

6. COSMOLOGICAL ENTREÉ

Matthew Baring – Lecture Notes for ASTR 360, Spring 2025

1 Olbers' Paradox and Mach's Principle

- Edmund Halley was noted as being concerned about why the sky was dark at night. Heinrich Olbers demonstrated (circa 1820) that for an infinite, homogenous and transparent universe, the sky should be as bright as the sun. This seeming contradiction came to be known as **Olbers' Paradox**.

C & O,
pp. 1145

- For stars of luminosity L_* and uniform space number density n_* , sampling out to large a distance d will eventually encounter stars in any given direction, the net flux for which will be

$$\mathcal{F} = \int_0^d \left(n_* \frac{L_*}{4\pi r^2} \right) 4\pi r^2 dr \quad . \quad (1)$$

This clearly diverges as $d \rightarrow \infty$, thereby quantifying Olbers' paradox.

- Olbers suggested that extinction of distant starlight could explain the dark night sky. However, the resolution came with the advent of cosmology, and the knowledge that the universe is *inhomogenous and expanding*.

- **Mach's Principle:** Inertial frames are determined relative to the motion of the rest of the matter in the universe. The bulk of the universe is either at rest or at constant velocity. (*Posed late 1800s*)

Longair,
pp. 8–9

* *Einstein's position:* There is no absolute inertial frame. Inertial frames are determined **by** the distribution and motion of matter in the universe.

2 The Cosmological Principle

- On the face of it, assuming that spacetime is flat in most of the universe was anti-Machian: the influential matter is far from the observation point.

- * De Sitter proposed introducing a boundary condition of “infinite” matter at “infinite” distance so as to dominate the determination of $g_{\mu\nu}(x)$.

- Einstein didn’t like this proposal, probably on aesthetic grounds, so he posited his **Cosmological Principle** (a beautiful symmetry principle):

**C & O,
Sec. 29.1**

The Universe is Homogeneous and Isotropic on average.

- * All observed deviations from this large scale behaviour must be explained in physically-plausible ways. The Cosmological Principle is a remarkable leap given the sky distribution of “nebulae” known in Einstein’s time:

Plot: Charlier map of nebulae in 1922.

- **Evidence for Isotropy:** the radio galaxy sky map with 30,000 galaxies provides a clear indicator of isotropy on large scales.

