Midterm Exam II Math 212 Section 4

February 28,2008

Instructions: You have **3 hours** to complete the exam. Write the time you start and end on the exam, along with the honor code. You may **NOT** use any books, notes, calculator, or other people. Do not discuss the exam with anyone except your instructor until all exams are turned in.

There are 7 questions, plus one bonus. Show all work to receive full credit. Partial credit will be given, so it is best to turn in all work. Indicate final answer clearly and draw a picture will be a plus. The exam will be due Thursday, March 13rd, 11am. No late exams will be accepted.

Good luck and have a nice spring break!

Formulas you can use:
1. Cylindrical coordinates: $x = rcos\theta$, $y = rsin\theta$,$z = z$, $0 \leq r$, $0 \leq \theta \leq 2\pi$.
2. Spherical coordinates: $x = \rho sin\phi cos\theta$, $y = \rho sin\phi sin\theta$, $z = \rho cos\phi$, $0 \leq \rho$,$0 \leq \phi \leq \pi$,$0 \leq \theta \leq 2\pi$.

Honor code:

1.Evaluate $\int_0^2 \int_{x^3}^8 x^5 e^{y^3} dydx$
Hint: Change order of integration.

2.Calculate the volume of the region under the graph of $f(x,y) = ysinx$ and on the region bounded by $y = 0; y = cosx; x = 0; x = \pi/2$.

3.Find the volume enclosed by the cylinder $x^2 + y^2 = 4$, bounded below by the plane $z = 0$ and above by $z = x^2 + y^2$ (Hint: what coordinates should you use?).

4. Set up the integral to calculate the volume enclosed by the surfaces $x^2 + y^2 + z^2 = 2$ and $x^2 + y^2 = z$. Evaluate for extra credit.

5.A tank is in the shape of a half-cylinder of radius 2 and height 3. It is situated in R^3 by the following inequalities:
$\sqrt{x^2 + y^2} \leq 2, y \geq 0, 0 \leq z \leq 3$
The temperature at a point inside the tank is given by $T(x,y,z) = 2yz^2\sqrt{x^2 + y^2}$.

(a) What is volume of the tank? You can do this in multiple ways.
(b) What is $\iiint_S T(x,y,z)dxdydz$ where S is the tank? (Hint: change to cylindrical coordinates).
(c) Calculate the average temperature inside the tank, i.e. $\frac{answer(b)}{answer(a)}$

6.Evaluate

$$\iiint_W \frac{z^2}{x^2 + y^2 + z^2} dxdydz$$

where W is the solid shell bounded by the two spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$

7.Consider the region D bounded between the circle $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$, $x \geq 0, y \geq 0$.
(a) Write the double integral of a general function f(x, y) over the region D as an iterated integral over x and y.
(b) Write an equivalent integral after changing to polar coordinates
(c) Calculate

$$\iint_D \frac{sin(\pi\sqrt{x^2 + y^2})}{\sqrt{x^2 + y^2}} dA$$

8(**bonus**)Find the volume of the ellipsoid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$$

(Hint: Change coordinates twice, first by a suitable scaling of each variable)