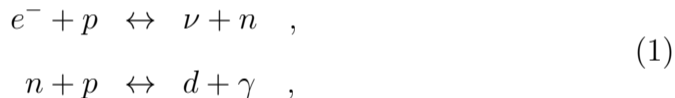


- Alpha, Bethe and Gamow ( $\alpha\beta\gamma$  1948) calculated dynamic (time-evolving) nucleosynthetic evolution in the  $T > 10^9$  K epoch of the early Universe. This temperature is necessary in order to generate appreciable nuclear fusion rates, based on the knowledge of  $pp$  chain physics for stellar interiors.
- The hydrogen fusion in the  $pp$  chain yields effective fusion reactions (two-way quasi-equilibrium) along the lines of



$\sigma \sim 10^{-28} \text{ cm}^2$ , and Gamow obtained the estimate

$$\langle \sigma \frac{v}{c} \rangle n_b R_{\text{caus}} \sim 1 \quad (2)$$

for nuclear opacity of unity. Here,  $R_{\text{caus}} = vt_{\text{nucl}}$  is the “radius” of causal connection of the Universe at the nucleosynthetic epoch, which we will presume to be at an age of  $t_{\text{nucl}}$ . The minimum temperature required to trigger nuclear reactions is  $T \sim 10^9 \text{ K}$ , and this establishes  $v/c \sim 0.008$  for protons. The baryonic density is just  $n_b \sim (\rho_0/m_p)(1+z)^3 \sim 2.5 \times 10^{-7} (1+z)^3 \text{ cm}^{-3}$ . The age of the Universe can be obtained from the familiar redshift integral, evaluated in an epoch of radiation dominance:

$$H_0 t_{\text{nucl}} \approx \int_z^\infty \frac{dz'}{(1+z')\sqrt{\Omega_{\text{rad}}(1+z')^4}} = \frac{1}{2\sqrt{\Omega_{\text{rad}}}} \frac{1}{(1+z)^2} \quad (3)$$

Inserting the various bits and pieces into Eq. (2) yields the estimate

$$8 \times 10^{-31} (2.5 \times 10^{-7}) \frac{0.008c(1+z)}{2H_0\sqrt{\Omega_{\text{rad}}}} \sim 1 \quad \Rightarrow \quad z \sim 3 \times 10^8 \quad , \quad (4)$$

for  $\Omega_{\text{rad}} \approx 5 \times 10^{-5}$  now. The redshift of this epoch yields  $R \sim a_0/(1+z) = c/H_0/(1+z) \sim 15 \text{ pc}$  for the size of the Universe; the causally-connected nucleosynthetic portion is actually a factor of  $\sim 100$  smaller! The baryonic density in the nucleosynthesis epoch is

$$n_b \sim \frac{\rho_0}{m_p} (1+z)^3 \sim 6 \times 10^{18} \text{ cm}^{-3} \quad \text{for } z \sim 3 \times 10^8 \quad . \quad (5)$$

These are *sub-nuclear densities*, inferior to those encountered in neutron star interiors. Using Eq. (3), the age of this fledgling Universe is estimated to be around 6 minutes. The Big Bang nucleosynthetic epoch was very brief!

• Since  $T \propto 1/R \propto n_b^{1/3}$ , and  $n_b = \rho_0/m_p \sim 2 \times 10^{-7} \text{ cm}^{-3}$  now, it follows that the Universe should be filled with a **relic radiation of temperature**