Plot: 6cm Radio Galaxy maps

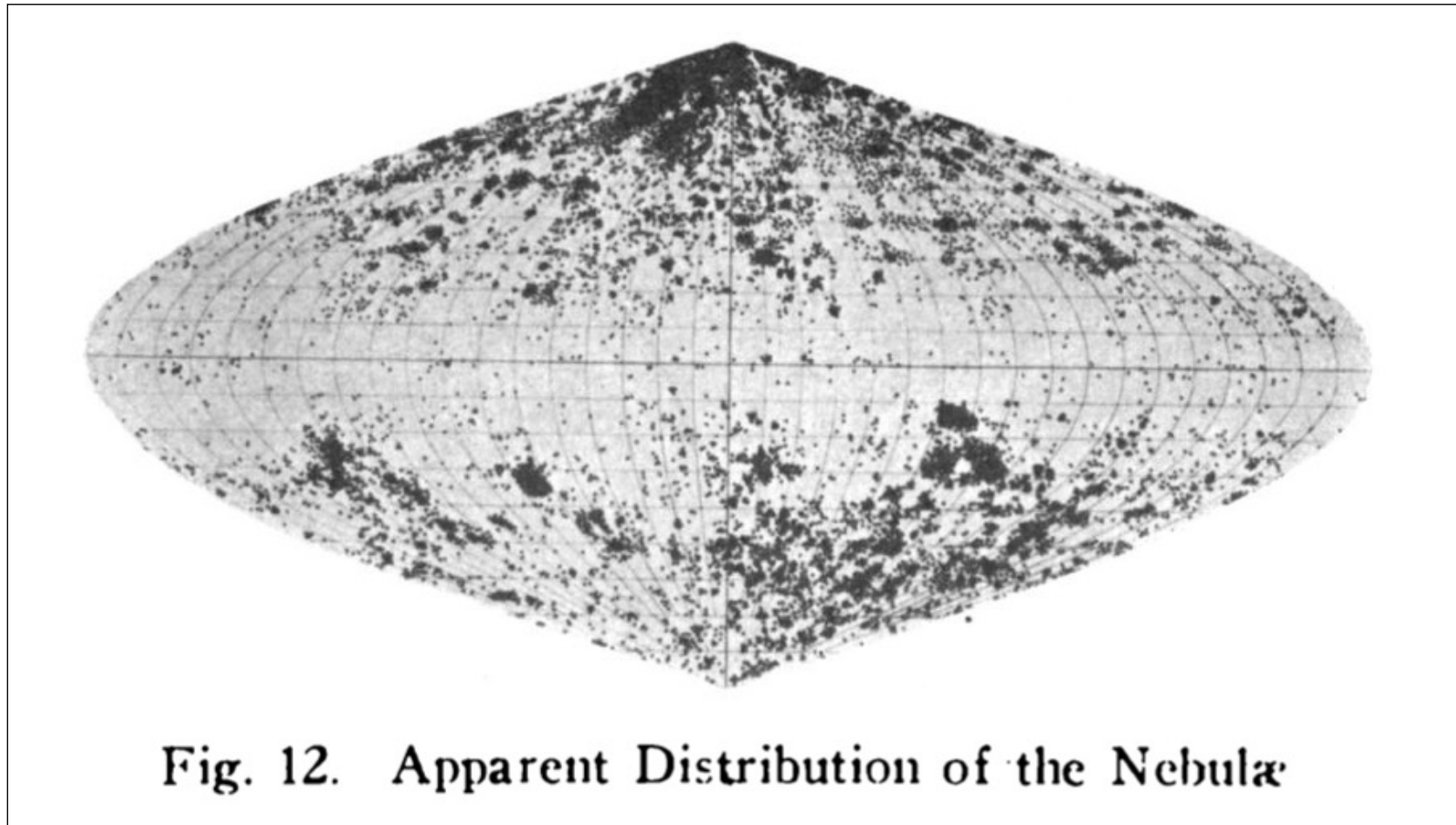
- The COBE microwave background at high redshifts represents an even more compelling, very smooth dipole signal from superposition of galactic peculiar velocity on an isotropic background.

- * small amplitude ($\sim 10^{-5}$) fluctuations superposed on this on 3-10 degree angular scales signify the precursor of large scale structure formation.

Plot: COBE fluctuation map.

