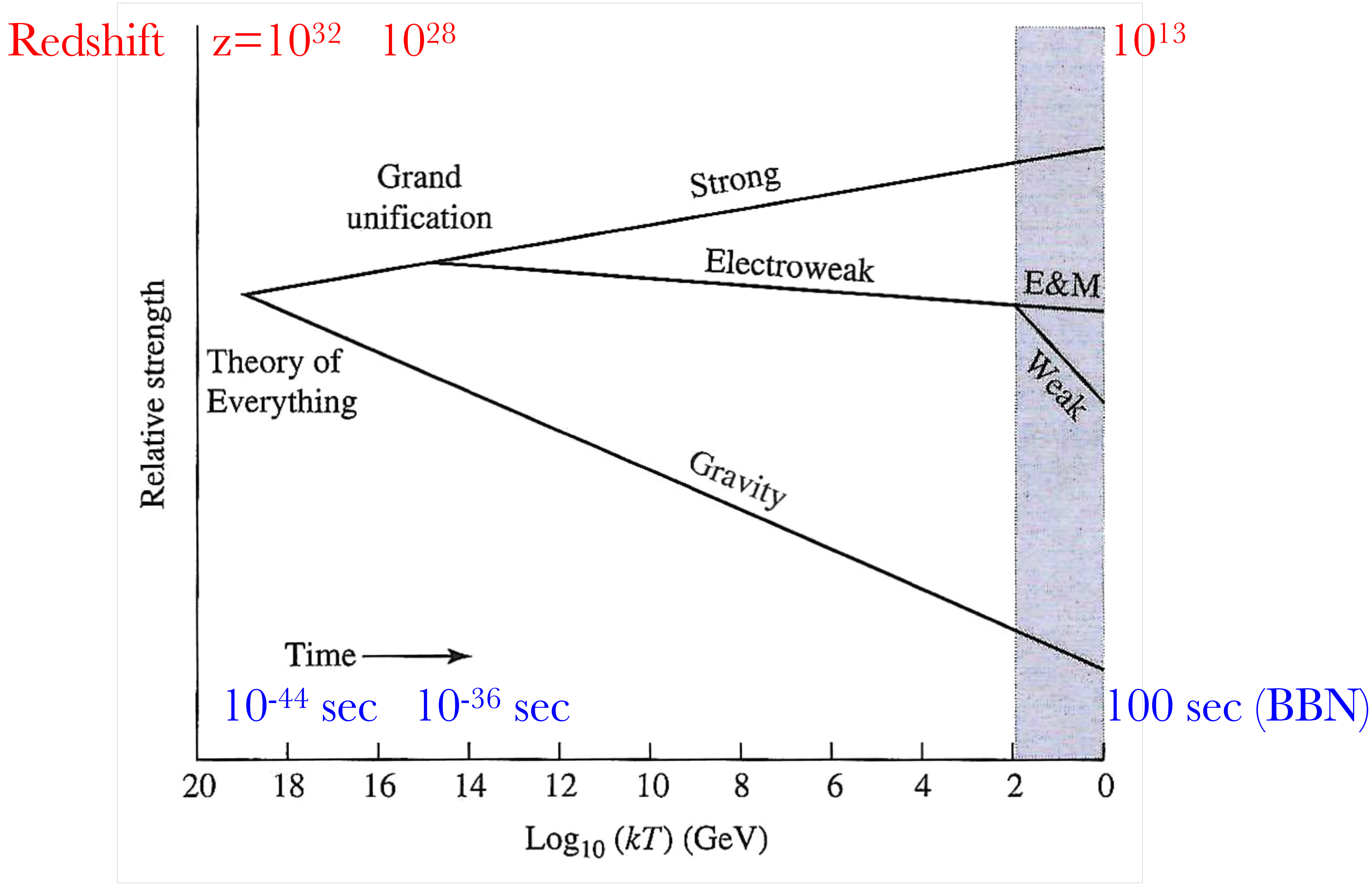
Prior to Big Bang Nucleosynthesis, the universe did not consist of nuclei as we know them. As z increases, the matter transitioned first to a quark-gluon plasma in a dense phase beyond what might currently exist in neutron star interiors. The current understanding of **The Standard Model** (SM) of particle physics suggests that as one goes back in time, a major phase transition will arise when the *electromagnetic and weak interactions unify* in becoming of comparable strength. Prior to this, the physics is unknown and there is a general expectation that the three interactions, strong, electromagnetic and weak, will merge in strength in a **Grand Unified Theory** (GUT). This is still, as yet, poorly understood as it is well beyond the scope of exploration in terrestrial laboratories. Add in gravity, decoupling at a much earlier timescale, and one has a **Theory of Everything** (TOE).

