9. BIG BANG NUCLEOSYNTHESIS

Matthew Baring – Lecture Notes for ASTR 360, Spring 2024

1 Primordial Nucleosynthesis by the pp Chain

One of the key markers of the Big Bang model is primordial nucleosynthesis of low mass elements. It was well known that stellar nucleosynthesis models underproduced helium and other light elements relative to observed average abundances. Compact, high temperature epochs in the early universe are inevitable in a Big Bang scenario $\Rightarrow$ rampant nuclear burning.

• During the first few minutes of evolution of the universe, nucleosynthesis is prolific and succeeds in generating the primordial abundances of light elements that we observe today via a complex network of nuclear interactions.

Plot: Network of Primordial Nucleosynthetic Reactions

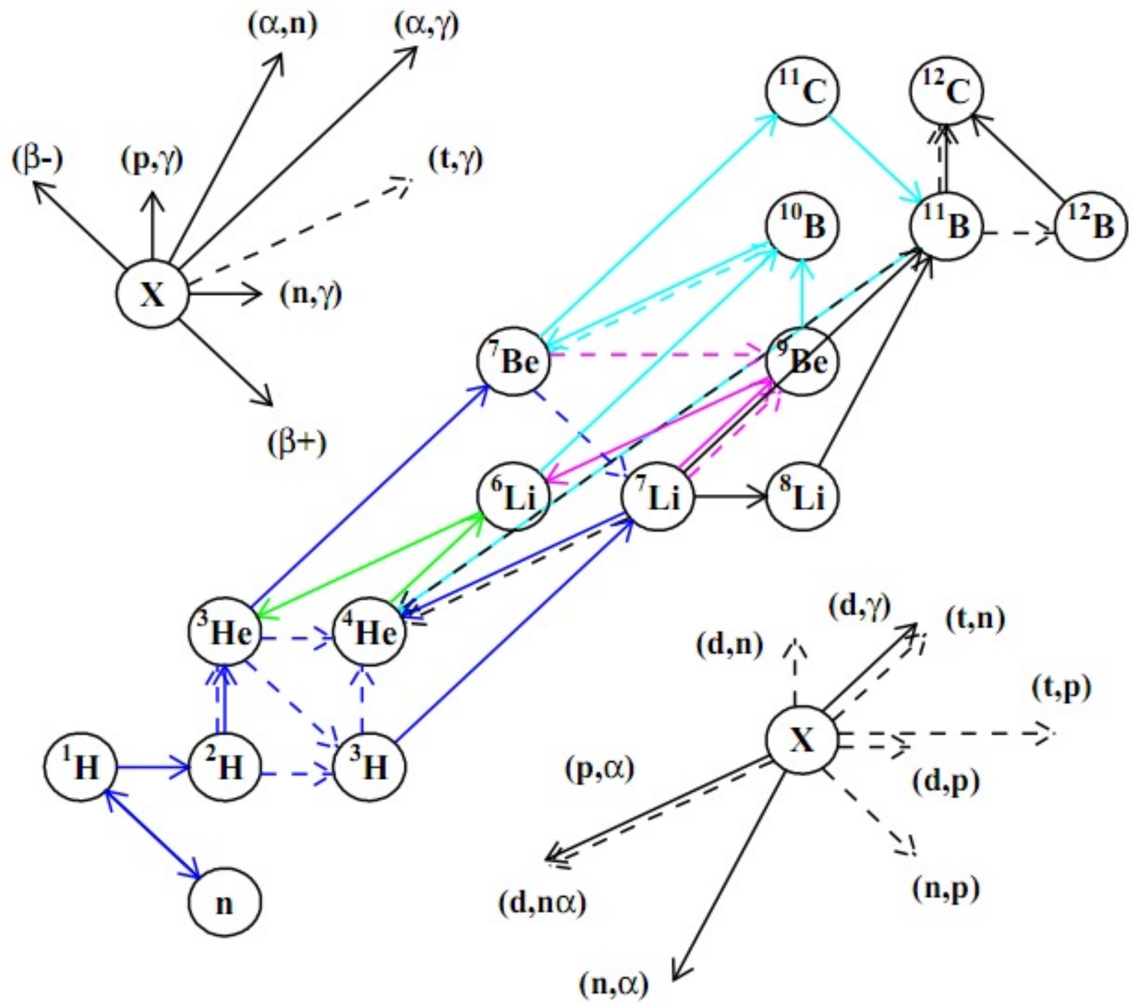
• Numerical computations of light element abundances in an evolving early universe generally take into account many paths in this network up to isotopes of C, N and O.