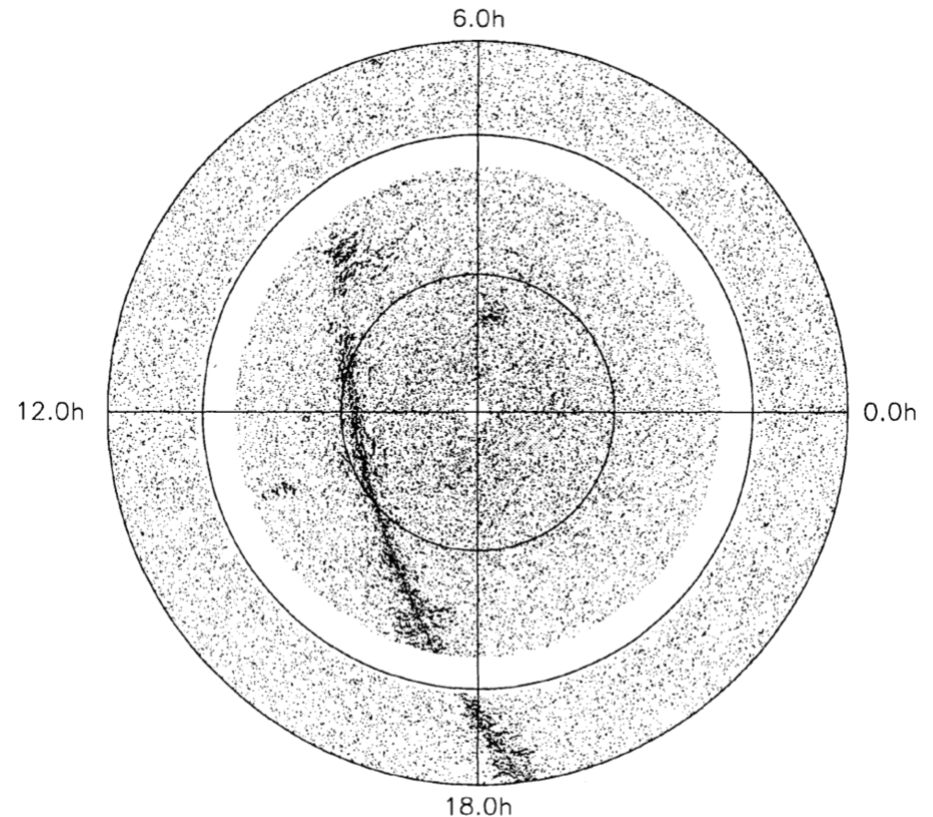
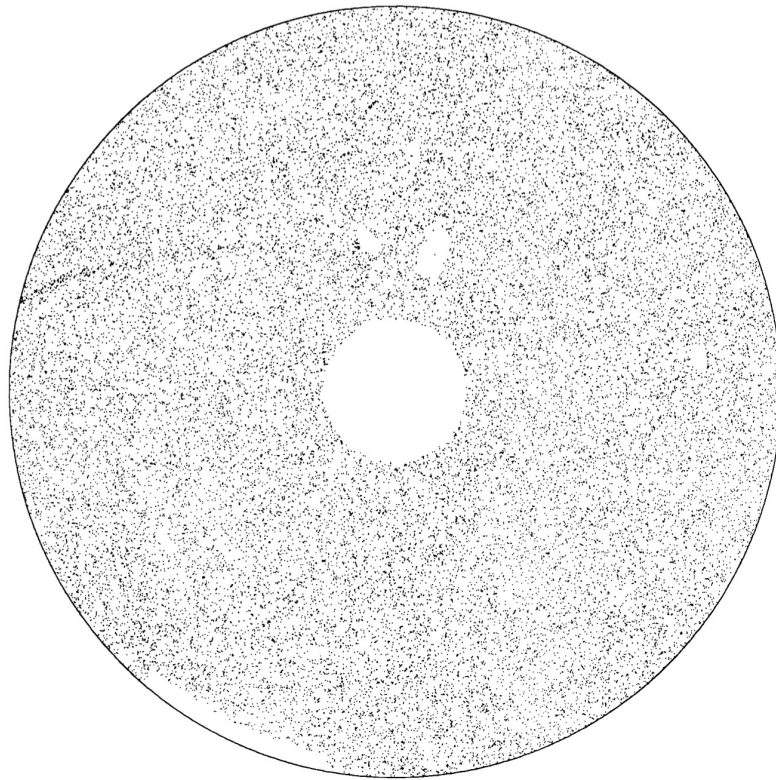
- **Evidence for Homogeneity:** $\log N - \log S$ distributions. We have already learnt that these adopt a well-known form for isotropic and uniformly-distributed *standard candles* of luminosity L . The flux observed from these sources is then $S \equiv f = L/(4\pi d^2)$.

