A Run On A Financial Market*

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January 30, 2002

Abstract

Our paper offers a minimalist model of a run on a financial market. The prime ingredient is that each investor fears having to liquidate after a run, but before prices can recover back to fundamental values. During the run, only the market-making sector is willing to absorb shares. To avoid having to possibly liquidate shares at the marginal post-run price—in which case the market-making sector will already hold a lot of share inventory and thus be more reluctant to absorb additional shares—all investors may prefer selling their shares into the market today at the average run price, thereby causing the run itself. Liquidity runs and crises are not caused by liquidity shocks per se, but by the fear of future liquidity shocks.

*This is a preliminary version. Please do not circulate. The most recent version of this paper will be available from http://welch.som.yale.edu/academics/. We thank Harry Mamaysky for useful conversations.
In contrast to the financial institutions literature, in which investors receive a commitment for repayment, runs on financial markets have not been a prime subject of inquiry. Our paper offers a minimalist model of a run on a financial market. The prime ingredient of our model is that investors fear (but do not necessarily experience) future liquidity shocks. This creates two scenarios.

In the good scenario, a risk-neutral public holds most of the risky shares. Investors hit by a liquidity shock in the future will sell to the risk-averse market-making sector at a “low-inventory price,” which will be close to the risk-neutral value of the asset. In the good scenario, the market-making sector provides the public with low-cost insurance against liquidity shocks.

In the bad scenario, every investor conjectures that other investors intend to sell today, thus causing a “run.” By joining the pool of selling requests today, an individual investor can expect to receive the average price that is necessary to induce the market-making sector to absorb all tendered shares today. The investor's alternative is to not enter the pool and instead to hold onto the shares. In making this decision, this investor is better off if he can wait out the storm and realize the eventual expected asset value. However, if he were randomly hit by the possible liquidity shock, this investor would need to sell his shares behind the rest of the public. But, with the market-making sector already holding the shares of other tendering investors, this post-run price will be worse than the average in-run price today. If the average in-run price is greater than the expected payoff achieved by waiting, this investor will join the herd and also sell into the run. If other investors act alike, the conjecture that other investors sell today ends up being true.

Our bad scenario relies either on random or batch execution. However, if execution is sequential, investors cannot expect to avoid a later place in line by joining
the selling pool. Thus, the last investors (who now know they are last) are better off waiting rather than joining the herd and the bad scenario unravels.

In reality, financial markets lack perfectly sequential execution in at least three circumstances. First, there is often no sequential execution after a market closure: for example, at the stock market opening or after a trading halt, markets are often conducted in a “batch” mode where all orders are crossed at the same price—and, indeed, fears of stock market runs seem higher around the NYSE opening period. Second, even during normal trading, sequential execution may break down under the load of orders flowing in, and investors' orders' executions could become random. A tendering investor, not knowing his place in the queue, would expect to receive some average price.\(^1\) There is a lot of anecdotal evidence that sequential execution broke down in the 1987 stock market crash. This could lead to an immediate transition from a situation in which liquidity shocks are not a major concern (as in a sequential market) to a situation in which they become paramount (as in a batch market). Third, in many over-the-counter financial markets, counterparties need to be found, and when multiple sellers are searching for counterparties, there is randomness as to who will find the potential buyers first.

It is important to point out that our model is not driven by the liquidity shocks themselves. Instead, prices and market-making inventories are driven by the fear of future liquidity shocks. Thus, the liquidity shocks might loom in the distant future and cause a run today. If underlying exogenous parameters change, high volatility and runs (low prices, high market-making inventory) can appear and disappear many times before the liquidity shocks themselves. An empiricist might not even

\(^1\)Yet another interpretation would have a seller be unaware whether he received information about overvaluation/undervaluation before or after other investors, as in Brunnermeier and Abreu (2001).
necessarily recognize the relevance of actual liquidity constraints.

In a sense, the outcome of our model is perplexing. There are no transaction or search costs or asymmetric information. Investors are numerous, risk-neutral, and homogeneous. The market-making sector can be very deep with with only slight risk aversion (small discounts to absorb liquidity). Unlike much feedback trading literature, any liquidity shocks can loom far away in the distance and can be correlated or uncorrelated among investors. Unlike in the financial institutions run literature, in our financial markets setting, there are no commitments to repay promised deposits, no technology convertability losses, and no need for investors to join in a run in order to get anything. Rather, our investors can attempt to “wait out the storm,” and thereby perhaps do better. And, yet, our financial market can produce very inefficient allocation outcomes, as well as an accelerator effect in which every investor wants to sell to avoid selling behind the average investor. The intuition that runs can be driven by investors fearing “to come in last” is solid, and resonates with many who witnessed the 1987 crash.

The main assumptions and insights of the model seem both realistic and robust. Indeed, the analytics of the model are simple, relying only on situations in which sequential execution breaks down, and on a split of participants into a (potentially slightly) risk-averse market-making sector and an outside sector living in fear of potential future liquidity shocks. Not requiring much machinery, the model hints that run equilibria may not be esoteric but intrinsic to financial markets (just as they are intrinsic for financial institutions). This is not to argue that runs are frequent (indeed, they are not!), but that their occurrence is not logically far-fetched.

Our paper now proceeds as follows: Section I lays out the model. The model's emphasis is on simplicity. Section II describes the equilibrium under CARA and
CRRA market-making utility. (It also offers an equilibrium model for the market-making sectors’ inventory in ordinary times.) Section III adds margin constraints to our model, which de-facto endogenizes the liquidation probability and causes multiple equilibria. Section IV discusses the economics of the equilibrium. Section V relates our work to earlier papers, particularly the bank-run literature (Diamond and Dybvig (1983)). And Section VI concludes.

I The Basic Model Setup

We consider a model with three dates \( t = 0, 1, 2 \) and two assets: a risk-free bond in infinitely elastic supply with a gross payoff of $1 at date two and a risky asset (henceforth, “stock”) with gross random payoff of \( \tilde{Z} \) at date two. For simplicity, we normalize the date 0 and date 1 price of the bond to be $1. The date 0 and date 1 price of the stock is determined endogenously.

Shares in the stock trade at date 0 and date 1. There are two types of traders in our market: atomistic individual investors and a competitive market-making sector.

Market-makers constitute an entire sector which encompasses not just the specialist, but all traders willing to absorb shares upon demand, i.e., regardless of the (fear of) liquidity shocks. Still, it is reasonable to attribute a finite risk absorption capacity to this sector and thus we assume the market-making sector is risk-averse.
in aggregate. For example, many institutions and traders do not seem willing to absorb shares during a financial markets crash, and instead prefer to wait it out.\footnote{Dennis and Strickland (2002) find that institutions are more likely than retail investors to sell into a dropping market. Thus, the market-making sector may be smaller than often assumed, and suffer a steeper and quicker price drop than sketched by our model.} We also assume that the market-making sector is competitive and is characterized by a “representative” market-maker with date 0 wealth $W_0$ and date 0 inventory of zero shares.

**Individual investors** are identical and endowed with shares which sum to the total supply of shares (normalized to one). Individual investors are assumed to be risk-neutral. Importantly, individual investors face a potential liquidity shock at date 1. We model liquidity shocks in various ways. In Section II, we assume that each individual investor may be forced to liquidate her shares at date 1 with an exogenous probability $s$. After showing that this leads to a non-zero market-making inventory at date 0 in Subsection II.A, we parameterize the liquidity shocks and market-making utility function. The liquidity shocks are assumed to be perfectly correlated across investors in Subsection II.B and independent across investors in Subsections II.C and II.D. In Section III, we endogenize the date 1 liquidation probability to depend on the date 0 stock price.

