1. **Answer:** 668
   Note that 111 = 3 \cdot 37. It follows that $m_i$ is divisible by 37 for all $i = 3, 6, 9, \ldots, 2004$. The others will clearly leave remainders of 1 or 11.

2. **Answer:** 10
   The expression can be written as $(x - 2)^2 + (x - y)^2 + (y - 2z)^2 + (z - 1)^2 + 10$. This clearly must be at least 10. Indeed, if $x = 2$, $y = 2$, $z = 1$, this value is achieved.

3. **Answer:** $(1 + 2i)(2 + 3i)$
   We write $-4 + 7i = (a + bi)(c + di)$. The solution can be intuitive after the first line of expansion, in the same way as factoring of polynomials. However, we can assume $a = 1$ and then move factors from $(c + di)$ back to $(a + bi)$ if we don’t end up with integers (fortunately, in this case we’re lucky).

   We then know $c$ should be positive (and not too large), so we can try $c = 1$, giving $1 - bd = -4$ and $b + d = 7$, which clearly has no rational solution. We then try $c = 2$, giving $6 = bd$ and $2b + d = 7$, which is easily solved giving the final solution.

4. **Answer:** $a + b + c$

   $$\frac{a^3}{(a - b)(a - c)} + \frac{b^3}{(b - a)(b - c)} + \frac{c^3}{(c - a)(c - b)} = \frac{a^3(c - b) + b^3(a - c) + c^3(b - a)}{(a - b)(b - c)(c - a)}$$

   $$= \frac{a^3(c - b) + a(b^3 - c^3) + bc^3 - cb^3}{(a - b)(b - c)(c - a)}$$

   $$= \frac{(c - b)(a^3 - a(b^2 - bc + c^2) + b^2c + c^2b)}{(a - b)(b - c)(c - a)}$$

   $$= \frac{-b^2(c - a) + b(c^2 - ac) + a^3 - ac^2}{(a - b)(c - a)}$$

   $$= \frac{- (c - a)(b^2 + bc - ac - a^2)}{(a - b)(c - a)}$$

   $$= \frac{(a - b)(a - b) + c(a - b)}{a - b}$$

   $$= a + b + c$$

5. **Answer:** 352
   Let $N$ represent the number of remaining pebbles after Kramer eats the second. Then $N$ is divisible by 10, and $N + 1$, which must end in 1, is divisible by 9. Put $N + 1 = 100a + 10b + 1$, where $a$ and $b$ are digits summing to 8 or 17 (so the sum of the digits will be divisible by 9 - hence the number will be divisible by 9). Now we need $N + 2$ to be divisible by 8. Try 82, 172, 262, and 352 to get 352 as the answer.
6. Answer: 31, −25
From the first equation:
\[
ab - a = b + 119
\]
\[
a(b - 1) = (b - 1) + 120
\]
\[
(a - 1)(b - 1) = 120
\]
Similarly, \((b - 1)(c - 1) = 60\) and \((a - 1)(c - 1) = 72\). Therefore \(\frac{a-1}{b} = 2\), and so \(2(c-1)^2 = 72\). This gives \(c = 7\), and then it is easy to find \(a = 13\) and \(b = 11\). The other solution is \(c = -5\), so \(a = -11\), and \(b = -9\). The sums are 31 and -25.

7. Answer: (6, 5)
Since \(11 | a abb, a abb = 11 \cdot 0b\). Factor \(n^4 - 6n^3 = (n - 6)n^3\), so clearly \(n > 6\), as \(a abb > 0\). Also, \(a0b < 1000\), so unless \(n = 11, n < 10\). Trying \(n = 7, 8, 9\) yields no solutions, so \(n = 11\) must be the only solution, if it exists. Indeed we get \(6655 = (11 - 6) \cdot 11^3\).

8. Answer: \(\frac{27}{55}\)
Let \(f(x) = \frac{1}{x(x-1)}\). Then:
\[
\sum_{x=2}^{10} \frac{2}{x(x-1)} = \sum_{x=2}^{10} (f(x) - f(x+1)) = \sum_{x=2}^{10} f(x) - \sum_{x=3}^{11} f(x) = f(2) - f(11) = \frac{1}{2} - \frac{1}{110} = \frac{27}{55}
\]

9. Answer: 169
Let \(A\) be the value of the expression. We have: \(m^2 + n^2 - 13m - 13n - mn + A = 0\). Multiplying by 2 yields:
\[
(m - n)^2 + (m - 13)^2 + (n - 13)^2 = 2 \cdot 13 \cdot 2 - 2A
\]
In order for there to be a single solution, the sum of the squares must equal zero, yielding \(A = 169\). If instead the sum is a positive integer with a solution \((m, n)\), then \((n, m)\) will provide an additional solution unless \(m = n\). In that case, \((26 - m, 26 - n)\) is an additional solution. Hence, it is both sufficient and necessary that the sum of the squares equal zero in order that the solution be unambiguous.

10. Answer: \(\left(\frac{a}{a-1}\right)^2\)
\[
\sum_{k=1}^{\infty} \frac{k}{a^{k-1}} = \frac{1}{1} + \frac{2}{a} + \frac{3}{a^2} + \frac{4}{a^3} + \cdots
\]
\[
= \left(\frac{1}{1} + \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \cdots\right) + \left(\frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \cdots\right) + \left(\frac{1}{a^2} + \frac{1}{a^3} + \cdots\right) + \cdots
\]
\[
= \left(\frac{1}{1} + \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \cdots\right) + \frac{1}{a} \left(\frac{1}{1} + \frac{1}{a} + \frac{1}{a^2} + \cdots\right) + \frac{1}{a^2} \left(\frac{1}{1} + \frac{1}{a} + \cdots\right) + \cdots
\]
\[
= \left(\frac{1}{1} + \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \cdots\right) \left(\frac{1}{1} + \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \cdots\right)
\]
\[
= \left(\frac{1}{1 - 1/a}\right)^2
\]
\[
= \left(\frac{a}{a - 1}\right)^2
\]