$$T_{\text{CBR}} \sim 3 - 4 \text{ K} \quad , \quad (6)$$

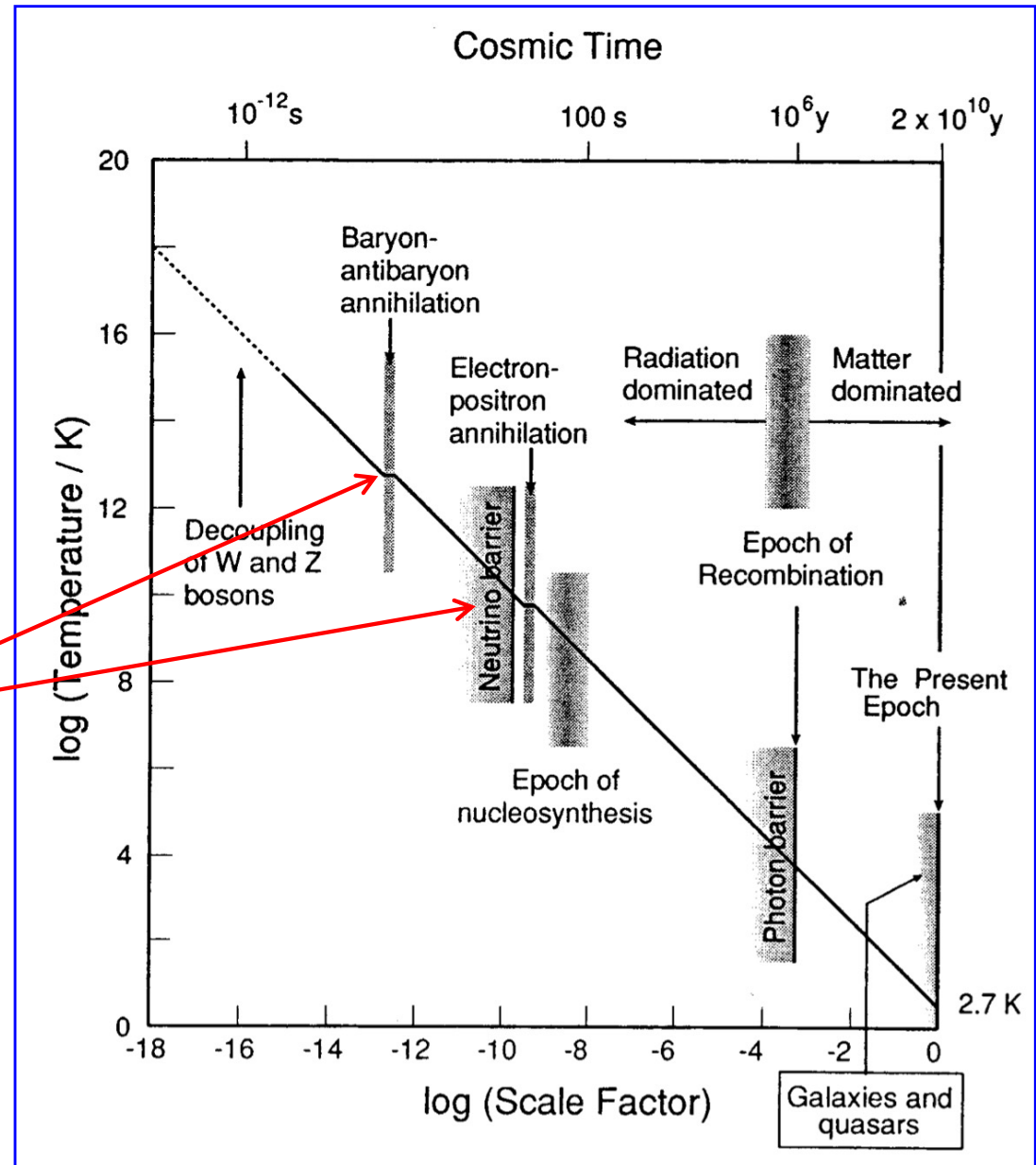
since  $z \gtrsim 10^8$  during **primordial nucleosynthesis**. This is in the microwave band (using Wien’s displacement law). Gamow made this prediction of the relic of the Big Bang in 1948, and the hunt for this background radiation was on!

**Plot:** Early Thermal History of the Universe [*Figure 9.2 of Longair*]

Details of such Big Bang nucleosynthesis will form the focus of Chapter 9.

# Thermal History of Universe

- Key epochs in the thermal evolution of the Universe. Each **bump** in the temperature decline is a phase transition due to matter annihilation.
- From Longair, Fig. 9.2.



## 1.2 Discovery

In 1965, Penzias & Wilson discovered this radiation field with their microwave radiometer tuned to  $\sim 7$  cm wavelength, and ascertained that it could be modeled by a blackbody with a temperature

$$T_{\text{CMB}} \sim 3.5 \pm 1.0 \text{ K} \quad . \quad (7)$$

They received the Nobel Prize in physics in 1978. Their result was soon confirmed by Roll & Wilkinson at the shorter wavelengths of around 3cm, obtaining  $T_{\text{CMB}} = 3.0 \pm 0.5$  K. The competition to discover the CMB was hot, including Peebles and Dicke at Princeton.

