Dynamic Corporate Capital Stocks: Cross-sectional and Inter-temporal Stock Return Patterns

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Abstract

We investigate a general multiple security equilibrium model in which firms adjust their capital stock in response to economic shocks. Asset values are determined by competitive risk-averse investors. When corporate capital goes up on value, firms react by creating more of it. This leads to additional risk that must be borne by investors. Overall, the model generates a vector VAR(1) structure for the state variables determining the cross-section of expected returns, and is broadly consistent with stylized facts (e.g., the value premium, size premium, earnings momentum, and investment premium). In addition, the paper tests a new prediction of the model and finds support for it in the data.

Stock returns appear to display a number of long-run cross sectional and inter-temporal patterns. One of the most studied is the tendency for returns to increase in a firm's book-to-market ratio and decrease in its size (Fama and French, 1992). Another is that stock returns appear to fall following equity issues and rise following repurchases.[1] This 'issuance puzzle' has since been reconfirmed in both the U.S. and international markets.[2] Importantly for this paper, Titman, Wei and Xie (TWX, 2004) observe that the equity issue and repurchase findings are in fact tied to a firm's investments. Firms that invest today tend to have lower returns going forward and vice versa.[3] Yet another return pattern is the 'earnings momentum' phenomenon (Bernard and Thomas, 1989), in which firms with unusually high earnings post higher abnormal returns than those with unusually low earnings. The goal of this paper is to provide an explanation for why stock returns, corporate investment and profitability are related. We do so in a tractable general equilibrium framework, capturing observed patterns and generating a number of new testable cross-sectional predictions.

In the model both firms and investors play an active role in the determination of equilibrium prices and thus expected returns. Firms dynamically create goods and services across a number of industries by employing industry specific capital that varies over time. One type of change to the stock of capital arises from an exogenous source, which we model as shocks to available 'skilled labor'. Capital can also be created or

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[1] See Marsh (1982), Asquith and Mullins (1986), Mikkelson and Partch (1986), Jung, Kim and Stulz (1996), and Baker and Wurgler (2002) for evidence on equity issues; and Ikenberry, Lakonishok, and Vermaelen (1995) for evidence on repurchases.

[2] See Fama and French (2007) and Pontiff and Woodgate (2008) for U.S. evidence and McLean, Pontiff, and Watanabe (2008) for international evidence.

[3] Lyandres, Sun, and Zhang (2008) find that an investment factor explains most of the new issues puzzle, reducing 75% to 80% of SEO and IPO underperformance. Motivated by the q-theory, Xing (2008) shows that an investment growth factor does about as well as the value factor, driving out the value effect.

dismantled through a discretionary and firm-specific proprietary technology. It is assumed that if a firm adds or subtracts from its capital stock in this manner it is relatively less expensive to do so slowly, meaning that capital adjustment to shocks is sluggish in equilibrium. The demand side of the model comes from risk-averse investors that own shares in the industries and trade them in a competitive market. By using an overlapping generations framework based on Spiegel (1998) (see also Watanabe, 2008; and Biais, Bossaerts, and Spatt, 2008) the model is simultaneously tractable and capable of capturing a great deal of heterogeneity in the level, volatility, persistence, and correlations of cash flows generated by the panel of model firms. Though dynamic, the CAPM holds period-by-period in the model and the return an investor can expect to earn by investing in an industry varies over time with the industry's profitability and investment activity.

For intuition, consider the steel industry. A unit of capital would be a steel mill. The number of mills a firm has depends on both its past stock and what it can and does create with its skilled and unskilled labor pool. In the model these changes result from a exogenous shocks and deliberate actions. The former may include things like a flood which washes away nearby mills. On the other hand, the fortuitous granting of a building permit or the immigration of a superior labor pool can be thought of as positive shocks. Of course, firms can also take deliberate actions to also increase or reduce the pace at which mills are constructed as well as sell off facilities. Presumably, managed growth involves adjustment costs and may not respond immediately to changes in the economic environment. Thus, an increase in steel prices does not immediately produce a corresponding increase in the number of steel mills. Rather, it results in the start of their construction which takes time.

As the above example indicates, investment takes place whenever profitability is unusually high, and/or whenever there is a shortage of mills relative to some steady-state level. In each of these cases, the price of a steel facility will be relatively high, as will be the market value of the industry. Firms will respond to shocks that increase the profitability of mills by (slowly) increasing their number through investment. This construction activity eventually drives down the value of a new mill until construction costs and mill values are once again equilibrated. However, as a result of this process, the new construction ultimately leads to a negative equilibrium association between investment and expected returns.[4] The relationships just described between profitability shocks, investments, and expected returns reflect patterns identified in the literature. That is, high stock returns are accompanied by an immediate increase in capital accumulation. Afterwards returns are below normal and capital accumulation in the industry tapers off. As in TWX the return phenomena are tied to changes in corporate capital levels and not financial issues or retirements *per se*.

Supplementing the existing evidence on investment, we provide empirical evidence for the predicted relationship between profitability and expected returns. We measure profitability by the ratio of earnings per unit capital to the cost of creating unit capital. We find that the zero-cost portfolio that goes long high-profitability growth firms and short low-profitability growth firms earns a value-weighted average return of 0.84% per month. The risk-adjusted alpha from the standard four-factor model is 0.91% per month. Both of these numbers are economically as well as statistically significant (at the

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[4] Our calibration exercise demonstrates that the negative association can also hold unconditionally.

1% level). This profitability effect is robust to various portfolio sorting and weighting schemes and is also confirmed using cross-sectional regressions of individual stock returns on lagged firm characteristics when outliers are appropriately trimmed.

The remainder of the paper is structured as follows. After a brief literature review, Section 1 presents the model. Section 2 contains the analysis, followed by some empirical evidence for its predictions in Section 3. Section 4 concludes.

Literature Review: This paper is not the first to theoretically examine the relationship between stock returns and both real and financial corporate capital adjustments. A number of authors have proposed behavioral explanations in which managers take advantage of overvalued shares to raise capital.[5] By contrast, Pastor and Veronesi (2005), Dittmar and Thakor (2007), and Li, Livdan, and Zhang (2009) propose rational models which is the paradigm used here. In both Pastor and Veronesi and Li, Livdan, and Zhang the pricing kernel is time-varying along various dimensions that lead to both time varying investment and expected returns. In Dittmar and Thakor a firm's managers and the investing public may not agree on the value associated with a new investment. When the divergence is large firms finance with debt, otherwise equity is used.

This paper contributes to the above line of work by seeking to explain the phenomena between investment and returns documented in TWX via a general equilibrium framework. That allows the model to examine not only time variation in returns, but betas, cross sectional patterns, and the relationship these all bear to variables like industry productivity. In both Pastor and Veronesi (2005) and Li, Livdan, and Zhang (2009), market conditions are exogenous and firms react to them, while here they are

[5] See Loughran, Ritter, and Rydqvist (1984), Ritter (1991), Loughran and Ritter (1995), Rajan and Servaes (1997), Pagano, Panetta, and Zingales (1998), Baker and Wurgler (2000) and Lowry (2003)

endogenous and influenced by the firms. This interplay allows the model to also make some predictions regarding how overall capital investment impacts the future trajectory of the economy. Also, whereas Dittmar and Thakor (2007) look at how heterogeneous beliefs influence returns, in this article everyone has identical beliefs.

Other related models are those by Berk, Green and Naik (1999), and Carlson, Fisher, and Giammarino (2004, 2006). These authors use real options models to examine how a firm's expected return will vary over time. Part of their focus is on explaining and generating the relationship in the data between a firm's book-to-market and size.[6] The firms in this paper have a much simpler investment problem than those in the articles cited above, yet they still generate similar book-to-market return patterns. Another difference is in the data needed to corroborate each model's predictions. Using commonly available data sources it is often difficult to know where and to what degree real option values are influencing a firm's current stock price. Our model only requires information about the firm's current capital stock, profitability, and investment levels. While that does not make the model any more or less likely to be "right" it does make it potentially easier to test and refute. Finally, as both Berk, Green and Naik (1999) and Carlson, Fisher, and Giammarino (2004) acknowledge, their models are set up in a partial equilibrium framework with either the pricing kernel or the demand function exogenously given. In contrast, ours is a general equilibrium model in which prices equilibrate supply and demand endogenously through market clearing.

Another related approach can be found in Gomes, Kogan, and Zhang (2003), Gala

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[6] Other partial equilibrium models that looks at real options and time varying risk are Sagi and Seasholes (2007) and Hackbarth and Morellec (2008). In both, the exercise of options on assets affects a firm's risk characteristics. The latter examine how real options interact with merger and acquisition activity. They find that, among others, a firm's beta can even increase with the exercise of a call option on real assets.

(2005), and Kogan (2004). Gomes, Kogan, and Zhang modify the CCAPM model to allow for corporate heterogeneity induced by the random arrival of new projects. Gala extends this so that firms may have a temporary competitive advantage. In both investments are irreversible. Both papers also model the economy as having one economic industry. Multiple firms with varying attributes then arise due to the historical path of random factors that impacted their past investments. In Gala there is also the additional possibility that a firm may have a temporary competitive edge in making new investments. Kogan's paper examines a two consumption good – two industry model with irreversible real investment. In contrast to the above papers, the model presented here includes as multiple industries, a single consumption good and reversible (though costly) investment among other differences.

1. *A Competitive Model with Capital Adjustments*

1.1 Setting

Technology: The economy has K production factors corresponding to distinct industries in a discrete-time and infinite horizon setting. The date-t profit *per unit of capital* for each of the K industries is represented by a $K\times1$ vector, D_t, that evolves via:

$$D_t = D_{t-1} + G\left(\bar{D} - D_{t-1}\right) - HN_t + \delta_t. \tag{1}$$

G and H are constant $K\times K$ matrices, $\bar{D}$ is a $K\times1$ constant vector, N_t is a $K\times1$ vector whose k^{th} entry is the date-t supply of capital in industry k, and δ_t is a $K\times1$ (exogenous) normally distributed random vector with zero mean and variance-covariance matrix Σ_δ. The vector $\bar{D}$ represents the long run level to which profits tend if aggregate supply is 'small,' which is akin to the intercept in the equilibrium price function. The vector HN_t in

Eq. (1) captures the impact of aggregate supply on an asset's per-unit cash flows.[7] When h_{ij}, the ij^{th} element of H, is positive the size of industry j adversely affects the profitability of industry i; if h_{ij} is negative then the effect is complementary. We will assume that G, $I - G$ and H are positive definite so that profitability is mean-reverting and an industry's profitability decreases with its size.

Investors: A continuum of price-taking investors with unit mass is born at date-t, consumes and dies next period.[8] Investors have negative exponential utility functions with risk aversion parameter θ. Each investor begins life with an endowment of industry-specific human capital, and immediately sells their human capital to firms (who subsequently convert it to corporate capital) at a fair market value: If investors sell a total of $\eta_{k,t}$ units of capital to industry k at date t, and the date-t price per unit of capital is $p_{k,t}$, then investors are paid $\eta_{k,t} p_{k,t}$ for their industry-k endowment. Investors then buy or sell securities to fund their end-of-period consumption. Because investors have negative exponential utility functions, initial wealth is immaterial for portfolio choice. Thus, the only material effect of the human-capital endowments is the associated change in aggregate *corporate* capital, and this is captured by the $K\times 1$ vector, η_t. We assume that η_t is exogenously specified, and specifically a normally distributed random vector with mean zero and variance-covariance matrix Σ_η.[9]

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[7] We thank S. Viswanathan and Gur Huberman for suggesting that the model include N_t in Eq. (1).

[8] The assumption that investors live for one period of time is made only for tractability. There is no *a priori* reason to believe that this has a qualitative impact on how to calibrate the model to data. "Units of time" are determined by the speed at which capital can be accumulated and are not really dependent on the length of an agent's "life" (any more than the CAPM depends on investors living for one period).