To recap, there are two important differences between the market-making sector and individual investors. First, individual investors are assumed to be risk-neutral and the market-making sector is risk-averse. The former assumption is not crucial to the analysis but captures the fact that the investing public has considerably more risk absorption capacity than the market-making sector, and that, in a Pareto efficient outcome, shares should be held by the investing public (the most efficient bearers of risk). Second, only individual investors face a potential liquidity shock at date 1.
date 1. This provides the motivation for trade between individual investors and the market-making sector at both dates 0 and 1. Moreover, this ensures that only the market-making sector (in equilibrium) is willing to buy shares in a run situation, at a price that depends on the market-making sector’s risk-tolerance. An investor who learns that he is not subject to any future forced liquidation can join and thereby deepen the market-making sector.\(^3\)

To close the model, we assume that the equilibrium price is determined by a zero-utility condition on the representative market-maker. Specifically, in our batch execution model, we assume that the price of the stock at each date is set so that the representative market-maker is indifferent between buying the entire batch and holding his inventory. Because the representative market-maker is risk averse, this price is typically decreasing in his inventory. Our zero-utility condition is analogous to Kyle’s zero-expected-profits condition for a risk-neutral market-making sector and can be justified by the joint assumptions that the market-making sector is competitive and market-makers are free to enter or exit after each date.

There are two equivalent interpretations to the market microstructure which determines the equilibrium price—and both of them can be shown to lead to the same market-maker demand function and thus identical solutions for our model. One interpretation is that all orders are batched and executed at an identical “average” price. For simplicity of exposition, we proceed using this specific assumption only. Sell orders from individual investors at each date are batched and then executed at an average price that yields zero utility for the representative market maker. Another interpretation is that orders are executed sequentially (with lower prices for

\(^3\)Naturally, this investor must have some risk-aversion or limited capital. Otherwise, the market-making sector itself becomes infinitely deep when the first investor appears. Section IV.A discusses what happens when the aggregate risk tolerance changes in a crisis.
subsequent trades) but in random order. In both cases the investor submits “mar-
ket orders” and is unsure of the exact price at which her shares will be executed.
In the first interpretation, we must impose a zero-utility condition in each period
while in the second interpretation we must impose a zero-utility condition on each
trade. Thus, our model applies equally to batch auction markets (e.g., at the NYSE
stock market opening and after a trading halt); and to over-the-counter markets and
to stock market crashes, when limited communication lines to the market-making
sector can change the typical deterministic sequential execution into random exe-
cution.

II The Equilibrium

A A Theory of Market-Making Inventory

In what follows, we assume that individual investors are endowed with the entire
supply of shares at date 0 (consistent with an efficient allocation of risk). We an-
alyze only the situations in which individual investors may wish to sell shares at
date 0 due to the fear of a liquidity shock at date 1. These situations are the most
interesting because we intend to demonstrate that the fear of liquidity shocks can
lead to substantial market-maker inventories and thus inefficient allocations of risk.
Therefore, we ignore situations in which individual investors want to buy shares at
date 0.

Consider an individual investor who conjectures that a total of $\alpha$ shares will be
sold by individual investors to the market-making sector at date 0 and let $p_0(\alpha)$
denote the date-0 price set by the market makers when $\alpha$ sell orders arrive at date 0.
If this investor also sells her shares at date 0, she will expect to receive the price \( p_0(\alpha) \). However, if this investor chooses not to sell her shares at date 0 then either (i) she will be forced to liquidate her shares with probability \( s \) at date 1 or (ii) she will not be forced to liquidate her shares with probability \( 1 - s \) at date 1 and will optimally wait to receive the expected value of the stock, \( \mu \), at date 2. If liquidity shocks are perfectly correlated, the remaining proportion \( (1 - \alpha) \) of shares will be liquidated at date 1 if the liquidity shock occurs. (We show in Subsection C that if liquidity shocks are independent the proportion \( s(1 - \alpha) \) shares will be liquidated.)

Let \( p_1(1 - \alpha; \alpha) \) denote the date-1 price set by the market-makers when they hold \( \alpha \) shares of inventory and \( (1 - \alpha) \) new sell orders arrive at date 1. If this investor does not sell at date 0 she will expect to receive \( s \cdot p_1(1 - \alpha; \alpha) + (1 - s) \cdot \mu \). Thus, it will be optimal for this investor to sell if and only if

\[
p_0(\alpha) \geq s \cdot p_1(1 - \alpha; \alpha) + (1 - s) \cdot \mu.
\]  

Notice that if an investor is forced to liquidate at date 1, she receives a lower selling price than if she had sold at date 0. A risk-averse market-making sector implies that \( p'(\cdot) < 0 \), i.e., the market-making sector will require a lower price (greater risk premium) if it has to buy a greater number of shares. However, if she is not forced to liquidate at date 1, she receives the expected value of the stock which is greater than the selling price at date 0. The decision to sell at date 0 depends critically on the investor's beliefs about whether other investors will choose to sell at date 0.
We consider only symmetric Nash equilibria.

**Definition 1** Let \( F(\alpha) \) denote the expected net benefit of selling shares at date 0 (compared to not selling) when the investor conjectures that \( \alpha \) shares will be sold at date 0. If liquidity shocks will be perfectly correlated across investors,

\[
F(\alpha) = \begin{cases} 
  p_0(\alpha) & \text{if tender today} \\
  s \cdot p_1((1 - \alpha);\alpha) & \text{if forced to liquidate tomorrow} \\
  (1 - s) \cdot \mu & \text{if liquidation not necessary}
\end{cases}
\]

If liquidity shocks will be independent across investors,

\[
F(\alpha) = \begin{cases} 
  p_0(\alpha) & \text{if tender today} \\
  s \cdot p_1((1 - \alpha);\alpha) & \text{if forced to liquidate tomorrow} \\
  (1 - s) \cdot \mu & \text{if liquidation not necessary}
\end{cases}
\]

Then (i) waiting \((\alpha^* = 0)\) is a pure strategy Nash equilibrium iff \( F(0) \leq 0 \); (ii) selling \((\alpha^* = 1)\) is a pure strategy Nash equilibrium iff \( F(1) \geq 0 \); and (iii) \( \alpha^* \in (0, 1) \) is a mixed strategy Nash equilibrium iff \( F(\alpha^*) = 0 \).

We can immediately demonstrate that \((\alpha^* = 0)\) is not a symmetric Nash equilibrium if the probability of a liquidity shock is positive.

**Theorem 1** Although market-makers are risk-averse and investors are risk-neutral and not yet liquidity-shocked, the market-making sector holds inventory at date 0 if there is a positive probability of a liquidity shock \((s)\) at date 1.

**Proof:** For \( s > 0 \), \( F(0) > 0 \), so \( \alpha^* = 0 \) is not an equilibrium. \( \Box \)

The intuition is that if the market-making sector holds zero inventory, it would be willing to accept the first share at its risk-neutral valuation today. Thus, the first seller would avoid the liquidation risk tomorrow without any price penalty today.
Unfortunately, there is little more we can say without parametrizing returns and the representative market-making sector’s utility function in order to determine the \( p_t(\cdot) \) functions. Thus, we now consider two cases. In the first case, we assume that the market-making sector has constant absolute risk aversion (CARA) preferences and the stock payoff is distributed normal. We solve for the symmetric Nash equilibria with perfectly correlated liquidity shocks in Subsection \( B \) and independent liquidity shocks in Subsection \( C \). The CARA plus normality assumptions allow us to obtain simple closed-form solutions but at the expense of rich comparative statics. In the second case, we assume that the market-making sector has constant relative risk aversion (CRRA) preferences and the stock payoff is distributed binomial. (We also assume independent liquidity shocks.) The CRRA example yields richer comparative statics, but it can only be solved numerically.