\* Gamow's predicted temperature was very good. Subsequent refinements confirmed the blackbody spectral shape and established  $T = 2.726$  K.

- The discovery of the CMB as **relic radiation** from a past era corresponding to a hot, dense phase was the triumphal vindication of the **hot Big Bang model**, and the death knell for steady-state cosmologies.
- The energy density of the CMB actually exceeds that of all other wavebands, namely radio, IR (dust), optical (starlight), X-ray (unresolved Seyferts, cosmic ray bremsstrahlung) and gamma-rays (unresolved AGNs).

\* This ultimately connects to the relative scales of  $T_{\text{CMB}}$  and  $\Delta T$  (to be discussed). The total temperature expresses the energy content of the background radiation. The perturbation  $\Delta T$  reflects that apportioned into peculiar matter fluctuations that ultimately become galaxies, from which all the other backgrounds emerge.

**Plot:** Broadband Cosmic Background Radiation Spectrum

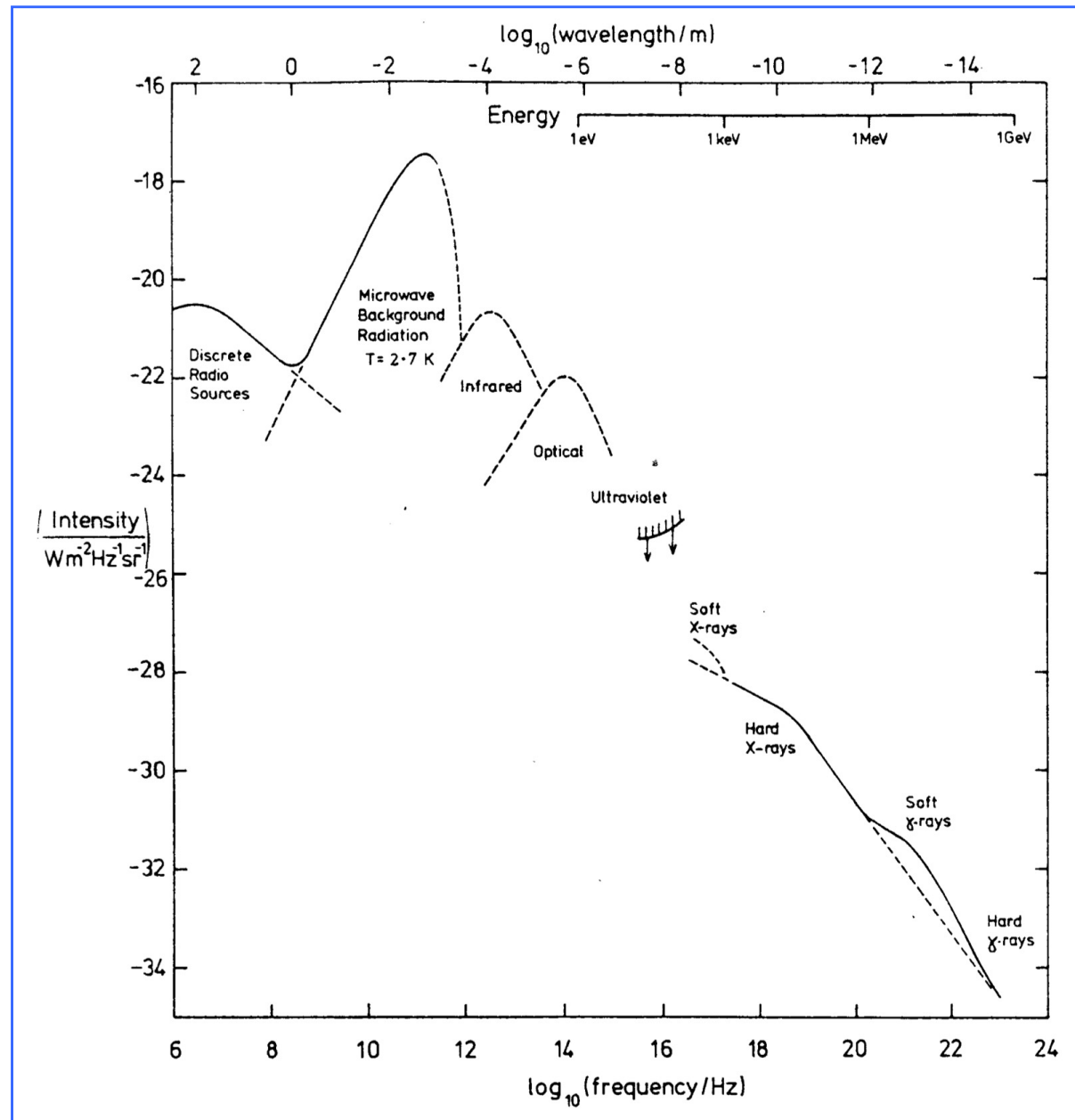
From  $U_{\text{rad}} = (4\sigma/c)T^4$ , we quickly find that  $n_\gamma = U_{\text{rad}}/(3kT)$  for CMB temperature  $T = 2.725$  K yields a current number density  $n_\gamma = 370 \text{ cm}^{-3}$  of photons in the universe. Hence it follows that