[9] The mean-zero assumption for η_t is made for simplicity and does not affect the qualitative results.

Each share of an industry represents one unit of capital. There is a single risk free asset that pays r per period and serves as the numeraire with a constant value of 1. Let the corresponding vector of K prices at date-t be P_t and $R = 1 + r$. Define

$$Q_{t+1} = P_{t+1} + D_{t+1} - RP_t \tag{2}$$

to be the excess payoff vector from a unit position in each type of capital, and let $\text{var}_t\left[Q_{t+1}\right]$ be the corresponding variance-covariance matrix. Assuming that prices are normally distributed, investors choose their portfolios so as to maximize a myopic mean-variance objective function.[10] In this setting, optimality and market clearing are achieved if and only if,

$$\theta \, \text{var}_t\left[Q_{t+1}\right] N_t = E_t\left[Q_{t+1}\right]. \tag{3}$$

Note that N_t enters in Eq. (3) because the supply of shares must equal the supply of capital in equilibrium.

Firms: There is a continuum of competitive, all-equity, value maximizing, and price-taking firms with mass of unity. Each firm's output comes from a single production factor and its profits derive from the units of capital that the firm deploys.[11] Changes in a firm's capital base come from two sources. As discussed earlier, labor exogenously affects the supply of real assets. Additionally, executives can order the firm to create or dismantle capital via a quadratic cost function. Assume that managers seek to maximize their wages and that like regular labor they capture all the rents from their employment

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[10] We will soon establish conditions under which an equilibrium with normally distributed prices exists.
[11] In principle, firms can produce more than a single type of capital output. Assuming that the cost of building or liquidating capital is assessed at the firm level for each production factor separately, there is no loss of generality in considering firms that specialize only in a single type of output.

(implying executive talent is in short supply). In this case a price taking manager of firm f in industry k will then seek to:[12]

$$\max_{y_{fk,t}} y_{fk,t} p_{k,t} - c_{1k} y_{fk,t} - \frac{1}{2} c_{2k} y_{fk,t}^2 . \tag{4}$$

Here $y_{fk,t}$ is the capital created through the quadratic cost technology, the positive constants c_{1k} and c_{2k} represent the firm's cost (assumed to be the same across all firms in an industry) of adding (or equivalently speeding up the creation) of a unit of capital to the k^{th} production factor. If $p_{t,k} > c_{1k}$ then positive value is generated by the creation of *some* capital. The optimal amount is determined by the adjustment cost, c_{2k}. Thus, one can view the optimal choice of $y_{fk,t}$ as a measure of the positive NPV investment opportunities available to the firm.[13]

Solving for the optimal $y_{fk,t}$ in (4) and integrating over the unit-mass distribution of firms yields an optimal capital addition for each industry of:

$$y_{k,t} = \left(p_{k,t} - c_{1k} \right) / c_{2k} , \tag{5}$$

where $y_{k,t} = \int y_{fk,t} \, df$ is the total amount of new capital deployed in industry k. Notice that (5) yields the type of investment behavior one expects. When capital values are high firms create more, when low they divest. The more expensive it is to accelerate the production of new capital the slower it is added. Thus, if chicken prices rise one might see an increase in poultry farms within a year (high $p_{k,t}–c_{1k}$ and a low c_{2k}). On the other

[12] We show in an Appendix that the manager's objective function is also consistent with firm value maximization. The key assumption being that inter-firm competition causes labor to be paid an amount equal to the value of the product it creates.
[13] If one associates any amount of costs with the issuance or repurchase of equity, then it stands to reason that equity issuance or repurchase is associated with positive NPV opportunities, or $y_{fk,t}$, from the point of view of management.

hand if steel prices increase it may be years before the first new mills go on line (high $p_{k,t}-c_{1k}$ and a high c_{2k}).

Defining Y_t to be the $K\times1$ vector of $y_{k,t}$'s we can write (5) in vector form as:

$$Y_t = C_{2D}^{-1}(P_t - C_1), \tag{6}$$

where C_1 is the $K\times1$ vector of linear costs with the k^{th} element c_{1k}, and C_{2D} is a $K\times K$ matrix with the k^{th} diagonal element equal to c_{2k} and zeros elsewhere.

Aggregate supply of capital: Changes to the total supply of capital come from the contribution of labor (i.e., η_t) and from firms' use of the quadratic capital creation technology (i.e., Y_t). Thus

$$N_t = N_{t-1} + \eta_t + Y_t \tag{7}$$

where again, N_t is the $K\times1$ vector asset supply vector.

Summarizing, there are two types of exogenous shocks affecting firms: supply shocks (η_t) and profitability shocks (δ_t). Firms invest or divest (Y_t) in response to these, and the resulting surplus or shortage of risky capital feeds back into profitability (see Eq. (4)) and must be borne by investors (see Eq. (3)).

1.2 Equilibrium

Investors conjecture that prices are determined via the following formula:

$$P_t = A_0 + A_1 N_t + A_2 D_t \tag{8}$$

where A_0 is a $K\times1$ vector, while A_1 and A_2 are $K\times K$ matrices to be determined. At this point one can set up the set of simultaneous equations needed to derive the equilibrium parameters. However, the system is sufficiently complex that it yields few insights on its own and thus its derivation is left to Section 5.2 in the Appendix. More importantly, for much of the analysis that follows it turns out that, even without explicit solutions for the model's endogenous parameters, it is possible to say quite a bit about prices and returns.

The rest of the paper restricts attention to equilibria in which A_1 is negative definite and A_2 is positive definite. One can establish that this is the case in various instances including the following.[14]

Proposition 1: If G is symmetric positive definite and H and $I - G$ are positive definite, in each of the following limits an equilibrium exists in which A_2 is a finite positive definite matrix, while A_1 is a negative definite matrix.

1. $C_{2D}^{-1} \rightarrow 0$ for $\| H \|$ sufficiently small.[15]
2. $C_{2D} \rightarrow 0$ for $\| H \|$ sufficiently small.
3. Σ_δ, Σ_η, and G approach diagonal matrices and $\| H \|$ is sufficiently small.
4. $\theta\Sigma_\delta \rightarrow 0$, $(rI + G)^{-1}H$ is positive definite, and $\| H \|$ is sufficiently small.
5. $\theta\Sigma_\delta \rightarrow \infty$.

Proof. See the Appendix for the proof of this and all other propositions.

It is straightforward to confirm that the equilibria with finite A_1 and $H = 0$ converge to those of Spiegel (1998) as $C_{2D}^{-1} \rightarrow 0$.

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[14] Requiring A_1 to be negative definite is equivalent to requiring that $(P_t - E[P_t])' \cdot (N_t - E[N_t]) < 0$ whenever $D_t = E[D_t]$ and $N_t \neq E[N_t]$; i.e., if payoffs are at their steady state, prices and capital stock always move in opposite directions. Requiring A_2 to be positive definite is equivalent to requiring that $(P_t - E[P_t])' \cdot (D_t - E[D_t]) > 0$ whenever $N_t = E[N_t]$ and $D_t \neq E[D_t]$; i.e., if capital stocks are at their steady state, prices and payouts always move in the same direction.

[15] For a matrix X, we define $\| X \|$ to be the absolute value of the largest eigenvalue of X. As $\| X \|$ approaches zero, the matrix X approaches the zero matrix. It is possible to show that $\| F^{-1} \| < 1$ in cases 2,3, and 5 of Proposition 1 ($\| F^{-1} \| \rightarrow 1$ in the other two cases).

2. Analysis

2.1 Prices

Updating the time subscripts in (8) to t+1 and then plugging equations (1), (6) and (7) into equation (8) allows us to solve for P_{t+1} in terms of the parameter values known at time t and the unknown t+1 shocks:

$$P_{t+1} = \left(I-\left(A_1-A_2H\right)C_{2D}^{-1}\right)^{-1}\left\{A_0+\left(A_1-A_2H\right)\left(N_t+\eta_{t+1}-C_{2D}^{-1}C_1\right)+A_2\left[D_t+G\left(\bar{D}-D_t\right)+\delta_{t+1}\right]\right\}. \quad (9)$$

Letting $F \equiv I-(A_1-A_2H)C_{2D}^{-1}$, and again using (1), (6) and (7) after a bit of manipulation, one arrives at the following Proposition.

Proposition 2: The prices P_t and cash flows $\widehat{D}_t \equiv G(D_t+G^{-1}HN_t-\bar{D})$ follow the vector VAR(1) process,

$$\begin{aligned} P_{t+1}-C_1 &= F^{-1}\left\{(P_t-C_1)-A_2\widehat{D}_t+\left(I-F\right)C_{2D}\eta_{t+1}+A_2\delta_{t+1}\right\}, \\ \widehat{D}_{t+1} &= (I-G)\left\{\left[I-HC_{2D}^{-1}F^{-1}A_2\right]\widehat{D}_t+HC_{2D}^{-1}F^{-1}(P_t-C_1)\right\} \\ &\quad +(I-G)\left[HC_{2D}^{-1}F^{-1}\left(I-F\right)C_{2D}+H\right]\eta_{t+1}+\left[G+(I-G)HC_{2D}^{-1}F^{-1}A_2\right]\delta_{t+1}. \end{aligned} \quad (10)$$

The VAR(1) process associated with prices is behind many of the model's empirical predictions. If the diagonal elements of $I-F$ are negative, a positive supply (η) shock to the k^{th} industry will negatively impact $p_{k,t}$. Likewise, if as assumed earlier, A_2 is positive definite then a positive payout (δ) shock to the k^{th} industry will positively impact $p_{k,t}$.[16]

Eq. (10) allows one to write the price as a function of past supply and payoff shocks as:

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[16] It is possible to show that $I-F$ satisfies this property in the limits considered in Proposition 1.

$$P_t = C_1 + \sum_{s=0}^{\infty} F^{-s-1}\left[\left(I-F\right)C_{2D}\eta_{t-s} + A_2\left(\delta_{t-s} - \widehat{D}_{t-s-1}\right)\right] \qquad (11)$$

implying that the impulse response τ periods after a date-t η-supply shock is given by $F^{-\tau-1}(I-F)C_{2D}\eta_t$. Similarly, the impulse response τ periods after a date-t payout change is given by $F^{-\tau-1}A_2\left(\delta_t - \widehat{D}_{t-1}\right)$. As long as $\| F^{-1} \| < 1$, the effect of past shocks on prices eventually decays.

Equation (11) provides a number of empirical predictions. If a date-0 shock creates a large positive price move in stock prices, then in view of Eq. (6), the high stock price will be accompanied by investment. Eq. (11) suggests that the subsequent price series will decline, and this will in turn be accompanied by reduced or even negative investment. The process continues on like this until a steady state is reached. Although prices may decline towards an equilibrium level, returns are not negative because investors continue to receive a cash flow stream from the assets.

2.1.1 Returns and Betas

Generally, in a model with a downward sloping demand curve, a negative supply shock increases the current price. In the model, this is additionally associated with an observable change in capital investment in the same direction as the price change. The presence of adjustment costs ensures that shocks are not fully undone by capital investment. Intuitively, the net supply shortage will mean that investors can bear more risk than before, thereby leading to lower expected returns. Because net supply shortages are associated with positive capital investments, one might expect to see a negative relation between expected stock returns and capital investment.

To formally analyze the above scenario define industry k's excess return as

$$r^e_{k,t+1} = q_{k,t+1} / p_{k,t} \, , \tag{12}$$

where $q_{k,t+1}$ is the k^{th} element of Q_{t+1}. The next proposition asserts that the expected excess return decreases with capital investment and increases with profitabilty.

