1. Answer: \[
\begin{pmatrix}
1 & 2500 \\
0 & 1
\end{pmatrix}
\]
First note that \[
\begin{pmatrix}
1 & a \\
0 & 1
\end{pmatrix}\begin{pmatrix}
1 & b \\
0 & 1
\end{pmatrix} = \begin{pmatrix}
1 & (a + b) \\
0 & 1
\end{pmatrix}.
\]
This implies the given product is
\[
\begin{pmatrix}
1 & (1 + 3 + 5 + \ldots + 99) \\
0 & 1
\end{pmatrix} = \begin{pmatrix}
1 & 50^2 \\
0 & 1
\end{pmatrix} = \begin{pmatrix}
1 & 2500 \\
0 & 1
\end{pmatrix}.
\]

2. Answer: \(.010\overline{3}_6\).\]
\[
\frac{4}{5} - \frac{1}{10} + \frac{1}{7} - \frac{1}{2} = \frac{73}{315}\]. Note this is all base 10. To convert to a base 6 decimal, we get each digit by multiplying the fraction by 6 and taking the integer portion. This is the reverse of converting integers to base 6. Note that the decimal begins to repeat after the first five places. The answer is \(.010\overline{3}_6\).

3. Answer: \(\sin(80^\circ)\)
The expression is equivalent to \(\cos(10^\circ) + \cos(10^\circ) \cdot \sin(100^\circ) + \sin(280^\circ) + 2 \cdot 360^\circ\) = \(\cos(10^\circ) + \cos(10^\circ) \cdot \sin(100^\circ) + \sin(280^\circ) + \sin(280^\circ) = \cos(10^\circ) + \cos(10^\circ) + \cos(10^\circ) - \cos(10^\circ) = \cos(10^\circ) = \sin(80^\circ)\).

4. Answer: \(3\sqrt{2}\)
Using the law of cosines in the right triangle, we find \(9^2 = 6^2 + 3^2 - 2(5)(6) \cos a\) where \(a\) is the angle the triangles have in common. Thus \(\cos a = \frac{1}{3}\). Since \(\sin^2 a + \cos^2 a = 1\), \(\sin a = \frac{\sqrt{2}}{3}\). Using the law of sines in the left triangle yields \(\frac{x}{\sin 30^\circ} = \frac{8}{\sin a}\). Thus, \(x = 3\sqrt{2}\).

5. Answer: 1680
There are 8! orders for RICEOWLS and each of the 4! orders of WISE are equally likely within them, so \(\frac{8!}{4!} = 8 \cdot 7 \cdot 6 \cdot 5 = 1680\) have the correct order.

6. Answer: \(2F_{2005}\).
\(F_0 = 1 = F_1\). Then \(F_1 + F_2 = F_3\). Then \(F_3 + F_4 = F_5\). And so on until we get \(F_{2006} + F_{2003}\). This equals \(F_{2005} + F_{2004} + F_{2003} = 2F_{2005}\).

7. Answer: \(\frac{17}{4}\)
Let \(X\) be the number of days until Chris is king if Adam is king and \(Y\) be the number of days until Chris is king if Bill is king. Then \(X = 1 + \frac{X}{3} + \frac{X}{2} + \frac{0}{4}\) and \(Y = 1 + \frac{X}{2} + \frac{X}{3} + \frac{0}{4}\). Solving for \(X\) we get \(\frac{17}{4}\) but we want expected day, not "days until" so we add 1 day to get \(\frac{21}{4}\).

8. Answer: 0
\((i+1)^4\) is in the direction of -1. Also, \((i-1)^4\) is in the direction of -1 and they have the same magnitude. Call \((i+1)^4 = n\). Then this is \(n^{501} - n^{501} = 0\).

9. Answer: 30
It’s best to rewrite it as \(\cos x = \frac{x^2}{2004}\) and first to consider only positive values. Clearly, \(x < \sqrt{2004}\) in agreement with the range of \(\cos x\). We’ll definitely have 2 solutions for every interval \([2\pi*(n-1), 2\pi*n]\) for \(n = 1, 2, \ldots, m\) for some \(m\). It’s not hard to see that \(m\) is the largest integer that does not exceed \(\sqrt{\frac{2004}{2x}}\). Since \(44^2 < 2004 < 45^2\), \(44 < x < 45\). The expression is hence between \(\frac{22}{7}\) and \(\frac{22.5}{7}\). Note that \(\frac{22}{7} > 3.142 > \pi\), so \(\frac{22}{7} > 7\) and \(m = 7\) (since \(\frac{22.5}{7} < 8\)). But at the end of the seventh interval,
\[ x = 14\pi, \text{ and } \frac{x^2}{2004} = \frac{49\pi^2}{901} < \frac{\pi^2}{10} < \frac{3.15^2}{10} < 1 = \cos 14\pi. \] Hence there must be at least one more solution. There cannot be more than one in \([14\pi, 15\pi]\) since \(\cos x\) decreases and \(\frac{x^2}{2004}\) increases. Note also that \(\frac{(15\pi)^2}{2004} = \frac{225\pi^2}{2004} > \frac{225\pi^2}{2025} = \frac{\pi^2}{9} > 1\), so \(x \geq 15\pi\) yields no solutions. There are a total of \(2(7) + 1 = 15\) solutions for positive \(x\) and hence 30 overall.

10. Answer: \(\frac{433}{833}\)

Instead of thinking about people picking cards, we will place the aces in the deck. So we assign a number between 1 and 52 to each of the aces. There are \(\binom{52}{4}\) ways to do this. Now we examine what happens if the first ace is in an odd numbered slot. If the first ace is number 1, we have \(\binom{51}{3}\) possibilities for the other 3 aces. Similarly, for slot 3 we have \(\binom{49}{3}\) and so forth. So the probability can be written as \(\binom{51}{3} + \binom{49}{3} + \cdots + \binom{3}{3}\). We can put the numerator into summation form as

\[
\sum_{n=1}^{\infty} \frac{1}{6} (2n + 1)(2n)(2n - 1).
\]

This is \(\frac{2 \times 25 \times 26^2 - 25 \times 26}{6} = \frac{844350}{6} = 140725\). Then \(\frac{140725}{\binom{52}{4}} = \frac{433}{833}\).