$$\frac{n_b}{n_\gamma} \approx 3.04 \times 10^{-8} \Omega_b h^2 \quad (8)$$

is the baryon to photon *number ratio*, where  $h = H_0/[100 \text{ km/sec/Mpc}]$ , and  $n_b = 1.12 \times 10^{-5}(\Omega_b h^2) \text{ cm}^{-3}$  is the baryonic number density. In what follows, we will use  $h = 0.7$  from the WMAP determinations.

# Cosmic Background Spectrum

- The spectrum of extragalactic background radiation as it was known in 1969 (Longair & Sunyaev 1971); this version approximates our current view.
- Dashed lines are theoretical estimates for discrete sources.
- The microwave background dominates the total energy density, since all other bands have been reprocessed through cooler matter after the recombination epoch.



Since each non-relativistic baryon has  $\sim 938$  MeV in energy, compared with photons (of energy  $3kT$  with  $T = 2.73$  K) with  $\sim 7.05 \times 10^{-10}$  MeV, the energy budget at present between radiation and matter is described by

$$\Omega_b/\Omega_{\text{rad}} \approx 4.0 \times 10^4 \Omega_b h^2 \quad . \quad (9)$$

We now replace  $\Omega_b \rightarrow \Omega_m$  to include all matter content that influences the Universe's dynamics. Then, from this the redshift  $z_{\text{eq}}$  of matter-radiation equipartition, at which  $\Omega_{\text{rad}} \sim \Omega_m$ , is simply deduced to be

$$z_{\text{eq}} \sim \frac{\Omega_m}{\Omega_{\text{rad}}} = 4.0 \times 10^4 \Omega_m h^2 \quad . \quad (10)$$

Observe that this uses  $U_\gamma \propto (1+z)^4$  and  $\rho_m \propto (1+z)^3$  to derive these quantities at arbitrary redshift. This yields  $z_{\text{eq}} \sim 5300$  using  $\Omega_m \sim 0.27$  and  $h = 0.7$ , a result that could be directly derived using the fact that the CMB temperature establishes  $\Omega_{\text{rad}} = 5 \times 10^{-5}$ . Note that conventionally, equipartition is defined to be a little later,  $z_{\text{eq}} \sim 3200$ , when the radiation is starting to fall off in dynamical influence.

## 2 COBE Results and Implications

In 1990, NASA's **Cosmic Background Explorer** (COBE) microwave background mission obtained the first results to prove unambiguously that the CMB was a *beautiful blackbody*, with

$$T_{\text{CMB}} = 2.726 \text{ K} \quad . \quad (11)$$

It provided a clear demonstration of isotropy to  $\ll 1\%$ , but the anisotropic components were clearly detected with impressive precision. The COBE discoveries led to the 2006 Nobel Prize in physics for Smoot & Mather.