• At redshifts $z \gtrsim 10^{10}$, when $kT \gtrsim 3\,\text{MeV}$, photodissociation breaks up heavy elements into their constituent elementary particles: $n, p, \nu, \gamma, e^{\pm}$ are the predominant species. Subsequent to this, the ν background forms.

• The primary processes in this **proto-nuclear soup** are the relatively slow *weak interactions* that do not build nuclei:

$$\begin{aligned} e^- + p &\leftrightarrow n + \nu \\ \bar{\nu} + p &\leftrightarrow n + e^+ \\ n &\leftrightarrow p + e^- + \bar{\nu} \ . \end{aligned} \tag{1}$$

Primordial Nucleosynthesis Network



Adapted from depiction in Fig. 2 of Schramm & Wagoner (1977)

- Reduced nuclear synthesis network displaying important reactions for D, He, Li, Be and beyond. Reaction types are indicated in the upper left and lower right.
- From Coc et al., ApJ 744, 158 (2012) [arXiv: 1107.1117], Fig. 22 therein.

1.1 Neutrino Decoupling

Early in the era of primordial nucleosynthesis, once quarks have morphed into . hadrons, the first species to **decouple** or **freeze out** are the neutrinos, principally due to their weakly interacting character. Electron-positron pairs still abound so that the dominant weak interactions are

C & O, pp. 1172-3

$$e^- + e^+ \leftrightarrow \nu_e + \overline{\nu}_e \ , \quad e^\pm + \nu_e \leftrightarrow e^\pm + \nu_e \ , \quad e^\pm + \overline{\nu}_e \leftrightarrow e^\pm + \overline{\nu}_e \quad . \quad (2)$$

The cross sections generally scale as $\sigma \sim 3 \times 10^{-45} (E_\nu / m_e c^2)^2 \ \text{cm}^2$. The neutrinos are relativistic in this epoch, so $E_\nu \sim 3kT \approx 3k \times 2.73(1+z)$ since their temperature will be tied to radiation then.

The pairs are in chemical equilibrium with the neutrinos and photons, and so their density traces that of the photons, i.e. $n_\pm \sim n_\gamma \sim 420(1+z)^3 \,\text{cm}^{-3}$, remembering the present CMB photon density that was obtained from the Planck spectrum: $n_\gamma = 0.244\, \Theta^3 / \lambda\!\!\!^-{}^3$ for $\Theta = kT/m_e c^2$.

The timescale for the weak interactions is $t_{\text{w}} \sim (\sigma n_\pm c)^{-1}$, with $n_\pm \propto a^{-3} \propto (1+z)^3$. Accordingly, this **neutrino decoupling timescale** has a strong dependence on redshift $z = 10^{10} z_{10}$:

$$t_{\text{w}} \ \approx \ \frac{1}{n_\pm \sigma c} \ \approx \ \frac{1.4 \times 10^{49}}{(1+z)^5} \ \text{sec} \ \sim \ \frac{0.14}{(z_{10})^5} \ \text{sec} \quad . \qquad (3)$$

Observe that since $\rho_c / m_p \approx 5.8 \times 10^{-6}$ protons per cc (now, with $\rho_c = 3H_0^2 / 8\pi G$) for $h = 0.72$, the number density of pairs in this early epoch exceeds that for baryons by a factor of $420/[5.8 \times 10^{-6}] = 7.2 \times 10^7$.

- In this early epoch that is dynamically radiation-dominated, the age $t(z)$ of the universe is given by

$$H_0 t(z) \ = \ \int_z^\infty \frac{dz'}{(1+z')\, E(z')} \quad , \quad E(z') \ \approx \ \sqrt{\Omega_{\text{rad}}}\, (1+z')^2 \quad , \qquad (4)$$

where the mass/energy evolution factor $E(z)$ simplifies considerably. Thus,

$$t(z) \ \approx \ \frac{1}{2H_0 \sqrt{\Omega_{\text{rad}}}} \, \frac{1}{(1+z)^2} \ \sim \ \frac{0.31}{(z_{10})^2} \ \text{sec} \quad , \qquad (5)$$

where $\Omega_{\text{rad}} \sim 4.8 \times 10^{-5}$. The Universe ages more slowly as z drops than the weak interaction timescale grows, and thus decoupling does occur.

