# The Cross-Section of Volatility and Expected Returns*

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## Abstract

We examine how volatility risk, both at the aggregate market and individual stock level, is priced in the cross-section of expected stock returns. We estimate a significantly negative cross-sectional price of risk for systematic volatility, with a cross-sectional volatility factor earning -0.87% per month. We find that stocks with high idiosyncratic volatility earn abysmally low returns. The quintile portfolio with the highest idiosyncratic volatility does not even earn an average positive total return, and the difference in Fama-French (1993) alpha's between quintile portfolios with the lowest and highest idiosyncratic risk is -1.31% per month. We find that the low returns earned by stocks with high exposure to systematic volatility risk and the low returns of stocks with high idiosyncratic volatility are not priced by the standard size, value or momentum factors and are not subsumed by liquidity or volume effects.

# 1 Introduction

The volatility of stock returns, both at the individual level and at the aggregate level, varies over time. While there has been extensive study of the relation between aggregate volatility and expected returns (see, among others, Campbell and Hentschel, 1992; Glosten, Jagannathan and Runkle, 1993; Scruggs, 1998; Goyal and Santa-Clara, 2003), the question of how volatility affects the cross-section of expected stock returns has received little attention.

We provide a systematic study of how stochastic volatility is priced in the cross-section of expected stock returns. Our goals are twofold. First, we cross-sectionally estimate a price of risk for aggregate market volatility. If the volatility of the market return is a systematic risk factor, an APT or factor model indicates that aggregate volatility should also be priced in the cross-section of stocks. We find that innovations to aggregate volatility have a statistically significant negative price of risk. Economically, a negative premium for systematic volatility risk implies that assets with positive exposures to aggregate volatility risk pay off in times when market returns are low. Since equity prices react negatively to positive shocks in aggregate volatility, investors are willing to pay premiums to hold assets with high exposure to systematic volatility risk. Hence, assets with high sensitivities to fluctuations in aggregate volatility earn low returns. Our findings are consistent with many option pricing studies that have also documented negative prices of aggregate volatility risk.[1] However, all of these option pricing studies estimate the price of aggregate volatility risk using, at most, only the time-series and a cross-section of options on an aggregate market index, in addition to returns on the market portfolio.

There are several advantages of using a cross-section of returns on stocks, rather than a cross-section of options on the market, to estimate the price of risk of aggregate volatility. First, using the cross-section of returns allows us to create a useful hedging, or mimicking factor portfolio for aggregate volatility risk. If the price of volatility risk is negative, the zero-cost hedge portfolio will have average returns that are consistently negative. The portfolio is easy to construct and reflects only exposure to innovations in aggregate volatility. The second reason for using the cross-section of stock returns is to gauge the strength of exposure to volatility risk in individual stocks or portfolios. This approach creates a new set of assets with exposure to volatility risk that are not options. Hence, performing tests on this new set of assets with exposure to volatility innovations complements and confirms the findings of the option pricing

[1] See, for example, Jackwerth and Rubinstein (1996), Bakshi, Cao and Chen (2000), Chernov and Ghysels (2000), Burashi and Jackwerth (2001), Coval and Shumway (2001), Benzoni (2002), Jones (2002), Pan (2002), Bakshi and Kapadia (2003) and Eraker, Johannes and Polson (2003).

studies. Finally, using the cross-section allows us to estimate the price of volatility risk controlling for other standard cross-sectional effects, such as the size and value effects of Fama and French (1993), the momentum effect of Jegadeesh and Titman (1993), and the liquidity effect of Pástor and Stambaugh (2003). Estimating volatility risk controlling for other cross-sectional factors cannot be done using only a cross-section of options on the market portfolio.

We find strong evidence that systematic volatility risk is priced in the cross-section of stocks. The difference in average returns between the highest and lowest quintile portfolios sorted by exposure to volatility innovations is -1.04% per month, and is still statistically significant at -0.83% per month controlling for the Fama and French (1993) factors. The cross-sectional volatility risk effect is robust to liquidity effects and is not priced by a momentum factor. We find that our mimicking factor created to represent exposure to systematic volatility risk is significantly priced in the cross-section of stock returns.

A second related goal of this paper is to examine patterns in cross-sectional expected returns of portfolios formed by ranking on idiosyncratic volatility, measured relative to standard models of systematic risk. In contrast, recent studies focus only on the average level of firm-level volatility. For example, Campbell et al. (2001) and Xu and Malkiel (2001) document that idiosyncratic volatility, relative to the market or to the Fama-French (1993) three-factor model, has increased over time. Goyal and Santa-Clara (2003) demonstrate that idiosyncratic risk has positive predictive power for excess market returns. In contrast, we focus on how idiosyncratic risk is cross-sectionally reflected in expected returns.

Standard asset pricing models predict that idiosyncratic volatility is not priced and thus cannot influence cross-sectional average returns. However, recent economic theory indicates that idiosyncratic risk may be positively related to expected returns, if investors demand compensation for not being able to diversify risk (see Malkiel and Xu, 2002; Jones and Rhodes-Kropf, 2003). Merton (1987) suggests that in an information-segmented market, firms with larger firm-specific variance require higher returns to compensate for imperfect diversification. Recent behavioral models, like Barberis and Huang (2001) also predict that higher idiosyncratic volatility stocks should earn higher expected returns. Our results are directly opposite to these theories. We find that stocks with low idiosyncratic risk deliver high average returns. There is a strongly significant pattern of over -1.06% per month in the average return difference between quintile portfolios of lowest and highest idiosyncratic risk, computing idiosyncratic volatility relative to the Fama-French (1993) model.

Our findings are totally the opposite of Tinic and West (1986) and Malkiel and Xu (2002).

These authors find that portfolios with higher idiosyncratic risk have higher average returns. However, they do not directly sort stocks based on the measure of interest, idiosyncratic volatility, nor do they tabulate any significance levels for their idiosyncratic volatility premiums. Instead, Tinic and West (1986) work only with 20 portfolios sorted on market beta, while Malkiel and Xu work only with 100 portfolios sorted on market beta and size.[2] Hence, Tinic and West and Malkiel and Xu miss the strong negative relation between idiosyncratic volatility and expected returns.

The very low returns we find for high idiosyncratic volatility represent somewhat of a puzzle. We outline some potential explanations and investigate if they can explain these puzzling results. Our results are robust to controlling for value, size, liquidity, volume, dispersion of analysts' forecasts, and momentum effects. In particular, we find that the effect is common to stocks of all sizes, but is strongest among middle-sized, not the smallest-sized, stocks. The effect persists in both bull and bear markets, recessions and expansions, and volatile and stable periods. Moreover, we find the effect robust to different formation periods for computing idiosyncratic volatility and for different holding periods. We also find that the portfolios sorted by idiosyncratic volatility have little exposure to systematic volatility risk.

The rest of this paper is organized as follows. In Section 2, we examine how systematic volatility is priced in the cross-section of stock returns. Section 3 documents that firms with high idiosyncratic volatility have very low average returns. Finally, Section 4 concludes.

# 2 Pricing Systematic Volatility in the Cross-Section

## 2.1 A Simple Model

To motivate the empirical analysis that follows, we start by presenting a model to illustrate how stochastic volatility might affect the cross-section of equity returns. This simple model is not intended to provide a complete structural explanation of how aggregate volatility is priced by agents in the cross-section. Rather, we use the simple model as motivation to illustrate how stochastic market volatility implies cross-sectional differences in expected returns.

[2] Malkiel and Xu (2002) do consider a cross-sectional regression on individual stocks, but instead of using a measure of an individual stock's idiosyncratic volatility, they assign a stock's residual standard deviation to be the idiosyncratic risk of one of the 100 beta/size portfolios to which that stock belongs each month.

Consider a two-factor model that allows the market volatility to be stochastic:

$$
\begin{aligned}
\frac{dS_t^m}{S_t^m} &= \mu_t^m dt + \sigma_t^m dW_t \\
d\sigma_t^m &= a_t^m dt + b_t^m dV_t
\end{aligned}
\tag{1}
$$

where $dS_t^m/S_t^m$ represents the aggregate market return and $\sigma_t^m$ represents the aggregate market volatility. We assume that the two Weiner processes $dW_t$ and $dV_t$ are correlated, $dW_t dV_t = \rho dt$.

The formulation of the drift and volatility of $\sigma_t^m$ can be very general. For example, we can specify $\sigma_t^m$ to be mean-reverting if we set $a_t^m = \kappa(\theta - \sigma_t^m)$ and $b_t^m = b$. In order to derive closed-form option pricing models, a common specification is to model $d(\sigma_t^m)^2$ as a square-root process, as in Heston (1993). Since our focus is not on deriving a closed-form option pricing model, we can allow $a_t^m$ and $b_t^m$ to take very general forms. Similarly, we do not take a stand on the functional form of the drift of the market return $\mu_t^m$ as our focus is on deriving cross-sectional, rather than aggregate, pricing implications.

Suppose markets are complete and the pricing kernel $m$ takes the form:

$$
\frac{dm_t}{m_t} = -r_f dt - \eta_t^m dW_t - \eta_t^v dV_t, \tag{2}
$$

where we have assumed a constant risk-free rate $r_f$ without loss of generality. To model an individual stock, we set the return on stock $i$, $dS_t^i/S_t^i$ to also follow a two-factor model:

$$
\begin{aligned}
\frac{dS_t^i}{S_t^i} &= \mu_t^i dt + \sigma_t^i dW^i \\
d\sigma_t^i &= a_t^i dt + b_t^i dV^i
\end{aligned}
\tag{3}
$$

where $dW dW^i = \rho_{mi} dt$ and $dV dW^i = \rho_{vi} dt$. The drift $a_t^i$ and volatility $b_t^i$ functions can be functions of stock $i$'s volatility $\sigma_t^i$. In equation (3), shocks to a stock's own volatility are correlated with shocks to the stochastic volatility factor in the pricing kernel (2). This implies that a stock's volatility exposure to systematic volatility plays a part in determining that stock's expected return.[3]

By definition of the pricing kernel, the drift term of $d(mS_t^i)$ must be zero, therefore, Ito's lemma gives the following relation:

$$
\frac{\mu_t^i - r_f}{\sigma_t^i} = \eta_t^w \rho_{mi} + \eta_t^v \rho_{vi}, \tag{4}
$$

[3] A related specification of (1) to (3) is Brennan, Wang and Xia (2002), who specify the Sharpe ratio of the market, $\eta_t^m$, to assume its own Ornstein-Uhlenbeck process.

In the case of asset returns that can proxy for $dS^m_t/S_t$ and $d\sigma^m_t$, then:[4]

$$\begin{aligned}\eta^w_t + \eta^v_t \rho &= \frac{\mu^m_t - r_f}{\sigma^m_t} \\ \eta^w_t \rho + \eta^v_t &= \frac{a^m_t - \sigma^m_t r_f}{b^m_t}.\end{aligned} \tag{5}$$

Substituting into equation (4), we obtain:

$$\begin{aligned}\frac{\mu^i_t - r_f}{\sigma^i_t} = \frac{1}{(\rho^2-1)} \left( \frac{a^m_t - \sigma^m_t r_f}{b^m_t} \rho - \frac{\mu^m_t - r_f}{\sigma^m_t} \right) \rho_{mi} \\ + \frac{1}{(\rho^2-1)} \left( \frac{\mu^m_t - r_f}{\sigma^m_t} \rho - \frac{a^m_t - \sigma^m_t r}{b^m_t} \right) \rho_{vi}.\end{aligned} \tag{6}$$

This can be re-written as:

$$\mu^i_t - r_f = \beta^m_{it}(\mu^m_t - r_f) + \beta^v_{it}(a^m_t - \sigma^m_t r_f), \tag{7}$$

where

$$\beta^m_{it} = \frac{\rho - \rho_{mi}}{\rho^2 - 1} \frac{\sigma^i_t}{\sigma^m_t} \quad \text{and} \quad \beta^v_{it} = \frac{\rho - \rho_{vi}}{\rho^2 - 1} \frac{\sigma^i_t}{b^m_t}.$$

We can interpret $\beta^m_{it}$ to be the multivariate beta of stock $i$, measuring exposure to fluctuations in the market shock $dW_t$. Similarly, $\beta^v_{it}$ measures exposure of stock $i$ to innovations in the market volatility $dV_t$. Note that expected returns depend on the sensitivity to innovations, not the sensitivity to the level of either the market or volatility per se.[5]

In our empirical work, we examine if there is any relationship between a stock's exposure to innovations in systematic volatility, as in equation (7). Note that in deriving equation (7), we assume that market volatility is a traded asset, allowing us to specify the price of risk of volatility. In our empirical work, we use several proxies to measure innovations in volatility. Equation (7) implies that stocks with different exposures to innovations in volatility, through different $\beta^v_{it}$ coefficients, have different expected excess returns. We will build a mimicking factor for volatility risk to estimate its risk premium. A further goal is to control not only for the effect of the market but also control for other known cross-sectional factors (for example, the Fama and French, 1993, value and size factors).

Equation (7) is in the standard form of an APT or factor model (see Ross, 1976), so that market volatility risk, through $\beta^v_{it}$, is explicitly priced in the cross-section of stock returns. The

[4] For at-the-money options, very short holding period returns are approximately linear functions of changes in implied volatility.