**B Example 1: CARA utility, normally distributed payoffs, and perfectly correlated liquidity shocks**

In this example we assume that (i) the stock payoff \( \tilde{Z} \) is normally distributed with mean \( \mu \) and variance \( \sigma^2 \), (ii) the market-making sector has the negative exponential utility function \( u(w) = -e^{-\gamma w} \) where \( \gamma \) is the coefficient of absolute risk aversion, and (iii) liquidity shocks are perfectly correlated.

First, we derive the equilibrium pricing function, \( p_0(\alpha) \). By assumption, this is the price at which the representative market-maker is indifferent between buying \( \alpha \)
shares at date 0 and maintaining zero inventory of shares. Under the assumptions of CARA preferences and normally distributed stock payoffs, the price $p_0(\alpha)$ solves

$$
E[-e^{-\gamma \tilde{W}_2}] = E[e^{-\gamma W_0}]
\Leftrightarrow E[\tilde{W}_2] - \gamma \cdot \text{Var}[\tilde{W}_2]/2 = W_0.
$$

(4)

where $\tilde{W}_2 = W_0 + \alpha \cdot (\bar{Z} - p_0)$. Solving yields $p_0(\alpha) = \mu - \gamma \sigma^2 \alpha/2$.

Now we derive the price that obtains at date 1 in the event of a liquidity shock. Obviously, if no liquidity shock occurs the price at date 1 is the expected payoff of the stock, $\mu$. Let $p_1(1 - \alpha; \alpha)$ denote the price at which the representative market-maker is indifferent between buying $(1 - \alpha)$ new shares at date 1 and maintaining an inventory of $\alpha$ shares. Under the assumptions of CARA preferences and normally distributed stock payoffs, the price $p_1(1 - \alpha; \alpha)$ solves:

$$
E[\tilde{W}_2 + (1 - \alpha)(\bar{Z} - p_1)] - \gamma \cdot \text{Var}[\tilde{W}_2 + (1 - \alpha)(\bar{Z} - p_1)]/2
= E[\tilde{W}_2] - \gamma \cdot \text{Var}[\tilde{W}_2]/2
\Rightarrow p_1(1 - \alpha; \alpha) = \mu - (1 + \alpha)\gamma \sigma^2/2
$$

(5)

Substituting $p_0(\alpha)$ and $p_1(1 - \alpha; \alpha)$ into our definition of $F(\alpha)$ yields the following result:

**Theorem 2** If liquidity shocks are perfectly correlated across investors there is a unique symmetric Nash equilibrium with

$$
\alpha^* = \begin{cases} 
(\frac{s}{1-s}) & \text{if } s \leq 1/2 \\
1 & \text{if } s > 1/2 
\end{cases}
$$

(6)
Proof: Substitute the pricing functions into equation 2. Note that \( F(0) > 0 \) for all \( s > 0 \) and \( F_\alpha^* \), the derivative of \( F \) with respect to \( \alpha^* \), is negative. Thus, there are two possibilities. If \( F(1) \geq 0 \) then there is a unique pure strategy equilibrium \( \alpha^* = 1 \) and if \( F(1) < 0 \) there is a unique mixed strategy, \( \alpha^* \), where \( F(\alpha^*) = 0 \). For \( s > 1/2 \), \( F(1) > 0 \) thus \( \alpha^* = 1 \). For \( s \leq 1/2 \) solving for \( \alpha^* \) yields the result. \( \square \)

The model features an “accelerator” effect: fear of other investors liquidating has an immediate influence on each investor's own decision to liquidate. For very small values of \( s \), i.e., very little chance of future liquidity shocks, an investor sees other investors waiting and thus does not mind waiting herself. The market-making sector needs to hold almost no shares today (\( \alpha^* \) close to zero) and the outcome is close to the Pareto-optimum. With increasing \( s \), the fraction of tendering investors rises ever more quickly since the first derivative of \( \alpha^* \) with respect to \( s \) is \( \frac{\partial \alpha^*}{\partial s} = \frac{1}{(1 - s)^2} \) which is increasing in \( s \). In fact, even if there is “only” a 50-50 chance of investors facing a future liquidity shock, and even if the market-making sector is extremely risk-averse (\( \gamma \to \infty \)), risk-neutral investors find themselves unwilling to hold any stock today. Naturally, this is an extremely inefficient outcome.

Although these are not distinct equilibria, there is a flavor of two distinct scenarios here: a good scenario, in which the probability of liquidation is low, and the market-making sector is not holding much inventory; and a bad (or run) scenario, in which the probability of individual liquidation is average, and the risk-averse market-making sector has to absorb all shares in the economy.\(^4\)

\(^4\) It is straightforward to show if \( s \leq 1/2 \) the volatility of stock returns (\( \tilde{R}_{0,1} = \frac{p_1 - p_0}{p_0} \)) is given by

\[
\sigma(\tilde{R}_{0,1}) = \frac{y \sigma^2 s}{2(1-s)\mu - y \sigma^2 s}.
\]

As expected, the underlying value volatility \( \sigma^2 \) is distinct from the stock return volatility. The volatility of stock returns increases in \( s \), \( y \), and \( \sigma^2 \) and decreases in \( \mu \). Moreover, the change in volatility increases in \( s \). Thus, seemingly small changes in liquidation probability \( s \) can significantly change market volatility.
Interestingly, with CARA utility, the risk-absorption capacity of the market-making sector ($\gamma$) and the riskiness of the stock ($\sigma$) play no role in the equilibrium outcome ($\alpha^*$). Expanding the market-making sector in both good and bad times would not solve the allocation problem created by the fear of facing a liquidity shock.\footnote{Of course, when the market-making sector is deep, prices are close to risk-neutral even if no risk-neutral investor is willing to hold shares and thus the welfare loss is small.} The reason is that there are two countervailing forces when the market-making sector is deep (or payoff variance is low): On the one hand, the average in-run price is higher because the market-making sector is close to risk neutral. On the other hand, the marginal price obtained after the run is also higher. In the case of constant absolute risk aversion preferences, these two effects exactly offset each other in the investors’ selling decision. With CARA preferences, the market-making price is linear in inventory. Although risk aversion and payoff variance affect the slope of the linear demand curve, they do not affect the relation between average and marginal prices. Thus, the tradeoff between tendering today and waiting is independent of these parameters. The prime ingredient in this version of our model is investors’ fear of future liquidation, $s$.

C Example 2: CARA utility, normally distributed payoffs, and independent liquidity shocks

We continue with CARA utility and normal returns but now allow liquidity shocks to occur independently with exogenous probability $s$ at date 1. The key difference between the perfectly correlated and independent liquidity shock cases is that in the former case, all investors who did not sell at date 0 must liquidate with probability $s$ at date 1 whereas in the latter case, proportion $s$ of investors who did not sell at
date 0 must liquidate with probability 1 at date 1.