1.2 Neutron Freeze-out

Neutrons are the next species to decouple from the soup of nuclear interactions. The n/p ratio is controlled by thermodynamic equilibrium via a Boltzmann factor:

$$\frac{n_n}{n_p} \;=\; e^{-Q_n/kT} \quad , \quad Q_n \;=\; (m_n - m_p)c^2 \;=\; 1.2934 \;\; \text{MeV} \quad . \tag{6}$$

The balance energetically favors proton generation due to their lower energy/mass (i.e. chemical potential).

$*$ Hence **neutron freeze-out**, or decoupling, from the proto-nuclear soup occurs when kT drops below 1 MeV, i.e. for $T \lesssim 10^{10}\,$K.

Prior to the creation of light elements, the neutron abundance is described by a differential equation that expresses a non-equilibrium Saha-type calculation. The seminal work on this topic is the paper by Schramm & Wagoner (SW). The neutron number density evolution satisfies

$$\frac{dn_n}{dt} \;=\; \lambda_p n_p - \lambda_n n_n \quad , \tag{7}$$

where the λ coefficients are given by

$$\lambda_n \;=\; \Lambda_{ne^+} + \Lambda_{n\nu} + \frac{1}{\tau_n} + \frac{1}{\tau_{exp}} \tag{8}$$

for the neutron absorption terms, decay (e-lifetime of τ_n) and expansion losses (through τ_{exp}), and

$$\lambda_p \;=\; \Lambda_{pe^-} + \Lambda_{p\bar{\nu}} \tag{9}$$

for the proton attrition (=neutron production) terms. In what follows, the expansion dilution term in Eq. (8) will be neglected. However, note that it is generally significant $\Rightarrow$ a dynamic, non-equilibrium evolution is appropriate.

$*$ The production of deuterium via $n + p \leftrightarrow d + \gamma$ adds another term to both Eqs. (8) and (9); this is not detailed here, but discussed shortly.

• Schramm and Wagoner give asymptotic forms for multitudinous nuclear reactions in the exponential (**quantum tunneling**) regime appropriate to relatively "low" temperatures. For example, with

$$\mu \;=\; \frac{m_e c^2}{kT} \;=\; \frac{5.93}{T_9} \quad , \tag{10}$$

with T_9 being the universe's temperature in units of $10^9\,$K, the interaction rate coefficients can be written

$$\begin{aligned} \lambda_n &\approx \frac{0.98}{\tau_n}\left\{1+\frac{0.565}{\mu}-\frac{6.382}{\mu^2}+\frac{11.108}{\mu^3}+\frac{36.492}{\mu^4}+\frac{27.512}{\mu^5}\right\}\ \mathrm{s}^{-1} \\ \lambda_p &\approx \frac{0.98}{\tau_n}\left\{\frac{5.252}{\mu}-\frac{16.229}{\mu^2}+\frac{18.059}{\mu^3}+\frac{34.181}{\mu^4}+\frac{27.617}{\mu^5}\right\}e^{-2.531\mu}\ \mathrm{s}^{-1} \end{aligned} \tag{11}$$

These both display $\approx 1/\tau_n$ character that reflects neutron decay, though λ_p contains no neutron decay and so possesses an exponential factor.

Plot: The Nucleosynthetic Era: Wagoner, Fowler & Hoyle (1967)

• Clearly, as $\mu \to \infty$, the decay rate coefficients simplify, and we can write

$$\frac{\lambda_p}{\lambda_n} \;\approx\; \frac{5.25}{\mu}\, e^{-2.531\mu} \quad . \tag{12}$$

This is the equilibrium solution for a non-evolving n/p density ratio, from Eq. (7). It follows that neutron decline or freeze-out occurs for

$$\frac{n_n}{n_p} \lesssim 0.5 \quad \Leftrightarrow \quad \frac{\lambda_p}{\lambda_n} \lesssim 0.5 \quad \Leftrightarrow \quad T_9 \lesssim 6 \quad . \tag{13}$$