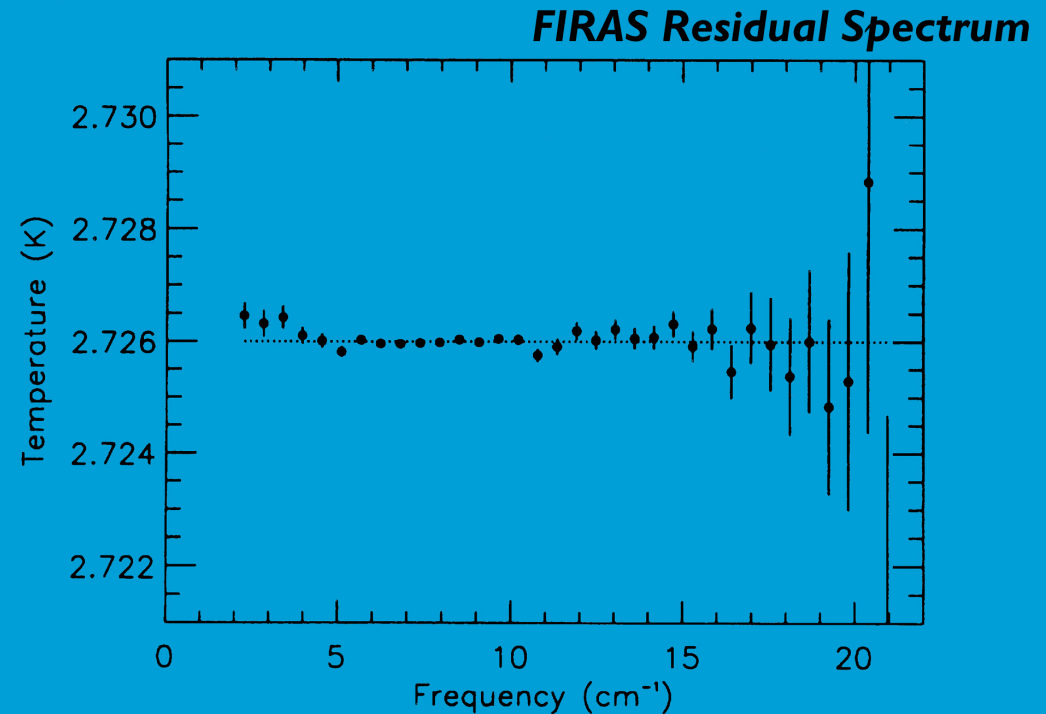
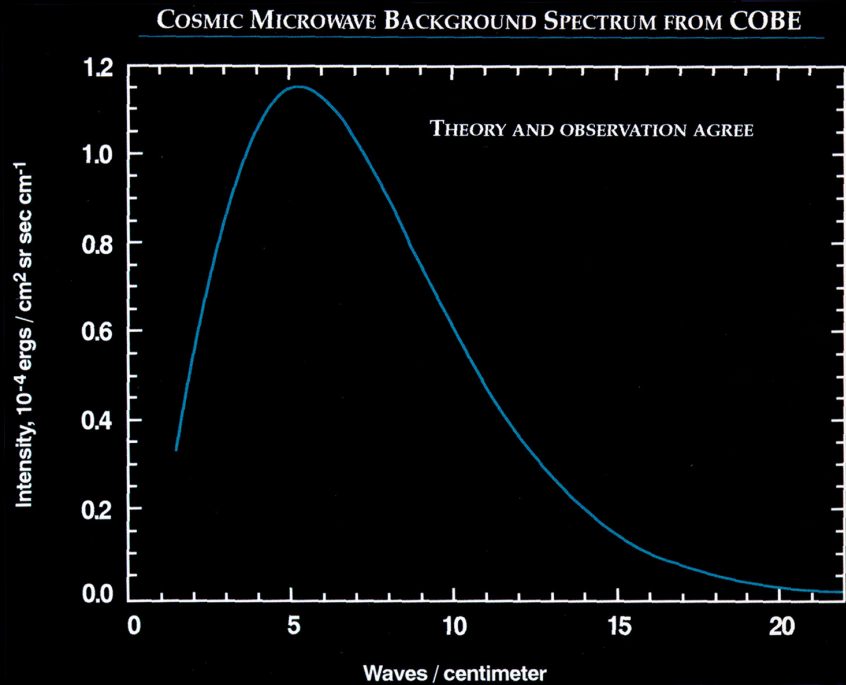
**Plot:** COBE Spectrum of Cosmic Microwave Background Radiation

- The first **dipole component** was seen at the  $\Delta T \sim 3$  mK level, and provided a Doppler estimate of  $(v/c) \cos \theta \approx \Delta T/T \approx 1.2 \times 10^{-3} \Rightarrow v \sim 380$  km/sec for the peculiar motion of the sun. Subtracting off the Earth/sun motion then gave the first accurate determination of the peculiar motion of the Milky Way w.r.t the CMB of **600 km/sec in the direction of Leo**.