The derivation of the equilibrium price function at date 0 is the same in both cases. Thus, as we demonstrated above, if an individual investor conjectures that a total of $\alpha$ shares will be sold by individual investors to the market-making sector at date 0 she will receive the price $p_0(\alpha) = \mu - \gamma \sigma^2 \alpha / 2$ if she sells at date 0. However, because liquidity shocks are independent across individual investors we know with probability one (by the Law of Large Numbers) that a proportion $(1 - \alpha)s$ of shares will be liquidated at date 1. Let $p_1((1 - \alpha)s; \alpha)$ denote the date-1 price set by the market makers when $(1 - \alpha)s$ new sell orders arrive at date 1 and the market-making sector already held $\alpha$ shares. By assumption, $p_1((1 - \alpha)s; \alpha)$ is the price at which the representative market-maker is indifferent between buying $(1 - \alpha)s$ new shares at date 1 and maintaining an inventory of $\alpha$ shares. Under the assumptions of CARA preferences and normally distributed stock payoffs, the price $p_1((1 - \alpha)s; \alpha)$ solves:

\[
\begin{align*}
E[\tilde{W}_2 + s(1 - \alpha)(\tilde{Z} - p_1)] - \gamma \text{Var}[\tilde{W}_2 + s(1 - \alpha)(\tilde{Z} - p_1)])/2
&= E[\tilde{W}_2] - \gamma \text{Var}[\tilde{W}_2]/2 \\
\Rightarrow p_1(s \cdot (1 - \alpha); \alpha) &= \mu - [2\alpha + (1 - \alpha)s] \gamma \sigma^2 / 2
\end{align*}
\]

(8)

Replacing the perfect correlation shocks $p_1((1 - \alpha); \alpha)$ from the previous section with its independent shocks equivalent $p_1((1 - \alpha)s; \alpha)$ in our definition of $F(\alpha)$ and substituting the equilibrium price functions yields the following result:
Theorem 3 If liquidity shocks are independent across investors there is a unique symmetric Nash equilibrium with

\[ \alpha^* = \begin{cases} \left( \frac{s}{1-s} \right)^2 & \text{if } s \leq 1/2 \\ 1 & \text{if } s > 1/2 \end{cases} \]  

(9)

Proof: Substitute the pricing functions into equation 3 Note that \( F(0) > 0 \) for all \( s > 0 \) and \( F_{\alpha^*} \), the derivative of \( F \) with respect to \( \alpha^* \), is negative. Thus, there are two possibilities. If \( F(1) \geq 0 \) then there is a unique pure strategy equilibrium \( \alpha^* = 1 \) and if \( F(1) < 0 \) there is a unique mixed strategy, \( \alpha^* \), where \( F(\alpha^*) = 0 \). For \( s > 1/2, F(1) > 0 \) thus \( \alpha^* = 1 \). For \( s \leq 1/2 \) solving for \( \alpha^* \) yields the result. \( \square \)

Again, the model features an “accelerator” effect: fear of other investors liquidating has an immediate influence on each investor’s own decision to liquidate. The first derivative of \( \alpha^* \) with respect to \( s \) is \( \frac{\partial \alpha^*}{\partial s} = \frac{2s}{(1-s)^3} \). Thus, around \( s = 0.23 \), the fraction of investors that unload their shares onto the market-making sector changes one-to-one with changes in \( s \). Above \( s = 0.23 \), even small changes in the perceived fraction of investors can cause large changes in market-making inventory and equilibrium pricing. Again, even if there is “only” a 50-50 chance of investors facing a future liquidity shock, and even if the market-making sector is extremely risk-averse (\( \gamma \rightarrow \infty \)), risk-neutral investors find themselves unwilling to hold any stock today.

Unlike in the perfectly correlated shocks version of our model, in this independent liquidity shock version of our model we know with certainty that the price at date 1 will be lower than the price at date 0. This creates an arbitrage opportunity. One might reasonably ask why a market-maker would buy shares at date 0 when she knows for sure that the price will be lower at date 1. In reality (unlike our model) there may be sources of uncertainty that make it possible that the price at date 1
will be higher than the price at date 0. For example, a change in the environment which make liquidity shocks unlikely for some investors will cause them to buy the stock at date 1 and drive up its price. One might also reasonably ask why investors do not short the stock at date 0 and buy it back at date 1. Shorting stock, however, might be extremely difficult during such runs. Alternatively, uncertainty about the timing of the stock price bounce back can potentially introduce a source of risk (costly margin calls) that limits the aggressiveness of short positions at date 0 (see, e.g., Liu and Longstaff (2000)).

Another pertinent question is why noone simply waits to be a standby investor to buy only at the bottom of the crash? But, this question is bigger than just our model. What prevented an investor from becoming rich during the 1987 (or any other) crash? There are a number of answers. First, there may be (unmodelled) uncertainty about the timing of the end of the crash and about a simultaneous revision in the expected return of the assets. Indeed, we sometimes see persistent market calamities following financial market crashes, we sometimes see sharp drops followed by immediate reversals—presumably with investors recognizing that their liquidity constraints will not bind, that they can safely join the market-making sector, and that the good state is about to return. Such investors should do well. Second, it is costly to create stand-by liquidity, i.e., a large investor who carries zero-inventory most of the time (so as to be almost risk-neutral) and who is lurking around only for the opportunities presented in a crash. If crashes are rare, this may not be a profitable use of resources. Third, execution in the final stage, the termination of the run, may be as uncertain as it is in the run initiation. A standby investor may try to wait until the bottom of the market to buy shares for a song,

6Most U.S. crashes indeed showed an immediate bounceback.
but he may not be sure whether his buy orders will be executed at the immediate last-investor run price or whether he may miss this opportunity altogether. After all, the price in our model drops sharply and then rebounds sharply back to $\mu$.

D Example 3: CRRA utility

Although the CARA equilibrium illustrates the importance of the fear of liquidity shocks, the linearity of the market-maker's demand function reduces the richness of its comparative statics. To obtain a non-linear demand curve, we now assume that the representative market-maker has constant relative risk aversion (CRRA) preferences of the form $u(w) = \left( \frac{w^{1-\gamma}}{1-\gamma} \right)$ where $\gamma$ is the coefficient of relative risk aversion. We further assume that the stock payoff $Z$ takes on one of two values: $U$ with probability $\pi$ and $D$ with probability $1 - \pi$. Absence of arbitrage requires $U > p_t > D$.

Again, the equilibrium pricing function, $p_0(\alpha)$, ensures that the representative market-maker is indifferent between buying $\alpha$ shares at date 0 and maintaining zero inventory of shares. Let $W_2(z) \equiv W_0 + \alpha \cdot (z - p_0)$ for $z = U, D$. In this example, the price $p_0(\alpha)$ solves

$$\left( \frac{1}{1-\gamma} \right) \cdot \left[ \pi \cdot [W_2(U)]^{1-\gamma} + (1 - \pi) \cdot [W_2(D)]^{1-\gamma} \right] = \left( \frac{1}{1-\gamma} \right) W_0^{1-\gamma}. \quad (10)$$

This price function is the same whether liquidity shocks are perfectly correlated or independent across investors.
If a liquidity shock occurs and they are perfectly correlated across investors the price $p_1(1 - \alpha; \alpha)$ solves

$$
\left(\frac{1}{1 - \gamma}\right) \cdot \left\{ \pi \cdot [W_2(U) + (1 - \alpha)(U - p_1)]^{1 - \gamma} + (1 - \pi) \cdot [W_2(D) + (1 - \alpha)(D - p_1)]^{1 - \gamma} \right\} \\
= \left(\frac{1}{1 - \gamma}\right) \cdot \left\{ \pi \cdot [W_2(U)]^{1 - \gamma} + (1 - \pi) \cdot [W_2(D)]^{1 - \gamma} \right\}.
$$

