1. **Answer:** \(\frac{-1}{x^2+1}\)

Notice that as \(t \to 0\), both the numerator and the denominator approach 0. Thus, applying L'Hopital's rule on \(t\) (keeping \(x\) constant):

\[
\frac{d}{dt} \tan^{-1} \left( \frac{1}{x+t} \right) \bigg|_{t=0} = -\frac{1}{1+x^2}
\]

2. **Answer:** 1

Let \(f(x) = e^x - x - \frac{x^2}{2}\). Then \(f'(x) = e^x - 1 - x^2\). When \(x < 0\), \(e^x < 1\) and \(1 + x^2 > 1\), so \(f'(x) = e^x - (1 + x^2) < 0\). Thus, \(f\) is decreasing on \((-\infty, 0)\). When \(x = 0\), \(f'(x) = f'(0) = e^0 - 0 - 0^2 = 1 - 1 = 0\). Finally, for \(x > 0\), \(f'(x) = e^x - 1 - x^2 > 0\) by a Maclaurin series expansion, so \(f\) is increasing on \((0, \infty)\). Thus, \(f\) must attain its minimum when \(x = 0\), at which point \(f\) has the value \(e^0 - 0 - \frac{0^2}{2} = 1\).

3. **Answer:** \(\sqrt{2}\)

Consider:

\[
\frac{d}{dt} \sin^{-1}(t-\sqrt{1/2}) \bigg|_{t=0} = \frac{d}{dt} \int_{-\infty}^{x} e^{tx} f(x) dx \bigg|_{t=0} = \int_{-\infty}^{\infty} xe^{tx} f(x) dx - \int_{-\infty}^{0} xe^{tx} f(x) dx = \int_{0}^{\infty} xe^{tx} f(x) dx
\]

\[
\frac{d}{dt} \sin^{-1}(t-\sqrt{1/2}) \bigg|_{t=0} = \frac{1}{\sqrt{1 - (\sqrt{1/2} - t)^2}} \bigg|_{t=0} = \frac{1}{\sqrt{1 - (1/2)}} = \sqrt{2}
\]

4. **Answer:** \(x = -\frac{2}{3}\) and \(x = 0\)

Notice that \(f(x) \to 0\) as \(x \to \pm\infty\). Since \(9x^2 + 6x + 2\) has no real roots, the maximum value of \(f(x)\) is attained at the maximum of the absolute values of the critical points of \(\frac{3x+1}{9x^2+6x+2}\).

The extrema of \(\frac{3x+1}{9x^2+6x+2}\) occur at \(x = -\frac{2}{3}\) and \(x = 0\). It is easily checked that maxima of \(f(x)\) occur at both of these points.

5. **Answer:** \(\frac{128\sqrt{3}}{27}\)

Let the circular island be a circle of radius 2 centered at the origin. Without loss of generality, let the length of the rectangular base be from \(-x\) to \(x\) and the width from \(-y\) to \(y\). Notice that by the equation of a circle, \(x^2 = 4 - y^2\). Then

\[
V = \frac{1}{3} (2x)^2 (2y) = \frac{8}{3} x^2 y = \frac{8}{3} (4-y^2)y = \frac{8}{3} (4y - y^3)
\]

\[
\frac{dV}{dy} = \frac{8}{3} (4-3y^2) = 0 \to y = \sqrt{\frac{4}{3}}
\]

\[
V = \frac{8}{3} \left( \frac{4}{3} \right) \sqrt{\frac{4}{3}} = \frac{128\sqrt{3}}{27}.
\]

6. **Answer:** 13

This is the evaluation of the mean of a Poisson distribution: for any \(\lambda\),

\[
\sum_{k=0}^{\infty} ke^{-\lambda} \frac{\lambda^k}{k!} = \sum_{k=1}^{\infty} ke^{-\lambda} \frac{\lambda^k}{k!} = \lambda \sum_{k=1}^{\infty} e^{-\lambda} \frac{\lambda^{k-1}}{(k-1)!} = \lambda e^{-\lambda} \sum_{m=0}^{\infty} \frac{\lambda^m}{m!} = e^{-\lambda} e^\lambda = \lambda.
\]
7. **Answer:** $-2\cos(t^2)$

By the Leibniz integral rule, the above integral becomes

\[\int_{\ln 1/t}^{\ln 1/t} -e^x \sin(te^x)dx + \cos(te^{\ln(1/t)})(-1/t) - \cos(te^{-\ln(1/t)})(1/t) = \frac{\cos(te^x)|_{\ln 1/t} - \cos(1) + \cos(t^2)}{t}\]

\[= \frac{-2\cos(t^2)}{t}.

8. **Answer:** $\ln 3$

The partial sums of this sum are equal to

\[\left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{3n}\right) - 3\left(\frac{1}{3\cdot 1} + \frac{1}{3\cdot 2} + \cdots + \frac{1}{3\cdot n}\right)\]

\[= \frac{1}{n + 1} + \frac{1}{n + 2} + \cdots + \frac{1}{3n} = \frac{1}{n}\left(\frac{1}{1 + \frac{1}{n}} + \frac{1}{1 + \frac{2}{n}} + \cdots + \frac{1}{1 + \frac{3n}{n}}\right).

This is a Riemann sum, so as $n \to \infty$ the partial sums converge to

\[\int_0^2 \frac{1}{1 + x} dx = \ln 3.

9. **Answer:** 3

Since the parabola $f(x) = x(4 - x) - k$ is symmetric about $x = 2$, the problem is equivalent to minimizing $\int_0^2 f(x)dx$. The vertex of the parabola equals $(2, f(2)) = (2, 4 - k)$. When $k = 4$, $f(x)$ lies completely below the x-axis in the interval $[0,2]$ and hence $k > 4$ would only translate $f(x)$ down and increase the integral. Similarly, at $k = 0$, $f(x)$ lies completely above the x-axis so $k < 0$ would only increase the integral. Thus, we can split the integral into two regions

\[a = \int_0^{\sqrt{4-k}} (x^2 - 4x + k) dx = -\frac{16}{3} + \frac{8\sqrt{4-k}}{3} + 2k - \frac{2}{3}\sqrt{4-k}k\]

\[b = \int_{\sqrt{4-k}}^{2} (-x^2 + 4x - k) dx = \frac{2}{3}(4 - k)^{3/2}\]

We want to solve for the critical point of $a + b$

\[\frac{d(a + b)}{dk} = 2 - \frac{4}{3\sqrt{4-k}} - \frac{5\sqrt{4-k}}{3} + \frac{k}{3\sqrt{4-k}} = \frac{2(-4 + \sqrt{4-k} + k)}{\sqrt{4-k}}\]

The numerator equals 0 when $k = 3$. It is clear that a global minimum results since this is a global minimum on $(-\infty, 4]$ and $F(k)$ is clearly increasing for $k > 4$.

10. **Answer:** $y = -4x^2 + 5x - 7$

Such a parabola intersects $f(x)$ precisely where $f'(x) = 0$. Hence, the value of the intersection points do not change when we replace $f(x)$ by $f(x) + g(x)f'(x)$ for any $g(x)$. Therefore, since $f'(x) = 6x^5 - 12x + 6$, we must have that $f(x) - 1/6x f'(x) = -4x^2 + 5x - 7$ passes through the three critical points. Since three points determine a parabola uniquely, this must be the unique parabola passing through the three critical points.