Charlier Map of Nebulae: 1922



- 11475 optical objects in the sky (Galactic coordinates), mostly extragalactic “nebulae.” The **zone of avoidance** along the midline (latitude \sim zero) due to **obscuration by dust** is evident.
- C. V. L. Charlier, *Arkiv. för Mat. Astron. Fys.* **16**, 1 (1922)

Extragalactic 6cm Radio Surveys

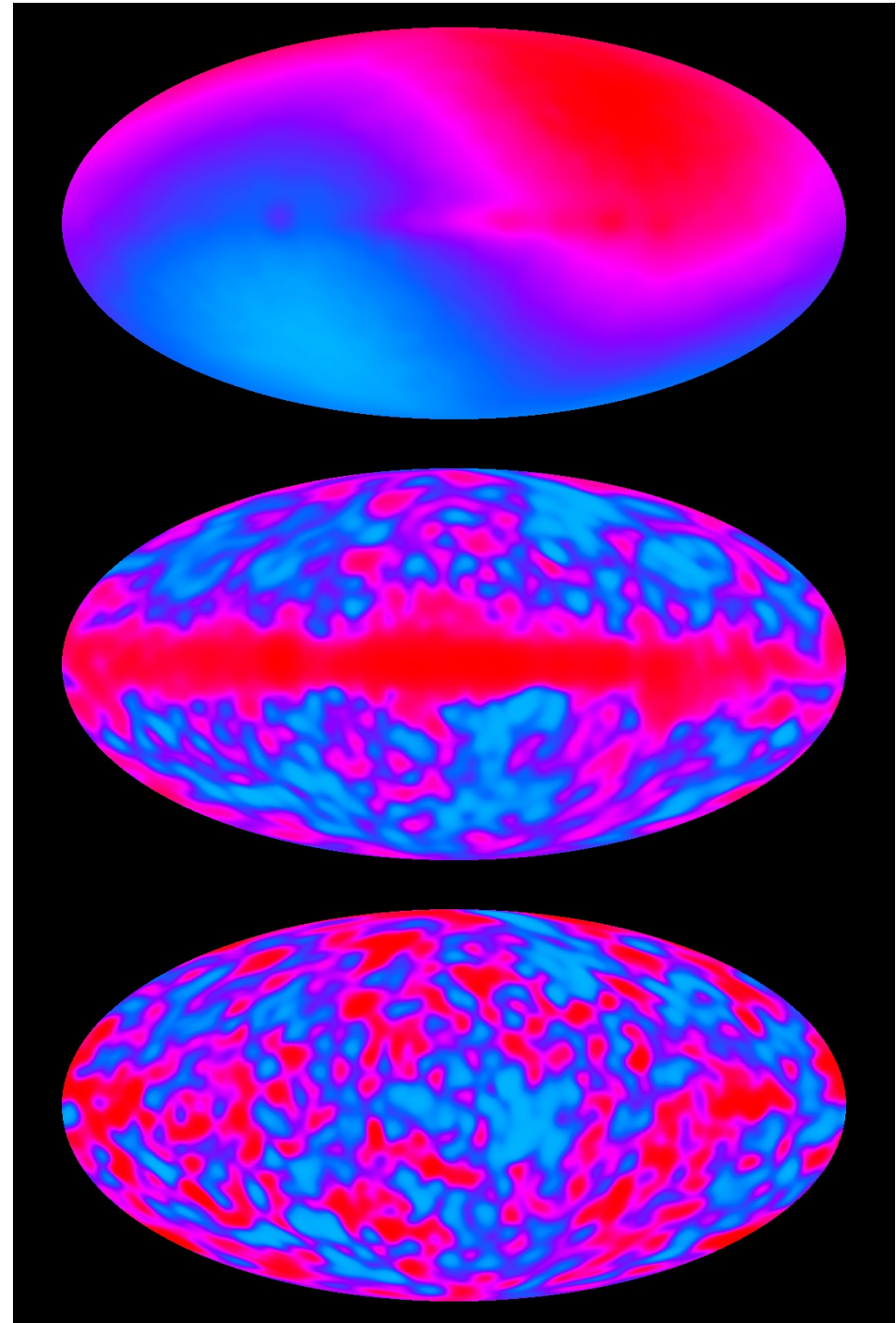


- *Left Panel:* **Green Bank NRAO Telescope** survey sky map of around 31,000 radio sources (essentially all galaxies, MW subtracted) illustrating their uniform angular distribution (data from [Gregory & Condon ApJS 75, 1011; 1991](#))
- *Right Panel:* **Parkes 64m Telescope** map of 36,640 sources in southern sky in two declination bands with RA denoting the “clock” configuration. The plane of the Milky Way is evident as the arc: [Griffith & Wright, Astron. J. 105, 1666 \(1993\)](#).

COBE DMR Skymap

- Three sky maps of the **primordial CMB** obtained by the Cosmic Background Explorer (COBE), the science of which garnered a Nobel Prize in Physics in 2006.
- DMR=Differential Microwave Radiometer; 31, 53, 90 GHz channels
- *Top Panel*: raw warm/cool dipolar map with peculiar velocity influence of solar neighborhood included.
- *Middle Panel*: map with local peculiar velocity subtracted, and also isotropic 2.73K Planck spectrum subtracted.
- *Bottom Panel*: fully-processed **CMB fluctuation map** with Galactic disk signal subtracted. Notice the **structure on scales of 2-5 degrees**, the precursor to large scale structure formation.