Plot: Timeline for the Unification of the 4 Fundamental Forces

The epoch the TOE is also known as the **Planck Era**. It is the epoch where **quantum gravity** should be operable, superseding the classical theory developed by Einstein. The content of the universe would then be exotic particles that are likely relativistic, and of much greater mass than those known in the SM. The **de Broglie wavelength** (Compton wavelength) $\hbar/mc$ of these particles would satisfy

$$\frac{\hbar}{mc} \sim R_{\rm s} = \frac{2Gm}{c^2} \quad , \tag{1}$$

Decoupling and Evolution of the Four Fundamental Forces



- Carroll & Ostlie, Figure 30.2.

i.e., would be of the order their Schwarzschild radius, in order for gravity to be in the quantum domain. This establishes a characteristic mass for the scale of quantum gravity:

$$m_P \;=\; \sqrt{\frac{\hbar c}{G}} \;\approx\; 2.177 \times 10^{-5}\,\mathrm{g} \quad , \tag{2}$$

known as the **Planck mass**. This is 19 orders of magnitude larger than the proton mass, so that the natural energy scale is given by

$$m_P c^2 \;\equiv\; \sqrt{\frac{\hbar c^5}{G}} \;\approx\; 1.96 \times 10^{16}\,\mathrm{erg} \;=\; 1.22 \times 10^{19}\,\mathrm{GeV} \quad , \tag{3}$$

leading to a temperature scale of $T \gtrsim 10^{32}\,\mathrm{K}$, i.e. a redshift of $z \sim 10^{32}$!

$*$ Since $a(t) \propto t^{1/2}$ in a cosmology with a relativistic equation of state, one deduces that $t \propto (1+z)^{-2}$ describes the age of the nascent universe. This implies that the Planck (TOE) epoch was at an age $\sim 10^{-58}$ of the 300,000 year value realized at recombination ($z_{\rm rec} \sim 10^3$), i.e., of the order of 10^{-45} sec. With a refined calculation, it is actually a factor of ~ 50 larger.

Plot: Timeline of the Universe

- The corresponding Compton wavelength is

$$\ell_P \;=\; \frac{\hbar}{m_P c} \;=\; \sqrt{\frac{\hbar G}{c^3}} \;\approx\; 1.62 \times 10^{-33}\,\mathrm{cm} \quad . \tag{4}$$