[5] If we assume that volatility risk is not priced ($\eta^v_t = 0$), then equation (7) simplifies to the traditional CAPM $\mu^i_t - r_f = \beta^m_{it}(\mu^m_t - r_f)$.

model is also in the spirit of a Merton (1973) Intertemporal CAPM (I-CAPM). Since market volatility affects the dynamics of the market return in equation (1), systematic volatility has implications for asset prices in the cross-section, as in equation (7).[6] An I-CAPM model implies joint time-series as well as cross-sectional predictability. The goal in our empirical work is to only examine the cross-sectional pricing implications of equation (7) directly. We do not examine joint time-series and cross-sectional predictability of asset returns by systematic volatility because we do not take a stand on the utility function of a representative agent or parameterize the time-series process of volatility.

## 2.2 Estimating Aggregate Volatility

We proxy systematic (market-wide) volatility using three estimators. The first estimator is the standard French, Schwert and Stambaugh (1987) and Schwert and Seguin (1990) measure, which is the sum of squared daily returns over the past $N_t$ days, adjusted for first-order autocorrelations:[7]

$$\hat{\sigma}_t^2 = \frac{1}{N_t}\left[\sum_{i=0}^{N_{t-1}} r_{t-i}^2 + 2\sum_{i=1}^{N_t} r_t r_{t-i}\right], \tag{8}$$

where $r_t$ is the return on the market portfolio. We denote the volatility measure $\hat{\sigma}_t$ in (8) by $SVOL$ (sample volatility) and compute $SVOL$ using daily returns on the market index from CRSP. We compute $SVOL$ at a daily frequency by using the last $N_t = 22$ trading days. The use of past daily data over the previous month to estimate volatility at time $t$ means that the $SVOL$ estimates do not reflect the true market volatility at $t$, rather they represent an average of the daily volatility from month $t-1$ to month $t$. Nevertheless, $SVOL$ should pick up broad trends in true volatility movements.

Our second proxy for market volatility is a range-based estimate, following Alizadeh, Brandt and Diebold (2002):

$$\hat{\sigma}_t = \log\left(\sup_{0<\tau\leq 1} S_\tau \Big/ \inf_{0<\tau\leq 1} S_\tau\right), \tag{9}$$

where $S_\tau$ is the level of the S&P500 index over day $t$. We denote this range-based estimate for aggregate volatility as $RVOL$. While easy to compute, $RVOL$ suffers from several drawbacks. First, $RVOL$ is biased downwards because the range on a discrete grid of prices is always less

[6] For example, Chen (2002) extends Campbell's (1993 and 1996) log-linear approximation of Merton (1973), by allowing for time-varying covariances and stochastic market volatility. Chen shows that any variable that forecasts future market returns or future market variances must be priced cross-sectionally.

[7] Our results are unchanged if we omit the autocorrelation terms, as in Schwert (1989).

than the range of a true continuous sample path. Second, the use of equation (9) assumes that the volatility of the market is constant each day, but changes from day to day. Third, the log range estimator relies on the assumption that a log volatility process is a good approximation for the underlying true volatility process. Finally, even if the true volatility process follows a log process, Andersen and Bollerslev (1998) and Alizadeh, Brandt and Diebold (2002) show that the efficiency of $RVOL$ is similar to the efficiency of estimates which use intra-day realized volatility forecasts of 4-6 hour windows, which provide at most two observations per trading day. Since $SVOL$ uses only one observation per trading day, the $RVOL$ measure should be better than, but may not be a substantial improvement on, using $SVOL$.

Our last proxy for volatility is the $VIX$ index, which is a Black-Scholes (1973) implied volatility index constructed by the Chicago Board of Exchange from eight S&P100 index puts and calls. The $VIX$ index takes into account the American features of the option contracts, discrete cash dividends and microstructure frictions such as bid-ask spreads. The $VIX$ index is constructed so that it represents the implied volatility on a synthetic at-the-money option contract that has a one month maturity. Whaley (2000) provides further details on the construction of the $VIX$ index.

At first glance, since $VIX$ is representative of traded option securities whose prices directly reflect volatility risk, $VIX$ might seem to be the most natural measure of changes in aggregate volatility. However, there are three main caveats with using $VIX$ to represent observable market volatility. First, the $VIX$ index is Black-Scholes implied volatility, rather than the true unobservable volatility process. However, we would expect that Black-Scholes volatilities would be highly correlated with the true volatility process. The second caveat is that $VIX$ may also reflect an interaction of a jump and a diffusion (see Eraker, Johannes and Polson, 2003). However, Bates (1991 and 2000) argues that implied volatilities computed taking into account jump risk are very close to Black-Scholes implied volatilities.

The third, but most serious, reservation about the $VIX$ index is that $VIX$ combines both stochastic volatility itself and the stochastic volatility risk premium. Only if the risk premium is zero or constant would $\Delta VIX$ represent only an innovation in volatility. Decomposing $\Delta VIX$ into the true innovation in volatility and the risk premium can only be done by writing down a formal model. The form of the risk premium depends on the parameterization of the volatility price of risk, the number of factors and the evolution of those factors. Each different model specification implies a different risk premium. For example, many stochastic volatility option pricing models parameterize the volatility risk premium to be a linear function of volatility (see, for example, Chernov and Ghysels, 2000; Benzoni, 2002; Jones, 2002; Pan, 2002). Rather than imposing a structural form, we use an unadulterated $VIX$ series. This has the additional advantage that our analysis is simple to replicate.

Other common methods of estimating volatility include GARCH-based models and methods based on intra-day, or high frequency, data (see, for example, Andersen et al., 2003). We do not use a GARCH model because the parameters of the GARCH process must be estimated before computing the implied innovations in the variances. Hence, this method entails a look-ahead bias if the full sample is used. When we form portfolios, it is important that we form portfolios only using only data available as of the formation date. If a rolling GARCH estimator is used to avoid look-ahead bias, the time-periods near the beginning of the sample suffer from very poor estimates of the GARCH process. While Andersen et al. (2003) formally justify the use of the realized sample volatility measured with intra-day data as a highly efficient volatility proxy, intra-day data on market returns are not readily available, making this estimation method hard to implement. In particular, intra-day data are collected only for individual stocks and the main source of these data, the TAQ database, starts only in 1993.

We concentrate on using the sample period from January 1986 to December 2000. This is because the data for the $VIX$ series begins in January 1986, and we would like to compare all our series on a common sample period. Nevertheless, we also comment on the sample period July 1963 to December 2000 for $SVOL$ and $RVOL$.

Table 1 presents some summary statistics for $SVOL$, $RVOL$ and $VIX$ at a daily frequency. The annualized mean of $SVOL$ ($RVOL$) is $0.0099 \times \sqrt{250} = 16\%$, ($0.0120 \times \sqrt{250} = 19\%$). The mean of $VIX$ is higher than both these two measures, at 21%. The higher average of $VIX$ volatility indicates that it is a biased forecast of realized future volatility. The bias may reflect a risk premium for stochastic volatility, a market inefficiency or a Peso-problem.[8] The annualized standard deviations of $SVOL$, and $VIX$ are approximately equal, at 9% and 8%, but the annualized standard deviation of $RVOL$ is higher, at 14%. All three series are negatively correlated with the market return, with $RVOL$ ($VIX$) having a -23% (-18%) correlation. The correlation of $SVOL$ with the market is noticeably less, at only -4%. The low correlation of $SVOL$ is due to the fact that a large negative movement in returns has only a 1/22th weight in the computation of $SVOL$ from equation (8), whereas the increase in volatility is reflected more immediately by $RVOL$ and $VIX$.

[8] See, among many others, Day and Lewis (1992), Canina and Figlewski (1993), Lamoureux and Lastrapes (1993), Blair, Poon and Taylor (2001), Poteshman (2000) and Chernov (2002).

We graph the three volatility measures in Figure 1, which annualizes each volatility measure so that they are comparable. Overall, all three measures share the same trends. In particular, each series has two noticeable spikes. The first spike shows the increase in implied volatilities after the 1987 crash, and the second spike occurs in 1998 during the Russian default, the emerging markets crises, and the bailout of Long Term Capital Management. Figure 1 shows that while $SVOL$ and $VIX$ are fairly smooth series (autocorrelations of 98% and 94%, respectively at at a daily frequency), the $RVOL$ measure is much less smooth (the daily autocorrelation of $RVOL$ is 49%). The more volatile range-based measure also magnifies the movements in volatility measured by $SVOL$ and $VIX$.

To measure daily innovations in aggregate volatility, we compute daily changes in $SVOL$, daily changes in $RVOL$ or daily changes in $VIX$. We denote these measures as $\Delta SVOL$, $\Delta RVOL$ or $\Delta VIX$, respectively. From equation (8) (and ignoring the autocorrelation term), $\Delta SVOL$ effectively takes the difference between the squared market return at $t$ and the squared market return 22 trading days prior to time $t$. Hence, the time-series of daily $\Delta SVOL$ effectively measures monthly innovations in volatility at time $t$. In contrast, $\Delta RVOL$ and $\Delta VIX$ reflect daily changes in volatility movements and may be better estimates of changes in true market volatility. In particular, $\Delta VIX$ reflects a daily change in implied option volatilities. Nevertheless, $\Delta SVOL$, $\Delta RVOL$ and $\Delta VIX$ are all quite highly correlated with each other. For example, Table 1 reports that the correlation of $\Delta VIX$ with $\Delta SVOL$ ($\Delta RVOL$) is 45% (39%).

In Table 1, all the estimates for daily innovations in volatility have strong negative correlations with the market return. The correlations with the market and $\Delta SVOL$, $\Delta RVOL$ and $\Delta VIX$ are -25%, -26% and -64%, respectively. Hence, when a positive volatility shock arrives, the market excess return decreases. The best example of this effect is the increase in volatility over 1987 in Figure 1, coinciding with the large negative returns of the market over this period. Table 1 shows one source of discrepancy between $\Delta RVOL$ and the two other estimators $\Delta SVOL$ and $\Delta VIX$. While $\Delta SVOL$ and $\Delta VIX$ have very low autocorrelations (7% and -7%, respectively), $\Delta RVOL$ has a strong negative autocorrelation of -43%. In fact, $RVOL$ and $\Delta RVOL$ also have almost the same standard deviation. This is due to the large movements in $RVOL$, which is shown in Figure 1.

## 2.3 Portfolios Sorted by Exposure to Systematic Volatility

Equation (7) predicts that firms with different sensitivities (measured by betas) to innovations in systematic volatility should have different expected excess returns. Based on this implication, we sort firms into portfolios according to their sensitivities to systematic volatility. If stochastic volatility risk is priced, the average returns on these volatility sensitivity-sorted portfolios should be different. Our first step is to check that firms with different sensitivities to market volatility innovations indeed have different average returns.

Equation (7) suggests estimating the sensitivities of stock $i$ to systematic volatility in the following regressions:

$$
\begin{aligned}
r_t^i &= \alpha^i + \beta_{MKT}^i \cdot MKT_t + \beta_{\Delta SVOL}^i \cdot \Delta SVOL_t + \varepsilon_t^i \\
r_t^i &= \alpha^i + \beta_{MKT}^i \cdot MKT_t + \beta_{\Delta RVOL}^i \cdot \Delta RVOL_t + \varepsilon_t^i \\
r_t^i &= \alpha^i + \beta_{MKT}^i \cdot MKT_t + \beta_{\Delta VIX}^i \cdot \Delta VIX_t + \varepsilon_t^i \qquad (10)
\end{aligned}
$$

where $r_t^i$ is firm $i$'s excess return and $MKT$ is the market excess return. Equation (10) proxies the innovation in market volatility ($dV_t$ in equation (1)) by $\Delta SVOL$, $\Delta RVOL$, or $\Delta VIX$. The coefficients $\beta_{\Delta SVOL}^i$, $\beta_{\Delta RVOL}^i$ and $\beta_{\Delta VIX}^i$ represent the sensitivity of firm $i$'s returns to innovations in market volatility, measured by these proxies. Note that, as equation (7) suggests, we control for the effect of the market in computing the volatility betas in equation (10).

To form portfolios, we run regression (10) on daily excess returns over the previous month for each firm with more than 17 daily observations within that month on all stocks on AMEX, NASDAQ and the NYSE. At the end of each month, we sort the stocks into quintiles, based on the value of the $\beta_{\Delta SVOL}$, $\beta_{\Delta RVOL}$ or $\beta_{\Delta VIX}$ coefficients. Firms in quintile 1 (5) have the lowest (highest) coefficients. Within each quintile portfolio, we value-weight the stocks. If volatility risk is priced cross-sectionally, the average returns of these quintile portfolios should be different.

Table 2 reports various summary statistics for quintile portfolios sorted by exposure to aggregate volatility shocks. If the negative price of systematic volatility risk found by the option pricing studies is reflected in the cross-section, we should see lower average returns with higher coefficients of $\beta_{\Delta SVOL}^i$, $\beta_{\Delta RVOL}^i$ or $\beta_{\Delta VIX}^i$. We turn first to the portfolios sorted by $\beta_{\Delta SVOL}$. The $\beta_{\Delta SVOL}$ portfolios have little differences in spreads or alpha's, relative to the CAPM or to the Fama-French (1993) model (FF-3 hereafter), reported in the last two columns. This is not surprising since we know that $\Delta SVOL$ is potentially a poor measure for daily changes in stochastic volatility.