Proposition 3: Assume that $E[r^e_{k,t+1}] > 0$. In each of the limits described in Proposition 1, an equilibrium exists in which, for every k, holding everything else constant industry k's expected excess returns decrease with $y_{k,t}$ and increase with $d_{k,t}$:

$$\frac{\partial E\,[r^e_{k,t+1}]}{\partial y_{k,t}} < 0 \text{ and } \frac{\partial E\,[r^e_{k,t+1}]}{\partial d_{k,t}} > 0. \tag{13}$$

The first inequality is consistent with the empirical relationship described in TWX and the related literature on new stock issuances and repurchases. The firms issue or retire securities to buy or sell capital in response to shocks to the real economy that also impact equity prices. Although it may seem unintuitive that supplying the economy with a risky asset will *reduce* the equilibrium risk premium, the point is that such investment is generally a partial response to contemporaneous economic shocks that lead to an increase in the risk bearing capacity of the economy (e.g., negative shocks to the stock of capital). If firms did not invest, the risk premium would be even lower. We emphasize that the partial derivatives in Proposition 3 should be interpreted to state a *conditional* cross-sectional relationship. As demonstrated in Section 2.5, the model can be parameterized so that the relationships can also hold unconditionally.

The second inequality in (13) is consistent with Bernard and Thomas (1989) and the earnings momentum literature. In the model, firms issue more capital in response to higher profitability, given the current capital stock level. This is balanced by the increased risk premium investors require for bearing more capital stock risk.

Because the model's random variables are normally distributed and the stock market is assumed to be competitive and frictionless, the CAPM must hold period by period. The following result follows immediately:

Corollary to Proposition 3: Assume that $\beta_{k,t}$, the date-t beta of industry k with respect to the market portfolio, is positive. In each of the limits described in Proposition 1, an equilibrium exists in which for every k:

$$\frac{\partial E\,[\beta^{e}_{k,t+1}]}{\partial y_{k,t}} < 0 \text{ and } \frac{\partial E\,[\beta^{e}_{k,t+1}]}{\partial d_{k,t}} > 0. \tag{14}$$

Lewellen and Nagel (2006) find that the conditional CAPM cannot account for the magnitude of many asset pricing anomalies because most of the variation derives from the covariance between a stock's beta and the market risk premium. This result does not apply here. The return process in (13) derives from endogenous corporate investment policies that induce firm betas to change over time. This process need not be linked to the current market risk premium. As in Berk, Green and Naik (1999) the source of the time variation in individual stock returns derives from a firm's deliberate, and thus somewhat predictable, changes to its underlying asset base.

2.2 The Steady State

The economy is defined to be in a steady state in period t if firms do not actively seek to change their capital stock and if the payout per unit of capital is expected to remain unchanged. This is a useful base case as it yields the model's predictions regarding unconditional moments, which can be compared to those in the data. From the steady state benchmark, one can see how various shocks to the system will impact estimated returns, risk factors and other financial and economic variables of interest.

Assuming the equilibrium state variables are mean-stationary, a steady state corresponds to a situation when the state variables coincide with their long-term means. Taking the unconditional means of the equations in (10) and imposing $E[y_t] = 0$ yields $E[P_t] = C_1$ and $E[\widehat{D}_t] = 0 = E[D_t] + G^{-1}HE[N_t] - \overline{D}$. Hence, the long-term mean of P_t is C_1, the long-term of mean of Y_t is zero, while the long term mean of D_t is :

$$E[D_t] \equiv \breve{D} = \overline{D} - G^{-1}H\overline{N}, \tag{15}$$

where $\overline{N} \equiv E[N_t]$, and can be expressed in terms of the model endogenous parameters (i.e., A_0, A_1 and A_2) through Eq. (8).

Assume here and throughout the rest of the paper that $p_{k,t} > 0$.[17] The conditional expected return to an investor from holding a claim in one unit of corporate asset k at date t equals

$$E_t\left[r_{k,t+1}\right] = \frac{E_t\left[p_{k,t+1} + d_{k,t+1}\right] - p_{k,t}}{p_{k,t}}, \tag{16}$$

where $d_{k,t}$ represents the k'th element of the vector D_t. Employing the steady state condition that $p_{k,t} = c_{1k}$ and $d_{k,t} = \breve{d}_k$ from Eq.(16), along with Proposition 2, leads to the following:

Proposition 4: If the economy is in steady state at date t then

$$E_t\left[r_{k,t+1} \mid \text{steady state at } t\right] = r + \frac{\theta\{V\overline{N}\}_k}{c_{1k}} = \frac{\breve{d}_k}{c_{1k}} = \frac{\overline{d}_k - \sum_{j=1}^{K} z_{k,j}\overline{n}_j}{c_{1k}} \equiv ss_k, \tag{17}$$

[17] While both the price and supply are normally distributed in our model, one can arbitrarily reduce the probability of their assuming negative values. The distribution of the ratio of two Normal distributions is called the Fieller distribution and its application is extensively discussed in the statistics literature.

where $\bar{n}_j$ represents the j^{th} element of $\bar{N}$, Z denotes the matrix $G^{-1}H$, with $z_{k,j}$ as its kj^{th} element. The variance $V \equiv \text{var}_t\left[Q_{t+1}\right]$ is independent of time and explicitly given in the Appendix via Eq. (25). The notation $\{\bullet\}_k$ corresponds to the k^{th} element in the vector expression inside the curly bracket.

Proposition 4 implies that if a firm in industry k is in its steady state, the firm's stock returns will equal the long run ratio of the asset's per-unit profitability to its unit creation cost. Further, the right side of (17) can (at least in principle) be calculated with commonly available data. For a firm one can use the long run average earnings ($\breve{d}_k$) divided by the book value of assets or similar measures of a firm's cash flow and costs of productive assets.

The expression $E_t\left[r_{k,t+1} \mid \text{steady state at } t\right] = \dfrac{\breve{d}_k}{c_{1k}}$ in Eq. (17) indicates a firm's steady state expected returns are independent of its cash flow risk or investor risk aversion. The reason for this is that in the model firms can adjust the number of capital units they have over time. This leads to industries that add or subtract assets to the point where investors bear an optimal, or steady state, level of risk. Essentially, the stock of capital in equation (3) adjusts to the point where the benefits of bearing its risk is balanced with investors' risk aversion, and the steady state expected returns are equal to $r + \dfrac{\theta\{V\bar{N}\}_k}{c_{1k}}$. This emphasizes one of the model's fundamental properties: The long run productive economy is infinitely pliable. Industries can expand or contract as necessary based on costs and prices until the two are in line. Because prices reflect costs as well as risks, supply must change until investors, according to their risk bearing capacity, set prices equal to the creation cost of capital. Thus, from the perspective of long run returns, asset values (which ultimately must equal their cost of production) and profitability incorporate all one needs to know when calculating steady state expected returns.

2.2.1 The Market-to-Book Ratio

One can interpret $p_{k,t} / c_{1k}$ to be the market-to-book ratio, or Tobin's *q*, of a firm in industry *k*. This is justified by considering that c_{1k} is the cost of replacing a unit of capital in industry *k* as economically as possible. The firm's total book value in this case would be $c_{1k}\, n_{k,t}$, while its market value is $p_{k,t} n_{k,t}$. In the long run, the steady state requirement that $p_{k,t} = c_{1k}$ thus implies that the long run market-to-book ratio and thus Tobin's *q* for an industry should roughly equal 1. Consistent with this, the firm makes positive NPV investments in capital whenever the market-to-book ratio exceeds 1. Because of the adjustment costs, the book-to-market ratio deviates from 1 for potentially extended periods of time. For example, if the cash flow (d_k) to a particular type of capital goes up so will the market value of that asset. This will increase the market-to-book ratio and induce capital accumulation by firms in the industry.

Because Tobin's *q* for industry *k* in our model is equal to $\frac{c_{2k} y_{k,t}}{c_{1k}} + 1$, Proposition 3 and its corollary imply a cross sectional relationship between the book-to-market ratio and expected returns. A shock that decreases the book-to-market ratio today should be followed by future capital accumulation and lower than average expected returns to shareholders. This will continue until the "growth" stock sees its market-to-book (or equivalently Tobin's *q*) return to 1. The reverse will be true for "value" stocks.

The above analysis provides a rationale for the value-versus-growth return relationship that is both complementary to and separate from that in either Berk, Green and Naik (1999) or Carlson, Fisher and Giammarino (2004). In the prior models the premium results from firms altering their value through the exercise or expiration of growth options, whereas here the premium arises because surplus risky capital stocks tend to coincide with lower market value for capital.[18]

2.3 Return Dynamics Close to the Steady State

One can get a sense of the behavior of prices, and therefore expected returns, by examining the price evolution in terms of small deviations of the state variables around the steady-state. This is the purpose of the next Proposition:

Proposition 5: To linear order in Y_t and $D_t - \breve{D}$, the expected return on the stock of a firm in industry k is given by:

$$E_t\left[r_{k,t+1}\right] \approx ss_k - \frac{1}{c_{1k}}\left\{\left[(ss_k - r)I - \theta V A_1^{-1}\right]C_{2D}Y_t\right\}_k + \frac{1}{c_{1k}}\left\{(-\theta V A_1^{-1}A_2)(D_t - \breve{D})\right\}_k \quad (18)$$

where ss_k is defined in Eq. (17), and $\{\bullet\}_k$ corresponds to the k^{th} element in the vector expression inside the curly brackets.

If steady state expected excess returns (i.e., $ss_k - r$) are positive and, as is true under the conditions in Proposition 1, $-VA_1^{-1}C_{2D}$ is positive definite, then expected returns should be *negatively* related to investment (i.e., Y_t). Likewise, if $-VA_1^{-1}A_2$ is positive definite, expected returns are *positively* related to unusual profits. Proposition 5

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[18] It is worth noting that while there is considerable evidence for a value premium in stock returns there is some question as to whether or not it is concentrated primarily in securities shunned by institutional investors. See Houge and Loughran (2006) and Phalippou (2007) for evidence on this issue.

embodies the main message of the paper: The equilibrium cross-section of expected returns should depend on the cross-section of profitability and investment, both in the long run and dynamically.

In the model, as in practice, the long run cash flows an asset generates depends not only on long-run demand for the product ($\bar{d}_k$) but also on its supply and that of competing or complementary products ($\sum_{j=1}^{K} z_{k,j}\bar{n}_j$). Both of these are present in the steady state expected returns, ss_k. In addition, Eq. (18) provides guidance on how deviations from the long-run, or steady state, affect returns. An example, on the demand side, could be the coal industry which could be falling out of favor for environmental reasons. Currently, profits are higher than what the steady state profitability is predicted to be. In this case, equation (18) predicts that, going forward, expected returns will decline with time because of the expected drop in profits; this, however, could be somewhat offset to the extent that the coal industry shrinks through value-creating (positive NPV) divestment. Whichever effect dominates depends on specifics of the industry. The opposite effects would, presumably, be anticipated for those industries producing coal alternatives such as manufacturers of solar panels.

Eq. (18) also gives an indication of how the returns of one industry might be impacted by the investment and profitability of related industries. For instance, even if the investment and profitability of solar panel manufacturers does not change from one period to the next, their expected returns could change because of declining profits, and divestment in the coal industry. In other words, expected returns in one industry will be impacted by changes in the fundamentals of complementary or substitute industries.

The factors that (18) relates to investor returns may help explain some of the advice commonly given by financial pundits. Casual observation indicates that they encourage the purchase of shares in industries with increasing profits. Conversely, they recommend selling shares in industries that are declining because competing ones are on the rise. Assuming financial pundits do not incorporate risk (however defined) into their recommendations, they may simply tout stocks that they believe will have the highest expected returns.[19] Based on this, the model implies that advisors in the financial press will push stocks where (18) says the return is likely to be high and discourage ownership of those where (18) indicates they will be low. Casual observation and a consideration of what (18) implies about cross sectional returns in rising and declining industries seems to provide support for this hypothesis. Obviously, a more rigorous analysis would be needed to say anything truly definitive about this issue and there is no pretence that such has been carried out here.