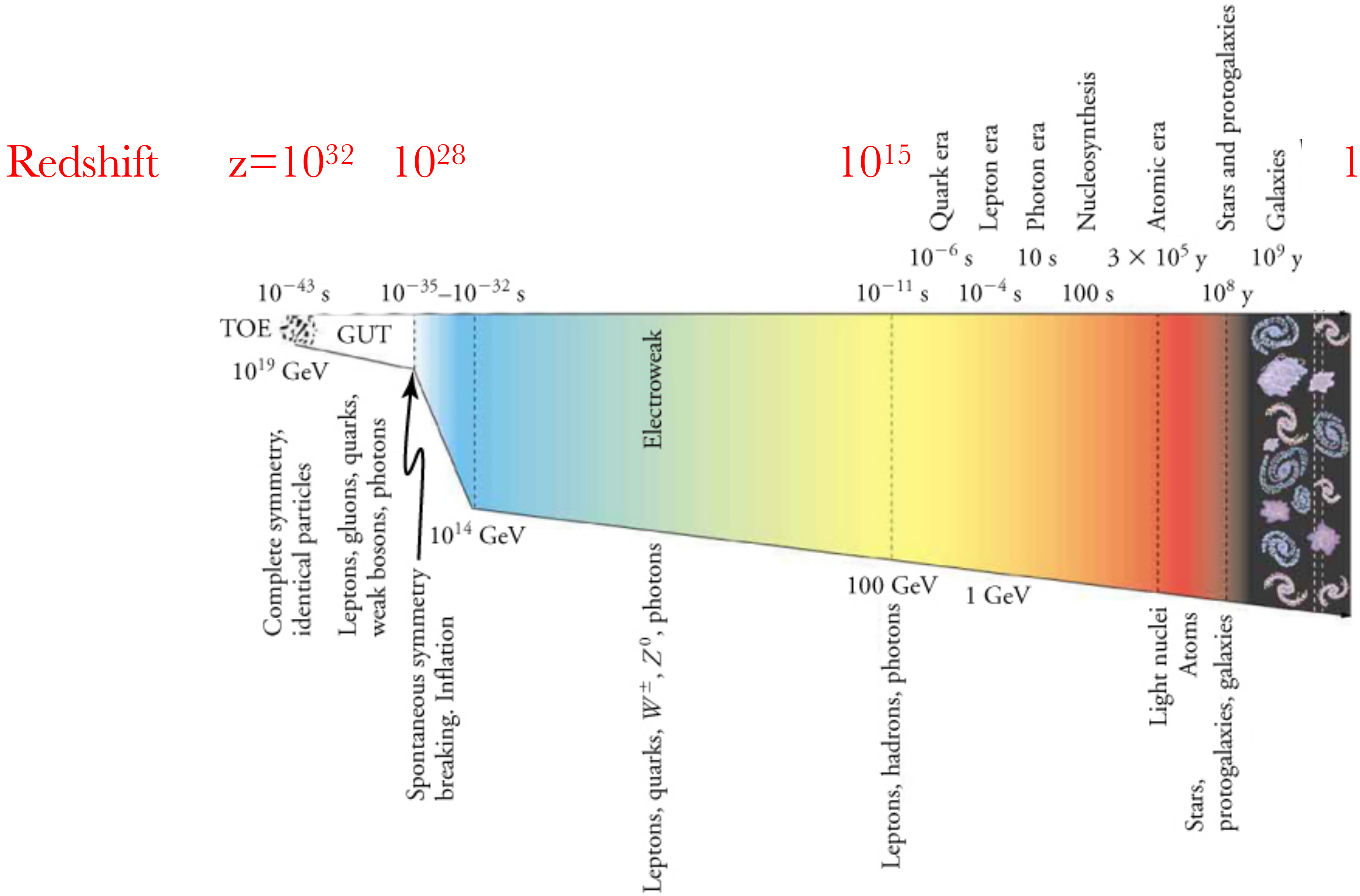
This quantum gravity lengthscale is know as the **Planck length**. All these scales define the epoch of **ultimate unification**, i.e. the TOE domain. The energy density of the very early universe can then be estimated:

$$U_P \;\sim\; \frac{m_P c^2}{\ell_P^3} \;=\; \frac{c^7}{G^2 \hbar} \;\approx\; 2.9 \times 10^{117}\,\mathrm{GeV\ cm^{-3}} \quad . \tag{5}$$

This is of the order of 10^{78} times the energy density of a proton!

- The decoupling of gravity first, and then the strong interaction, arises due to spontaneous symmetry breaking that constitutes changes in the field equations (unknown at these times) for the respective forces.

Timeline of the Universe



2 Inflationary Cosmology

2.1 Key Problems with Cosmology

Subsequent to the discovery of the CMB, and even prior to it, issues arose the standard theory of the Big Bang. These required a new picture to resolve them, and this was provided by Alan Guth's idea of **inflation**, born in 1980. Two of the key problems highlighted here are:

• *Why is the cosmic background radiation so smooth?* Our structure building studies have indicated that while matter+radiation pressure can enable communication within large portions of the pre-recombination universe, they cannot provide causal connection on the largest scales. The CMB looks the same on opposite sides of the sky, both in delivering $\delta T/T \sim 10^{-5}$, and also in the measures of the power spectra from WMAP and Planck. What prior manifestation in the Universe enabled this causal connection? This is known as the **horizon problem**.