We next turn to the quintile portfolios sorted by $\beta_{\Delta RVOL}$. Quintiles 1-4 all have higher average returns than quintile 5, and the 5-1 spread in average returns between the quintile portfolios with the lowest and highest $\beta_{\Delta RVOL}$ values is -0.42% per month. When we control for the Fama-French factors, the 5-1 alpha is -0.39% per month. While the negative point estimates of the 5-1 spread in average returns or alpha's are consistent with a negative price of volatility risk, the spreads are statistically insignificant at the 5% marginal level of significance using robust Newey-West (1987) t-statistics. The sample period of Table 2 is from January 1986 to December 2000. If we use more data from July 1963 to December 2000, for more power, we still cannot reject that the 5-1 difference in alphas or expected returns for both $\beta_{\Delta SVOL}$ and $\beta_{\Delta RVOL}$ portfolios are equal to zero.

We now turn to our last volatility proxy, $\Delta VIX$, which directly reflects the volatility of the market portfolio priced in option contracts. The average returns of the quintile portfolios are monotonically decreasing from 1.64% per month for low $\beta_{\Delta VIX}$ stocks to 0.60% per month for high $\beta_{\Delta VIX}$ stocks. The 5-1 spread in average returns between the quintile portfolios with the highest and lowest $\beta_{\Delta VIX}$ coefficients is -1.04% per month. This dramatic spread in average returns does not seem to be due to patterns in size or book-to-market characteristics. In the two last columns of Table 2, we compute alpha's relative to the CAPM and FF-3. Controlling for the $MKT$ factor only exacerbates the 5-1 spread (from -1.04% to -1.15% per month), while controlling for the FF-3 model decreases the 5-1 spread to -0.83% per month. Both the CAPM and FF-3 alpha's are significant at the 1% level using robust t-statistics.

One curious pattern about the average returns and the alpha's for the $\beta_{\Delta VIX}$ quintile portfolios is that the average returns and alpha's for quintiles 1-4 are approximately the same, with a slight downward trend. However, there is a dramatic fall in the average return and alpha for quintile 5. This implies that while there is a monotonic relation between increasing $\beta_{\Delta VIX}$ loadings and decreasing average returns and alpha's, the biggest effect is for stocks with the highest values of $\beta_{\Delta VIX}$, which have extremely low returns or alpha's. This category of stocks is not a small proportion of the market: the percentage market capitalization of quintile 5 is 7.4%. Quintile 5 portfolio's turnover is also not substantially higher than the other portfolios; its average turnover is 73%, of the same order of magnitude as decile portfolios sorted on book-to-market ratios.

The results of the sorts on $\beta_{\Delta VIX}$ confirm the negative price of volatility risk estimated by option pricing studies. The higher the $\beta_{\Delta VIX}$ coefficient, the higher is the exposure of a stock to systematic volatility risk. Since stocks with higher $\beta_{\Delta VIX}$ loadings have lower expected returns, this is consistent with stochastic volatility carrying a negative risk premium. However, before we construct a mimicking factor for cross-sectional volatility risk, we first ensure that the $\beta_{\Delta VIX}$ effect is robust to other known factors that affect the cross-section of average returns.

The FF-3 alpha's in Table 2 show that the very large spread in average returns between the highest and lowest $\beta_{\Delta VIX}$ quintiles is not due to size or book-to-market effects. However, periods of very high volatility tend to coincide with periods of market illiquidity. Chordia et al. (2001), Jones (2002) and Pástor and Stambaugh (2003) all comment that such periods often coincide with market downturns. For example, during the 1987 crash and the 1998 Russian debt and subsequent emerging markets crises, realized market returns and liquidity were low. Pástor and Stambaugh demonstrate that stocks with high liquidity betas have high expected returns. We now check that the spread in average returns reflecting sensitivities to volatility risk is not due to liquidity effects. Panel A of Table 3 reports the results.

To control for liquidity, we first sort stocks into five quintiles based on their historical liquidity betas, $\beta^L$, computed following Pástor and Stambaugh (2003). Then within each quintile, we sort stocks into five quintiles based on their $\beta_{\Delta VIX}$ coefficient loadings. These portfolios are rebalanced monthly and are value-weighted. After forming the $5 \times 5$ liquidity beta and $\beta_{\Delta VIX}$ portfolios, we average the returns of each $\beta_{\Delta VIX}$ quintile over the five liquidity beta portfolios. Thus, these quintile $\beta_{\Delta VIX}$ portfolios control for differences in liquidity.

Table 3, Panel A shows that controlling for liquidity reduces the 5-1 difference in average returns from -1.04% per month in Table 2 to -0.68% per month. In particular, after controlling for liquidity, we still observe the monotonically decreasing pattern of average returns of the $\beta_{\Delta VIX}$ quintile portfolios. The liquidity control also does not remove the sharp decrease in the average return of the fifth $\beta_{\Delta VIX}$ quintile. When we control for the CAPM (FF-3 model), the alpha becomes -0.73% (-0.55%) per month. Both these alpha's are significant at the 5% level. We also observe the same pattern of very low returns for the highest $\beta_{\Delta VIX}$ stocks within each liquidity beta quintile, before averaging across the liquidity beta portfolios, but do not report these results to save on space. Hence, liquidity effects cannot account for the spread in returns resulting from sensitivity to aggregate volatility risk.

Panel B reports the same exercise except we control for volume effects rather than liquidity. Gervais, Kaniel and Mingelgrin (2001) find that stocks with high trading volume earn higher average returns than stocks with low trading volume. It could be that the low average returns (and alpha's) we find for stocks with high $\beta_{\Delta VIX}$ loadings are just stocks with low volume. Panel B shows that this is not the case. In Panel B, we control for volume the same way that we control for liquidity in Panel A, except we first sort stocks into quintiles based on their trading volume (rather than Pástor-Stambaugh liquidity betas). Before averaging across the volume portfolios, we also observe the same pattern of low returns to high $\beta_{\Delta VIX}$ stocks within each volume quintile (not reported). Hence, the volume effect is also not responsible for the large spread in average returns and alpha's between stocks with low and high $\beta_{\Delta VIX}$ sensitivities.

## 2.4 A Cross-Sectional Volatility Factor

### Constructing the $VOL$ Factor

The pricing equation (7) from our simple model motivates a linear factor model for stock $i$ of the form:

$$\mathrm{E}(r_t^i) = \alpha^i + \beta_{MKT}^i \cdot \lambda_{MKT} + \beta_{\Delta VIX}^i \cdot \lambda_{VOL}, \tag{11}$$

where $\mathrm{E}(r_t^i)$ is the expected excess return of stock $i$, $\lambda_{MKT}$ is the market risk premium and $\lambda_{VOL}$ is a risk premium for the aggregate volatility risk factor. If the model is correctly specified, then $\alpha^i$ should be zero. In this section, we build a mimicking factor for stochastic volatility exposure, allowing us to cross-sectionally estimate the price of risk for stochastic volatility. The advantage of constructing a mimicking factor, which we call $VOL$, rather than just using $\Delta VIX$ is that $\Delta VIX$ does not represent the realized return on a tradable asset. By creating a tradeable factor, we can interpret alpha's from standard time-series factor regressions as well as directly estimate the volatility risk premium cross-sectionally.

Our volatility factor, $VOL$, is formed as follows. Each month, we rank stocks based on their $\beta_{\Delta VIX}$ coefficients into three groups: low, medium and high $\beta_{\Delta VIX}$ groups with 33.3% and 66.7% cutoffs. We calculate monthly value-weighted returns for each of these three portfolios. The $VOL$ factor is formed as the return difference between the high $\beta_{\Delta VIX}$ group and the low $\beta_{\Delta VIX}$ group. Hence, the $VOL$ factor goes long stocks with high volatility innovation sensitivities, which have low expected returns, and shorts stocks with low volatility innovation sensitivities, which have high expected returns.

Table 4 lists some summary statistics for the $VOL$ factor. The $VOL$ factor has a monthly mean return of -0.58% per month, and the mean is statistically significant at the 1% marginal level of significance. Table 4 also lists the correlation of $VOL$ with the excess market return $MKT$, the Fama and French (1993) size and value factors $SMB$ and $HML$, and $UMD$, a momentum factor constructed by Kenneth French. The momentum factor $UMD$ is constructed in a similar way to Carhart (1996)'s momentum factor, which goes long stocks with past high returns and shorts stocks with past low returns. The correlation of $VOL$ with the $MKT$ is 16%, which is smaller in magnitude than the respective correlations of $SMB$, $HML$ and $UMD$ with $MKT$ over our sample period. The low correlation results from controlling for the $MKT$ factor in our initial computation of $\beta_{\Delta VIX}$ in the regression (10). However, our $VOL$ factor is relatively highly correlated with $SMB$, at 48%, and $HML$ at -40%. This is consistent with the results in Table 2, where the alpha's from the FF-3 model for the quintile $\beta_{\Delta VIX}$ portfolios are slightly smaller than the raw average returns.

Table 4 also reports the results of regressing $VOL$ onto various factors in a time-series regression. Controlling for the $MKT$ factor decreases the $\alpha$ from -0.58% per month to -0.68% per month. The FF-3 model reduces this magnitude to -0.46% per month. Nevertheless, the alpha is still significant at the 5% level. When we add the $UMD$ momentum factor, the loading on $UMD$ is zero, and the point estimate of the alpha is almost unchanged, decreasing by only 1 basis point to -0.47% per month. However, the extra noise added by $UMD$ causes the $VOL$ alpha to be borderline significant at the 5% level.

In Figure 2, we plot the cumulative returns of the $VOL$ factor from January 1986 to December 2000. Over the sample, no particular time period drives the significantly negative mean (-0.58% per month) of the $VOL$ factor. The large increases in $VIX$ after the 1987 crash and during 1998 do coincide with negative returns of the $VOL$ factor, but these are not unusually large.

**Pricing $\beta_{\Delta VIX}$ Sorted Portfolios**

As a check on whether the $VOL$ factor captures the return premium between stocks with high $\beta_{\Delta VIX}$ loadings and stocks with low $\beta_{\Delta VIX}$ loadings, Table 5 examines if $VOL$ can price the quintile portfolios sorted on $\beta_{\Delta VIX}$. The table reports the portfolio alpha's and the p-value from a Gibbons-Ross-Shanken (1989) (GRS) joint test that the alpha's are equal to zero. The alpha's from the CAPM and FF-3 model are repeated from the last two columns of Table 2.

Table 5 shows that the standard models, the CAPM, FF-3, and FF-3 augmented with a $UMD$ momentum factor cannot account for the spread in returns of the $\beta_{\Delta VIX}$ portfolios. For all these models, we systematically reject the hypothesis that the alpha's of the portfolios are jointly equal to zero at a 5% level. In contrast, all the models with $VOL$ pass the GRS test at a 10% marginal level of significance. The alpha's of the difference between the highest and lowest $\beta_{\Delta VIX}$ portfolios are -1.15% per month for the CAPM regression. We can decrease this magnitude to -40 basis points per month by including the $VOL$ factor. However, the 5-1 difference is still statistically significant with the $VOL$ factor, despite the failure to reject a GRS test.

It is instructive to examine the factor loadings in Panel B of Table 5 for the most comprehensive factor specification with all $MKT$, $SMB$, $HML$, $UMD$ and $VOL$ factors. The market loadings, although highly significant, are fairly flat across the portfolios. The $SMB$ loadings have a U-shape, picking up the lowest and the highest $\beta_{\Delta VIX}$ portfolios. Similarly, $HML$ has an inverted U-shape, with lowest loadings on the lowest and highest $\beta_{\Delta VIX}$ portfolios. Hence, neither $SMB$ nor $HML$ can account for the spread between the 5-1 portfolios. The point statistics of the momentum factor loadings are almost zero and have no pattern. In contrast, the loadings on $VOL$ increase monotonically from -0.54 for the lowest $\beta_{\Delta VIX}$ portfolio to 0.57 to the highest $\beta_{\Delta VIX}$ portfolio. Hence, exposure to the $VOL$ factor accounts for $1.11 \times -0.58\% = -0.64\%$ per month of the raw -1.04% per month average return. $VOL$ is the only factor whose (increasing) loadings reflect the pattern in (decreasing) returns from low to high $\beta_{\Delta VIX}$ quintile portfolios.