http://lambda.gsfc.nasa.gov/product/cobe/dmr_image.cfm



- Then, the differential number $dN(> f)$ of these sources per unit solid angle $d\Omega$ with observed flux greater than f is given by

$$\frac{dN(> f)}{d\Omega} = \frac{n}{3} d^3 = \frac{n}{3} \left(\frac{L}{4\pi f} \right)^{3/2}, \quad (2)$$

for Euclidean geometry, i.e. flat spacetime. This formula provides a cogent test of homogeneity for any source population. For small redshifts, the $-3/2$ power-law is borne out in field galaxies, clusters and radio galaxies.

* Flattening of the $\log N - \log S$ distribution for faint sources can reflect sampling of the edge of their space distribution; it can also signal the systematic effects due to the sensitivity limitations of the observation technique.

Plot: Log N-Log S for Galaxies

- At moderate redshifts, spacetime curvature can influence the $\log N - \log S$ distribution, as can evolution in the sources: The $f^{-3/2}$ dependence is relinquished when the accessible volume in Eq. (2) is no longer Euclidean or spherical, or both.

- **Evolution** is defined to be the difference in luminosity and other radiative properties in a source class between past epochs and the current one ($z = 0$).

* e.g. Current paradigms favor the evolutionary sequence of quasars and starburst galaxies \rightarrow active galaxies \rightarrow normal spiral and elliptical galaxies, in going from the star formation epoch at redshift $z > 1$ to the present.