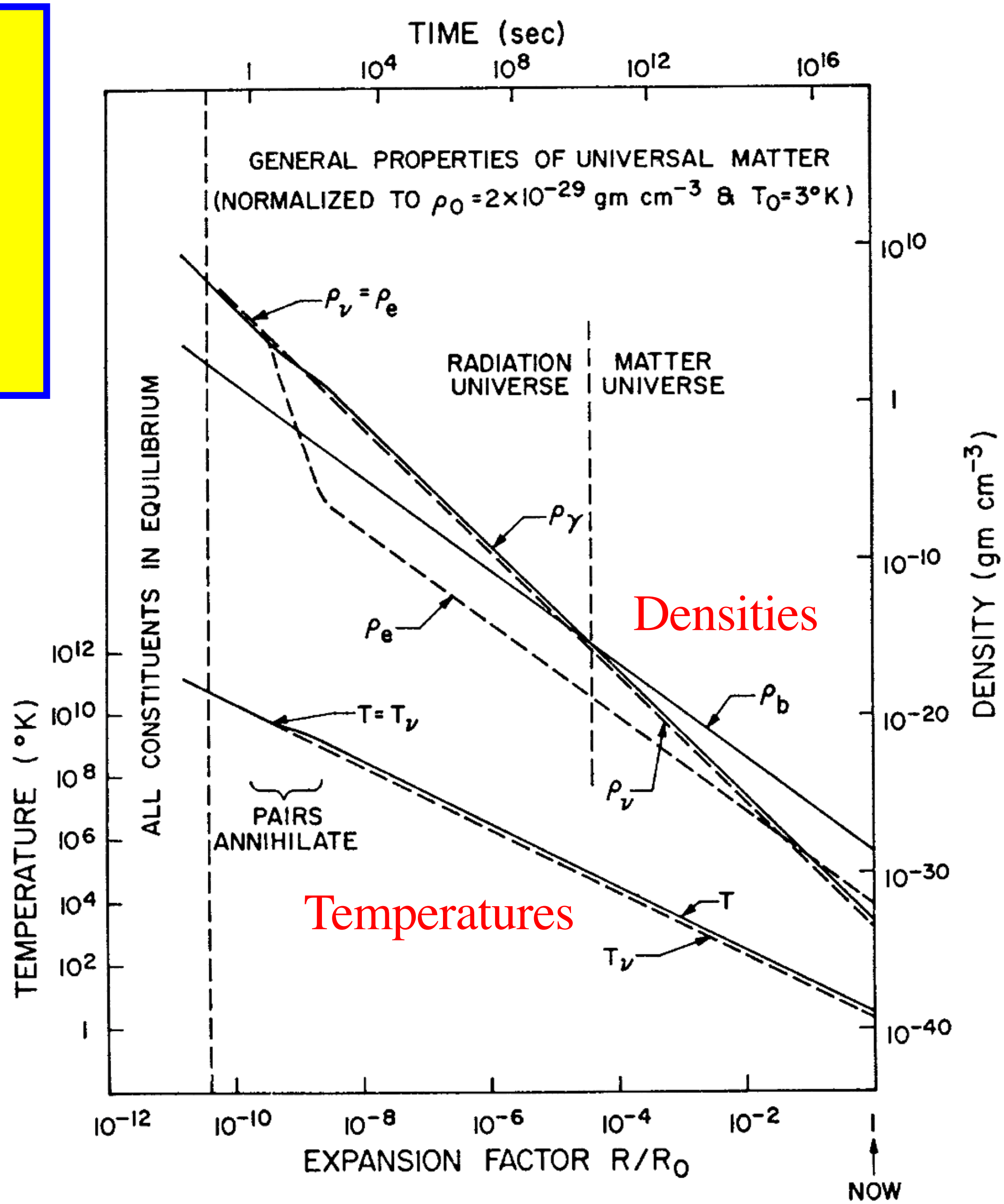
A more accurate dynamic calculation with full rate coefficients gives a lower value, $T_9 \sim 0.9$, corresponding to $t \sim 200\,\mathrm{sec} < \tau_n$.