(11)

If a liquidity shock occurs and they are independent across investors the price $p_1((1 - \alpha)s; \alpha)$ solves

$$
\left(\frac{1}{1 - \gamma}\right) \cdot \left\{ \pi \cdot [W_2(U) + s(1 - \alpha)(U - p_1)]^{1 - \gamma} + (1 - \pi) \cdot [W_2(D) + s(1 - \alpha)(D - p_1)]^{1 - \gamma} \right\} \\
= \left(\frac{1}{1 - \gamma}\right) \cdot \left\{ \pi \cdot [W_2(U)]^{1 - \gamma} + (1 - \pi) \cdot [W_2(D)]^{1 - \gamma} \right\}.
$$

(12)

Unfortunately, there are no closed-form expressions for the price functions, $p_0(\alpha)$, $p_1((1 - \alpha); \alpha)$, and $p_1((1 - \alpha)s; \alpha)$. However, a simple numerical algorithm can find them exactly. We can then use a simple search algorithm to find the equilibrium inventory of the market-making sector ($\alpha^*$).

Although we do not have a general proof of uniqueness for $\alpha^*$ nor for the monotonicity of the comparative statics for $\alpha^*$ with respect to important parameters of the model, an examination of a large region of the relevant parameter space yields consistent results. For the numerical examples that follow, we have chosen reasonable base-case parameters to represent market-maker wealth (roughly 1/10 of the value of the risky asset) and risk-aversion ($\gamma = 3$). For all cases, it can be shown that $\alpha^*$ is unique. Figure 1 graphs the market-making sector’s equilibrium holdings ($\alpha^*$) as a function of exogenous parameters for the case of independent liquidity.
shocks across investors. The numerical results are qualitatively similar when liq-
uidity shocks are perfectly correlated across investors. Typically, we find that the
market-making sector holds more inventory ($\alpha^*$)

- when the market-making sector has greater wealth;

- when the market-making sector has greater risk-absorption capacity (risk-
aversion coefficient $\gamma$ is lower);

- when the asset is less risky ($U - D$ is smaller holding the mean payoff $\pi U + (1 - \pi)D$ constant)

- when the probability of a liquidity shock ($s$) is higher.

We already know from the CARA case that it is not the steepness of the de-
mand curve itself (i.e., the “depth”) that matters to market-making inventory: higher
risk-capacity for the market-making sector does not only allow investors to unload
shares at an attractive price in a run, but it also allows them to enjoy a better price
after a run. Instead, what matters is the second derivative of the demand curve.
Just as in the CARA case, fear of liquidity can cause the run, and the sensitivity of
$\alpha$ to $s$ increases in $s$. But, in the CRRA case, the other parameters (such as wealth,
risk-aversion, and riskiness) matter for the relative share allocations to the extent
that they bend the market-making demand function.
III Margin Constraints

Margin calls, which force investors to sell more shares if the share price declines, are well-known to be important during financial market crashes (see, e.g., Chowdhry and Nanda (1998)). Our model does not require margin calls, but margin calls can endogenize liquidity constraints, produce multiple equilibria, and very high-frequency phase transitions. This can amplify the already existing accelerator effect. We can model how a low market price forces an investor to sell in a very simple fashion. Let the (endogenous) probability of liquidation be given by

\[ s \equiv s(p_0) \equiv \left(1 - \frac{p_0}{\mu}\right)^m \quad \text{for} \quad 0 \leq p_0 \leq \mu, \quad (13) \]

where \( m \) measures the severity of the margin constraints during a crash. The greater \( m \), the smaller the changes in stock price forcing a margin call. Regardless of \( m \), for a low enough price \( p_0 \rightarrow 0 \), the investor must liquidate his entire shareholdings. For a high enough price \( p_0 \rightarrow \mu \), the investor has no margin constraints.\(^7\) We focus on the CARA utility case with normally distributed payoffs which yields closed-form solutions when \( m = 1 \). (There is no principal difference if we use CRRA utility instead.)

\(^7\)There is some natural ambiguity in terminology here: a tighter initial margin constraint before a crash (that is, one where investors have to put up more money up-front) may reduce the need to liquidate positions during a crash (the maintenance margin) and thus be a more lax margin constraint later on.
**Theorem 4** Let the asset payoff be normally distributed and the representative market-maker have CARA utility so $p_0(\alpha) = \mu - \gamma \sigma^2 \alpha / 2$; and let investors’ liquidity constraints be $s(p_0) = \left( 1 - p_0 / \mu \right)$ (i.e., $m = 1$).

**Perfectly correlated liquidity shocks:** If $(\mu / \gamma \sigma^2) < 1/2$ or $(\mu / \gamma \sigma^2) > 1$ there is a unique equilibrium, $\alpha^* = 0$. If $1/2 \leq \mu / \gamma \sigma^2 \leq 1$, there are three equilibria:

1. The Pareto-Optimal Pure-Strategy Equilibrium:
   $$\alpha^* = 0;$$
   \hfill (14)

2. The Inefficient Pure-Strategy Equilibrium:
   $$\alpha^* = 1$$
   \hfill (15)

3. The Mixed-Strategy Equilibrium:
   $$\alpha^* = \left( \frac{2\mu - \gamma \sigma^2}{\gamma \sigma^2} \right)$$
   \hfill (16)

**Independent liquidity shocks:** If $(\mu / \gamma \sigma^2) > 1$ there is a unique equilibrium, $\alpha^* = 0$. If $0 \leq \mu / \gamma \sigma^2 \leq 1$, there are three equilibria:

1. The Pareto-Optimal Pure-Strategy Equilibrium:
   $$\alpha^* = 0;$$
   \hfill (17)

2. The Inefficient Pure-Strategy Equilibrium:
   $$\alpha^* = 1$$
   \hfill (18)

3. The Mixed-Strategy Equilibrium:
   $$\alpha^* = \left( \frac{1}{2} + \frac{2\mu}{\gamma \sigma^2} \right) - \sqrt{\left( \frac{1}{4} + \frac{2\mu}{\gamma \sigma^2} \right)}$$
   \hfill (19)

**Proof:** Consider the perfectly correlated case. Substitute $p_0(\alpha) = \mu - \gamma \theta \sigma^2 / 2$ into and $p_1(1-\alpha; \alpha)$ into equation 2 where $s = \left( 1 - p_0 / \mu \right) = \gamma \alpha^* \sigma^2 / 2\mu$. First, note that $F(0) = 0$ so $\alpha^* = 0$ is an equilibrium for all values of $\mu / \gamma \sigma^2$. Second, $F(1) \geq 0$ if and only if $\mu / \gamma \sigma^2 \leq 1$ but $s \leq 1$ requires $1/2 \leq \mu / \gamma \sigma^2$ thus $\alpha^* = 1$ is an equilibrium if and only if $1/2 \leq \mu / \gamma \sigma^2 \leq 1$. Finally, $F(\alpha^*) = 0$ has one solution in the interval $[1/2, 1]$ which is the mixed-strategy equilibrium. The independent liquidity shock case is proved similarly. □
All prior insights remain: there can be negative externalities and our investors can be caught in a prisoner’s dilemma. The pure-strategy equilibrium $\alpha^* = 0$ is Pareto-optimal: risk-neutral investors hold all shares and no liquidation occurs. The mixed-strategy equilibrium is a mixed blessing, with low equilibrium market-making inventory when market-making risk tolerance is high. The pure-strategy equilibrium $\alpha^* = 1$ is a total market collapse, in which the risk-averse market-making sector is forced to hold all shares, and no investor is willing to take the risk of trading last.

