1. Answer: \( \frac{7}{4} \)

\[
\lim_{x \to \infty} \left( \frac{\sqrt{4x^2 + 7x} - 2x}{\sqrt{4x^2 + 7x} + 2x} \right) = \lim_{x \to \infty} \left( \frac{7}{4x} \right) = \frac{7}{4}.
\]

2. Answer: 19

The derivative of \( f(x) - f'(2x) \) is \( f'(x) - 2f'(2x) \). So \( f'(1) - 2f'(2) = 5, f'(2) - 2f'(4) = 7 \). Thus

\[
f'(1) - 4f'(4) = (f'(1) - 2f'(2)) + 2(f'(2) - 2f'(4)) = 5 + 2 \cdot 7 = 19,
\]

the answer.

3. Answer: \([3,4]\) or \((3,4)\) or from \(t=3\) to \(t=4\)

The velocity of the object is given by \( v(t) = x'(t) = 20t^3 - 5t^4 \), and the acceleration function is \( a(t) = v'(t) = 60t^2 - 20t^3 \). The object is slowing down when the velocity is positive and the acceleration is negative, or vice versa. \( v(t) \) is positive from \( t = 0 \) to \( t = 4 \) and is negative after that. \( a(t) \) is positive from \( t = 0 \) to \( t = 3 \) and negative afterward. These only differ in sign from \( t = 3 \) to \( t = 4 \).

4. Answer: 1

Let \( g(x) = \log f(x) = x \log x \). Then \( \frac{f'(x)}{f(x)} = g'(x) = 1 + \log x \). Therefore \( f(x) = f'(x) \) when \( 1 + \log x = 1 \), that is, when \( x = 1 \).

5. Answer: \( 5 - \sqrt{3} \) miles

Let \( x \) be the amount of old road restored. Then the length of the new road is \( \sqrt{9 + (5 - x)^2} \) using the Pythagorean Theorem. Thus the total cost of the plan is \( C(x) = 200000x + 400000\sqrt{x^2 - 10x + 34} \). The minimum cost occurs at one of the critical points which are \( x = 5 \pm \sqrt{3} \). Clearly \( 5 + \sqrt{3} \) is not a valid answer and one can check \( 5 - \sqrt{3} \) is indeed a minimum.

6. Answer: 2

The two graphs intersect at \( x^2 - 2x^2 + 8x^2 = 28 \) or rather \( x = \pm 2 \) with \( y = \pm 4 \). At \( x = +2, m_1 = 2 \) and \( m_2 = g'(2) \). Using implicit differentiation on the second graph, we find \( g'(x) = \frac{y - 2x}{4y - x} \) and plugging in \((2,4)\) gives a slope of 0. If \( \alpha \) is the angle between the graphs then \( |\tan(\alpha)| = |\frac{m_2 - m_1}{1 + m_1 m_2}| \). Plugging in the values yields the answer 2. \( x = -2 \) yields the same value.

7. Answer: \( \pi/6 \)

The mouse can wait some amount of time while the table rotates and then spend the remainder of the time moving along that ray at 1 m/s. He can reach any point between the starting point and the furthest reachable point along the ray, \((1 - \theta/\pi)\) meters out. So the area is

\[
\int_0^{\pi} \frac{(1/2)(1 - \theta/\pi)^2}{2} \, d\theta = (1/2)(1/\pi)^2 \int_0^{\pi} \theta^2 \, d\theta = \pi/6.
\]

8. Answer: 27500 foot-pounds

Let \( x \) indicate the distance the cow has yet to travel. Then the work for a distance \( dx \) is \( (2x + 200 - \frac{1}{2}(100 - x)) \, dx \). Thus the total work is \( \int_0^{100} (\frac{1}{2}x + 150) \, dx = 27500 \) foot-pounds.
9. Answer: \(rac{3\sqrt{3}}{2}\)

The base region is bounded on the left by \(x = y^2\) and on the right by \(2y^2 = 3 - x\). The intersection points are \((1, 1)\) and \((1, -1)\). Each cross-section, say \(x = a\), is an equilateral triangle. The length of a side is \(2y\) where \(y = \sqrt{x}\) for \(a \leq 1\) but it is \(y = \sqrt{\frac{3-x}{2}}\) for \(a \geq 1\). The area of an equilateral triangle is \(\sqrt{3}s^2/4\) where \(s\) is the side length. Thus the volume is \(\int_0^1 \sqrt{3} \left(\frac{2\sqrt{3}}{4}\right) dx + \int_1^3 \frac{\sqrt{3}}{4} \left(2\sqrt{\frac{3-x}{2}}\right)^2 dx = \sqrt{3} \int_0^1 x dx + \frac{\sqrt{3}}{2} \int_1^3 (3-x) dx = \frac{3\sqrt{3}}{2}\).

10. Answer: \(\frac{1}{e}\)

The ratio test tells us that the series converges if

\[
\lim_{n \to \infty} \frac{(n+1)!}{(c(n+1))^{n+1}} \cdot \frac{n^n}{n!} = \frac{1}{e} \lim_{n \to \infty} \left(\frac{n}{n+1}\right)^n
\]

is less than one and diverges if it is greater than one. But

\[
\lim_{n \to \infty} \left(\frac{n}{n+1}\right)^n = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{-n} = \frac{1}{e}.
\]

Then the limit above is just \(\frac{1}{ce}\), so the series converges for \(c > \frac{1}{e}\) and diverges for \(0 < c < \frac{1}{e}\).