**Fama-MacBeth (1973) Estimates of the Price of Volatility Risk**

Equations (7) and (11) of the linear factor model imply a standard cross-sectional regression. If excess returns of assets are regressed on the $\beta_{\Delta VIX}$ coefficients of those assets, then there should be a significant coefficient on the $\beta_{\Delta VIX}$ loadings. This coefficient, $\lambda_{VOL}$, is the price of risk of stochastic volatility. To estimate $\lambda_{VOL}$ in the cross-section, equations (7) and (11) suggest the need to create a set of assets whose market betas and $\Delta VIX$ betas are sufficiently disperse. We construct 25 portfolios sorted by $\beta_{MKT}$ and $\beta_{\Delta VIX}$ as follows. At the end of each month, we sort stocks based on $\beta_{MKT}$, computed by a univariate regression of excess stock returns on excess market returns over the past month using daily data. We compute the $\beta_{\Delta VIX}$ loadings using the bivariate regression (10) also using daily data over the past month. Stocks are ranked first into quintiles based on $\beta_{MKT}$ and then within each $\beta_{MKT}$ quintile into $\beta_{\Delta VIX}$ quintiles.

Panel A of Table 6 reports FF-3 alpha's of these 25 $\beta_{MKT} \times \beta_{\Delta VIX}$ set of portfolios. There is some heterogeneity in the alpha's, but the 5-1 difference in the $\beta_{\Delta VIX}$ quintiles are always negative. For the larger $\beta_{MKT}$ quintiles 4 and 5, the alpha's are almost monotonic.[9] Across each $\beta_{MKT}$ quintile, it is always the fifth $\beta_{\Delta VIX}$ quintile that has the steepest drop in returns. Hence, this finer sort of stocks based on $\beta_{MKT}$ and $\beta_{\Delta VIX}$ coefficients has the same qualitative pattern

[9] The pattern in the means of raw returns is qualitatively similar to the the pattern of FF-3 alpha's.

of alpha's as the quintile $\beta_{\Delta VIX}$ portfolios in Table 2, which do not control for the market beta.

We use the base assets of Panel A to estimate factor premiums in Panel B, following the two-step procedure of Fama-MacBeth (1973). In addition to the standard FF-3 and $UMD$ factors, we include the Pástor-Stambaugh (2003) liquidity factor, $LIQ$. Although the $LIQ$ factor is non-traded, we can still include it in the cross-sectional regression and examine the statistical significance of its premium. Panel B shows that the premiums of the standard factors ($MKT$, $SMB$, $HML$ and $UMD$) are estimated very imprecisely with this set of base assets. The premium on $SMB$ is consistently estimated to be negative because the size strategy has performed poorly from the 1980's onwards. The low and insignificant premiums of $UMD$ and $LIQ$ illustrate that the spreads in expected returns of the $\beta_{\Delta VIX}$ portfolios are not related to momentum or liquidity effects.

When $VOL$ is included in the cross-sectional regressions, it is the only factor estimated to have a significant loading. Its premium of around -0.83% per month is of the same order of magnitude as the time-series mean of $VOL$ (-0.58% per month). The $VOL$ premium is significant in all the various specifications of including different factors. The cross-sectional $R^2$'s also increase significantly once $VOL$ is included. For example, the $R^2$ of the FF-3 specification is 50%, and it increases to 67% when the $VOL$ factor is included. Hence, the $VOL$ premium is robust to size, value, momentum and liquidity effects. We consider the $VOL$ factor to be a new cross-sectional factor representing systematic volatility risk.

### The $VOL$ Factor and Option Returns

The $VOL$ factor reflects exposure to systematic volatility risk and is constructed using the cross-section of stock returns. An alternative way to construct a traded asset reflecting volatility risk is to consider option returns. To construct a mimicking factor for systematic volatility risk from cross-sectional options is infeasible because of low liquidity and large bid-ask spreads. However, it is possible to construct a zero-delta straddle position in options on the aggregate market (S&P 100 options) which has zero market exposure but provides exposure to systematic volatility. This is precisely what Coval and Shumway (2001) do. They approximate daily at-the-money straddle returns by taking a weighted average of the zero-beta straddle returns corresponding to strike prices immediately above and below each day's opening level of the S&P 100 and cumulate these daily returns each month. We denote this factor as $STR$ (for "straddle returns").

It is reassuring that over the 1986 to 1995 sample period used by Coval and Shumway, the

$STR$ and $VOL$ factors have a positive correlation of 19%. Since $STR$ and $VOL$ both measure systematic volatility exposure, a time-series regression of $STR$ on $VOL$, or vice versa, should yield significant loadings. Unfortunately, Table 7 shows that while $STR$ and $VOL$ load on each other positively, the coefficient loadings are insignificant. The adjusted $R^2$'s of the regressions are also only 3%. The reason for the poor correspondence is that the $STR$ returns are extremely volatile, compared with the low volatility of $VOL$. The volatility of $STR$ is 35.48% per month (122.9% per annum), whereas Table 4 shows that the volatility of $VOL$ is only 3.29% per month (11.40% per annum).

Table 7 shows that when $VOL$ is regressed onto $STR$, the constant (-0.28%) is insignificant. This is what we would expect if $VOL$ and $STR$ are able to price each other. However, a regression of $STR$ onto $VOL$ only reduces the magnitude of the raw $STR$ average return of -11.02% per month to -9.96% per month, which is still significant at the 5% level. Since the zero-beta straddle positions are only approximately delta-neutral, because the approximations rely on a Black-Scholes (1973) formula to compute the weights in the option positions, it is likely that the $STR$ returns still incorporate some residual $MKT$ exposure. When the $MKT$ factor is added, the alpha becomes insignificant and the adjusted $R^2$ increases to 15%.

While $STR$ has a very impressive negative return, its large volatility means a person selling straddles can easily go bankrupt, which would have happened during the 1987 crash where the monthly return over October 1987 was 285%. In contrast, the low volatility of $VOL$ makes it a less risky trading strategy. Another advantage of $VOL$ over $STR$ is that taking straddle positions requires daily or weekly rebalancing (done by Coval and Shumway, 2001), whereas $VOL$ is re-balanced at a monthly frequency. Finally, the $VOL$ factor is easy to construct as the $VIX$ index is publicly available. The main source of option data, the Berkeley Option Database has reliable data only from the late 1980's and stops in 1995, and is no longer made available for research purposes.

# 3 Pricing Idiosyncratic Volatility in the Cross-Section

So far, we have examined how systematic volatility risk affects cross-sectional average returns. In this section, we investigate if the idiosyncratic volatility of stocks generates cross-sectional patterns of average returns. Naturally, if the factors driving systematic risk are correctly specified, we should see no reward for bearing idiosyncratic risk. While we concentrate our analysis on using the Fama-French (1993) model for systematic risk, we also examine idiosyncratic volatility relative to the traditional CAPM and total volatility (without decomposing total volatility into systematic and idiosyncratic components).

## 3.1 Estimating Idiosyncratic Volatility

To measure idiosyncratic volatility for an individual stock, we run either a CAPM or Fama-French (1993) regression:

$$\begin{aligned} r_t^i &= \alpha^i + \beta_{MKT}^i MKT_t + \varepsilon_t^i \\ r_t^i &= \alpha^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \varepsilon_t^i. \end{aligned} \tag{12}$$

Hence, we measure idiosyncratic volatility ($\sqrt{\text{var}(\varepsilon_t^i)}$) relative to the CAPM or FF-3. Given the failure of the CAPM to explain cross-sectional returns and the ubiquity of FF-3 in empirical financial applications, we concentrate on idiosyncratic volatility measured relative to the Fama-French model.

To examine trading strategies based on idiosyncratic volatility, we describe trading strategies based on a formation period of $L$ months, a waiting period of $M$ months and then a holding period of $N$ months. We can describe an $L/M/N$ strategy as follows. At month $t$, we compute idiosyncratic volatilities from the regression (12) on daily data over an $L$ month period from month $t-L-M$ to month $t-M$. At time $t$, we construct value-weighted portfolios based on these idiosyncratic volatilities and hold these portfolios for $N$ months. We concentrate our analysis on the $1/0/1$ strategy, but examine robustness to various choices of $L$, $M$ and $N$. For the $1/0/1$ strategy, we simply sort stocks into quintile portfolios based on their level of idiosyncratic volatility computed using daily returns over the past month, and hold these value-weighted portfolios for 1 month. The portfolios are rebalanced each month.

The construction of the $L/M/N$ portfolios for $L > 1$ and $N > 1$ is similar to Jegadeesh and Titman (1993), except our portfolios are value-weighted. For example, to construct $12/1/6$ quintile portfolios, each month we construct a value-weighted portfolio based on idiosyncratic volatility computed on 12-months of returns ending one month prior. Similarly, we form a value-weighted portfolio based on 12-months of returns ending two months prior, three months prior, and so on up to six months prior. Each of these portfolios is value-weighted. We then take the simple average of these six portfolios. Hence, each quintile portfolio changes 1/6th of its composition each month, where each 1/6th part of the portfolio consists of a value-weighted portfolio. The first (fifth) quintile portfolio consists of 1/6th of the lowest value-weighted (highest) idiosyncratic stocks from one month ago, 1/6th of the value-weighted lowest (highest) idiosyncratic stocks two months ago, etc.

## 3.2 Patterns in Average Returns for Volatility Risk

Table 8 reports average returns of total and idiosyncratic volatility sorted portfolios, using a $1/0/1$ strategy. We turn first to the portfolios sorted by total volatility, without any control for systematic risk. Table 8 shows that average returns increase from 1.06% per month going from quintile 1 (low total volatility stocks) to 1.22% per month for quintile 3. Then, average returns drop. Quintile 5, which comprises stocks with the highest total volatility, experiences a dramatic decrease in average total returns (only 0.09% per month). A FF-3 alpha, reported in the last column, for quintile 5 is -1.16% per month, and while highly significant, it is the only portfolio that has a significant alpha. The large spread in average returns between quintiles 1 and 5 (-0.97% per month) may just be due to inappropriate controls for systematic risk.

The next two panels of Table 8 report average returns of stocks sorted by idiosyncratic volatility measured relative to the CAPM and FF-3 model, respectively.[10] An interesting pattern is that there is a reward in raw average returns for increasing idiosyncratic volatility, but this does not hold for stocks with the highest idiosyncratic volatilities in quintiles 4 and 5. In both the CAPM and FF-3 cases, the low average returns of quintiles 4 and 5 are exacerbated, compared to the sorts on total volatility, and their alpha's are highly statistically significant. In particular, the average returns of quintile 5 are -1 basis point (-2 basis points) for idiosyncratic volatility relative to the CAPM (FF-3). Stocks with high idiosyncratic risk have abysmally low average returns.

Let us focus attention on sorts by idiosyncratic volatility relative to FF-3, which is reported in the last panel of Table 8. The difference in raw average returns between quintile 1 and 5 is a very large -1.06% per month. Controlling for the CAPM (FF-3) model increases the difference in magnitude to -1.38% (-1.31%) per month. Clearly, the FF-3 model cannot account for all systematic risk.

Table 8 shows distinct patterns in the size and book-to-market ratios of the FF-3 idiosyncratic volatility portfolios. Stocks with low (high) idiosyncratic volatility are generally large (small) stocks and have low (high) book-to-market ratios. The very low returns of quintile 5

[10] If we compute idiosyncratic risk relative to a factor model with $MKT$ and $VOL$ factors, the patterns in average returns and alpha's in Table 8 are qualitatively preserved. The 5-1 alpha's are smaller in magnitude than for the case of idiosyncratic volatility relative to the CAPM or FF-3 models, but the alpha's are still significant.

are opposite to what the FF-3 model predicts. Hence, the stocks with the highest idiosyncratic volatilities, since they are small and are value stocks, should have the highest average returns. Nevertheless, there are three major concerns major concerns with the anomalously low average returns of quintile 5. First, although quintile 5 contains 20% of the stocks sorted by idiosyncratic volatility, quintile 5 represents only a small proportion of the market (only 1.9% on average). Hence, stocks with high idiosyncratic risk consist, on average, of a small fraction of the market. Are these patterns repeated if we only consider large stocks, or only stocks traded on the NYSE? Second, illiquidity distortions among small stocks are pervasive, so we should control for liquidity. Third, are these patterns robust to different formation and holding periods? We now check how robust our findings are.

## 3.3 Robustness

*Using Only NYSE Stocks*

Table 9 examines robustness of our $1/0/1$ portfolio formation strategy for FF-3 idiosyncratic volatility portfolio sorts, controlling for various effects. The table reports FF-3 alpha's, and the difference in FF-3 alpha's between the quintile portfolios with the lowest and highest idiosyncratic risks. First, we rank stocks based on idiosyncratic volatility using only NYSE stocks. Excluding NASDAQ and AMEX has no effect on our results. The highest quintile of idiosyncratic volatility stocks has a FF-3 alpha of -0.60% per month and the 5-1 difference is still high, at -0.66% per month, which is significant at the 1% level.

*Controlling for Size and Book-to-Market*

We control for size by first forming quintile portfolios ranked on size and then within each size quintile, we sort stocks based into quintile portfolios ranked on FF-3 idiosyncratic volatility. Within each size quintile, quintile 5 with the highest idiosyncratic volatility stocks, still has a dramatically lower alpha. The effect is not most pronounced among the smallest stocks. Rather, quintiles 2-4 have the largest 5-1 differences in FF-3 alpha's, at -1.91%, -1.61% and -0.86% per month, respectively. The average market capitalization of quintiles 2-4 is, on average, approximately 21% of the market. The t-statistics of these alpha's are all above 4.5 in absolute magnitude. The 5-1 alpha's for the smallest and largest quintiles are actually statistically insignificant at the 5% level. Hence, it is definitely not small stocks that are driving these results. We can control for size by averaging the returns of the quintile idiosyncratic volatility portfolios over the five size portfolios. Controlling for size, the 5-1 difference in FF-3 alpha's is still -1.04% per month.