Plot: Schramm and Wagoner History of Light Element Abundances

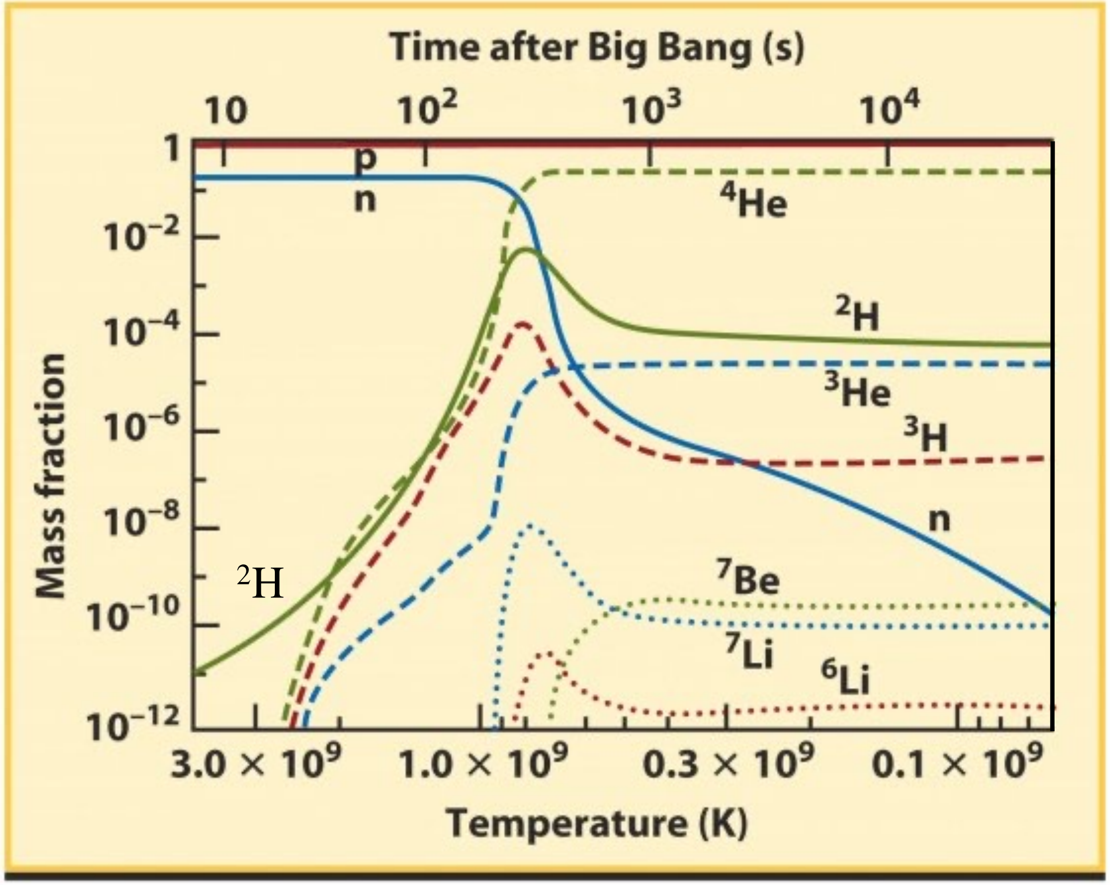
• Sub-exponential fall-off of n_n with replenishment from heavier elements (principally deuterium) precipitates an **interaction-driven decline**.

The Nucleosynthetic Era

- Densities and temperatures of baryons, electrons, neutrinos and photons in the nucleosynthetic epoch.
- From Wagoner, Fowler & Hoyle (1967, ApJ **148**, 3).



Big Bang Light Element Synthesis



- Fractional abundances of light elements and their evolution through the nucleosynthetic era. Relative abundances depend on $\Omega_b h^2$ parameter so that measured final apportionment probes the baryonic matter content.
- Adapted from the original version in the seminal review of Schramm & Wagoner (1977; Ann. Rev. Nucl. Sci. 27, 37), Figure 3 therein.

1.3 Deuterium Synthesis

Deuterium forms primarily via

$$n + p \;\leftrightarrow\; d + \gamma \quad , \tag{14}$$

and so its abundance is generated in the epoch of neutron freeze-out. The Saha equation-type thermal equilibrium value for the abundance of deuterium via this reaction is given by

$$\frac{n_p n_n}{n_d n} \;=\; \frac{4}{3n}\left(\frac{m_p m_n}{m_d}\right)^{3/2}\left(\frac{kT}{2\pi\hbar^2}\right)^{3/2}\exp\left\{-\frac{E_{B,d}}{kT}\right\} \quad , \tag{15}$$

where $n = n_p + n_n + n_d$ is the total baryonic number density, and $E_{B,d} \equiv (m_n + m_p - m_d)c^2 = 2.23\,\text{MeV}$ is the deuterium binding energy. **C & O, pp. 1177-9**

$*$ Observe that $E_{B,d} > Q_n$, thereby providing an added reason why neutron freeze-out terminates deuterium production: energetics favors it, so when the neutron supply fades, the deuterium production halts.