Naturally, when there are margin constraints, investors need to make additional judgments as to which equilibrium they are in. Slight changes in beliefs may trigger not only steep changes in market-maker inventory ($\alpha$)—they may even trigger tendering changes which are outright discontinuous.

Evaluating the influence of our parameter $m$, the bite of the margin constraints, is a bit more tricky. First, in each case, interior equilibria live within their own feasible domains. Second, higher powers on $m$ yield multiple polynomial solutions. Third, there is no general closed-form solution.  

Figure 2 plots the equilibrium inventory holdings as a function of $\mu/\gamma\sigma^2$ for three different margin constraints. Higher values of $m$ (with their lower probabilities of a future liquidity shock) shift the mixed equilibrium left, increasing the in-equilibrium market-making inventory. From a policy-making perspective, forcing investors to put up more collateral (increasing $m$ by fiat) thus may reduce the inci-

---

\[\text{For example, if shocks are perfectly correlated, there is closed-form solution for } m = 2. \text{ If } \mu/\gamma\sigma^2 > 1/\sqrt{2} \text{ there is a unique equilibrium } \alpha^* = 0. \text{ If } \mu/\gamma\sigma^2 \leq 1/\sqrt{2} \text{ there are three equilibria: } \alpha^* = 0, \alpha^* = 1, \text{ and } \alpha^* = -1/2 + \sqrt{1/4 + 4x^2} \text{ where } x = \mu/\gamma\sigma^2. \text{ If liquidity shocks are independent, there is a closed-form solution for } m = 1/2, \text{ the equivalent triple equilibria are given by } \alpha^* = 0, \alpha^* = 1, \text{ and } \alpha^* = 1 + 2 \left(\mu/\gamma\sigma^2\right) - \sqrt{8 \left(\mu/\gamma\sigma^2\right)}.\]
dence of forced liquidation (liquidity shocks), but also may increase in-equilibrium inefficient market-making inventory.

IV Discussion of The Equilibrium

A Preventing Runs and Front-Running: Time-Varying Market Depth

The obvious question is what mechanisms could prevent the need for the market-making sector to absorb run inventory from the public.

The first answer lies in the enforcement of perfect sequentiality. With sequential execution the last investors (who now know they are the last investors!) would be better off just waiting it out instead of being the last in-the-run investors. This can unravel the tendering equilibrium. In response to the 1987 crash, the NYSE massively expanded its communication infrastructure, a mechanism to prevent the conversion of the sequential market into a random-execution market in times of declines.

Interestingly, a belief that one can front-run others (get their share sales executed with higher priority) can encourage run equilibria because it increases the expected payoff to tendering early.\(^9\) Naturally, in an equilibrium with homogeneous agents, noone can expect to front-run anyone else. However, in a real-world context, some heterogeneous investors may rationally or irrationally believe in their ability to front-run.

\(^9\)Within the context of the informational cascades literature (Bikhchandani, Hirshleifer, and Welch (1992), Welch (1992)), Chen (1995b) has modelled such informational interactions in a banking run context.
The second answer lies in providing liquidity during runs. If the market-making sector were to expand only in “bad” situations but offer lousy execution in “good” situations, then each investor would be relatively better off not trading into the market today, and instead would be relatively more eager to try to wait it out. This gives a natural interpretation to government intervention: if correctly done, standby liquidity in market runs could help prevent runs in the first place. (It is unlikely that the private sector could provide unusually good liquidity only in bad scenarios, but not in good scenarios.) Interestingly, non-intervention in good markets is as important as intervention in bad markets! Indeed, this is the equivalent of the national petroleum reserves, which are rarely released, but whose presence may in itself prevent runs.

It is straightforward to solve a model that proves this point. For simplicity, we analyze the case in which the representative market-maker has CARA utility and the stock payoff is normally distributed. We also assume that the liquidity shocks are perfectly correlated across investors (the independent liquidity shocks case yields similar qualitative results). In this example, we allow the market-making sector to be deeper at date 1 than at date 0 (i.e., \( y_{t=1} \leq y_{t=0} \)). Again, we assume that prices are set at each date by a zero-utility condition. However, the new prices reflect the different market-making depth at each date. Thus, \( p_0(\alpha) = \mu - y_0 \sigma^2 \alpha / 2 \) and \( p_1(1 - \alpha; \alpha) = \mu - (1 + \alpha) y_1 \sigma^2 / 2 \). Substituting \( p_0(\alpha) \) and \( p_1(1 - \alpha; \alpha) \) into our definition of \( F(\alpha) \) yields the following result:
Theorem 5 If the market-making sector is deeper at date 1 (i.e., $\gamma_1 < \gamma_0$) and liquidity shocks are perfectly correlated there is a unique symmetric Nash equilibrium with

$$\alpha^* = \begin{cases} \left( \frac{s}{\gamma_0/\gamma_1 - s} \right) & \text{if } s \leq \gamma_0/(2\gamma_1) \\ 1 & \text{if } s > \gamma_0/(2\gamma_1) \end{cases}.$$  \hspace{1cm} (20)

Proof: Substitute the pricing functions into equation 2. Note that $F(0) > 0$ for all $s > 0$ and $F_{\alpha^*}$, the derivative of $F$ with respect to $\alpha^*$, is negative since $\gamma_1 < \gamma_0$. Thus, there are two possibilities. If $F(1) \geq 0$ then there is a unique pure strategy equilibrium $\alpha^* = 1$ and if $F(1) < 0$ there is a unique mixed strategy, $\alpha^*$, where $F(\alpha^*) = 0$. For $s > \gamma_0/(2\gamma_1)$, $F(1) > 0$ thus $\alpha^* = 1$. For $s \leq \gamma_0/(2\gamma_1)$ solving for $\alpha^*$ yields the result. □

Market-making inventory $\alpha^*$ decreases in $\gamma_0$ and increases in $\gamma_1$. If early on, market makers are more risk-averse ($\gamma_0$ is high), investors are less eager to tender to market-makers at time 0 ($\alpha^*$ is low) and more inclined to take the chance of being forced to sell if personally hit by a subsequent liquidity shock. Conversely, if the subsequent “standby liquidity” in a crisis is low, because the market-making risk aversion $\gamma_1$ is then unusually high, the post-run price will be lower, which prompts investors to be more eager to sell at date 0. Casual empiricism suggests that, if anything, the market-making sector becomes intrinsically more risk-averse during runs than it is in ordinary times. Thus, government intervention which commits to provide market-depth in “bad” but not in “good” times might usefully mitigate run inefficiencies.

B A Multi-Period View

The single-period setting of our model is not important: Even if there are multiple periods before investors face their (random) real liquidity shock, the exact same liquidity run as that described in our model would appear—and appear immediately.

25
That is, investors would still want to offload their shares to avoid having to trade behind other investors. In periods between this first period and the period of the potential future shock, investors voluntarily do not trade. Thus, a liquidity run can be persistent even in the presence of multiple trading rounds.