The remainder of Table 9 repeats the explicit double-sort characteristic controls for book-to-market ratios, liquidity, volume, dispersion in analysts' forecasts, and momentum. In each case, we first sort stocks into quintiles based on the characteristic and then, within each quintile we sort stocks based on FF-3 idiosyncratic volatility. To control for the characteristic, we average the returns over each of the five characteristic portfolios.

We turn next to the book-to-market control. The value effect is concentrated among small stocks. Perhaps our idiosyncratic volatility portfolios are primarily composed of growth stocks, with lower average returns than value stocks. This is not the case. When we control for the book-to-market effect, stocks with the highest idiosyncratic volatility still have very low FF-3 alpha's, and the 5-1 difference in alpha's is -80% per month, and highly significant.

*Controlling for Liquidity and Volume*

We use the historical liquidity betas of Pástor and Stambaugh (2003) to proxy for liquidity. Controlling for liquidity does not remove the low average returns of high idiosyncratic volatility stocks. Quintile 5 still has very low average returns, with a FF-3 alpha of -1.01% per month. The 5-1 difference in alpha's is -1.08% per month, only slightly less in magnitude than the 5-1 difference in alpha's without the liquidity control in Table 8 (-1.31% per month). We control for volume because Lee and Swaminathan (2000) argue that high volume proxies for differences in opinion, which predicts lower returns. When we control for volume, the 5-1 difference in alpha's remains significant at the 1% level at -1.22% per month. Hence, the low returns on high idiosyncratic risk stocks are robust to controlling for liquidity and volume.

*Controlling for Dispersion in Analysts' Forecasts*

Diether, Malloy and Scherbina (2002) provide evidence that stocks with higher dispersion in analysts' earnings forecasts have lower average returns than stocks with low dispersion of analysts' forecasts. Stocks with high dispersion in analysts' forecasts tend to be more volatile stocks. If we use the sample period 1983-2000, similar to Diether, Malloy and Scherbina, we can test this hypothesis by performing a characteristic control for the dispersion of analysts' forecasts. We take the quintile portfolios of stocks sorted on increasing dispersion of analysts' forecasts (Table VI of Diether, Malloy and Scherbina, 2002, p2128) and within each quintile sort stocks on idiosyncratic volatility. Note that this universe of stocks are mostly large firms, where the idiosyncratic volatility effect is weak, because analysts usually do not make forecasts for small firms.

The line labelled 'Controlling for Dispersion of Analysts' Forecasts' in Table 8 presents the results for averaging the idiosyncratic volatility portfolios across the forecast dispersion quintiles. The 5-1 difference in alpha's is still -0.39% per month, with a robust t-statistic of -2.09. While the shorter sample period may reduce power, the dispersion of analysts' forecasts reduces the non-controlled 5-1 alpha considerably (from -1.31% per month). However, dispersion in analysts' forecasts cannot account for all of the low returns to stocks with high idiosyncratic risk.

We can also turn the question around and ask if the low average returns of stocks with high dispersion of analysts' forecasts is due to the low returns of stocks with high idiosyncratic risk. We first sort stocks into quintiles on the basis of idiosyncratic volatility, and then within each quintile sort stocks into portfolios ranked by forecast dispersion, using the set of firms used by Diether, Malloy and Scherbina. We compute the difference in FF-3 alpha's for stocks with high and low forecast dispersion, controlling for idiosyncratic volatility by averaging stocks over the idiosyncratic volatility quintiles. The 5-1 FF-3 alpha for forecast dispersion, controlling for idiosyncratic volatility, is -0.36% per month, which is insignificant at the 5% level (the robust t-statistic is -1.47).

*Controlling for Momentum*

One possibility of the low returns of high idiosyncratic risk stocks could be due to momentum. In particular, stocks with very low returns have very high volatility, by definition, and these stocks continue to have low returns (see Jegadeesh and Titman, 1993). Of course, stocks that are past winners also have very high volatility, but loser stocks could be over-represented in the high idiosyncratic risk quintile. The last row of Table 8 shows that this is not the case. Controlling for returns over the past month does not remove the very low FF-3 alpha of quintile 5 (-0.59% per month), and the 5-1 difference in alpha's is still -0.66% per month, which is statistically significant at the 1% level. What is surprising is that even the 5-1 difference in the raw average returns is very large in magnitude, at -0.84% per month, with a t-statistic of -3.76. Clearly, momentum cannot account for these patterns.

## 3.4 Can we Explain the Negative Premium for Idiosyncratic Risk?

A possible explanation for the large negative returns of high idiosyncratic volatility stocks is that stocks with large idiosyncratic risk relative to FF-3 have larger exposure to movements in systematic volatility. This is not unreasonable, since if market volatility increases, individual stock volatility might also increase. Table 10 tries to price the FF-3 idiosyncratic volatility quintiles with the $VOL$ factor. Panel A shows that including the $VOL$ factor into the standard linear factor specifications reduces the difference in alpha's between portfolios with the highest and lowest idiosyncratic risk. For example, for the FF-3 model (FF-3 model augmented with $UMD$), the 5-1 alpha is -1.43% (-1.36%) per month. This is reduced in magnitude to -1.34% (-1.26%) per month including $VOL$. However, the reductions are small and amount to approximately 10 basis points per month. All of the factor specifications fail to pass a GRS test, with p-values of less than 1%.

We report factor loadings of the full $MKT$, $SMB$, $UMD$ and $VOL$ specification in Panel B. Only $HML$ and $VOL$ have factor loadings that go in the correct direction from quintile 1 (high returns) to 5 (low returns). The spread in $HML$ factor loadings is $0.17 - (-0.42) = 0.59$. However, the average return of $HML$ over the 1986-2000 sample period is -4 basis points per month, so $HML$'s contribution to explaining the large negative 5-1 spread is negligible. While the mean of $VOL$ is -0.58% per month (see Table 4), the spread in the factor loadings of $VOL$ is only $-0.13 - .07 = -0.20$, so $VOL$ only reduces the large negative alpha by $-0.20 \times -0.58\% = 0.12\%$ per month. Hence, idiosyncratic volatility risk accounts for some, but cannot remove, the anomalous low returns of stocks with high idiosyncratic risk.

If standard factor models cannot price idiosyncratic volatility risk and exposure to systematic volatility also cannot explain the low returns to high idiosyncratic risk stocks, are there other explanations? To help disentangle various stories, Table 11 reports FF-3 alpha's of other $L/M/N$ strategies, with an $L$-month formation period that ended $M$ months ago prior to time $t$ and is held for $N$ months. First, we can rule out possible contemporaneous measurement errors through forming the portfolios using data ending one-month prior ($M = 1$). In the $1/1/1$ strategy, the 5-1 difference in FF-3 alpha's is still -0.82% per month, which is significant at the 1% level.

One possible behavioral explanation for our results is that higher idiosyncratic volatility does earn higher returns, but short-term over-reaction forces returns to be low in the next month. If we hold high idiosyncratic risk stocks for a long horizon ($N = 12$ months), we might see a positive relation between idiosyncratic risk and average returns. The second row of Table 11 shows that this is not the case. For the $1/1/12$ strategy, we still see very low FF-3 alpha's for quintile 5, and the 5-1 difference in alpha's is still -0.67% per month, which is significant at the 1% level.

By restricting the formation period to $L = 1$ month, our previous results may just be capturing various short-term events that impact idiosyncratic volatility. For example, the portfolio of stocks with high idiosyncratic volatility may be largely composed of stocks that have just made, or are just about to make, earnings announcements. To ensure that we are not capturing specific short-term corporate events, we extend our formation period to $L = 12$ months. The third row of Table 11 reports FF-3 alpha's for a $12/1/1$ strategy. Using one entire year of data to compute idiosyncratic volatility does not remove the anomalous low return-high idiosyncratic risk pattern: the 5-1 difference in alpha's is -1.12% per month. Similarly, the patterns are robust for the $12/1/12$ strategy, which has a 5-1 alpha of -0.77% per month.

While the low returns to high idiosyncratic risk stocks are amazingly robust to different formation and holding periods, a further possibility is that volatility, by definition, symmetrically treats gains and losses. Since volatility is asymmetric (and larger with downward moves), high idiosyncratic risk stocks may have average returns during normal or bull markets but their low returns may be driven largely by bear market periods. We can check this hypothesis by examining the returns of high idiosyncratic volatility stocks conditioning on observations which have the lowest 20% of market returns and comparing them to bull markets that have the highest 20% of market returns. For the periods of lowest (highest) 20% of market returns, the FF-3 alpha of quintile 5 is -2.83% (-2.98%) per month, both highly significant at the 1% level. Hence, stocks with high idiosyncratic risk earn low returns in both bull and bear markets.

We also find that the low returns of quintile 5 are robust over NBER recessions and expansions. During NBER expansions (recessions), the FF-3 alpha of quintile 5 is -1.19% (-1.88%). Both the expansion and recession FF-3 alpha's are significant at the 1% level. There are more negative returns to high idiosyncratic volatility stocks during recessions, but the fact that the t-statistic in NBER expansions is -7.07 shows that the low returns to high idiosyncratic risk is also a phenomenon that thrives during expansions. A final possibility is that this effect is concentrated during the most volatile periods in the market. To test for this possibility, we compute FF-3 alpha's of quintile 5 conditioning on periods with the lowest or highest 20% of absolute moves of the market return. These are ex-post periods of low or high market volatility. During stable (volatile) periods, the FF-3 alpha of quintile 5 is -1.70% (-0.89%) per month, both significant at the 5% level. Hence, the most negative returns of the high idiosyncratic risk strategy are earned during periods when the market is stable. Hence, these results indicate that it is not bull or bear market periods, asymmetries across the business cycle, or the clustering of periods of volatility that is driving the low returns to high idiosyncratic risk exposure.

# 4 Conclusion

Stocks with high exposure to innovations in systematic volatility earn low returns. We proxy for innovations in aggregate volatility by using changes in the $VIX$ index, an index of Black-Scholes (1973) implied volatilities constructed by the Chicago Board of Exchange. The low returns earned by stocks with high coefficient loadings to changes in the $VIX$ index is consistent with systematic volatility carrying a negative price of risk. We estimate that a mimicking factor for market volatility risk has a significant negative mean of -0.87% per month in the cross-section and is robust to controlling for size, value, momentum and liquidity factors.

According to a standard asset pricing framework, idiosyncratic volatility should not be priced. Recent theories predict that stocks with high idiosyncratic volatility may earn high expected returns to compensate for imperfect diversification. A puzzling result that we uncover is that stocks with high idiosyncratic volatility have abysmally low returns. In particular, using the Fama-French (1993) model to adjust for systematic risk, a quintile portfolio with stocks with the highest idiosyncratic risk earns total returns of just -0.02% per month. The results are surprisingly robust to controlling for size, value, size, liquidity, volume and momentum effects and the effect persists in bull and bear markets, NBER recessions and expansions, and volatile and stable periods. Accounting for exposure to aggregate volatility helps to price, but cannot remove, the anomalous low returns of stocks with high idiosyncratic risk.

We also find that the low returns of high idiosyncratic volatility stocks persist for different formation and holding periods as long as one year. This rules out stories of short-term overreaction or reaction to short-term corporate events. It is also unlikely that economic agents actually prefer stocks with high idiosyncratic risk, leading to their low returns. If this were the case, agents would exhibit no home bias. Hence, our results on the cross-sectional expected return patterns to idiosyncratic volatility present something of a puzzle.

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Table 1: Daily Sample Moments

| | Mean | Stdev | Auto | Correlations $MKT$ | $SVOL$ | $\Delta SVOL$ | $RVOL$ | $\Delta RVOL$ | $VIX$ |
|---|---|---|---|---|---|---|---|---|---|
| $MKT$ | 0.0004 | 0.0097 | 0.08 | | | | | | |
| $SVOL$ | 0.0099 | 0.0056 | 0.98 | -0.04 | | | | | |
| $\Delta SVOL$ | 0.0001 | 0.0148 | 0.07 | -0.25 | 0.08 | | | | |
| $RVOL$ | 0.0120 | 0.0087 | 0.49 | -0.23 | 0.54 | 0.43 | | | |
| $\Delta RVOL$ | -0.0000 | 0.0088 | -0.43 | -0.26 | -0.00 | 0.40 | 0.50 | | |
| $VIX$ | 0.2052 | 0.0785 | 0.94 | -0.18 | 0.79 | 0.15 | 0.68 | 0.03 | |
| $\Delta VIX$ | 0.0000 | 0.0265 | -0.07 | -0.64 | 0.00 | 0.45 | 0.29 | 0.39 | 0.16 |

We report daily sample moments of the excess market return $MKT$, sample volatility $SVOL$, range-based volatility measure $RVOL$, and the daily volatility $VIX$ index from the CBOE. $\Delta SVOL$, $\Delta RVOL$ and $\Delta VIX$ refer to daily changes in $SVOL$, $RVOL$ and $VIX$, respectively. 'Auto' denotes daily autocorrelation. There are 3784 daily observations from January 1986 to December 2000.