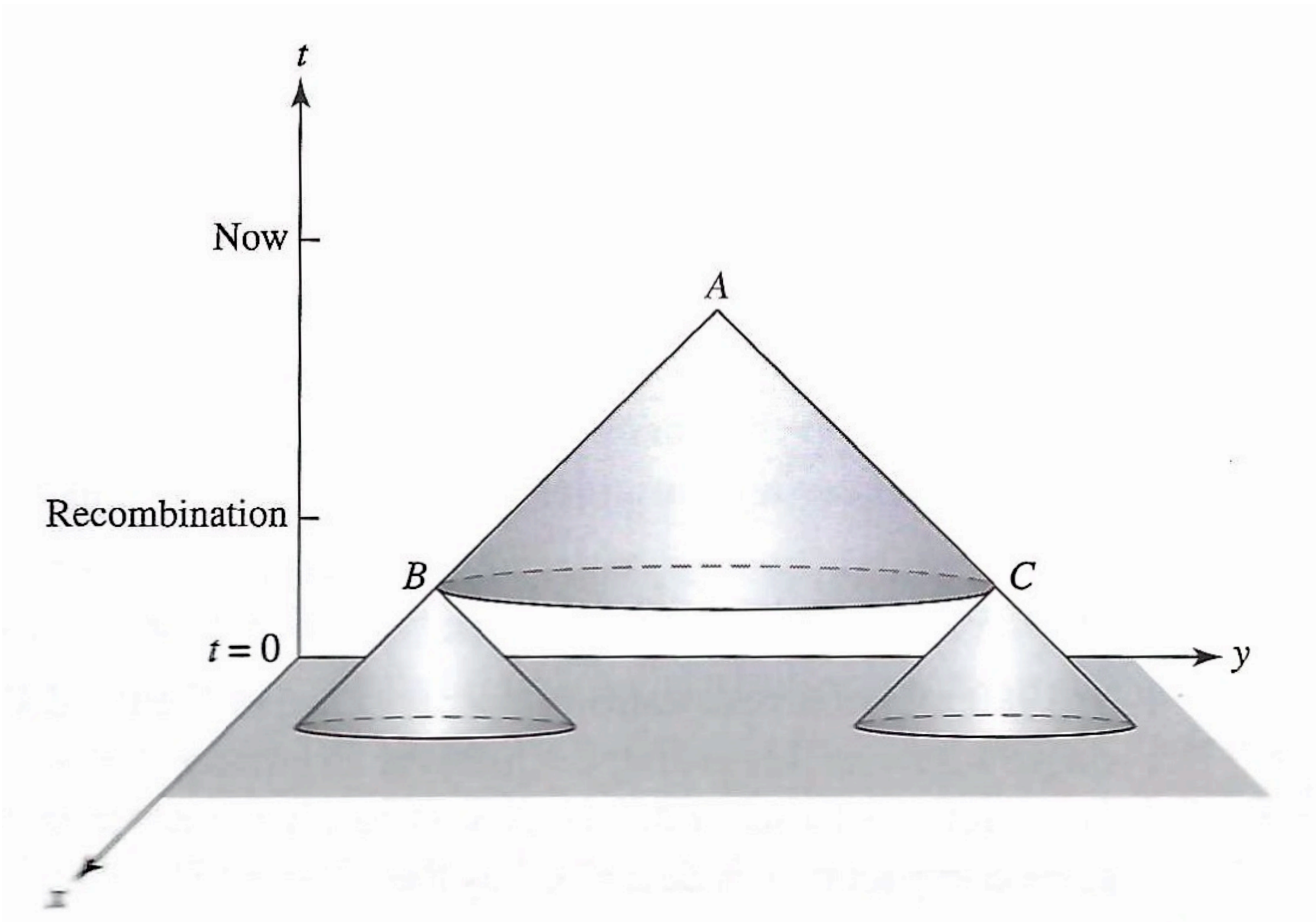
Plot: Cones of Causality and the Horizon Problem

• *Why is the Universe so nearly flat ($\Omega_0 \approx 1$)?* As has been identified in a homework problem, the Friedmann equation can be expressed in the form

$$\frac{1}{\Omega_0} - 1 \;=\; \left(\frac{1}{\Omega} - 1\right)(1+z) \quad . \tag{6}$$

It is then simply deduced that if $\Omega \neq 1$ at some early epoch with $z \gg 1$, then either $\Omega_0 \ll 1$ or $\Omega \gg 1$ at the present. This implies that the Universe was incredibly fine-tuned to $\Omega \approx 1$ well prior to big bang nucleosynthesis. The alternative is such under dense or over dense circumstances that would respectively prohibit galaxy formation or would rapidly close the Universe and drive it quickly to a Big Crunch. This is know as the **flatness problem**.