2.4 Why Profitability and Investment Affect Expected Returns

As far as investors are concerned, only the price and supply of risky capital matter for expected returns. To see this, use Eq. (3) to write

$$E_t\left[Q_{t+1}\right] = \theta \,\mathrm{var}_t\left[Q_{t+1}\right] N_t = \theta V\bar{N} + \theta V(N_t - \bar{N}),$$

and then divide the k^{th} element of this vector by $p_{k,t}$ to get the expected excess return:

$$E_t[r_{k,t+1}] = \frac{\left\{\theta V\bar{N} + \theta V\left(N_t - \bar{N}\right)\right\}_k}{p_{k,t}}. \qquad (19)$$

[19] Essentially, this assumption amounts to positing that analysts do not consider the stock's riskiness or correlation with other stocks when issuing recommendations. Indeed, some television shows track the returns from holding their expert's picks but (so far as this paper's authors know) none track the volatility of the resulting portfolio.

To understand how and why profitability and investment affect expected returns, it is useful to consider an economy in its steady state at date-(t-1) that is subsequently perturbed by an exogenous shock. Using Eq. (7) one can rewrite Eq. (19) as

$$E_t[r_{k,t+1}] = \frac{\left\{\theta V\bar{N} + \theta V\left(y_t + \eta_t\right)\right\}_k}{p_{k,t}} \tag{20}$$

There are two shocks to consider: supply (η) shocks and payoff (δ) shocks. Consider, first, the impact on the steady state of a one-time positive supply shock to industry k. The oversupply of capital will prompt firms to divest (i.e., $y_{k,t} < 0$). If adjustment costs are non-zero, the divestment will not completely counter the supply shock (i.e., $y_t + \eta_t > 0$), leading to a contemporaneous price decline. Thus, according to Eq. (20), expected returns will be higher than in the steady state because the numerator is greater than in the steady state while the denominator is smaller than in the steady state. As time passes and the effect of the shock slowly diminishes (as divestment continues), the same effect prevails but to a lesser degree: The oversupply of capital stock is still only partially offset by divestment while prices are still below their steady state value, leading to higher expected returns than in the steady state. The reverse takes place in response to a negative supply shock.

Summarizing, the presence of adjustment costs prevents firms from fully compensating for positive (negative) supply shocks via divestment (investment), forcing investors to bear more risk and leading to a higher (lower) risk premium. Thus, at least in reaction to supply shocks, divestment over time is unambiguously associated with a higher risk premium while investment is associated with a lower risk premium.

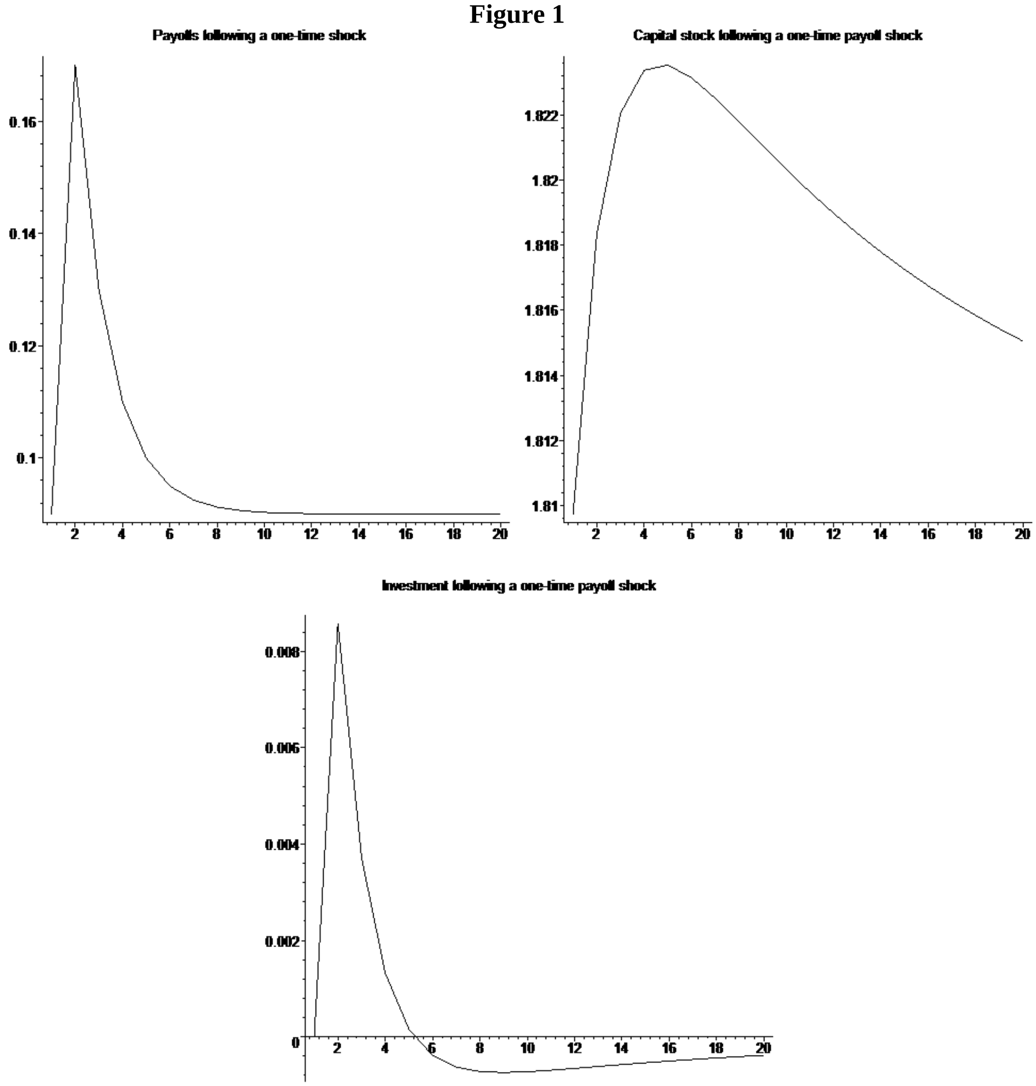
Consider now the impact on the steady state of a one-time positive *payoff* shock to industry k, turning off the η shocks so as to isolate the impact of a payoff shock on returns. Such a one-time shock makes owning capital more profitable and will therefore prompt firms to invest (i.e., $y_{k,t} > 0$). In this case, Eq. (20) does not unambiguously characterize the impact on expected returns.[20] The higher price of capital (in the denominator) will tend to reduce expected returns while the increase in capital stock will force investors to bear more risk and tend to increase expected returns. One can recast this tradeoff, however, as one between the response of capital stock to higher profitability (which is positively related to $D_t - \breve{D}$) and the negative response of expected returns to an increase in prices (which is negatively related to Y_t), just as in Eq. (18). This emphasizes the point that, in general, profitability and investment are not independent. In particular, when one assesses the sensitivity of expected returns to one of profitability or investments, one should control for the other (e.g., see Proposition 3, where the sensitivities are calculated as *partial* derivatives).

As time passes, profitability will revert to its long-term mean. At that point, the industry will have to slowly divest the extra capacity built to take advantage of the unusually high profits. Thus, the initial positive shock to payoffs eventually translates into an effective positive supply shock (once profits revert) with the same impact on returns identified earlier for supply shocks. As an illustration, Figure 1 plots the impact on payoffs, capital stock and investment of a one-time date-2 shock, assuming the economy is in a steady state at date-1, and using the parameters in Section 2.5. In the

[20] For example, if C_{2D} is very small, prices adjust very quickly to shocks, and nearly all of the effect on expected returns is in the numerator of (20). On the other hand, if C_{2D} is very large, capital stocks do not change significantly and nearly all of the effect on expected returns is in the denominator of (20).

illustration, the initial increase in capital stocks and investment (i.e., price) can have an ambiguous impact on expected returns. However, it is evident that once the rationale for having more capital stock diminishes with declining profitability (after about five years in the figures), the long-term impact of the payoff shock is akin to a supply shock.

Figure 1



Summarizing, investments are made for two reasons in the model: To mitigate supply shocks and to take advantage of increasing profitability. The profitability motive leads to a higher supply of risky capital and, therefore, a higher-than-usual risk premium or expected returns. When one controls for the profitability motive, investment is driven by supply shocks, and the presence of adjustment costs means supply shocks are only partially mitigated in equilibrium. In particular, positive investment takes place when there is an undersupply of risky capital, and is therefore followed by a lower-than-usual risk premium and hence expected returns. This explains why investment and profitability enter into Eq. (18) with a negative and positive impact, respectively.

2.5 Cross-sectional Returns – A Rough Calibration Exercise

The analytic results in the preceding sections provide a qualitative sense that the model can be consistent with a number of stylized facts in the literature. In order to illustrate this better we choose a set of parameters highlighting the cross-sectional effects. Our intention is not to perform a full scale calibration to industry data and cross-sectional moments at this stage (as is done for cross-sectional moments in Carlson, Fisher, and Giammarino, 2004). Rather it is simply to show that the model can potentially produce quantitative results in line with the data as well as qualitative ones.

We consider the case of ten iid industries, where Σ_δ, Σ_η, *G,* and *H* are proportional to the identity matrix. We focus on ten industries so that we can construct the equivalent of cross-sectional deciles when calculating return moments. Each period corresponds to a year. Without loss of generality, we normalize the steady-state book value of one unit of capital to be 1. The risk free rate is chosen to be $R = 1.01$, consistent with the realized real rate of return over the past half century, while $\overline{d}$ is chosen to be 0.09 and $H = hI$ with $h = 0.005$. We set the volatility of payoffs and supply to be $\sigma_\delta = \sigma_\eta = 8\%$ and the rate of payoff mean-reversion is $g = 0.5$ (where $G = gI$). Thus the steady state excess rate of return is

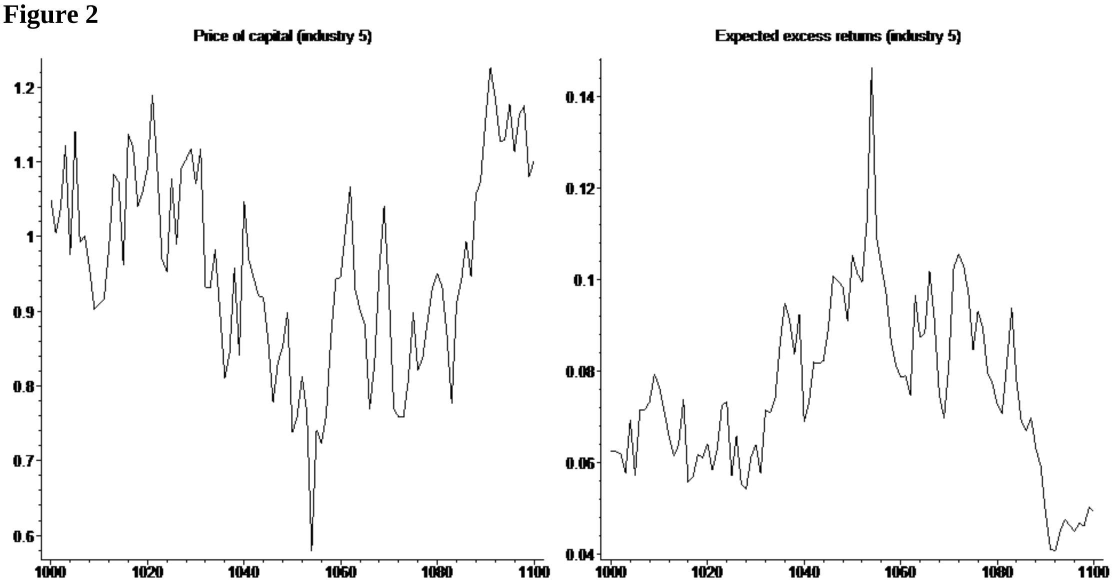
$$\frac{\breve{d}_k}{c_1} - r = \frac{\bar{d} - g^{-1} h \bar{n}_k}{c_1} - r = 0.08 - 0.01 \times \bar{n}_k, \qquad (21)$$

with $\bar{n}$ determined endogenously. The remaining parameters are chosen to help arrive at 'reasonable' magnitudes for the stylized cross-sectional moments. The risk aversion coefficient is set $\theta = 2$. In order to distinguish between the dispersion in market-to-book ratios (i.e., the dispersion in elements of P_t) and the dispersion in investment (i.e., the dispersion in elements of Y_t), we choose a heterogeneous set of adjustment costs. Elements of C_{2D} are symmetrically set according to the deciles of the normal distribution, $\mathcal{N}(8,4)$, with mean 8 and standard deviation 4. Explicitly, $c_{2,1}$ to $c_{2,4}$ are the first through fourth deciles of $\mathcal{N}(8,4)$, $c_{2,5}$ and $c_{2,6}$ are 8, and $c_{2,7}$ to $c_{2,10}$ are the seventh through the last deciles of $\mathcal{N}(8,4)$. These parameters completely determine the model.