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# COBE FIRAS CMB Spectrum



Precision blackbody cosmic microwave spectrum from the **FIRAS instrument** on **COBE** with residuals at the 0.1% level.

From: [http://lambda.gsfc.nasa.gov/product/cobe/slide\\_captions.cfm](http://lambda.gsfc.nasa.gov/product/cobe/slide_captions.cfm)

Subtracting off the dipole component gave isotropy at the  $\Delta T/T \lesssim 3 \times 10^{-4}$  level, but this was dominated by the galactic plane contribution.

**Plot:** COBE Sky Maps

This signal was observed to be unpolarized at the  $< 2 \times 10^{-4}$  % level (but later detected cleanly using Planck), implying that the cosmological expansion was uniform in the past, i.e. principally at redshifts from recombination through the epoch of last scattering.

- After subtraction of the disk foreground, COBE determined  $\Delta T/T \sim 6 \times 10^{-6}$  fluctuations that signal overdensities, predominantly on the  $7^\circ$  scales that corresponded to its FWHM angular resolution. This limiting scale meant that COBE was not empowered to measure the angular power spectrum of CMB fluctuations; this was the task of the next generation missions WMAP and Planck.

- \* This detection was sufficient to rule out the topical **hot dark matter** models of structure formation; for example, relativistic neutrinos tend to smear out structure on small angular scales because they are weakly interacting and therefore transfer their energy on larger scales.

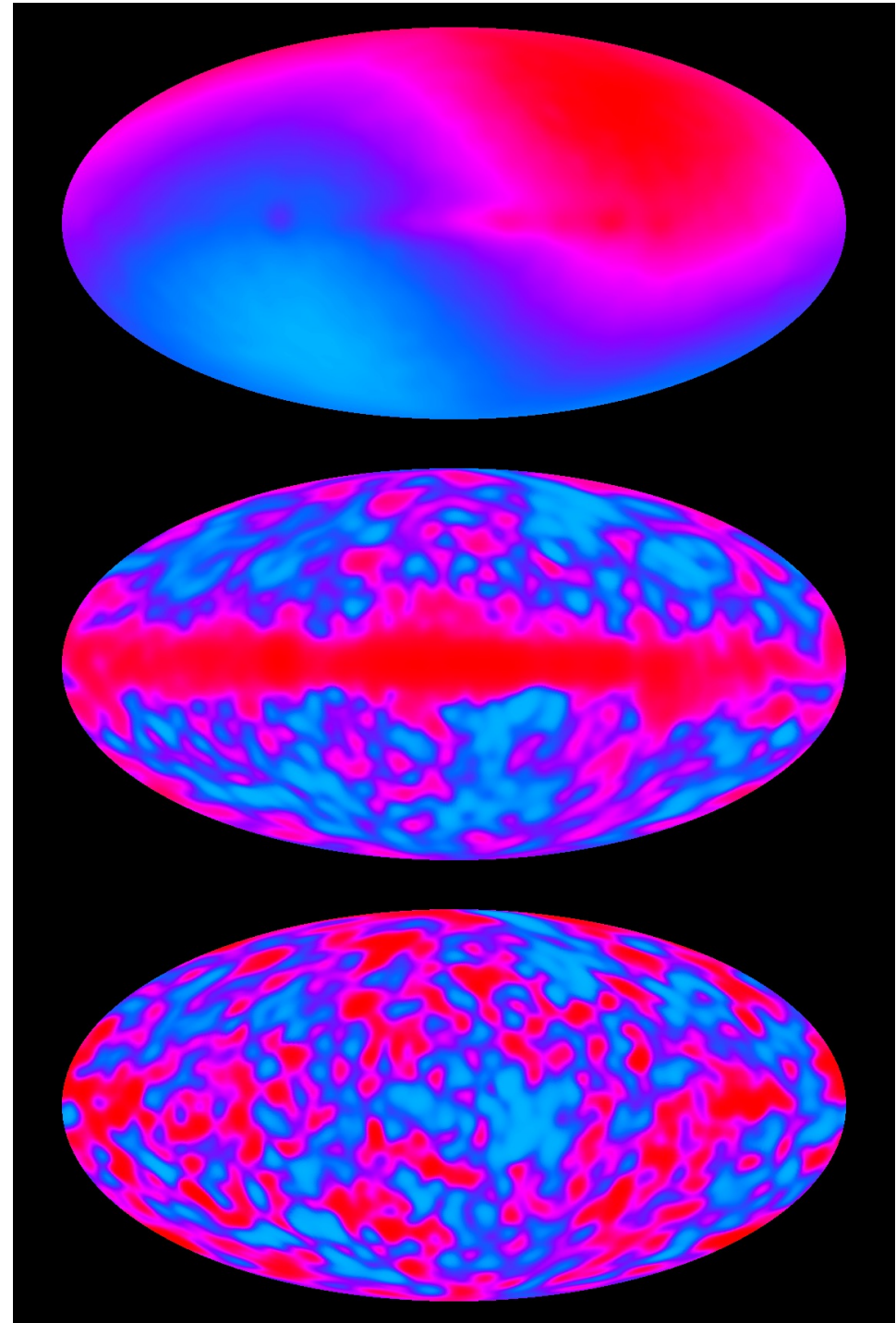
- The sky variations of temperatures and by association densities are naturally explained as **acoustic fluctuations** at the epoch of recombination, i.e. variations  $\delta P$  and  $\delta \rho$  to the equation of state. The **amplitude** of these acoustic fluctuations implies  $M \sim 10^{12} M_\odot$  seeds that are consistent with gravitational initiation of galaxy formation. Moreover, the fluctuation spectrum was scale-invariant (Zeldovich-Harrison), i.e.  $|\delta_k|^2 \propto k^n$  with  $n = 1$  for  $\delta_k = \delta \rho / \rho$ , which was consistent with inflationary models. Here,  $k = |\mathbf{k}| = 2\pi/\lambda$  is the density fluctuation wavenumber that couples to the lengthscale  $\lambda$  of density/pressure perturbations.