Cones of Causality: the Horizon Problem



- Carroll & Ostlie, Figure 30.3.

2.2 The Inflation Picture

While there are many variants, the basic idea of inflation is that not long after the Planck epoch, when the Universe had established thermal equlibrium that was causally communicated throughout, the Universe underwent a period of rapid expansion, with $a(t)$ being exponential in time. This episode smoothed out the cosmos and drove it to $\Omega = 1$, thereby solving the flatness problem. As it rapidly made the Universe much larger, with *an increase in a by roughly 43 orders of magnitude*, since distant portions originated from a causally-connected progenitor universe, it thereby solved the horizon problem.

Plot: Scale Factor and Temperature Evolution during Inflation

The seed for the inflationary expansion could be the energy of the **quantum vacuum**. Such a vacuum was posited in early quantum theory to describe the minimum energy (**ground state**) of a system; e.g. the simple harmonic oscillator. This vacuum possesses a sea of virtual particles, and in special circumstances, they facilitate processes that would otherwise not be allowed in normal quantum mechanics. An example is the production of electron positron pairs in QED when the vacuum is polarized by a strong magnetic or electric field in neutron stars. The energy of the vacuum can thus be ephemerally tapped, and this is so for **Hawking radiation** near black holes.

- The baseline energy density of the vacuum for the Planck era has already been calculated:

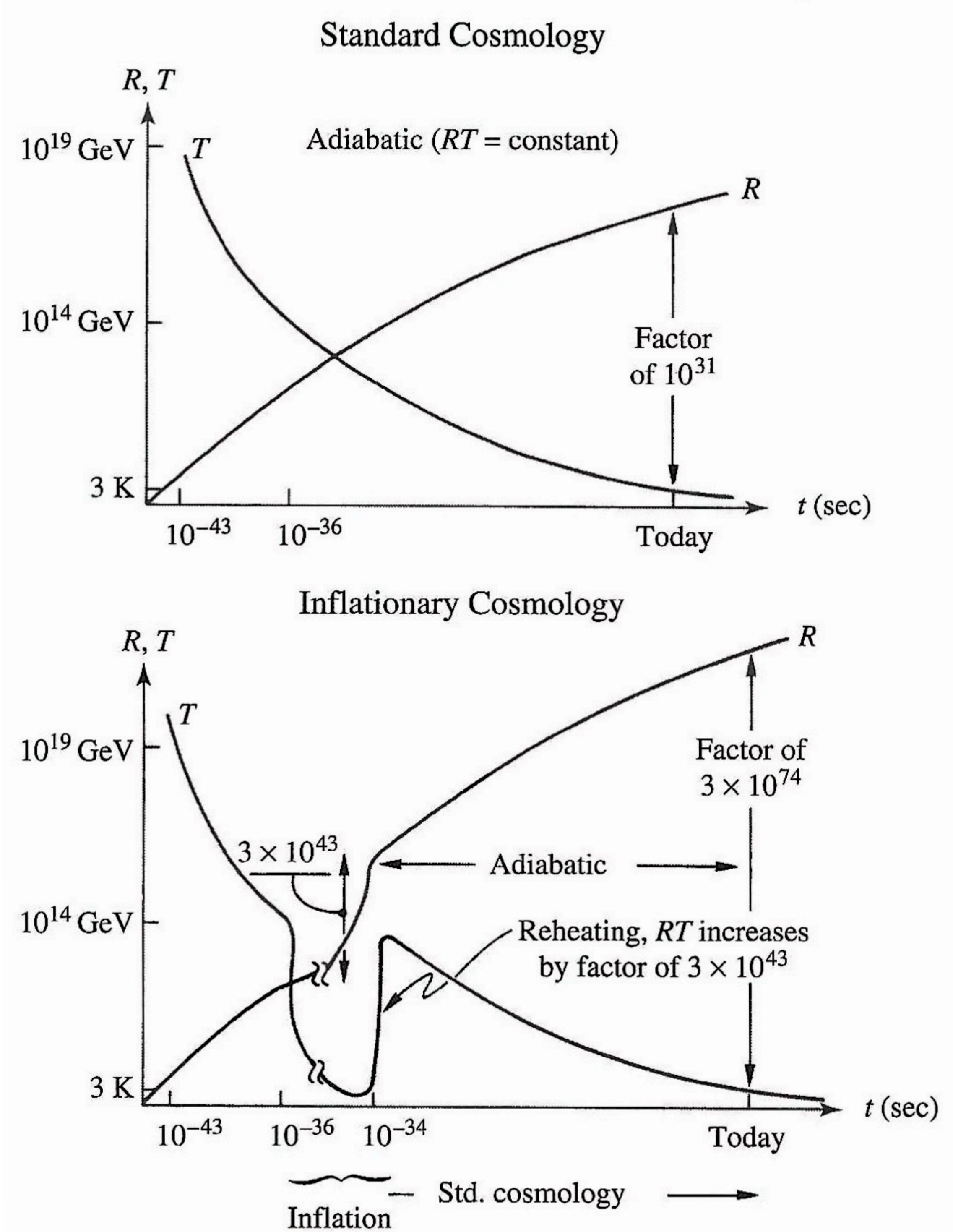
$$U_P \sim \frac{m_P c^2}{\ell_P^3} = \frac{c^7}{G^2 \hbar} \approx 2.9 \times 10^{117} \,\text{GeV cm}^{-3} \quad . \tag{7}$$