This is easy to show. Suppose investors now have two opportunities to sell, denoted date 0 and date 1, prior to the occurrence of a liquidity shock. Now, an equilibrium is a pair \((\alpha_0, \alpha_1)\), for which—given that \(\alpha_0\) proportion sell at date 0—it is optimal for an \(\alpha_1\) proportion to sell at date 1, and vice-versa. One condition for optimality is that someone who sells at one date does not have the incentive to deviate and sell at the other date. But there is only one case for which this is true: \(\alpha\) proportion sell at date 0 and no one sells at date 1! In this case, the date 0 price exceeds the date 1 price so no one has an incentive to deviate and sell at date 1. Moreover, the possibility of a liquidity shock in the future makes an investor indifferent between selling at date 0 and waiting if she conjectures that \(\alpha^*\) (as in our earlier model) proportion of investors will sell at date 0. Note that the opposite is not an equilibrium, i.e. no one sells at date 0 and \(\alpha\) proportion sell at date 1 because the date 0 price will be higher, permitting investors to front-run and thus profit by selling at date 0. In sum, the only equilibrium is the same \(\alpha^*\) as in our earlier model: first, active selling at date 0, followed by no more selling until the date at which the liquidity shock occurs.\(^\text{10}\)

Naturally, changes in model parameters might cause some readjustments as time goes by. We have already sketched the influence of time-varying market depths in Subsection A. Similarly, one could imagine time-varying probability assessments of

\(^{10}\text{Note that in this independent shocks version of our model, we must again resort to a competitive market-making sector to prevent an individual market-maker to exploit to wait for the vulture opportunity.}\)
future liquidity shocks, which could lead to active trading and time-varying market-making inventory adjustments, even in the absence of any current liquidity shocks. (Incidentally, such a model can easily explain relatively high trading volume in the presence of only mild news.)

C The Social Cost of Investor Fear

In our model, there is asymmetric information or trading costs—and yet the market outcome can be significantly worse than the Pareto-optimal allocation. In what follows, we analyze the social cost of investor fear when market-makers have CARA utility and the stock payoff is normally distributed. Our analysis considers exogenous liquidity shocks which are either perfectly correlated or independent across investors.

C.1 Perfectly correlated liquidity shocks

In the Pareto-optimal outcome, the risk-neutral investors hold all the shares at date 0 and sell to the market-making sector at date 1 only if they are actually hit by a liquidity shock. In this case, every investor would sell shares with probability \( s \) at a price \( p_1 = \mu - \gamma \sigma^2 / 2 \) (assuming that the market-maker sector executes these sell orders at a price that yields no utility gain for them) and would retain shares with probability \( 1 - s \) (with expected value \( \mu \)). Thus, investors’ utility would be

\[
\mu - \frac{\gamma s \sigma^2}{2} .
\]  (21)
In the batch-execution model, risk-neutral investor sell with probability $\alpha^*$ at date 0 at the average price $p_0 = \mu - (y\alpha^*\sigma^2)/2$, liquidate with probability $(1-\alpha^*)s$ at date 1 at the average price $p_1 = \mu - (1 + \alpha^*)y\sigma^2)/2$, and retain shares with probability $(1-s)(1-\alpha^*)$ at expected value $\mu$. Thus, investors’ utility is

$$
\begin{cases}
\mu - \frac{y\sigma^2}{2} \cdot \frac{s}{(1-s)} & \text{if } s \leq 1/2 \\
\mu - \frac{y\sigma^2}{2} & \text{if } s > 1/2
\end{cases}
$$

(22)

By assumption, the market making sector has zero expected utility gain thus a total welfare comparison only requires a comparison of the investors’ utility. Simple algebra shows that equilibrium welfare (expected selling price) is below the Pareto-optimal level of welfare by the amount:

$$
\begin{cases}
\frac{y\sigma^2}{2} \cdot \frac{s^2}{(1-s)} & \text{if } s \leq 1/2 \\
\frac{y\sigma^2}{2} \cdot (1 - s) & \text{if } s > 1/2
\end{cases}
$$

(23)

The Pareto-inferior outcome is caused by a prisoner’s dilemma among risk-neutral investors that cannot easily be overcome. The welfare loss is increasing in the market-maker’s risk-aversion and the payoff variance $\sigma^2$ since inefficient risk-sharing is exacerbated. Finally, the welfare loss is greatest when $s = 1/2$ because the market-making sector must absorb all shares in this case, not just those of the liquidity-shocked individuals. Because $\alpha^*$ increases at a faster rate as $s$ approaches 1/2, the welfare loss increases in $s$ for $s \in [0, 0.5)$. However, because (i) $\alpha^* = 1$ for all $s \geq 1/2$ and (ii) as $s$ increases the market makers would hold an increasing proportion of shares in the Pareto-optimal outcome, the welfare loss decreases in $s$ for $s \in (0.5, 1]$.
C.2 Independent liquidity shocks

In the Pareto-optimal outcome, the risk-neutral investors hold all the shares and sell to the market-making sector only if they are actually hit by a liquidity shock. In this case, investors sell shares with probability $s$ at a price $p_1 = \mu - \gamma s \sigma^2 / 2$ (assuming that the market-maker sector executes these sell orders at a price that yields no utility gain for them) and would retain shares with probability $1 - s$ (with expected value $\mu$). Thus, investors’ utility would be

$$\mu - \frac{\gamma s^2 \sigma^2}{2}.$$  \hfill (24)

In the batch-execution model, risk-neutral investor sell with probability $\alpha^*$ at date 0 at the average price $p_0 = \mu - (\gamma \alpha^* \sigma^2) / 2$, liquidate with probability $(1 - \alpha^*)s$ at date 1 at the average price $p_1 = \mu - [2 \alpha^* + s(1 - \alpha^*)] \gamma \sigma^2 / 2$, and retain shares with probability $(1 - s)(1 - \alpha^*)$ at expected value $\mu$. Thus, investors’ utility is

$$\begin{cases} 
\mu - \frac{\gamma \sigma^2 }{2} \cdot \frac{s^2}{(1-s)^2} & \text{if } s \leq 1/2 \\
\mu - \frac{\gamma \sigma^2 }{2} & \text{if } s > 1/2 
\end{cases}$$ \hfill (25)

As in the perfectly correlated case, the welfare loss is increasing in $\gamma$ and $\sigma^2$ and the welfare cost is greatest when $s = 1/2$. 

29
Contagion effects fall naturally out of the model. In the bad scenario, there are spillovers in the decisions of investors to sell their shares. This causes each individual investor to fear that he may have to sell (for exogenous reasons) behind every other investor. If selling late, he will get only the marginal price after everyone else has already sold to the market-making sector, which—already being burdened with the inventory of all other investors—can only offer a very low price.

The negative externalities among investors causes an accelerator effect, in which just small increases in the probability of future liquidity shocks cause a large layoff of risky shares onto the risk-averse market-making sector. Again, the accelerator effect does not amplify the effects of the actual liquidity shock! It amplifies the extent to which one investor's fear of a future liquidity shock has a negative spillover on other investors' fears.