Note that in the direction of clusters of galaxies, locally hotter electron gas heats the CMB slightly, at around the  $\sim 10^{-4}$  level. This leads to a blueshifting/distortion of the Planck spectrum. This is known as the **Sunyaev-Zeldovich effect** (see below), and must be corrected for when analyzing the microwave background fluctuations.

# COBE DMR Skymap

- Three sky maps of the **primordial CMB** obtained by the Cosmic Background Explorer (COBE), the science of which garnered a Nobel Prize in Physics in 2006.
- DMR=Differential Microwave Radiometer; 31, 53, 90 GHz channels
- *Top Panel*: raw warm/cool dipolar map with peculiar velocity influence of solar neighborhood included.
- *Middle Panel*: map with local peculiar velocity subtracted, and also isotropic 2.73K Planck spectrum subtracted.
- *Bottom Panel*: fully-processed **CMB fluctuation map** with Galactic disk signal subtracted. Notice the structure on scales of 3-7 degrees, the precursor to large scale structure formation.

[http://lambda.gsfc.nasa.gov/product/cobe/dmr\\_image.cfm](http://lambda.gsfc.nasa.gov/product/cobe/dmr_image.cfm)



## 2.1 Acoustic Oscillations: the Robust Cosmic Ruler

• An essential beauty of the CMB fluctuations is that they provide a precise ruler that can be used to calibrate the cosmology of the Universe. The principal reason for this is that at the recombination epoch, the photons contribute significantly to the total equation of state. Their fractional contribution can be assessed via

$$\frac{\Omega_{\text{rad}}(1+z_{\text{rec}})^4}{\Omega_m(1+z_{\text{rec}})^3} = \frac{1+z_{\text{rec}}}{1+z_{\text{eq}}} \sim \frac{1}{3} \quad , \quad (12)$$

by comparing the recombination era to the condition for equipartition, and using  $z_{\text{rec}} \sim 1100$  and  $z_{\text{eq}} \sim 3200$ .