Suppose some fraction f of this is included in the Friedmann equation as a constant term, i.e. via $U_{\rm vac} = f\, U_P$. This contribution therefore mimics that of the cosmological constant. The pressure of this vacuum is negative as it has a repulsive influence on cosmological dynamics due to Heisenberg's uncertainty principle. Thus $P_{\rm vac} = -f\, U_P$. The dynamical equation resulting from Einstein's field equations that leads to the Friedmann equation becomes

$$\frac{d^2 R}{dt^2} = -\frac{4\pi}{3}\, G \left(\frac{U_{\rm vac}}{c^2} + \frac{3P_{\rm vac}}{c^2} \right) R = \frac{8\pi G}{3c^2}\, U_{\rm vac}\, R \tag{8}$$

in a flat ($k = 0$) universe with a relativistic EOS.

Evolution due to Inflation



- Carroll & Ostlie, Figure 30.4. From the book by Kolb & Turner *The Early Universe*.

This yields an exponentially growing solution with an inflation timescale τ_i :

$$R(t) \;=\; R(t_i)\, e^{t/\tau_i} \quad , \quad \tau_i \;=\; \sqrt{\frac{3c^2}{8\pi G\, U_{\rm vac}}} \;=\; \sqrt{\frac{3}{8\pi f}}\, \frac{\ell_P}{c} \tag{9}$$

Thus we determine rapid expansion on a timescale of

$$\tau_i \;\approx\; \frac{1.8\times 10^{-44}}{\sqrt{f}}\, \text{sec} \quad . \tag{10}$$

The expansion is adiabatic and so cools the Universe incredibly rapidly, by around 32 orders of magnitude in temperature.

∗ At or near the end of inflation, there is a rapid (responsive) **reheating** of the cosmos by 27 orders of magnitude in temperature before the Universe sets itself on the more standard evolutionary track towards decoupling of the weak and electromagnetic interactions.

∗ Observe that the Planck energy density is 120 order of magnitude larger than what would describe dark energy, and so any connection between inflation and dark energy is not established.

• Inflationary cosmologists have posited that the true Planck vacuum is not what initiated inflation, but rather a **false vacuum** (FV) that is not in a minimum energy state. This FV can release latent heat in a manner analogous to the supercooling of liquids. This phase transition begins after the GUT epoch and so GUT theory determines that the appropriate value for the FV energy density corresponds to

$$f \;\sim\; 10^{-16} \quad\Rightarrow\quad \tau_i \;\sim\; 10^{-36}\, \text{sec.} \tag{11}$$

The textbook details a toy calculation for the duration Δt_i of inflation. Yet, solving the horizon problem dictates that the net expansion of 43 orders of magnitude should establish

$$\frac{R(t_i+\Delta t_i)}{R(t_i)} \;\equiv\; e^{\Delta t_i/\tau_i} \sim 10^{43} \quad\Rightarrow\quad \Delta t_i \;\sim\; 10^{-34}\, \text{sec} \quad . \tag{12}$$

Thus, the inflationary epoch is over in an instant, much shorter than the end of this course!