We have repeatedly pointed out that runs are not caused by liquidity shocks themselves, but by fears of future liquidity shocks. The probability of a future liquidity shock may constantly fluctuate, even though the liquidity shock itself can be off on the horizon. Consequently, an empiricist can observe dramatic price movements and market-making inventory changes without observing any actual liquidity shocks. And, for the rare empiricist able to measure the fear of liquidity shocks ($s$), depending on its value, seemingly small changes can cause large sudden changes in the desire of investors to unload shares onto the market-making sector.\footnote{Although our model has emphasized purely rational behavior, where the fear of liquidity shocks is rationally assessed or derived from margin constraints, our equilibrium could also be embedded in a world of “non-rational behavioral economics,” if the fear of a liquidity shock (the need to terminate an investment early during a market run) were itself non-rational.}
V Related Literature

Our financial markets runs model has both similarities and differences to the financial intermediation runs models, foremost Diamond and Dybvig (1983). Our model is also driven by investor liquidity shocks and the fact that the last investor is worst off. However, our model does not require specified promised payoffs, technological convertability issues, and a necessary total loss if an investor fails to join a run.\textsuperscript{12} Indeed, working out the endogenous pricing and market inventory is a major focus of our paper. Also, in both cases, a “lender of last resort” can prevent the run.

There is also related literature on stock market crashes. Grossman and Miller (1988) present a two trading period model in which all traders are not simultaneously present in the market. In the first period, there is a temporary order imbalance which must be absorbed by market makers. Between the first and second period, new information arrives about the security so the market-making sector is exposed to risk. However, the market-making sector is small and has low risk absorption capacity. Thus, the equilibrium price falls more than if all traders were available to absorb the imbalance. In the second period, the remaining traders arrive to buy some of the market makers inventory and the price rises. The key feature of their model which produces crashes is the asynchronous arrival of traders in the market, combined with the limited risk-bearing capacity of market makers. Greenwald and Stein (1991) extend the Grossman and Miller (1988) analysis by assuming that traders can only submit market orders in the second period of trade. This introduces transactional risk (uncertainty about the price at which their trades will

\textsuperscript{12}Allen and Gale (2000) and similar financial contagion models, though quite different, build on the Diamond and Dybvig framework and retain these two assumptions. The same can be stated for the liquidity crisis and international runs on currency reserves literature, e.g., Caballero and Krishnamurthy (2001). Geanakoplos (2001) embeds collateral crises into a broader model.
execute) which reduces the willingness of buyers to absorb the market makers inventory in the second period. Knowing this, the market makers demand a larger risk premium in the first period to absorb the temporary order imbalance, which causes prices to fall even further than in the Grossman and Miller analysis. Like Greenwald and Stein (1991), we permit only market orders and have an uncertain execution price.

Although our model is closest in spirit Greenwald and Stein (1991), it is quite different. First and foremost, runs in our model occur when investors think that others will tender and therefore choose to tender themselves. In Greenwald and Stein, there are no runs (defined as “I tender because I think you will tender”) in the spirit of bank runs. Rather, Greenwald and Stein (1991) have crashes in price because the uncertain execution price with batch orders introduces an extra risk which makes the price fall further than if investors could submit limit orders. Second, our financial market runs are endogenous and are not driven by asynchronous trading arrivals or exogenous supply shocks. Indeed, the Greenwald and Stein (1991) model is driven by the uncertainty in the number of arriving value traders, an uncertainty which does not even exist in our model. Third, the negative externality in their model is that value buyers may destroy the value opportunities for other buyers. In our model, the negative externality derives from investor selling, not buying. Further, this ever-present negative externality forces market-makers to hold a socially suboptimal inventory of shares. And finally, they argue that circuit-breakers might help: in our model, circuit-breakers are counterproductive.

There are also other areas of research more distant in spirit, but which also explain facets of financial market crashes. There is a large literature examining the impact of portfolio insurance (e.g., Grossman (1988), Brennan and Schwartz (1989),
Genotte and Leland (1990), Jacklin, Kleidon, and Pfleiderer (1992), Donaldson and Uhlig (1993), Grossman and Zhou (1994), Basak (1995). Portfolio insurers are usually modelled as agents who display positive feedback trading (of an accelerating kind) for exogenous (often assumed) reasons. This literature’s primary goal is to show that portfolio insurers can exacerbate crashes (discontinuous movements). Models differ in choosing discrete single-shot vs. continuous time modelling techniques, implications on what portfolio insurance does for general price volatility in ordinary markets, and in how asymmetric information matters. Our own model differs from this literature in that the reason for selling is not the (usually exogenous) consumption motive, but the direct negative externality arising endogenously from other investors’ trading.

Other papers have also presented ingenious mechanisms that can elicit large price changes. In Madrigal and Scheinkman (1997), an informed strategic market-maker attempting to control both the order flow she receives and the information revealed to the market by the prices she sets may choose an equilibrium price schedule that is discontinuous in order flow thus prompting large changes in price for arbitrarily small changes in market conditions. In Romer (1993), uncertainty about the quality of others’ information is revealed by trading, and large price movements, such as the October 1987 crash, may be caused not by news about fundamentals but rather by the trading process itself. In Sandroni (1998), market crashes can be a self-fulfilling prophecy when agents have different discount rates and different beliefs about the likelihood of rare events (even if these beliefs converge in the limit). Barlevi and Veronesi (2001) present a model in which uninformed traders precipitate a stock price crash because as prices fall they rationally infer that informed traders have negative information which leads them to reduce their demand for the
stock and drive its price even lower. The key feature of their model is that the uninformed traders have locally upward sloping demand curves which, when combined with the informed’s downward sloping demands, can generate an equilibrium price function discontinuous in fundamentals.

Finally, we are not the first to employ margin constraints to generate multiple equilibria. In Chowdhry and Nanda (1998), perhaps the paper most similar to our own endogenous liquidity constraint section, some investors engage in margin borrowing to obtain their desired investment portfolio. Because shares can be used as collateral there is a link between the price of the stock and the capacity to invest in it which introduces the possibility of multiple equilibria. For example, lower (higher) stock prices can be a self-fulfilling equilibrium because it diminishes (increases) the capacity for levered investors to purchases their desired amount of stock which in turn makes the price fall (rise) rational.13

VI Conclusion

Our paper has developed a theory of financial market runs: socially inefficiently large market-making inventory in batch or random-ordering financial markets. Batch markets are the standard stock market opening mechanism and auction mechanism on some foreign exchanges. Random-ordering markets are common in many over-the-counter markets. They also can occur (infrequently) after a large price drop, when limited communication channels between investors and the financial system fail and break down the perfect sequentiality of execution. In such cases, investors’

13For a more recent example, Yuan (2000) demonstrates that margin constraints can be beneficial because they may apply to informed investors and thus reduce the adverse selection problem with uninformed investors.
fears of future liquidity constraints can cause a prisoner’s dilemma among investors today. This destroys efficient risk-sharing and aggravates any fundamental price drops.

Aside from sequential execution and reasonable fear of liquidity shocks (but not necessarily actual liquidity shocks), our model required very little machinery. Thus, it is the (presumably rare) combination of breakdown of sequential execution and a common fear of liquidity shocks, perhaps caused by or related to margin constraints, that facilitates a run on a financial market.
Comparative statics when investors face independent liquidity shocks. Our base parameters are a down-stock-value of \( D = 10 \) and an up-stock-value \( U = 20 \) with equal probability \( \pi = 0.5 \), a risk aversion coefficient of \( \gamma = 3 \), and market-making wealth of \( W = 1.5 \) (i.e., roughly 1/10 of the value of the financial market).
Figure 2. ComparativeStatics under Market-Making CARA Utility and Margin Constraints

$m = 1.5$ represents a situation in which investors are less likely to be forced to liquidate (i.e., upon a smaller price drop). $m = 0.5$ represents a situation in which investors are more likely to be forced to liquidate. The figure shows that financial market runs [excessive market-making inventories] are more severe for higher $m$. 

37
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