Acoustic oscillations therefore propagate roughly at the sound speed  $c_s$  for a relativistic gas ( $P \propto \rho^{1/3}$ ) with adiabatic index  $\gamma = 4/3$ :

$$c_s^2 = \frac{\partial P}{\partial \rho} = \frac{P}{3\rho} = \frac{c^2}{3} \quad \Rightarrow \quad c_s = \frac{c}{\sqrt{3}} \quad . \quad (13)$$

The physical distance on the horizon for self-gravitating density fluctuations (and therefore temperature fluctuations due to virialization) is then

$$D = c_s t_{\text{rec}}(z) \quad . \quad (14)$$

This provides a robust ruler on the sky at this epoch – it is in practice a much tighter standard than the diameters of galaxy clusters provide.

• At recombination (atomic coupling), the universe’s age is

$$t_{\text{rec}}(z) = \frac{1}{H_0} \int_z^\infty \frac{dz'}{(1+z')E(z')} \quad , \quad (15)$$

where

$$E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_{\text{rad}}(1+z)^4 + \Omega_\Lambda + (1-\Omega)(1+z)^2} \quad . \quad (16)$$

We can safely neglect the  $\Omega_\Lambda$  and  $(1-\Omega)$  terms in  $E(z)$  at  $t_{\text{rec}}$ . At recombination, the radiation term is only moderately influential, so we neglect it and arrive at the matter-dominated result

$$t_{\text{rec}}(z) \approx \frac{2}{3(1+z)^{3/2}} \frac{1}{H_0 \sqrt{\Omega_m}} \quad , \quad (17)$$

which yields the acoustic fluctuation scale

$$D \approx \frac{2}{3\sqrt{3}} \frac{1}{(1+z)^{3/2}} \frac{c}{H_0\sqrt{\Omega_m}} \quad . \quad (18)$$

Using  $\Omega_m \sim 0.27$ , including both baryonic and dark matter, at  $z_{\text{rec}} \sim 1100$  this yields  $D \sim 90$  kpc. The universe was a much smaller place then!

- This physical scale of fluctuations translates to a volume  $V \sim 4\pi D^3/3$ . In this volume, the matter density perturbation might scale with the temperature fluctuation via  $\delta\rho/\rho \sim \delta T/T \sim 6 \times 10^{-6}$ . The mean matter density  $\rho$  couples to the present one via  $\rho = \Omega_m \rho_c (1+z)^3$ . Therefore, the total mass perturbation within an “overdensity bubble” would be of the order

$$\delta M \sim V \delta \rho \sim \frac{4\pi}{3} D^3 \frac{\delta T}{T} \Omega_m \rho_c (1+z_{\text{rec}})^3 \sim 9 \times 10^{11} M_\odot \quad (19)$$

for  $z_{\text{rec}} \sim 1100$ . In practice, since at recombination, matter is already dominating radiation, then  $\delta\rho/\rho$  exceeds  $\delta T/T$  by a factor of  $\sim 3/2$ . This arises because matter has a non-relativistic equation of state  $P \propto \rho^{2/3}$  so that for cold matter,  $\delta T/T = \delta P/P = (2/3)\delta\rho/\rho$ . This tweak pushes this mass estimate above  $10^{12} M_\odot$ , yet still on the scale of masses for large galaxies.

- Now, the angular diameter distance gives a corresponding angular scale

$$\theta = \frac{D}{d_A} \quad \text{with} \quad d_A \equiv \frac{c}{H_0(1+z)} \int_0^z \frac{dz'}{E(z')} \quad (20)$$

in the approximation that  $\Omega \approx 1$ , so that  $S_k(\Theta) \rightarrow \Theta$ . The integral is evaluated using only the  $\Omega_m$  term, since the low  $z$  contributions dominate, yielding approximately  $2/\sqrt{\Omega_m}$ . Combining with Eq. (18), it follows that

$$\theta \approx \frac{1}{3\sqrt{3}} \frac{1}{\sqrt{1+z_{\text{rec}}}} \sim 0.33^\circ \quad (21)$$

is the approximate largest angular scale (fundamental) for acoustic fluctuations in the recombination epoch, i.e. at  $z_{\text{rec}} \sim 1100$ .

- Better angular resolution than COBE affords was required to explore the principal and harmonics of such fluctuations  $\Rightarrow$  WMAP and Planck.