• The Saha equilibrium then becomes, for current baryonic number density $n = 1.12\times 10^{-5}(\Omega_{\rm b}h^2)\,\text{cm}^{-3}$ with $h = H_0/[100\,\text{km/sec/Mpc}]$,

$$\frac{n_p n_n}{n_d n} \;=\; \exp\left\{25.82 - \log_e\left(\Omega_{\rm b}h^2 T_{10}^{3/2}\right) - \frac{2.58}{T_{10}}\right\} \quad . \tag{16}$$

Observe that this highlights $n_d \propto \Omega_{\rm b}h^2$ character, which differs from the Rowan-Robinson plot (to come) where absorption of deuterium in creating helium is a big factor.

$*$ This destruction of deuterium is what yields the sensitivity of n_d to $\Omega_{\rm b}h^2$ so that it becomes a useful diagnostic.

• The peak abundance of deuterium is not as sensitive to $\Omega_{\rm b}h^2$; this arises because adjustments in this cosmological parameter are accompanied by adjustments to the temperature/redshift for neutron freeze-out.

Plot: Schramm and Wagoner History of Light Element Abundances

$*$ Also, the $n_d = n_n$ *crossover* occurs at $T \sim 8\times 10^8\,\text{K}$, i.e. at time $t \approx 3$ minutes. *This identifies the age of the Big Bang nucleosynthetic epoch.*

1.4 Helium and Lithium Production

As the mass of the light elements increases, the complexity of the network of involved reactions becomes greater. For helium, deuterium is destroyed, in a manner similar to in the pp chain in stellar interiors: **Longair, Sec. 10.3**

$$\begin{aligned} d + d &\leftrightarrow t + p \\ d + d &\leftrightarrow {}^3He + n \\ t + d &\leftrightarrow {}^4He + n \\ {}^3He + d &\leftrightarrow {}^4He + p \end{aligned} \tag{17}$$

The stability of alpha particles (7 MeV/nucleon as opposed to 1.1 MeV/nucleon for deuterium) drives the "equilibrium" toward ${}^4He$ production.

• As the universe evolves, while the eventual d abundance is sensitive to $\Omega_b h^2$, that for ${}^4He$ is not. The reason for this is essentially caused by the n_d/n_n ratio at the peak deuterium abundance being insensitive to $\Omega_b h^2$.

• In the earlier universe, ν decoupling at a temperature of $T_{10} \sim 3$ terminates "free" inverse beta decay reactions and establishes

$$\frac{n_n}{n_p} \sim 0.16 \approx e^{-Q_n/kT} \tag{18}$$

prior to the onset of rapid helium production. The eventual ${}^4He$ abundance dominates that of deuterium and tritium, so that it is **determined by n "conservation"** in either hydrogenic or helium form:

$$\begin{aligned} n_H &= n_p - n_n \\ n_{He} &= (n_p + n_n) - (n_p - n_n) = 2n_n \quad . \end{aligned} \tag{19}$$

This then yields a helium mass fraction of

$$Y = \frac{n_{He}}{n_p + n_n} = \frac{2n_n}{n_p + n_n} \approx 28\% \quad . \tag{20}$$

The observed value, from UV observations of the ISM, is 23 %.

• Lithium production is sensitive to $\Omega_{\rm b} h^2$, largely because it has a higher binding energy (not per nucleon). Hence, relevant reactions are in the $e^{-E_B/kT}$ tail more than for deuterium and helium.

Plot: Light Element Abundance Constraints on Baryonic Matter

The bottom line is that these light elements constraints concur and establish a tight bound on the density of baryonic matter in the universe:

$$\boxed{0.010 \lesssim \Omega_{\rm b} h^2 \lesssim 0.02 \quad ,} \tag{21}$$

which must be matched by subsequent determinations of the cosmological parameters of the universe. This was a test that WMAP was subjected to and passed admirably, since for $h = 0.71$ one gets $0.02 \lesssim \Omega_{\rm b} \lesssim 0.04$.

Primordial Nucleosynthesis Diagnostics

- Comparison of four light element abundances, observed (denoted by vertical sides of boxes) and predicted (colored curves), as functions of baryon density.
- From Kirkman et al. (2003: ApJS **149**, 1) [arXiv: 0302006] who reported D abundances (shaded region constraints) along line of site to quasar QSO 1243+3047.