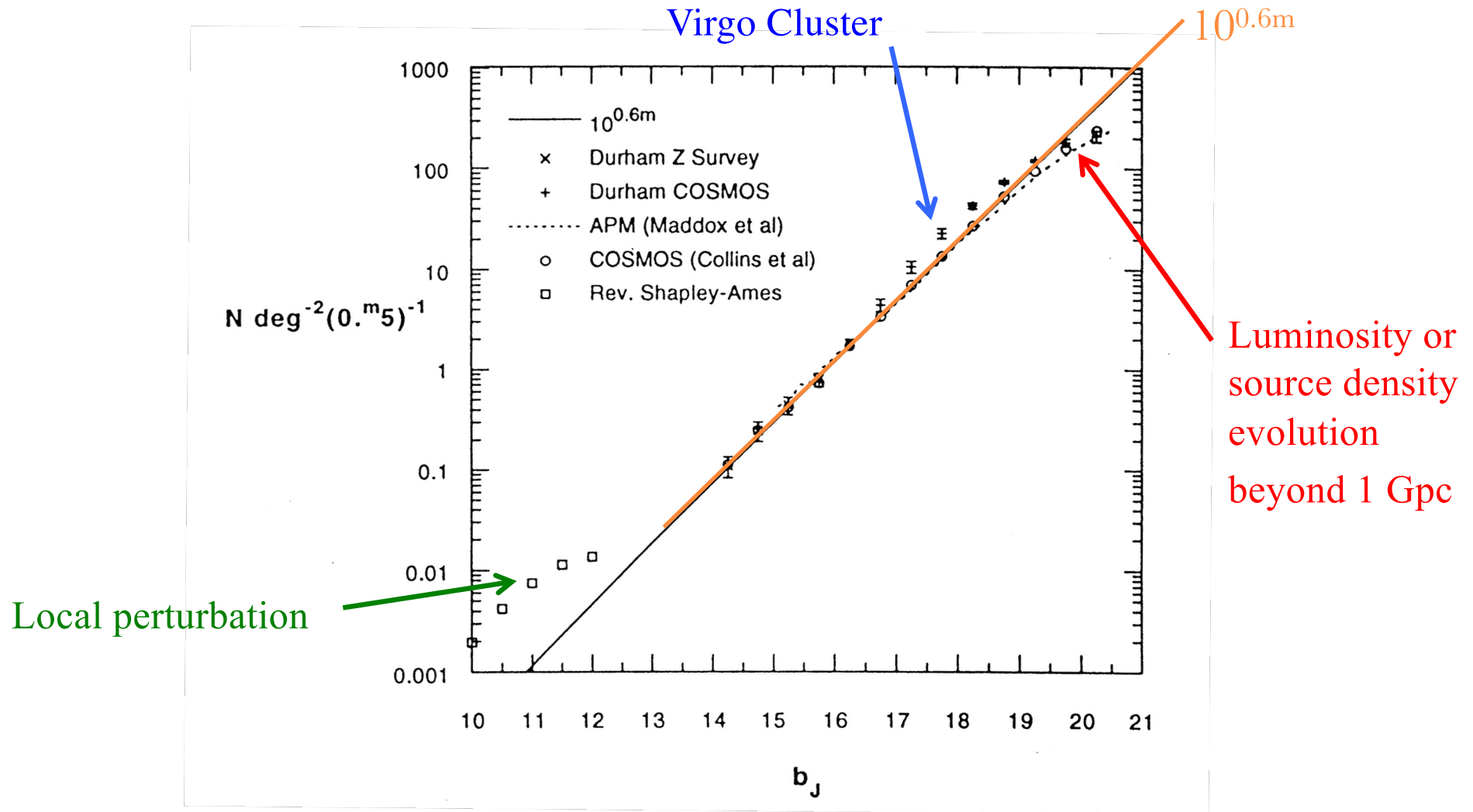
* Yet this scenario is being challenged since the quasars are too uncommon to provide an early re-ionization epoch (at $z > 6$) as indicated by Lyman α forest observations; normal, low luminosity or dwarf galaxies must have abounded in the early universe. Assessing this is an objective of JWST.

- If our sources are not standard candles, i.e. their *luminosity function* $dn(L)/dL$ is not a delta function, then

$$\frac{dN(> f)}{d\Omega} \propto f^{-3/2} \int \frac{dn}{dL} L^{3/2} dL, \quad (3)$$

which indicates that bright galaxies are strongly favored in determining the $\log N - \log S$ curve. Gamma-ray bursts provide a representative example.

Observer's Log N – Log S: Galaxy Counts



- Galaxy counts in optical surveys (Shanks 1991), as a function of apparent magnitude m .
- Straight orange line is the Euclidean homogeneity law $N \propto 10^{0.6m}$.

3 Cosmochronology

- The key timescale to emerge from the observation of Hubble's Law is the Hubble age, which in the age of precision cosmology is

$$t_H = \frac{1}{H_0} = 1.37 \times 10^{10} \text{ years} . \quad (4)$$

This timescale must be consistent with other independent estimates or bounds on the age of the universe, four of which are discussed below.