For each industry, the coefficient, $(A_1)_{k,k}$, solves a degree-five polynomial, which under our parameter specification has a unique negative real root (at -0.386 for the first industry and monotonically decreasing to -0.863 for the tenth industry); given that a negative real value for $(A_1)_{k,k}$ is the only sensible economic solution, this means that our particular parameter specification is not complicated by the presence of multiple equilibria. The remaining coefficients in the equation relating price to quantity and payoffs are: $(A_0)_{1,1}$ = 1.53, monotonically increasing to $(A_0)_{10,10}$ = 2.02, and $(A_2)_{1,1}$ = 0.876, monotonically increasing to $(A_2)_{10,10}$ = 0.924. Because the adjustment costs increase with the industry index, in the steady state the supply of industry capital monotonically decreases from $\bar{n}_1$=1.55 to $\bar{n}_{10}$=1.27. The annual steady state expected excess returns vary between 6.45% for industry 1 to 6.73% for industry 10. The standard deviation of the price of a unit of capital varies from 0.144 to 0.163, thus the unconditional probability of a negative price realization is less than one in 10^9 years. The rates of mean-reversion of the price are determined by $(F^{-1})_{1,1}$ = 0.88, monotonically increasing to $(F^{-1})_{10,10}$ = 0.94. Industries with greater adjustment costs have capital prices that exhibit slower rates of mean reversion.

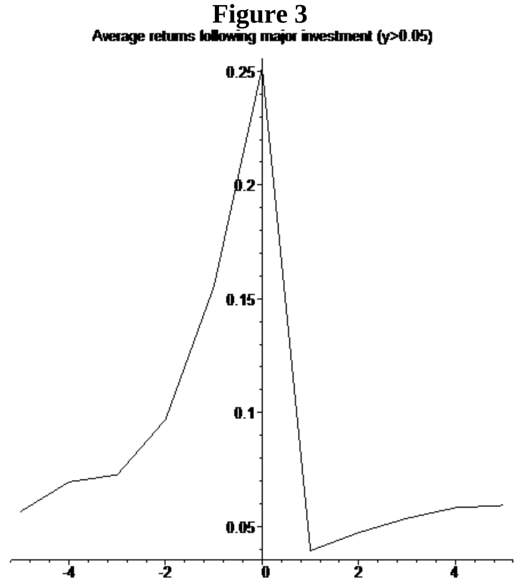
We simulate 10,000 years of the economy assuming that it is initially at the steady state. There was no instance in which the price fell below 0.29 or rose above 1.78. The average difference between the industry with the highest price per unit capital and that with the lowest price per unit capital is 0.48, with an average difference of about 6.4% in the amount of equity issued between the industry with the highest growth (i.e., positive y_t) and that with the lowest growth. Below is a plot of the price per unit capital in industry 5 after a 'burn-in' period of 1000 years; this is beside a plot of the corresponding expected excess returns for the same industry over the same period.

Figure 2



As explained earlier, when the price of capital is high, expected returns tend to be low, and vice versa. The model also produces the familiar run-ups preceding major issuance events, and which are subsequently followed by declining returns. An instance of this is plotted below. The figure illustrates the average returns 5 years before and five years after a 'major investment' made by the leading investing industry (i.e., the event is said to take place whenever the leading industry makes an investment of y>5%). While the graph plots average realized returns (and not cumulative abnormal returns, or 'CARs'), it should be clear that the expected returns prior to the event are higher than the expected returns following the event, thus using a market model to adjust for risk will result in the usual observed patterns in CARs.

Figure 3



We calculate the market capital of each industry by multiplying its date-t price per unit capital by the size of the industry. Consistent with the discussion in Section 2.2.1, we set the market-to-book ratio of an industry to be its price per unit capital. By sorting the industries with respect to size, market-to-book, investment, and payoffs, we can calculate the various asset-pricing moments, reported below along with moments from data:[21]

Portfolio	Average Excess Returns	Data
SMB	1.4%	3.0%
HML	6.3%	5.2%
Low Investment	10.0%	9.5%
High Investment	4.7%	1.6%
High Profitability	7.5%	5.8%
Low Profitability	6.4%	0.3%

The SMB returns are the time-series average difference between the annual *expected* returns of the smallest industry (in market value) and the largest industry at date *t*. We use expected returns rather than realized returns to improve the power of the procedure (and because we can calculate these in our model). We only use the latter half of the simulated sample (using the first half makes a negligible difference given the number of significant digits we keep). The HML returns are the time-series average difference between the annual expected returns of the highest book-to-market industry and the lowest book-to-market industry at date *t*. Both the SMB and HML returns are consistent with stylized cross-sectional evidence in magnitude and sign. The book-to-market effect is discussed in Section 2.2.1. The size effect enters our model through two channels. First, there is the channel discussed in Berk (1995): given two firms with identical cash flows, the smaller firm, tautologically, will be the one with the higher discount rate and

[21] The data column shows the annualized average excess monthly returns of the following portfolios over the period from July 1963 to December 2008: SMB and HML are the size and value factors downloaded from Kenneth French's website. Low and high investment portfolios are the two extreme-decile value-weighted portfolios sorted by Lyandres, Sun, and Zhang's (2008) investment to total asset ratio. The high and low profitability portfolios are the two extreme-decile value-weighted portfolios sorted by the profitability measure described in Section 3.

therefore higher expected returns. Second, the steady-state returns in Eq. (21) negatively depend on $\overline{n}_k$, further contributing to a size effect.

The 'Low Investment' portfolio returns correspond to the time-series average of the lowest *y* industry expected returns at date *t*. The 'High Investment' portfolio returns are similarly calculated. Both are consistent with the observed issuance puzzle and Proposition 3. The difference between these two portfolio returns, about 5.3%, is distinct from the book-to-market (i.e., HML) premium. This is because heterogeneous adjustment costs allow for the highest market-to-book firm to be different from the highest investment firm. Finally, the payoff portfolios report a similar time-series average for the industry that happens to post the highest (respectively lowest) change in payoffs between dates *t*-1 and *t*. The results here are consistent with the direction (though perhaps not the magnitude) of the earnings momentum phenomenon observed by Bernard and Thomas (1989): Firms that announce high (lower) earnings exhibit positive (negative) 'abnormal' returns relative to their pre-announcement risk-adjustment. This can be interpreted as a change in 'risk' subsequent to the announcement or that markets inadequately adjust for the impact of earnings announcements. Our model provides a risk-based explanation for this effect within an equilibrium framework: risk increases after large payoff news because of subsequent investment or divestment.

3. Empirical Evidence

The results in Section 2 imply that the book-to-market ratio (which is closely related to investments) and profitability are key variables to determining the cross-sectional variation in expected returns. We now examine this point empirically. Consistent with the model's implication, we find that average returns increase with proxies of these two quantities.

3.1 Data and Methodology

We obtain accounting variables from the Compustat annual file. To measure profitability, we are guided by Proposition 4 to look for a proxy for $\breve{d}_k / c_{1k}$. We compute this as the ratio of "Operating Income Before Depreciation" (Compustat Xpressfeed data item *OIBDP*, FTP data item 13) to contemporaneous "Property Plant and Equipment - Total (Gross)" (*PPEGT*, data item 7). This is consistent with our interpretation of c_{1k} as the steady state book value of capital. We denote this measure as *PROF*:

$$PROF_t = \frac{OIBDP_t}{PPEGT_t}. \tag{22}$$

The construction of the book-to-market ratio (*BM*) follows Fama and French (1993). Based on the firm characteristics at the end of fiscal year $t - 1$, we form portfolios in June of calendar year t and measure returns monthly from July through next June. The conservative six-month lag accounts for possible delay in the dissemination of accounting information and follows the usual practice. To save space our main analysis focuses on value-weighted returns because of practical relevance, but we also examine equally weighted portfolio returns and cross-sectional regressions of individual stock returns for robustness. The monthly returns and variables necessary to compute market capitalization are from the Center for Research in Security Prices (CRSP), which are matched to the Compustat data by the CRSP-Compustat Merged Database. We use only ordinary common shares (CRSP Share Code 10 or 11) of firms in non-financial industries (one digit SIC code not equal to 6), because investment of financial firms may be very different in nature from that of non-financial firms. We use only NYSE firms (CRSP Exchange Code 1) to compute breakpoints for ranking, but include NYSE, AMEX, and NASDAQ firms (CRSP Exchange Code 1, 2, and 3) in portfolio formation. Our final sample runs from July 1963 through December 2008.

3.2 Results

3.2.1 One Dimensional Sort on *PROF*

Table 1 shows the characteristics, excess returns, and risk-adjusted alphas of decile portfolios sorted by *PROF*. The second column tells us that firms in the lowest *PROF* decile incur losses on average. The market capitalization (*SIZE*) tends to increase, and *BM* to decrease, with *PROF*, but the relations are not monotonic. In fact, variations in *BM* within a given *PROF* quintile, and vice versa, will appear when portfolios are double sorted by these quantities in the next subsection. The table also indicates that there are relatively a large number of firms (*N*) in the top and bottom deciles; this implies that many NASDAQ firms fall in these two extreme *PROF* deciles, and that the point estimates of *SIZE* and *BM* may not properly represent the characteristics of firms in those deciles. To that extent, the excess value-weighted return (*EXRET*) may not exhibit a linear relationship with *PROF*. This appears to be the case in the column for *EXRET*. From this one might then incorrectly conclude that the underperformance of low *PROF* firms primarily comes from the lowest *PROF* decile only.

To account for the potential loadings on risk factors, we compute alphas from time series regressions of each excess portfolio return on the excess market return and the size, value, and momentum factors.[22] The estimated four-factor alpha (*ALPHA*) increases with *PROF* more monotonically than the excess returns, and tends to be negative for low profitability portfolios and positive for high profitability portfolios. The zero cost portfolio that goes long the highest profitability firms and short the lowest profitability firms earns a 0.44% average monthly return with a 0.62% risk-adjusted alpha, both of which are statistically significant at the 1% level. This demonstrates that, consistent with Proposition 3, firms with higher profitability earn higher expected returns and that this additional premium cannot be explained by existing risk factors. Another way to control for existing priced factors is to further sort firms by the characteristics to which the risk factors are related. This is the subject of the next two subsections.