Table 2: Portfolios Sorted by Exposure to Aggregate Volatility Shocks

| Rank | Mean | Std Dev | Turn-over | % Mkt Share | Size | B/M | CAPM Alpha | FF-3 Alpha |
|---|---|---|---|---|---|---|---|---|
| Portfolios Sorted by $\beta_{\Delta SVOL}$ | | | | | | | | |
| 1 | 1.11 | 5.65 | 0.74 | 7.9% | 3.42 | 0.92 | -0.27 | -0.16 |
| | | | | | | | [-1.44 ] | [-0.82] |
| 2 | 1.16 | 4.58 | 0.78 | 25.5% | 4.24 | 0.84 | -0.10 | -0.17 |
| | | | | | | | [-0.89 ] | [-1.48] |
| 3 | 1.41 | 4.31 | 0.72 | 30.5% | 4.20 | 0.88 | 0.20 | 0.15 |
| | | | | | | | [2.27 ] | [2.06 ] |
| 4 | 1.34 | 4.57 | 0.77 | 27.6% | 4.28 | 0.84 | 0.09 | 0.06 |
| | | | | | | | [1.37 ] | [0.90 ] |
| 5 | 1.12 | 6.13 | 0.75 | 8.4% | 3.44 | 0.94 | -0.32 | -0.07 |
| | | | | | | | [-1.48 ] | [-0.43] |
| 5-1 | 0.01 | | | | | | -0.04 | 0.09 |
| | [0.02] | | | | | | [-0.15 ] | [0.30 ] |
| Portfolios Sorted by $\beta_{\Delta RVOL}$ | | | | | | | | |
| 1 | 1.31 | 6.62 | 0.74 | 7.0% | 3.75 | 0.96 | -0.16 | 0.05 |
| | | | | | | | [-0.68] | [0.27] |
| 2 | 1.18 | 4.61 | 0.78 | 24.2% | 4.81 | 0.75 | -0.08 | -0.11 |
| | | | | | | | [-0.83] | [-1.06] |
| 3 | 1.28 | 4.37 | 0.73 | 31.4% | 4.90 | 0.77 | 0.06 | -0.03 |
| | | | | | | | [0.64] | [-0.46] |
| 4 | 1.54 | 4.50 | 0.78 | 28.9% | 4.90 | 0.75 | 0.29 | 0.28 |
| | | | | | | | [4.04] | [3.69] |
| 5 | 0.89 | 5.60 | 0.75 | 8.5% | 3.77 | 0.98 | -0.48 | -0.34 |
| | | | | | | | [-2.64] | [-2.04] |
| 5-1 | -0.42 | | | | | | -0.32 | -0.39 |
| | [-1.45] | | | | | | [-1.07] | [-1.31] |
| Portfolios Sorted by $\beta_{\Delta VIX}$ | | | | | | | | |
| 1 | 1.64 | 5.53 | 0.74 | 9.4% | 3.70 | 0.89 | 0.27 | 0.30 |
| | | | | | | | [1.66] | [1.77] |
| 2 | 1.39 | 4.43 | 0.78 | 28.7% | 4.77 | 0.73 | 0.18 | 0.09 |
| | | | | | | | [1.82] | [1.18] |
| 3 | 1.36 | 4.40 | 0.72 | 30.4% | 4.77 | 0.76 | 0.13 | 0.08 |
| | | | | | | | [1.32] | [1.00] |
| 4 | 1.21 | 4.79 | 0.78 | 24.0% | 4.76 | 0.73 | -0.08 | -0.06 |
| | | | | | | | [-0.87] | [-0.65] |
| 5 | 0.60 | 6.55 | 0.73 | 7.4% | 3.73 | 0.89 | -0.88 | -0.53 |
| | | | | | | | [-3.42] | [-2.88] |
| 5-1 | -1.04 | | | | | | -1.15 | -0.83 |
| | [-3.90] | | | | | | [-3.54] | [-2.93] |

We form value-weighted quintile portfolios every month from regressing excess stock returns of individual stocks on $\Delta SVOL$, $\Delta RVOL$ or $\Delta VIX$, controlling for the $MKT$ factor, as in equation (10). The regression is run on daily excess returns using data over the previous month. Stocks are sorted into quintiles based on the regression coefficients $\beta_{\Delta SVOL}$, $\beta_{\Delta RVOL}$ or $\beta_{\Delta SVOL}$ from lowest (quintile 1) to highest (quintile 5). The statistics in the columns labelled Mean and Std Dev are measured in monthly percentage terms. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The numbers in the Turnover column list the average proportion of firms that leave the quintile portfolio each month. The row 5-1 refers to the difference in monthly returns between portfolio 5 and portfolio 1. The Alpha columns report Jensen's alpha with respect to the CAPM or Fama-French (1993) three-factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

Table 3: Portfolios Sorted on $\beta_{\Delta VIX}$ Controlling for Liquidity or Volume

| Rank | Mean | Std Dev | CAPM Alpha | FF-3 Alpha |
|---|---|---|---|---|
| **Panel A: Controlling for Liquidity** | | | | |
| 1 | 1.57 | 5.47 | 0.21 | 0.19 |
| | | | [1.31] | [1.34] |
| 2 | 1.48 | 4.48 | 0.27 | 0.15 |
| | | | [2.25] | [1.68] |
| 3 | 1.40 | 4.54 | 0.15 | 0.09 |
| | | | [1.59] | [0.97] |
| 4 | 1.30 | 4.74 | 0.02 | -0.02 |
| | | | [0.21] | [-0.17] |
| 5 | 0.89 | 5.84 | -0.52 | -0.36 |
| | | | [-2.87] | [-2.09] |
| 5-1 | -0.68 | | -0.73 | -0.55 |
| | [-3.04] | | [-2.99] | [-2.15] |
| **Panel B: Controlling for Volume** | | | | |
| 1 | 1.10 | 4.73 | -0.11 | -0.13 |
| | | | [-0.58] | [-1.34] |
| 2 | 1.18 | 4.01 | 0.08 | -0.08 |
| | | | [0.46] | [-0.92] |
| 3 | 1.18 | 3.78 | 0.10 | -0.04 |
| | | | [0.66] | [-0.50] |
| 4 | 0.98 | 4.18 | -0.17 | -0.23 |
| | | | [-1.06] | [2.16] |
| 5 | 0.38 | 5.31 | -0.90 | -0.71 |
| | | | [-3.86] | [-4.84] |
| 5-1 | -0.72 | | -0.79 | -0.58 |
| | [-3.49] | | [-3.22] | [-3.03] |

In Panel A, we first sort stocks into five quintiles based on their historical liquidity beta, following Pástor and Stambaugh (2003). Then, within each quintile, we sort stocks based on their $\beta_{\Delta VIX}$ coefficient loadings into five portfolios. All portfolios are rebalanced monthly and value-weighted. The five portfolios sorted on $\beta_{\Delta VIX}$ are then averaged over each of the five liquidity beta portfolios. Hence, they are $\beta_{\Delta VIX}$ quintile portfolios controlling for liquidity. In Panel B, the same approach is used except we first sort stocks into five portfolios based on their past trading volume, and then within each quintile, stocks are sorted on $\beta_{\Delta VIX}$. The portfolios in Panel B are averaged over the five volume portfolios. Hence, they are $\beta_{\Delta VIX}$ quintile portfolios controlling for liquidity. The table reports alphas from a CAPM and Fama-French (1993) regression. The row 5-1 refers to the difference in monthly returns between portfolio 5 and portfolio 1. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

Table 4: A Cross-Sectional Volatility Factor

| | Mean | Std Dev | Auto |
|---|---|---|---|
| $VOL$ | -0.58 | 3.29 | -0.15 |

Correlation of Factors

| | $MKT$ | $SMB$ | $HML$ | $UMD$ |
|---|---|---|---|---|
| $SMB$ | 0.17 | | | |
| $HML$ | -0.50 | -0.50 | | |
| $UMD$ | 0.22 | 0.32 | -0.46 | |
| $VOL$ | 0.16 | 0.48 | -0.40 | 0.22 |

Regressing $VOL$ onto Various Factors

| const | $MKT$ | $SMB$ | $HML$ | $UMD$ | Adj $R^2$ |
|---|---|---|---|---|---|
| -0.68 | 0.11 | | | | 0.02 |
| [-2.74] | [1.03] | | | | |
| -0.46 | -0.01 | 0.34 | -0.21 | | 0.25 |
| [-2.12] | [-0.09] | [3.07] | [-0.21] | | |
| -0.47 | -0.01 | 0.34 | -0.21 | 0.01 | 0.25 |
| [-1.94] | [-0.09] | [3.24] | [-1.36] | [0.08] | |

The factor $VOL$ is formed by sorting all stocks into three portfolios based on $\beta_{\Delta VIX}$ from the regression (10) run at a daily frequency using data over the previous month and the portfolios are rebalanced every month. The three value-weighted portfolios have breakpoints set at one-third and two-thirds of the $\beta_{\Delta VIX}$ coefficients for all stocks. We take the return difference between the top third and bottom third portfolios to form $VOL$. We report monthly summary statistics of $VOL$ in percentage terms, and correlations of $VOL$ with respect to other factors $SMB$, $HML$, the size and value factors of Fama and French (1993), and the momentum factor $UMD$ from Kenneth French's web site. We also report regressions of $VOL$ onto various combinations of $MKT$, $SMB$, $HML$ and $UMD$. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

Table 5: Pricing Portfolios Sorted by $\beta_{\Delta VIX}$

**Panel A: Pricing $\beta_{\Delta VIX}$ Quintile Portfolios Using Various Factor Models**

| Model | | | | | GRS Test p-value | Portfolio Alpha's 1 | 2 | 3 | 4 | 5 | 5-1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $MKT$ | | | | | 0.007 | 0.27 | 0.18 | 0.13 | -0.08 | -0.88 | -1.15 |
| | | | | | | [1.66] | [1.82] | [1.32] | [-0.87] | [-3.42] | [-3.54] |
| $MKT$ | $SMB$ | $HML$ | | | 0.030 | 0.30 | 0.09 | 0.08 | -0.06 | -0.53 | -0.83 |
| | | | | | | [1.77] | [1.18] | [1.00] | [-0.65] | [-2.88] | [-2.93] |
| $MKT$ | $SMB$ | $HML$ | $UMD$ | | 0.023 | 0.44 | 0.11 | -0.00 | -0.01 | -0.48 | -0.92 |
| | | | | | | [2.50] | [1.20] | [-0.03] | [-0.12] | [-2.36] | [-2.86] |
| $MKT$ | | | | $VOL$ | 0.116 | 0.03 | -0.07 | 0.12 | 0.11 | -0.34 | -0.36 |
| | | | | | | [0.17] | [-1.47] | [-1.44] | [1.77] | [-2.19] | [-2.68] |
| $MKT$ | $SMB$ | $HML$ | | $VOL$ | 0.150 | 0.05 | -0.07 | 0.09 | 0.08 | -0.27 | -0.33 |
| | | | | | | [0.43] | [-1.36] | [1.25] | [1.45] | [-2.11] | [-2.43] |
| $MKT$ | $SMB$ | $HML$ | $UMD$ | $VOL$ | 0.106 | 0.18 | -0.05 | 0.01 | 0.14 | -0.22 | -0.40 |
| | | | | | | [1.55] | [-0.86] | [0.11] | [2.34] | [-1.67] | [-2.78] |

**Panel B: Factor Loadings**

| | $MKT$ | $SMB$ | $HML$ | $UMD$ | $VOL$ |
|---|---|---|---|---|---|
| 1 | 1.12 | 0.27 | -0.18 | -0.12 | -0.54 |
| | [43.8] | [5.06] | [-2.65] | [-3.38] | [-13.5] |
| 2 | 0.98 | -0.05 | -0.02 | -0.02 | -0.34 |
| | [69.9] | [-2.38] | [-0.69] | [-0.87] | [-16.3] |
| 3 | 0.99 | -0.06 | 0.12 | 0.07 | 0.02 |
| | [51.2] | [-1.83] | [2.11] | [2.64] | [0.44] |
| 4 | 1.04 | -0.04 | 0.05 | -0.05 | 0.32 |
| | [75.7] | [-1.93] | [2.60] | [-2.45] | [11.3] |
| 5 | 1.08 | 0.30 | -0.26 | -0.05 | 0.57 |
| | [30.2] | [6.62] | [-4.61] | [-1.60] | [12.1] |

In Panel A, we report a Gibbons-Ross-Shanken (1989) (GRS) test for pricing the quintile portfolios sorted by $\beta_{\Delta VIX}$, reported in Table 2, for various factor models using combinations of the factors $MKT$, $SMB$, $HML$, $UMD$ and $VOL$. $UMD$ is the momentum factor from Kenneth French's website and $VOL$ is the volatility factor. The columns labelled '1' through '5' report portfolio alpha's from each linear factor model. The column labelled '5-1' refers to the difference in monthly returns between portfolio 5 and portfolio 1. Panel B reports factor loadings from the most comprehensive $MKT$, $SMB$, $HML$, $UMD$ and $VOL$ factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