* Note that Hubble's original high value of $H_0 = 550 \text{ km/sec/Mpc}$ immediately caused conflict with known geological ages.

3.1 Gravitational Instability

- The dynamics of gravitational collapse provide a natural timescale for gravitational evolution and structure formation in the universe. Equating gravitational and centripetal accelerations yields $v^2/r \sim GM/r^2$. Then setting $v \sim r/t_g$ and $\rho = M/r^3$, the gravitational collapse timescale is then

**Peebles,
p. 68**

$$t_g \sim \frac{1}{\sqrt{G\rho}} , \quad (5)$$

which becomes $t_g \sim 10^{11}$ years for typical mean densities of $\rho \sim 10^{-31} \text{ g cm}^{-3}$ for matter in the universe. This result we have already encountered in our studies of galaxies and clusters. It is an overestimate as matter overdensities shorten the collapse timescale: *structure formation in the universe seeds structure enhancement*.

* Hence, fast stars evaporate from a galaxy and the bound matter in the galaxy collapses, both on a timescale of t_g .

- It is fortunate that t_g is not much less than t_H , otherwise we would live in a universe populated only with black holes or other condensed objects.

3.2 Nuclear Supply Ages

- The sun burns up nuclear fuel at roughly $\epsilon \approx 0.7\%$ efficiency for converting mass to radiative forms in the pp chain (more efficient than fission, but much less so than matter annihilation like $e^+e^- \rightarrow \gamma\gamma$).
- The sun, a typical star, emits luminosity $L_\odot = 4 \times 10^{33}$ erg/sec. Hence for burning at a constant rate, the nuclear (evolutionary) lifetime of a solar mass main sequence star is at most

$$t_{\text{nucl}} \sim \frac{\epsilon M_\odot c^2}{L_\odot} \sim 10^{11} \text{ years} \quad . \quad (6)$$

Again, this is agreeable: the evolutionary timescale is comparable (and larger than) to the Hubble time, so we should see evidence of significant stellar evolution, yet not be dominated by purely highly-evolved systems.

- The fact that $t_g \sim t_{\text{nucl}}$ is not purely coincidence. Gravity seeds star formation and stellar evolution precedes gravitational collapse, so they tend to equilibrate with each other:

* this establishes a stellar population whose most prominent members (solar mass objects) have comparable nuclear and gravitational timescales; i.e., the Milky Way is not predominantly made up of $50M_\odot$ stars.

3.3 Globular Cluster Constraints

- Globular clusters, whose masses can be as high as $10^6 M_\odot$, contain an array of main sequence and post-main-sequence stars. They can display a complete evolutionary trace. The nuclear burning or evolutionary timescale t_{nucl} establishes a strong dependence

$$L = L(M, T) \quad . \quad (7)$$

* Sub-solar mass stars are longest lived, while high mass stars with higher temperatures evolve the most rapidly.

* The strong dependence of $T(M)$ is a consequence of the Maxwellian exponential folding into nuclear reaction rates in the stellar cores.