3.2.2 Two Dimensional Sort on *BM* and *PROF*

Table 2 presents the characteristics of 25 portfolios formed as the cross section of independently sorted *PROF* and *BM* quintiles. Panel A indicates that average firms in the lowest *PROF* quintile again incur losses. Except for this quintile (and perhaps the highest-*PROF* fourth-largest *BM* portfolio), the level of profitability is controlled fairly well by the independent double sort. Panel B reports average size in million dollars. Firms in the lowest *PROF* quintile tend to be small, especially in growth quintiles. If this has any implication on our result, the size effect will work against us; if high profitability firms tend to be large in size, we would expect them to earn low average returns, rather than high returns implied by Proposition 3. Panel C demonstrates that the independent double sort controls for the book-to-market ratio quite well, as there is little variation in

[22] The four factors are *MKTRF*, *SMB*, *HML*, and *MOM*, respectively, downloaded from Kenneth French's web site.

BM along the columns. The number of stocks in Panel D assures us that each portfolio is well populated on average.

Panel E deserves attention. Excess value-weighted return generally increases in *PROF* controlling for *BM*. The profitability spread, given by the return on a zero cost portfolio that goes long the highest profitability firms and short the lowest profitability firms within a *BM* quintile, monotonically decreases with the level of *BM*. The long-short portfolio yields 0.84% per month among the growth firms, which is significant at the 1% level. On the other hand, the value spread is strongest among low profitability firms, yielding 1.05% per month. Interestingly, the value spread monotonically decreases with the level of *PROF*. The two numbers shown above are economically significant. A legitimate concern is that these spreads may partially reflect the reward for bearing known risks. The four-factor alphas in Panel F control for this possibility. Naturally, the value spread is significantly reduced after taking into account the loadings on the value and other factors. However, the profitability spread barely changes upon risk adjustment; the four-factor alpha of the zero cost profitability portfolio is 0.91% for growth firms. This magnitude of alpha is not only statistically significant (at the 1% level), but also economically significant. Like excess returns, the alpha decreases monotonically with *BM*. For concreteness, the next subsection further controls for size.

3.2.3 Three Dimensional Sort on Size, *BM* and *PROF*

Table 3 reports the characteristics of portfolios formed as the cross section of *SIZE*, *BM*, and *PROF* terciles. To ensure that each portfolio is well populated, stocks are first sorted independently by *SIZE* and *BM* into terciles. Within each of the nine *SIZE*-*BM* intersections, stocks are then sorted by *PROF* into terciles. Altogether this produces a 27 portfolios. For simplicity, we focus on the lowest and highest profitability terciles as we are interested in the profitability spread. Panel A shows the market capitalization of the nine *SIZE-BM* portfolios at the lowest and highest profitability levels. Again, if there is any bias resulting from size, it will work against us because the highest profitability firms tend to be larger than lowest profitability firms, thereby reducing the profitability spread. Again, the book-to-market ratio in Panel B appears to be well controlled. Panel C confirms that the value-weighted profitability spread is highest among small to mid size firms, with the smallest growth portfolio yielding a 0.54% spread, which is statistically significant at 1%. The profitability spreads barely change upon risk adjustment; in fact, Panel D shows that the four-factor alphas within the growth tercile are significant at all size, ranging from 0.33% to 0.60% per month.

3.2.4 Equal Weighting and Fama-MacBeth Cross-sectional Regressions

As a robustness check, the above analysis is repeated using equally weighted portfolios. Since these portfolios tend to put large weights on small firms with extreme returns, we trim returns in the top and bottom five percentiles within each portfolio. Generally, results are stronger for all the three sorting schemes. To save space, we only highlight some of the results here. The equally weighted profitability spread from one dimensional sorting is 0.75% (t = 5.10) with a four-factor alpha of 0.74% (t = 5.02). With double sorting by *PROF* and *BM*, both the profitability spread and the four-factor alpha are significant at 1% across all the *BM* quintiles, with the spread ranging from 0.55% (t = 4.25) to 1.09% (t = 5.40) and the alpha from 0.42% (t = 3.23) to 1.09% (t = 5.62). Finally, using the trimmed sample from the three-dimensional sorting Fama-MacBeth cross-sectional regressions are run. Following standard practice, individual stock returns are regressed on lagged firm characteristics, such as *PROF*, *SIZE*, *BM*, the momentum characteristic measured as the past one-year return skipping a month, and a constant. The time-series average coefficient on *PROF* from the second pass is positive and significant at the 1% level.

Overall, the empirical results presented in this section are consistent with Proposition 3, which says that high profitability firms should earn high returns. This profitability effect cannot be explained by existing risk factors.

4. Conclusion

Traditionally the asset pricing literature has taken the characteristics of the set of corporate assets as given when solving for the equilibrium returns demanded by investors. Recently a number of papers have begun to look at the problem when the characteristics of corporate assets change over time. Articles by Spiegel (1998), Watanabe (2008), Biais, Bossaerts, and Spatt (2008), Pastor and Veronesi (2005), Dittmar and Thakor (2007), Berk, Green and Naik (1999), and Carlson, Fisher, and Giammarino (2004, 2006) all fall into this category. This paper seeks to add to this literature a general equilibrium view of the problem. Both the pricing kernel and the equilibrium supply of productive capital are endogenous in our model. Corporate capital stocks are impacted by both random fluctuations and the investment decisions of firms that add and subtract from their capital base in response to market conditions. In turn, asset prices are determined period by period by risk averse investors via market clearing conditions. The end result is a tractable model that yields a number of empirical predictions, many of which are consistent with the data. Among these are the following:

- Stock returns should be positively correlated with a proxy for profitability of capital, such as the earnings yield on a firm's capital stock.
- Large returns (price moves) in one direction will be followed by a decaying series in the opposite direction.
- Capital expenditures will be negatively correlated with future returns.

Because the CAPM holds, period-by-period in the model, the above relationships regarding returns also hold for period-by-period betas. This, however, also implies that empirical models that do not allow time-varying betas will be incorrectly specified. In particular, the CAPM beta should be modeled as a decreasing function of capital investment.

5. Bibliography

Asquith, Paul, and David Mullins, 1986, "Equity Issues and Offering Dilution," *Journal of Financial Economics*, 15, 61-89.

Baker, Malcolm and Jeffrey Wurgler, 2000, "The Equity Share in New Issues and Aggregate Stock Returns," *Journal of Finance*, 55, 2219-2257.

Baker, Malcolm and Jeffrey Wurgler, 2002, "Market Timing and Capital Structure," *Journal of Finance*, 57, 1-32.

Berk, Jonathan, 1995, "A Critique of Size-Related Anomalies," *Review of Financial Studies*, 8, 275-286.

Berk, Jonathan, Richard Green, and Vasant Naik, 1999, "Optimal Investment, Growth Options, and Security Returns," *Journal of Finance*, 54, 1553-1607.

Bernard, V.L., Thomas, J.K., 1989, "Post-earnings Announcement Drift: Delayed Price Response or Risk Premium?" *Journal of Accounting Research* 27, 1–36.

Biais, Bruno, Peter Bossaerts, and Chester Spatt, 2008, "Equilibrium Asset Pricing and Portfolio Choice Under Asymmetric Information," *Review of Financial Studies*, forthcoming.

Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2004, "Corporate Investment and Asset Price Dynamics: Implications for the Cross-section of Returns," *Journal of Finance*, 59, 2577-2603.

Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2006, "Corporate Investment and Asset Price Dynamics: Implications for SEO Event Studies and Long-Run Performance," *Journal of Finance*, 61, 1009-1034.

Dittmar, Amy and Anjan Thakor, 2007, "Why Do Firms Issue Equity?," *Journal of Finance*, 62, 1-54.

Fama, Eugene F., and Kenneth R. French, 1992, "The Cross Section of Expected Stock Returns," *Journal of Finance*, 47, 427-466.

Fama, Eugene F., and Kenneth R. French, 1993, "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Financial Economics*, 33, 3-56.

Fama, Eugene F., and Kenneth R. French, 2007, "Average Returns, B/M, and Share Issues," *Journal of Finance*, forthcoming.

Hackbarth, Dirk, and Erwan Morellec, 2008, "Stock Returns in Mergers and Acquisitions," *Journal of Finance*, 63, 1213-1252.

Houge, Todd and Tim Loughran, 2006, "Do Investors Capture the Value Premium?," *Financial Management*, 35, 5-19.

Jung, Kooyul, Yong Cheol Kim, and Rene M. Stulz, 1986, "Timing, Investment Opportunities, Managerial Discretion, and the Security Issue Decision," *Journal of Financial Economics*, 42, 159-185.

Gala, Vito, 2005, "Investment and Returns," London Business School working paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=686889.

Gomes, Joao, Leonid Kogan, and Lu Zhang, 2003, "Equilibrium Cross Section of Returns," *Journal of Political Economy*, 111, 693-732.

Kogan, Leonid, 2004, "Asset Prices and Real Investment," *Journal of Financial Economics*, 73, 411-431.

Li, Erica, Dmitry Livdan, and Lu Zhang, 2009, "Anomalies," *Review of Financial Studies*, forthcoming.

Ikenberry, David, Josef Lakonishok, and Theo Vermaelen, 1995, "Market Underreaction to Open Market Share Repurchases," *Journal of Financial Economics*, 39, 181-208.

Lewellen, Jonathan, and Stefan Nagel, 2006, "The Conditional CAPM Does Not Explain Asset-Pricing Anomalies," *Journal of Financial Economics*, 82, 289-314.

Lyandres, Evgeny, Le Sun, and Lu Zhang, 2008, "The New Issues Puzzle: Testing the Investment-Based Explanation," *Review of Financial Studies* 21 (6),2826-2855.

Loughran, Tim, Jay Ritter, and Kristian Rydqvist, 1984, "Initial Public Offerings: International Insights," *Pacific-Basin Finance Journal*, 2, 165-199.

Loughran, Tim, and Jay Ritter, 1995, "The New Issues Puzzle," *Journal of Finance*, 50, 23-51.

Lowry, Michael, 2003, "Why Does IPO Volume Fluctuate So Much?," *Journal of Financial Economics*, 67, 3-40.

McLean, David R., Jeffrey Pontiff, and Akiko Watanabe, 2008, "Share Issuance and Cross-sectional Returns: International Evidence," *Journal of Financial Economics*, forthcoming.

Mikkelson, Wayne H. and Megan Partch, 1986, "Valuation Effects of Security Offerings and Issuance Process," *Journal of Financial Economics*, 15, 31-60.

Pagano, Marco, Fabio Panetta, and Luigi Zingales, 1998, "Why Do Companies Go Public? An Empirical Analysis," *Journal of Finance*, 53, 27-64.

Pastor, Lubos and Pietro Veronesi, 2005, "Rational IPO Waves," *Journal of Finance*, 60, 1713-1757.

Phalippou, Ludovic, 2007, "Institutional Ownership and the Value Premium," working paper: http://ssrn.com/abstract=360760.

Pontiff, Jeffrey, and Artemiza Woodgate, 2008, "Share Issuance and Cross-sectional Returns, *Journal of Finance*, 63 (2), 921-945.

Rajan, Raghuram, and Henri Servaes, 1997, "Analyst Following of Initial Public Offerings," *Journal of Finance*, 52, 507-529.

Ritter, Jay, 1991, "The Long Run Performance of Initial Public Offerings," *Journal of Finance*, 46, 3-27.

Sagi, Jacob S, and Seasholes, Mark S., 2007, "Firm Specific Attributes and the Cross-Section of Momentum," *Journal of Financial Economics*, 84, 389-434.

Spiegel, Matthew, 1998, "Stock Price Volatility in a Multiple Security Overlapping Generations Model," *Review of Financial Studies*, 11, 419-447.

Titman, Sheridan, K. C. John Wei, and Feixue Xie, 2004, "Capital Investments and Stock Returns," *Journal of Financial and Quantitative Analysis*, 39, 677-700.

Watanabe, Masahiro, 2008, "Price Volatility and Investor Behavior in an Overlapping Generations Model with Information Asymmetry," *Journal of Finance*, 63, 229-272.