Table 6: Estimating the Price of Volatility by Fama-MacBeth (1973)

**Panel A: Test Portfolio Fama-French (1993) Alpha's**

| | | Ranking on $\beta_{\Delta VIX}$ | | | | |
|---|---|---|---|---|---|---|
| | | 1 low | 2 | 3 | 4 | 5 high |
| Ranking | 1 low | -0.54 | 0.13 | -0.25 | 0.10 | -1.04 |
| on $\beta_{MKT}$ | | [-1.06] | [0.63] | [-1.13] | [0.51] | [-3.76] |
| | 2 | -0.22 | 0.02 | -0.39 | -0.17 | -0.57 |
| | | [-1.24] | [0.14] | [-2.20] | [-1.30] | [-2.41] |
| | 3 | -0.20 | -0.06 | -0.00 | -0.39 | -0.28 |
| | | [-1.08] | [-0.31] | [-0.01] | [-2.38] | [-1.56] |
| | 4 | 0.26 | -0.04 | -0.03 | -0.11 | -0.61 |
| | | [1.04] | [-0.33] | [0.19] | [-0.99] | [-2.61] |
| | 5 high | 0.31 | 0.30 | 0.10 | -0.10 | -0.66 |
| | | [0.95] | [1.42] | [0.48] | [-0.41] | [-1.92] |

**Panel B: Fama-MacBeth (1973) Factor Premiums**

| $MKT$ | $SMB$ | $HML$ | $UMD$ | $LIQ$ | $VOL$ | Adj $R^2$ |
|---|---|---|---|---|---|---|
| 0.22 | | | | | | -0.02 |
| [0.34] | | | | | | |
| 1.06 | -0.80 | 0.13 | | | | 0.50 |
| [1.59] | [-1.82] | [0.26] | | | | |
| 1.25 | -0.69 | 0.31 | 0.32 | | | 0.48 |
| [1.76] | [ -1.47] | [ 0.56] | [ 0.41] | | | |
| 1.27 | -0.77 | 0.60 | 0.11 | -0.03 | | 0.51 |
| [1.67] | [-1.48] | [1.00] | [0.13] | [-1.41] | | |
| 1.18 | | | | | -0.87 | 0.56 |
| [1.78] | | | | | [-2.46] | |
| 0.52 | -0.82 | -0.33 | | | -0.83 | 0.67 |
| [0.77] | [-1.85] | [-0.65] | | | [-2.37] | |
| 0.59 | -0.78 | -0.26 | 0.22 | | -0.83 | 0.65 |
| [0.86] | [-1.70] | [-0.49] | [0.29] | | [-2.37] | |
| 0.59 | -0.87 | 0.04 | -0.01 | -0.03 | -0.87 | 0.70 |
| [0.79] | [-1.71] | [0.07] | [-0.02] | [-1.86] | [-2.26] | |

In Panel A, we report Fama-French (1993) alpha's for the 25 portfolios sorted first on $\beta_{MKT}$ and then on $\beta_{\Delta VIX}$. These 25 portfolios are used as test assets in estimating the factor premiums using Fama-MacBeth (1973) in Panel B. $MKT$ is the excess return on the market portfolio, $SMB$ and $HML$ are the Fama-French (1993) size and value factors, $UMD$ is the momentum factor from Kenneth French's website and $LIQ$ is the aggregate liquidity measure from Pástor and Stambaugh (2003). $VOL$ is the volatility factor. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

Table 7: Zero-Beta Straddle $STR$ Returns and $VOL$ Factor Regressions

| | const | $MKT$ | $VOL$ | $STR$ | Adj $R^2$ |
|---|---|---|---|---|---|
| $VOL$ Regressions | -0.28 | | | 0.01 | 0.03 |
| | [-1.46] | | | [1.19] | |
| | -0.27 | -0.04 | | 0.01 | 0.03 |
| | [-0.22] | [-0.74] | | [1.15] | |
| $STR$ Regressions | -9.96 | | 3.73 | | 0.03 |
| | [-2.21] | | [0.78] | | |
| | -8.23 | -2.95 | 2.64 | | 0.15 |
| | [-1.77] | [-1.42] | [0.89] | | |

We regress $STR$, the monthly returns of zero-beta straddle positions constructed by Coval and Shumway (2001), onto $MKT$ and $VOL$ and regress $VOL$ onto $MKT$ and $STR$. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is from January 1986 to December 1995.

Table 8: Portfolios Sorted by Volatility

| Rank | Mean | Std Dev | Turn-over | % Mkt Share | Size | B/M | CAPM Alpha | FF-3 Alpha |
|---|---|---|---|---|---|---|---|---|
| Portfolios Sorted by Total Volatility | | | | | | | | |
| 1 | 1.06 | 3.71 | 0.40 | 41.7% | 4.66 | 0.88 | 0.14 | 0.03 |
| | | | | | | | [1.84] | [0.53] |
| 2 | 1.15 | 4.48 | 0.63 | 33.7% | 4.70 | 0.81 | 0.13 | 0.08 |
| | | | | | | | [2.14] | [1.41] |
| 3 | 1.22 | 5.63 | 0.67 | 15.5% | 4.10 | 0.82 | 0.07 | 0.12 |
| | | | | | | | [0.72] | [1.55] |
| 4 | 0.99 | 7.15 | 0.64 | 6.7% | 3.47 | 0.86 | -0.28 | -0.17 |
| | | | | | | | [-1.73] | [-1.42] |
| 5 | 0.09 | 8.30 | 0.41 | 2.4% | 2.57 | 1.08 | -1.21 | -1.16 |
| | | | | | | | [-5.07] | [-6.85] |
| 5-1 | -0.97 | | | | | | -1.35 | -1.19 |
| | [-2.86] | | | | | | [-4.62] | [-5.92] |
| Portfolios Sorted by Idiosyncratic Volatility Relative to the CAPM | | | | | | | | |
| 1 | 1.10 | 3.87 | 0.41 | 51.6% | 4.82 | 0.86 | 0.15 | 0.09 |
| | | | | | | | [2.17] | [1.88] |
| 2 | 1.13 | 4.70 | 0.63 | 28.5% | 4.71 | 0.80 | 0.07 | 0.05 |
| | | | | | | | [1.38] | [0.78] |
| 3 | 1.23 | 5.86 | 0.67 | 12.4% | 4.06 | 0.82 | 0.05 | 0.09 |
| | | | | | | | [0.54] | [1.11] |
| 4 | 0.94 | 7.07 | 0.64 | 5.5% | 3.41 | 0.87 | -0.32 | -0.25 |
| | | | | | | | [-2.03] | [-2.45] |
| 5 | -0.01 | 8.19 | 0.41 | 2.1% | 2.51 | 1.10 | -1.29 | -1.28 |
| | | | | | | | [-5.32] | [-7.84] |
| 5-1 | -1.11 | | | | | | -1.44 | -1.37 |
| | [-3.38] | | | | | | [-4.88] | [-7.25] |
| Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3 | | | | | | | | |
| 1 | 1.04 | 3.83 | 0.41 | 53.5% | 4.86 | 0.85 | 0.11 | 0.04 |
| | | | | | | | [1.57] | [0.99] |
| 2 | 1.16 | 4.74 | 0.64 | 27.4% | 4.72 | 0.80 | 0.11 | 0.09 |
| | | | | | | | [1.98] | [1.51] |
| 3 | 1.20 | 5.85 | 0.68 | 11.9% | 4.07 | 0.82 | 0.04 | 0.08 |
| | | | | | | | [0.37] | [1.04] |
| 4 | 0.87 | 7.13 | 0.65 | 5.2% | 3.42 | 0.87 | -0.38 | -0.32 |
| | | | | | | | [-2.32] | [-3.15] |
| 5 | -0.02 | 8.16 | 0.42 | 1.9% | 2.52 | 1.10 | -1.27 | -1.27 |
| | | | | | | | [-5.09] | [-7.68] |
| 5-1 | -1.06 | | | | | | -1.38 | -1.31 |
| | [-3.10] | | | | | | [-4.56] | [-7.00] |

We form value-weighted quintile portfolios every month by sorting stocks based on total volatility, idiosyncratic volatility relative to the CAPM and idiosyncratic volatility relative to the Fama-French (1993) model. Portfolios are formed every month, based on volatility computed using daily data over the previous month. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labelled Mean and Std Dev are measured in monthly percentage terms. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The numbers in the Turnover column list the average proportion of firms that leave the quintile portfolio each month. The row '5-1' refers to the difference in monthly returns between portfolio 5 and portfolio 1. The Alpha columns report Jensen's alpha with respect to the CAPM or Fama-French (1993) three-factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is July 1963 to December 2000.

Table 9: Portfolios Sorted on Idiosyncratic Volatility (FF-3) Controlling for Various Effects

| | | Ranking on Idiosyncratic Volatility | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 low | 2 | 3 | 4 | 5 high | 5-1 |
| NYSE Stocks Only | | 0.06 | 0.04 | 0.02 | -0.04 | -0.60 | -0.66 |
| | | [1.20] | [0.75] | [0.30] | [-0.40] | [-5.14] | [-4.85] |
| Size Quintiles | 1 small | 0.11 | 0.26 | 0.31 | 0.06 | -0.43 | -0.55 |
| | | [0.72] | [1.56] | [1.76] | [0.29] | [-1.54] | [-1.84] |
| | 2 | 0.19 | 0.20 | -0.07 | -0.65 | -1.73 | -1.91 |
| | | [1.49] | [1.74] | [-0.67] | [-5.19] | [-8.14] | [-7.69] |
| | 3 | 0.12 | 0.21 | 0.03 | -0.27 | -1.49 | -1.61 |
| | | [1.23] | [2.40] | [0.38] | [-3.36] | [-10.1] | [-7.65] |
| | 4 | 0.03 | 0.22 | 0.17 | -0.03 | -0.82 | -0.86 |
| | | [0.37] | [2.57] | [2.47] | [-0.45] | [-6.61] | [-4.63] |
| | 5 large | 0.09 | 0.04 | 0.03 | 0.14 | -0.17 | -0.26 |
| | | [1.62] | [0.72] | [0.51] | [1.84] | [-1.40] | [-1.74] |
| Controlling for Size | | 0.11 | 0.18 | 0.09 | -0.15 | -0.93 | -1.04 |
| | | [1.30] | [2.49] | [1.35] | [-1.99] | [-6.81] | [-5.69] |
| Controlling for Book-to-Market | | 0.61 | 0.69 | 0.71 | 0.50 | -0.19 | -0.80 |
| | | [3.02] | [2.80] | [2.49] | [1.47] | [-0.48] | [-2.90] |
| Controlling for Liquidity | | 0.08 | 0.09 | -0.01 | -0.16 | -1.01 | -1.08 |
| | | [1.71] | [1.53] | [-0.09] | [-1.62] | [-8.61] | [-7.98] |
| Controlling for Volume | | -0.03 | 0.02 | -0.01 | -0.39 | -1.25 | -1.22 |
| | | [-0.49] | [0.39] | [-0.32] | [-7.11] | [-10.9] | [-8.04] |
| Controlling for Dispersion | | 0.12 | -0.07 | 0.11 | 0.01 | -0.27 | -0.39 |
| in Analysts' Forecasts | | [1.57] | [-0.76] | [1.12] | [0.09] | [-1.76] | [-2.09] |
| Controlling for Momentum | | 0.07 | 0.08 | 0.09 | -0.05 | -0.59 | -0.66 |
| | | [0.43] | [0.94] | [1.26] | [-0.47] | [-3.60] | [-2.71] |

The table reports Fama and French (1993) alpha's, with robust Newey-West (1987) t-statistics in square brackets. All the strategies are $1/0/1$ strategies, but control for various effects. The column '5-1' refers to the difference in FF-3 alpha's between portfolio 5 and portfolio 1. In the panel labelled 'NYSE Stocks Only', we sort stocks into quintile portfolios based on their idiosyncratic volatility, relative to the FF-3 model, using only NYSE stocks. We use daily data over the previous month and rebalance monthly. In the panel labelled 'Size Quintiles', each month we first sort stocks into five quintiles on the basis of size. Then, within each size quintile, we sort stocks into five portfolios sorted by idiosyncratic volatility. In the panels controlling for size, liquidity volume and momentum, we perform a double sort. Each month, we first sort stocks based on the first characteristic (size, book-to-market, liquidity, volume, dispersion of analysts' forecasts, or momentum) and then, within each quintile we sort stocks based on idiosyncratic volatility, relative to the FF-3 model. The five idiosyncratic volatility portfolios are then averaged over each of the five characteristic portfolios. Hence, they represent idiosyncratic volatility quintile portfolios controlling for the characteristic. Liquidity represents the Pástor and Stambaugh (2003) historical liquidity beta, and momentum represents past 1-month returns. The sample period is July 1963 to December 2000 for all controls with the exceptions of liquidity (February 1968 to December 2000) and the dispersion of analysts' forecasts (February 1983 to December 2000). All portfolios are value-weighted.