Plot: Gaia Hertzsprung-Russell diagram: Open Clusters

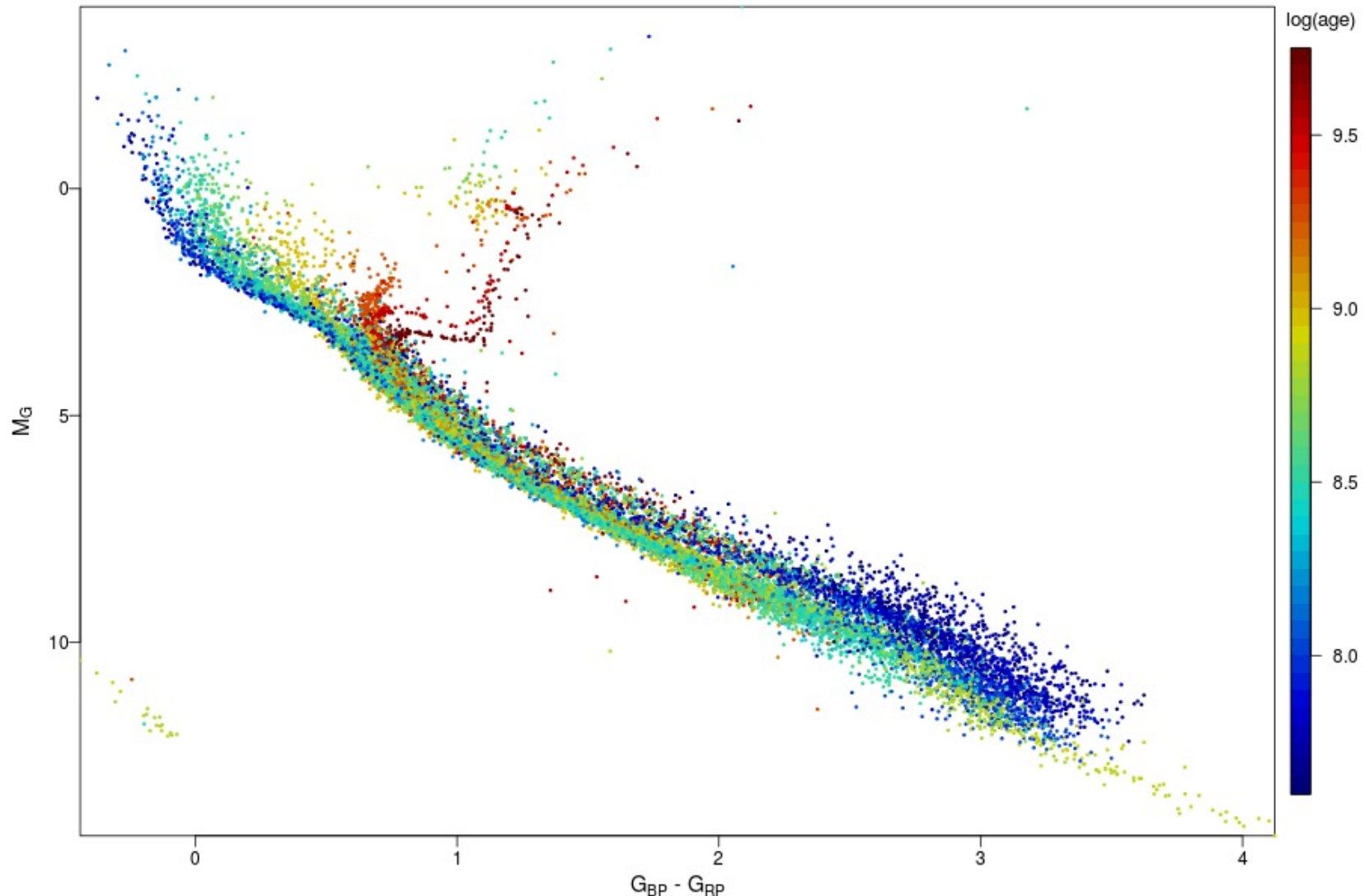
• Such $L-T$ diagrams can calibrate the age t_{gc} of a globular cluster: **spectroscopic parallaxes** or **main sequence fitting**. Van den Bergh quotes recent estimates as

C & O,
pp. 475-6

$$t_{gc} \sim (1.5 \pm 0.2) \times 10^{10} \text{ years} \quad . \quad (8)$$

This was a critical piece of evidence conflicting with Hubble's original estimates of the speed of the flow.

Gaia HR Diagram for Open Clusters



- Observational HR diagram from [Gaia Data Release 2 \(DR2\)](#) for 32 open clusters of main sequence and white dwarf stars. Cluster age is color-coded in years (log scale) according to bar on right. [Babusiaux et al. A&A **616**, A10 \(2018\)](#).