Xing, Yuhang, 2008, "Interpreting the Value Effect Through the Q-Theory: An Empirical Investigation," *Review of Financial Studies*,21 (4), 1767-1795.

Appendix

5.1 Equivalence between Maximizing Executive Value and Firm Value

The capital base of firm *f* at the end of date-*t* in industry *k* is worth $n_{fk,t}p_{k,t}$ (recall, $n_{fk,t}$ is the capital stock, and $p_{k,t}$ the market price of a capital unit). To get $n_{fk,t}$ units of capital, the firm adds (to its last period's capital base) $\eta_{fk,t}$ units from its employment of labor and $y_{fk,t}$ from management's decision to use additional capital resources. Thus, one has $n_{fk,t} = n_{fk,t-1} + \eta_{fk,t} + y_{fk,t}$. The additional capital units are not free and the model assumes that, due to competitive markets, labor captures the value of its input. For labor this yields a wage of $\eta_{fk,t}p_{k,t}$. Management's decisions adds $\xi_{fk,t} \equiv y_{fk,t}p_{k,t} - c_{1k}y_{fk,t} - \frac{1}{2}c_{2k}y_{fk,t}^2$ to the firm's value. This then equals the management's compensation. Thus, at the end of the period the firm's value equals:

$$\left(n_{fk,t-1} + \eta_{fk,t} + y_{fk,t}\right)p_{k,t} - c_{1k}y_{fk,t} - \frac{1}{2}c_{2k}y_{fk,t}^2 - \eta_{fk,t}p_{k,t} - \xi_{fk,t}. \tag{23}$$

The two $\eta_{fk,t}p_{k,t}$ terms cancel out and $\xi_{fk,t}$ exactly offsets the $y_{fk,t}p_{k,t} - c_{1k}y_{fk,t} - \frac{1}{2}c_{2k}y_{fk,t}^2$ term. Thus, the date-*t* firm value equals $n_{fk,t-1}p_{k,t}$. This figure is not impacted by management's choice of $y_{fk,t}$ because each firm is a price-taker. Therefore, maximizing managerial value equivalently maximizes the firm's market value, Eq. (23), and similarly the overall value to its constituent stakeholders.

It is worth noting that one can drop the assumption that management earns the full or partial value of its input to the firm and not alter the model's qualitative properties.

The important feature required by the model is that firm seeks to add assets when they have a high market value and subtract them when the value is low. Any modeling assumption that leads to this conclusion will yield similar results.

Here, at least, the assumption that labor and management capture the value of their input does keep the model's competitive markets setting consistent across all markets. Additionally, it also helps produce a closed form solution for the financial market's pricing function. In particular it makes the accounting particularly simple. An investor that owns X units of capital at the start of a period will own (in present value terms) the same number of units at the end of the period, since labor and management capture the value from changes to firm's capital base. Investors, by contrast, capture the value from fluctuations in the per unit capital values.

5.2 Derivation of the Equilibrium Conditions

Define $F \equiv \left(I - \left(A_1 - A_2 H\right) C_{2D}^{-1}\right)$. Use (2) and (10) to produce (after some algebraic manipulation):

$$
\begin{aligned}
E_t\left[Q_{t+1}\right] = {} & \left[\left(I - HC_{2D}^{-1}\right) F^{-1} - RI\right] P_t \\
& + \left(I - HC_{2D}^{-1}\right) F^{-1}\left[A_2 G\left(\bar{D} - D_t\right) + (F - I)C_1 - A_2 H N_t\right] \\
& + D_t + G\left(\bar{D} - D_t\right) - H\left(N_t - C_{2D}^{-1} C_1\right).
\end{aligned} \tag{24}
$$

Similarly,

$$
\begin{aligned}
\operatorname{var}_t\left[Q_{t+1}\right] & = \operatorname{var}_t\left[\left(I - HC_{2D}^{-1}\right) F^{-1}\left(\left(A_1 - A_2 H\right)\eta_{t+1} + A_2 \delta_{t+1}\right) - H\eta_{t+1} + \delta_{t+1}\right] \\
& = \left\{ \begin{array}{l} \left[\left(I - HC_{2D}^{-1}\right)\left(F^{-1} - I\right) C_{2D} - H\right] \Sigma_\eta \left[\left(I - HC_{2D}^{-1}\right)\left(F^{-1} - I\right) C_{2D} - H\right]' \\ \quad + \left[\left(I - HC_{2D}^{-1}\right) F^{-1} A_2 + I\right] \Sigma_\delta \left[\left(I - HC_{2D}^{-1}\right) F^{-1} A_2 + I\right]' \end{array} \right\} \\
& \equiv V.
\end{aligned} \tag{25}
$$

To solve for the equilibrium values of the A's, replace $E_t\left[Q_{t+1}\right]$ and $\operatorname{var}_t\left[Q_{t+1}\right]$ in equation (3) with the corresponding terms in equations (24) and (25). The coefficients of N_t and D_t must vanish separately as well as those that do not multiply a time varying parameter. For the terms that multiply neither N_t or D_t this yields,

$$\begin{aligned}&\left[\left(I-HC_{2D}^{-1}\right)F^{-1}-RI\right]A_0+\left(I-HC_{2D}^{-1}\right)F^{-1}\left[A_2G\bar{D}+\left(F-I\right)C_1\right]\\&\quad+G\bar{D}+HC_{2D}^{-1}C_1=0,\end{aligned}\tag{26}$$

while for the terms multiplying N_t,

$$\left[\left(I-HC_{2D}^{-1}\right)F^{-1}-RI\right]A_1-\left(I-HC_{2D}^{-1}\right)F^{-1}A_2H-H-\theta V=0,\tag{27}$$

and finally for the terms multiplying D_t,

$$\left[\left(I-HC_{2D}^{-1}\right)F^{-1}-RI\right]A_2-\left(I-HC_{2D}^{-1}\right)F^{-1}A_2G+I-G=0.\tag{28}$$

The equilibrium values of A_0, A_1, and A_2 can now be found by solving (26)-(28).

5.3 Proofs

Proposition 1: If G is symmetric positive definite and H and $I-G$ are positive definite, in each of the following limits an equilibrium exists in which A_2 is a finite positive definite matrix, while A_1 is a negative definite matrix.

1. $C_{2D}^{-1}\to 0$ for $\|H\|$ sufficiently small.
2. $C_{2D}\to 0$ for $\|H\|$ sufficiently small.
3. Σ_δ, Σ_η, and G approach diagonal matrices and $\|H\|$ is sufficiently small.
4. $\theta\Sigma_\delta\to 0$, $(rI+G)^{-1}H$ is positive definite, and $\|H\|$ is sufficiently small.
5. $\theta\Sigma_\delta\to\infty$.

Proof. Taking the limit $C_{2D}^{-1} \to 0$ while holding A_1 and A_2 finite, one can write equation (27) as $-rA_1 - A_2H - H - \theta V = 0,$ where V is the covariance matrix of excess payoffs defined in equation (25). Because V is positive definite by construction, for $\| H \|$ sufficiently small A_1 is negative definite. Next, taking the limit of (28) as $C_{2D}^{-1} \to 0$, $-rA_2 - A_2G + I - G = 0,$ implying that

$$A_2 = (I - G)(rI + G)^{-1}. \tag{29}$$

Here and throughout the rest of proofs, we will repeatedly use the fact that the product of two symmetric positive definite matrices that share common eigenvectors is symmetric positive definite (consider the spectral decomposition). Because $I - G$ and $rI + G$ are symmetric positive definite and share common eigenvectors (those of G), A_2 is symmetric positive definite.

Now, taking the limit $C_{2D} \to 0$ with A_1 and A_2 finite and $\| H \|$ sufficiently small, it's straight forward to show that Eq. (27) has an approximate solution $A_1 = -\frac{\theta}{R}V,$ which is negative definite. Eq. (28) in this limit becomes $A_2 = \frac{1}{R}(I - G),$ which is positive definite.

To establish case 3, first set H to zero, assume Σ_δ, Σ_η, and G are diagonal, and write equation (27) for the k^{th} diagonal element as $\left(\frac{1}{1 - a_{1,k}/c_{2,k}} - R\right)\frac{a_{1,k}}{c_{2,k}} = \theta\frac{V_{kk}}{c_{2,k}}.$

For $a_{1,k} \in (-\infty, 0)$, $a_{2,k}$ is bounded as is V_{kk}, so a solution to the above equation exists with $a_{1,k} \in (-\infty, 0)$. This proves that there exists an equilibrium in which A_1 is (diagonal) negative definite in this limit. In the same limit, given that $F_{kk} > 1$ and $0 < 1 - G_{kk} < R$, Eq. (28) implies A_2 is (diagonal) positive definite.

To establish the fourth case, first consider the limit $H, \theta\Sigma_\delta \to 0$. We look for a solution for which $A_1 \to 0$. Thus, as $\theta\Sigma_\delta \to 0$ and for H small, we look for a solution to A_1 of the form $A_1 \sim \xi H$. To linear order in H Eq. (27) becomes $-r\xi H - A_2 H - H = 0$, yielding $A_1 = -\frac{1}{r}(A_2 + I)H$. Applying the limit to Eq (28) yields A_2 from Eq. (29), which is positive definite. Plugging back into the expression for A_1, yields $A_1 = -\frac{R}{r}(rI + G)^{-1}H$, where $(rI + G)^{-1}H$ is assumed positive definite.

In the limit $\theta\Sigma_\delta \to \infty$, Eqs. (27) and (28) imply that A_1 diverges while A_2 approaches a finite quantity. In turn, this means that F^{-1} approaches 0. The consequent limiting behavior of Eqs. (27), and (28) can be summarized as

$$\begin{aligned} -RA_1 - H - \theta\left[C_{2D}\Sigma_\eta C_{2D} + \Sigma_\delta\right] = 0, \\ -RA_2 + I - G = 0. \end{aligned} \tag{30}$$

A_2 in Eq. (30) is positive definite, while A_1 approaches a negative definite matrix as $\theta\Sigma_\delta \to \infty$.

■

Proposition 4: If the economy is in steady state at date t then

$$E_t\left[r_{k,t+1} \mid \text{steady state at } t\right] = r + \frac{\theta\{V\bar{N}\}_k}{c_{1k}} = \frac{\breve{d}_k}{c_{1k}} = \frac{\bar{d}_k - \sum_{j=1}^{K} z_{k,j}\bar{n}_j}{c_{1k}} \equiv ss_k,$$

where $\bar{n}_j$ represents the j^{th} element of $\bar{N}$, Z denotes the matrix $G^{-1}H$, with $z_{k,j}$ as its kj^{th} element, and $V \equiv \text{var}_t\left[Q_{t+1}\right]$ is independent of time and explicitly given in Eq. (25), and where $\{\bullet\}_k$ corresponds to the k^{th} element in the vector expression inside the curly bracket.

Proof. The first equality follows from using Eq. (3)write

$$E_t[P_{t+1}+D_{t+1}]=E_t\left[Q_{t+1}\right]+RP_t=\theta\,\text{var}_t\left[Q_{t+1}\right]N_t+RP_t=\theta V\bar{N}+RP_t+\theta V(N_t-\bar{N}), \quad (31)$$

dividing elementwise by P_t, setting the date-t variables to their steady state values, and subtracting 1. The second equality comes from using Eq. (10) in the expression for $E_t[P_{t+1}+D_{t+1}]$, dividing by P_t and setting the date-t variables to their steady state values.