Table 10: Pricing Portfolios Sorted on Volatility

**Panel A: Pricing Idiosyncratic Volatility (Relative to FF-3) Quintile Portfolios Using Various Factor Models**

| Model | | | | | GRS Test p-value | Portfolio Alpha's 1 | 2 | 3 | 4 | 5 | 5-1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $MKT$ | | | | | 0.000 | 0.30 | -0.01 | -0.17 | -0.93 | -1.92 | -2.22 |
| | | | | | | [2.16] | [-0.13] | [-0.83] | [-3.02] | [-3.98] | [-3.73] |
| $MKT$ | $SMB$ | $HML$ | | | 0.000 | 0.13 | -0.83 | 0.05 | -0.48 | -1.31 | -1.43 |
| | | | | | | [1.64] | [-0.06] | [0.38] | [-2.82] | [-4.04] | [-3.91] |
| $MKT$ | $SMB$ | $HML$ | $UMD$ | | 0.000 | 0.12 | -0.11 | 0.03 | -0.48 | -1.28 | -1.36 |
| | | | | | | [1.63] | [-0.86] | [0.24] | [-2.74] | [-4.09] | [-3.37] |
| $MKT$ | | | | $VOL$ | 0.001 | 0.11 | -0.10 | -0.01 | -0.56 | -1.35 | -1.47 |
| | | | | | | [1.28] | [-0.83] | [-0.04] | [-1.99] | [-3.45] | [-3.23] |
| $MKT$ | $SMB$ | $HML$ | | $VOL$ | 0.000 | 0.07 | -0.11 | 0.03 | -0.48 | -1.28 | -1.34 |
| | | | | | | [1.04] | [-0.86] | [0.24] | [-2.74] | [-4.09] | [-3.90] |
| $MKT$ | $SMB$ | $HML$ | $UMD$ | $VOL$ | 0.003 | 0.06 | 0.06 | 0.14 | -0.40 | -1.21 | -1.26 |
| | | | | | | [0.98] | [0.63] | [1.13] | [-2.22] | [-3.49] | [-3.41] |

**Panel B: Factor Loadings**

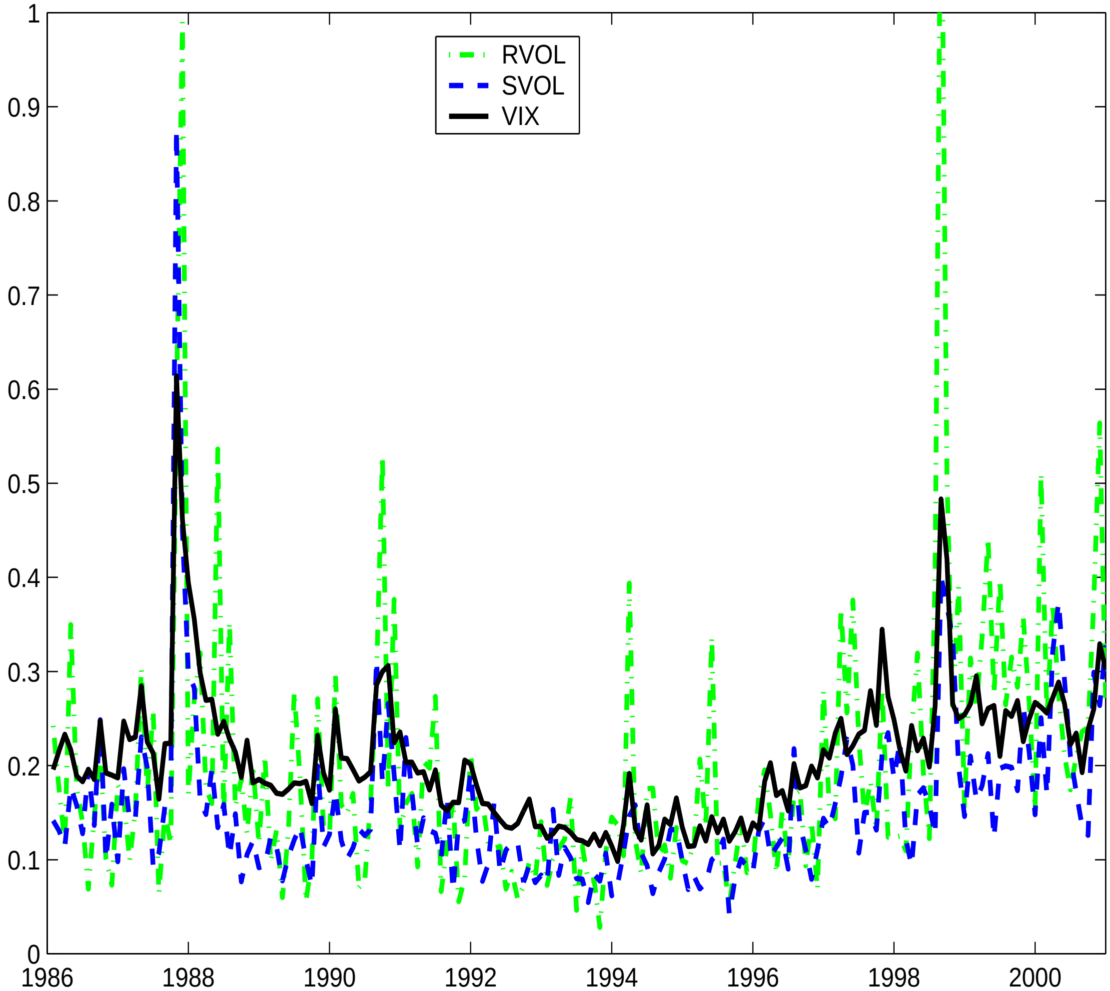
| | $MKT$ | $SMB$ | $HML$ | $UMD$ | $VOL$ |
|---|---|---|---|---|---|
| 1 | 0.96 | -0.19 | 0.17 | 0.01 | -0.13 |
| | [43.6] | [-4.92] | [2.93] | [0.31] | [-2.39] |
| 2 | 1.08 | 0.00 | -0.05 | -0.16 | -0.11 |
| | [35.4] | [0.01] | [-1.12] | [-3.44] | [-2.44] |
| 3 | 1.17 | 0.43 | -0.22 | -0.10 | -0.04 |
| | [33.9] | [6.81] | [-3.02] | [-2.24] | [-0.79] |
| 4 | 1.21 | 0.84 | -0.39 | -0.07 | -0.01 |
| | [22.4] | [9.22] | [-3.57] | [-1.18] | [-0.09] |
| 5 | 1.10 | 1.23 | -0.42 | -0.07 | 0.07 |
| | [13.2] | [9.30] | [-1.90] | [-0.64] | [0.37] |

In Panel A, we report a Gibbons-Ross-Shanken (1989) (GRS) test for pricing the quintile portfolios sorted by idiosyncratic volatility relative to FF-3, reported in the last panel of Table 8, for various factor models using combinations of the factors $MKT$, $SMB$, $HML$, $UMD$ and $VOL$. $UMD$ is the momentum factor from Kenneth French's website and $VOL$ is the volatility factor. The columns labelled '1' through '5' report portfolio alpha's from each linear factor model. The column labelled '5-1' refers to the difference in monthly returns between portfolio 5 and portfolio 1. Panel B reports factor loadings from the most comprehensive $MKT$, $SMB$, $HML$, $UMD$ and $VOL$ factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

Table 11: Quintile Portfolios of FF-3 Idiosyncratic Volatility of $L/M/N$ Strategies

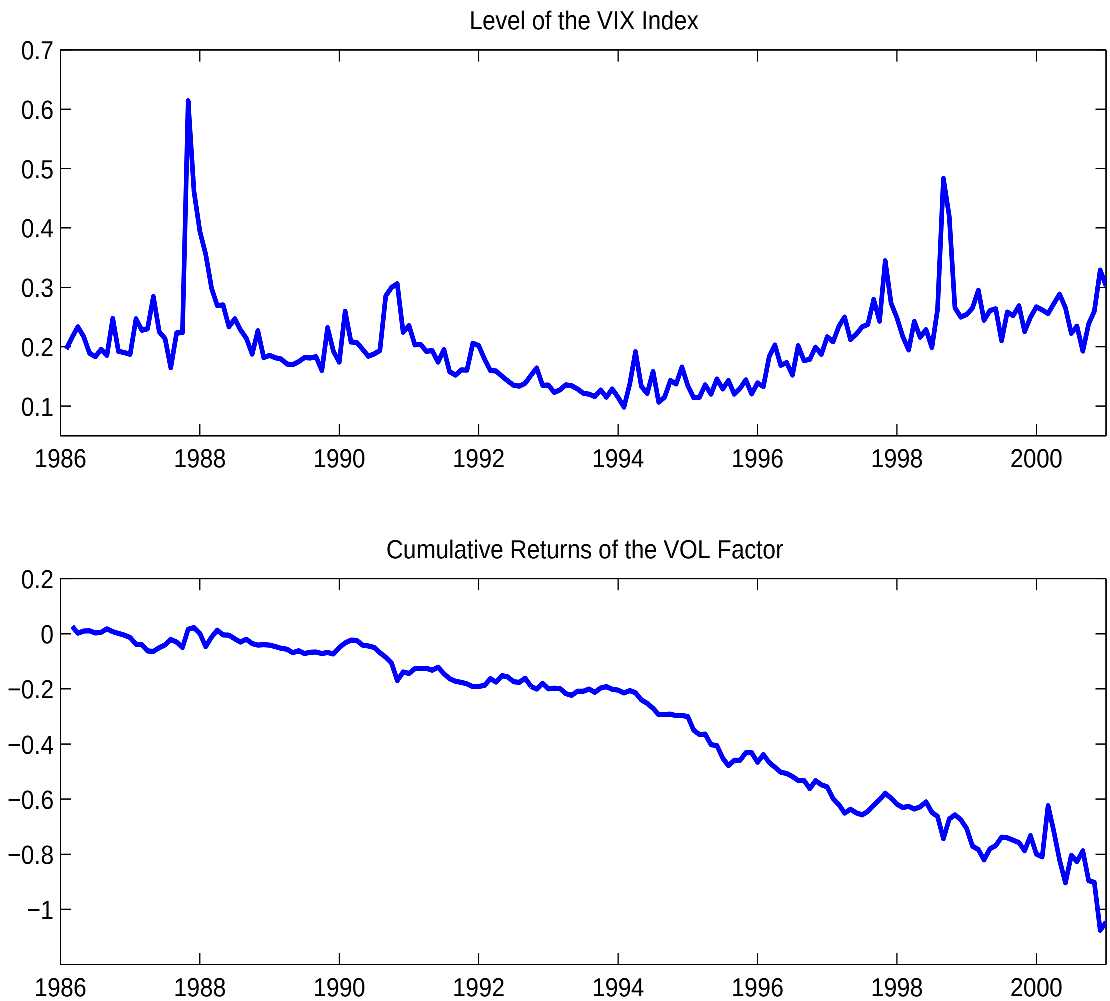
| | Ranking on Idiosyncratic Volatility | | | | | |
|---|---|---|---|---|---|---|
| Strategy | 1 low | 2 | 3 | 4 | 5 high | 5-1 |
| $1/1/1$ | 0.06 | 0.04 | 0.09 | -0.18 | -0.82 | -0.88 |
| | [1.47] | [0.77] | [1.15] | [-1.78] | [-4.88] | [-4.63] |
| $1/1/12$ | 0.03 | 0.02 | -0.02 | -0.17 | -0.64 | -0.67 |
| | [0.91] | [0.43] | [-0.37] | [-1.79] | [-5.27] | [-4.71] |
| $12/1/1$ | 0.04 | 0.08 | -0.01 | -0.29 | -1.08 | -1.12 |
| | [1.15] | [1.32] | [-0.08] | [-2.02] | [-5.36] | [-5.13] |
| $12/1/12$ | 0.04 | 0.04 | -0.02 | -0.35 | -0.73 | -0.77 |
| | [1.10] | [0.54] | [-0.23] | [-2.80] | [-4.71] | [-4.34] |

The table reports Fama and French (1993) alpha's, with robust Newey-West (1987) t-statistics in square brackets. The column '5-1' refers to the difference in FF-3 alpha's between portfolio 5 and portfolio 1. We rank stocks into quintile portfolios of idiosyncratic volatility, relative to FF-3, using $L/M/N$ strategies. At month $t$, we compute idiosyncratic volatilities from the regression (12) on daily data over an $L$ month period from months $t-L-M$ to month $t-M$. At time $t$, we construct value-weighted portfolios based on these idiosyncratic volatilities and hold these portfolios for $N$ months, following Jegadeesh and Titman (1993), except our portfolios are value-weighted. The sample period is July 1963 to December 2000.




The figure shows $SVOL$, $RVOL$ and $VIX$, plotted at a monthly frequency. We annualize $SVOL$ and $RVOL$ by multiplying the daily series by $\sqrt{250}$. The sample period is January 1986 to December 2000.

Figure 1: $SVOL$ and $VIX$




The figure shows the level of the $VIX$ index (top panel) and cumulative returns of the $VOL$ factor (bottom panel). The sample period is January 1986 to December 2000.

Figure 2: Cumulative Returns of the $VOL$ Factor