■

Proposition 3: Assume that $E[r^e_{k,t+1}]>0$. In each of the limits described in Proposition 1, an equilibrium exists in which, for every k, holding everything else constant industry k's expected excess returns decrease with $y_{k,t}$ and increase with $d_{k,t}$:

$$\frac{\partial E\,[r^e_{k,t+1}]}{\partial y_{k,t}}<0 \text{ and } \frac{\partial E\,[r^e_{k,t+1}]}{\partial d_{k,t}}>0. \quad (13)$$

Proof. Using Eq. (6), write

$$E_t\left[Q_{t+1}\right]=\theta\,\text{var}_t\left[Q_{t+1}\right]N_t=\theta V\bar{N}+\theta V(N_t-\bar{N})=\theta V\bar{N}+\theta VA_1^{-1}\left(C_{2D}Y_t-A_2(D_t-\breve{D})\right), (32)$$

where we have used

$$N_t-\bar{N}=A_1^{-1}\left((P_t-C_1)-A_2(D_t-\breve{D})\right)=A_1^{-1}\left(C_{2D}Y_t-A_2(D_t-\breve{D})\right). \quad (33)$$

Dividing the k^{th} element of Eq. (32) by $p_{k,t}=c_{2k}y_{k,t}+c_{1k}$ to get the expected excess return,

$$E\,[r^e_{k,t+1}]=\frac{\left\{\theta V\bar{N}+\theta VA_1^{-1}\left(C_{2D}Y_t-A_2(D_t-\breve{D})\right)\right\}_k}{c_{2k}y_{k,t}+c_{1k}}. \quad (34)$$

Taking the partial derivative with respect to $y_{k,t}$ yields

$-c_{2k}\dfrac{\left\{\theta V\bar{N}+\theta VA_1^{-1}\left(C_{2D}Y_t-A_2(D_t-\breve{D})\right)\right\}_k}{(c_{2k}y_{k,t}+c_{1k})^2}+\dfrac{\left\{\theta VA_1^{-1}C_{2D}\right\}_{kk}}{c_{2k}y_{k,t}+c_{1k}}$. The first term is negative as long as $E[r^e_{k,t+1}]>0$ and $p_{k,t} > 0$. It should be clear from the proof of Proposition 1 that the second term is negative in cases 1-3 and 5. For case 4, set H to zero and note that Eq. (27) implies that VA_1^{-1} is negative definite, so this will remain true whenever $\| H \|$ is sufficiently small.

Taking the derivative of (34) with respect to $d_{k,t}$ yields $-\dfrac{\left\{\theta VA_1^{-1}A_2\right\}_{kk}}{c_{2k}y_{k,t}+c_{1k}}$. Following the same line of argument as with $\dfrac{\partial E\,[r^e_{k,t+1}]}{\partial y_{k,t}}$, it is straight forward to show that $VA_1^{-1}A_2$ is negative definite.

■

Proposition 5: To linear order in Y_t and $D_t-\breve{D}$, the expected return on the stock of a firm in industry k is given by:

$$E_t\left[r_{k,t+1}\right]\approx ss_k-\frac{1}{c_{1k}}\left\{\left[(ss_k-r)I-\theta VA_1^{-1}\right]C_{2D}Y_t\right\}_k+\frac{1}{c_{1k}}\left\{(-\theta VA_1^{-1}A_2)(D_t-\breve{D})\right\}_k \quad (35)$$

where $\{\bullet\}_k$ corresponds to the k^{th} element in the vector expression inside the curly brackets and $V\equiv \text{var}_t\left[Q_{t+1}\right]$ is independent of time and explicitly given in Eq. (25).

Proof. Replace $N_t-\bar{N}$ in Eq. (31) with Eq. (33). Dividing the k^{th} component of this by the price $p_{k,t}=c_{1k}+c_{2k}y_{k,t}$, expanding the resulting expression to linear order in deviations from the steady state yields the desired result.

■

Table 1: Portfolios sorted on profitability. This table shows the characteristics of decile portfolios sorted on profitability. The profitability measure, *PROF*, is the ratio of Compustat annual item "Operating Income Before Depreciation" to "Property Plant and Equipment - Total (Gross)." *SIZE* is the average market capitalization of member firms in millions of dollars. *BM* is the average book-to-market ratio, constructed as in Fama and French (1993). *N* is the average number of firms. *EXRET* is the excess value-weighted return with the t-statistic in parentheses. *ALPHA* is the intercept from the time-series regression of the excess portfolio return on the excess market return and the size, value, and momentum factors, with the t-statistic in parentheses. *, **, and *** represent significance at 10, 5, and 1%, respectively. Based on the firm characteristics at the end of fiscal year $t-1$, we form portfolios in June of calendar year t and measure value-weighted monthly returns from July through next June. We use only ordinary common shares on NYSE, AMEX, and NASDAQ of firms in non-financial industries. Only NYSE firms are used to compute breakpoints for ranking. The sample runs from July 1963 through December2008.

PROF rank	*PROF*	*SIZE*	*BM*	*N*	*EXRET*	(t-stat)	*ALPHA*	(t-stat)
1	-1.087	195	1.12	720	0.0003	(0.11)	-0.0033 **	(-2.41)
2	0.100	905	1.20	207	0.0034 *	(1.90)	-0.0018 **	(-2.03)
3	0.130	1,056	1.12	220	0.0045 **	(2.48)	-0.0003	(-0.30)
4	0.164	1,290	1.07	244	0.0042 **	(2.20)	-0.0007	(-0.75)
5	0.205	1,160	0.98	257	0.0055 ***	(2.67)	0.0006	(0.66)
6	0.254	1,010	0.90	254	0.0043 **	(2.11)	0.0002	(0.31)
7	0.313	1,410	0.81	259	0.0029	(1.39)	0.0001	(0.14)
8	0.391	1,730	0.74	268	0.0048 **	(2.37)	0.0019 **	(2.52)
9	0.527	1,582	0.66	303	0.0050 **	(2.41)	0.0032 ***	(4.09)
10	1.704	1,202	0.58	474	0.0047 *	(1.88)	0.0029 ***	(3.69)
10-1					0.0044 ***	(2.65)	0.0062 ***	(3.85)

Table 2: Portfolios sorted on the book-to-market ratio and profitability. This table shows the characteristics of 25 portfolios formed as the cross section of the book-to-market ratio and profitability quintiles. The panels report the following quantities: Panel A: Profitability, the ratio of Compustat annual item "Operating Income Before Depreciation" to "Property Plant and Equipment - Total (Gross)"; Panel B: Size, average market capitalization in millions of dollars; Panel C: The book-to-market ratio, as described in Fama and French (1993); Panel D: The average number of firms; Panel E: Excess value-weighted return; Panel F: The four-factor alpha, computed as the intercept from the time-series regression of the excess portfolio return on the excess market return and the size, value, and momentum factors. *, **, and *** represent significance at 10, 5, and 1%, respectively. Based on the firm characteristics at the end of fiscal year $t-1$, we form portfolios in June of calendar year t and measure value-weighted monthly returns from July through next June. We use only ordinary common shares on NYSE, AMEX, and NASDAQ of firms in non-financial industries. Only NYSE firms are used to compute breakpoints for ranking. The sample runs from July 1963 through December 2008.

Panel A: Productivity

		BM				
		1	2	3	4	5
	1	-1.88	-0.94	-0.41	-0.29	-0.22
	2	0.15	0.15	0.15	0.15	0.15
PROF	3	0.23	0.23	0.23	0.23	0.23
	4	0.36	0.35	0.35	0.35	0.35
	5	1.27	1.03	1.04	1.75	1.17

Panel B: Size ($ million)

		BM				
		1	2	3	4	5
	1	219	355	445	502	276
	2	1,852	2,279	1,547	896	433
PROF	3	2,433	1,499	861	545	396
	4	3,640	1,550	625	375	400
	5	2,538	773	446	362	360

Panel C: Book-to-market ratio

		BM				
		1	2	3	4	5
	1	0.23	0.54	0.77	1.05	2.13
	2	0.28	0.55	0.77	1.05	1.87
PROF	3	0.29	0.54	0.77	1.04	1.80
	4	0.29	0.54	0.76	1.04	1.79
	5	0.26	0.53	0.76	1.04	1.84

Panel D: Number of stocks

		BM 1	2	3	4	5
	1	260	118	116	149	284
	2	52	65	87	114	147
PROF	3	82	105	107	107	111
	4	134	130	109	84	70
	5	336	176	115	82	67

Panel E: Excess returns

		BM 1	2	3	4	5	5-1
	1	-0.0040	0.0003	0.0014	0.0038 **	0.0065 ***	0.0105 ***
	2	0.0021	0.0036 *	0.0052 ***	0.0061 ***	0.0085 ***	0.0064 ***
PROF	3	0.0015	0.0055 ***	0.0064 ***	0.0078 ***	0.0089 ***	0.0074 ***
	4	0.0035 *	0.0041 *	0.0067 ***	0.0069 ***	0.0080 ***	0.0044 **
	5	0.0044 *	0.0072 ***	0.0064 ***	0.0077 ***	0.0082 ***	0.0038 *
	5-1	0.0084 ***	0.0068 ***	0.0050 ***	0.0040 *	0.0017	

Panel F: Four-factor alphas

		BM 1	2	3	4	5	5-1
	1	-0.0058 ***	-0.0044 **	-0.0028 **	-0.0016	-0.0010	0.0048 **
	2	-0.0015	-0.0009	0.0004	-0.0006	0.0002	0.0016
PROF	3	-0.0012	0.0005	0.0013	0.0007	0.0031 *	0.0043 **
	4	0.0013	0.0014	0.0011	0.0005	0.0005	-0.0008
	5	0.0033 ***	0.0022 **	0.0013	0.0017	0.0006	-0.0028
	5-1	0.0091 ***	0.0065 ***	0.0041 **	0.0033 *	0.0015	

Table 3: Portfolios sorted on size, the book-to-market ratio, and profitability. This table shows the characteristics of portfolios formed as the cross section of the size, book-to-market ratio, and profitability terciles. Profitability is measured by the ratio of Compustat annual item “Operating Income Before Depreciation” to “Property Plant and Equipment - Total (Gross).” The panels report the following quantities: Panel A: Size, average market capitalization in millions of dollars; Panel B: The book-to-market ratio, as described in Fama and French (1993); Panel C: The value-weighted return on a zero cost portfolio that goes long highest profitability firms and short lowest profitability firms; Penal D: The four-factor alpha, computed as the intercept from the time-series regression of the zero cost portfolio return on the excess market return and the size, value, and momentum factors. *, **, and *** represent significance at 10, 5, and 1%, respectively. Based on the firm characteristics at the end of fiscal year $t-1$, we form portfolios in June of calendar year t and measure value-weighted monthly returns from July through next June. We use only ordinary common shares on NYSE, AMEX, and NASDAQ of firms in non-financial industries. Only NYSE firms are used to compute breakpoints for ranking. The sample runs from July 1963 through December 2008.

Panel A: Size ($ million)

(i) Low profitability portfolios

		SIZE 1	2	3
	1	58	531	6,676
BM	2	57	560	4,994
	3	36	566	3,351

(ii) High profitability portfolios

		SIZE 1	2	3
	1	94	538	8,410
BM	2	79	525	4,879
	3	55	506	6,134

Panel B: Book-to-market ratio

(i) Low profitability portfolios

		SIZE 1	2	3
	1	0.29	0.36	0.39
BM	2	0.79	0.80	0.80
	3	2.02	1.62	1.49

(ii) High profitability portfolios

		SIZE 1	2	3
	1	0.33	0.29	0.26
BM	2	0.76	0.72	0.71
	3	1.57	1.35	1.33

Panel C: Zero-cost portfolio returns (high - low profitability)

		SIZE 1	2	3
	1	0.0054 ***	0.0032 **	0.0018
BM	2	0.0015	0.0033 ***	0.0015
	3	0.0028 **	0.0014	0.0001

Panel D: Four-factor alphas on zero-cost portfolios (high - low profitability)

		SIZE 1	2	3
	1	0.0060 ***	0.0033 **	0.0036 ***
BM	2	0.0018	0.0030 **	0.0019
	3	0.0030 **	0.0012	0.0008