Convertible Debt and Investment Timing*

Evgeny Lyandres[†]· Alexei Zhdanov[‡]

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Abstract

In this paper we provide an investment-based explanation for the popularity of convertible debt. Specifically, we demonstrate the ability of convertible debt to alleviate and potentially totally eliminate the underinvestment problem of Myers (1977). The conversion feature induces shareholders to accelerate investment. This effect arises from the incentive of equity holders to accelerate the issuance of new equity, used to finance investment. By investing early, shareholders dilute the value of convertible debt holders by reducing their proportional claims to the firm's cash flows. Since the underinvestment effect and the accelerated investment effect work in opposite directions, convertible debt allows, in many cases, to achieve the first-best investment strategy. In addition, we show that by choosing a right combination of straight and convertible debt, shareholders can, for a wide range of overall debt levels, commit to the first-best investment strategy.

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†Jesse H. Jones Graduate School of Management, Rice University, Houston TX 77005, lyandres@rice.edu, tel: (713) 348-4708.

‡School of Management, George Mason University, Fairfax, VA 22030, azhdanov@gmu.edu, tel: (703) 993-1866.

1 Introduction

Convertible debt occupies an important niche within firms' capital structures. For example, Stein (1992) reports that in more than 10% of Compustat firms convertible debt accounts for more than a third of total debt. Korkeamaki and Moore (2004) identify close to 4,000 convertible debt issues during a 17-year period 1980-1996.

Several theories have been proposed in the literature to explain why firms issue convertible debt. Stein (1992) argues that firms can issue convertible debt as an indirect way to issue ("backdoor equity"), while mitigating the information asymmetry problems associated with issuing common equity.[1] Mayers (1998) argues that convertible debt is advantageous for a firm with a multi-stage investment project. Just like straight debt, it helps control the free cash flow problem of Jensen (1986). Unlike straight debt, it does not require the firm to raise costly external funds to finance the second stage of the investment project.

In this paper we propose and examine an additional explanation for why convertible debt may prove superior to straight debt in certain cases. Our arguments are based on an idea that convertible debt helps to alleviate or even totally eliminate the underinvestment problem of Myers (1977), pertinent to straight debt. The reason is that while the debt-like features of a convertible debt contract result in underinvestment incentives, just like in the case of straight debt, the presence of the conversion option gives rise to an opposing accelerated investment effect. In a dynamic setting, the possibility of conversion provides the equity holders with an incentive to speed up the exercise of an investment opportunity, or "overinvest", if this opportunity is to be financed with external equity. By investing earlier, when the value of equity is lower, the equity holders are able to dilute the value accruing to the holders of convertible debt when (and if) they finally convert their claims into equity. Note that this effect is not related to (and not inconsistent with) the incentives of equity holders to "time the market" when issuing new equity. Market timing occurs in times of over-valuation (actual or perceived). In our model, equity is always correctly priced, and market timing considerations are beyond the scope of this paper.

[1] Nyborg (1995) argues that the benefit of convertible debt as means of obtaining delayed equity is limited, since in order to serve this purpose the conversion should be voluntary. In reality, however, many convertible bonds have call provisions, allowing issuing firms to force conversion.

We discuss various effects that convertible debt has on investment decisions. We first show that for a certain level of convertible debt these effects completely offset each other, resulting in the shareholders choosing the first-best investment policy. Then we analyze a scenario in which a firm has both straight and convertible debt in its capital structure and show that for a wide range of debt levels there exists a mix of straight and convertible debt that results in the first-best investment choices. This finding can serve as a potential explanation for why many firms choose to issue both straight and convertible debt. Straight debt may have stronger advantages (i.e., tax benefits), while the right amount convertible debt can offset the agency costs of straight debt (due to inefficient investment policy), while still providing certain benefits (i.e., tax benefits until the conversion option is exercised).

Our paper provides an investment-based explanation for issuing convertible debt. In a related work, Green (1984) argues that convertible debt has the potential to mitigate the asset substitution problem of Jensen and Meckling (1976). Unlike the payoff to the equity holders of a firm that issues straight debt, the payoff to the shareholders of a firm with outstanding convertible debt is not always convex in the value of the firm's assets, and, therefore, the equity holders have lesser incentives to engage in risk-shifting activities. While both in Green's model and in our model convertible debt serves to reduce or completely eliminate the agency costs of debt, there is a big difference between the two models. Green examines shareholders' risk shifting incentives, while our focus is on other agency problems among a firm's claim holders, in particular Myers' (1977) underinvestment problem. We purposely abstract from the effects of risk shifting (asset substitution) by assuming that new investment has the same risk as the firm's existing assets.

Many convertible debt contracts contain call provisions, allowing firms to call their debt if certain conditions are satisfied. For example, Korkeamaki and Moore (2004) report that only 13 out of 705 convertible bonds in their sample do not have a call provision.[2] Therefore, we incorporate call provisions into out analysis. Including

[2] Note, however, that most convertible debt contracts include call protection provisions. Korkeamaki and Moore (2004) categorize call protection into "soft" and "hard" groups. The former allow equity holders to call the debt contingent on a certain pattern of stock price behavior, while the latter prohibit calls for a certain period of time (up to five years and longer). Hard protection considerably reduces shareholders' ability to exercise their call options. Thus, convertible debt with

the possibility of calling debt does not change our main conclusions.[3] We show that both callable and non-callable convertible debt contracts are able to alleviate and, in some cases, completely eliminate the underinvestment problem of Myers (1977), and result in first-best investment strategies. We find that in general the ability of callable convertible debt to mitigate the underinvestment problem by providing the offsetting overinvestment incentives is lower than that of non-callable convertible debt. However, callable debt can lead to a situation in which all debt is called before investment, automatically leading to the first-best investment policy. Our conclusion is that regardless of whether convertible debt is callable or not, it allows firms to reduce the distortion in shareholders' investment incentives. Therefore, financial managers have a considerable degree of flexibility in their choice of financial instruments that can solve the problem of inefficient investment. Both callable and non-callable convertible debt can serve this purpose, as well as certain combinations of convertible and straight debt.

Our model is built in continuous time and relates to several articles in the literature. In a similar setting, Leland (1994) examines corporate debt values and capital structure in a unified analytical framework and derives closed-form results for the value of debt and yield spreads and for optimal capital structure. Anderson and Sundaresan (1996) construct a discrete time model capable of pricing various types of debt contracts. Fan and Sundaresan (2000), and Francois and Morellec (2002) incorporate the possibility of renegotiation between claimholders. The latter article is also designed to account for legal liquidation and reorganization procedures in the US. An interesting work by Morellec (2001) focuses on the role of secured debt as the device preventing the firm from curtailing its assets in economic recessions. Mauer and Triantis (1994) examine the interaction between dynamic investment and financing decisions in a model where a firm is endowed with an initial investment option, operating options, and the ability to recapitalize over time. Mauer and Sarkar (2005) examine a specific case of the asset substitution problem arising when a firm's debtholders commit to the terms of a debt contract in advance, prior to the actual issuance of debt.

hard protection resembles debt with no call provision at all. In that case our analysis of the case of non-callable convertible debt can also be applied to convertible debt with hard protection.

[3] Due to the technical nature of the problem we delegate it to the Appendix.

The remainder of the paper is organized as follows. The next section develops a framework for examining shareholders' investment incentives under a variety of capital structure scenarios. While our ultimate goal is to examine the investment incentives arising from issuing convertible debt, for the purpose of the clarity of exposition and in order to enable comparative statics analysis we also discuss investment policies of an all-equity firm, as well as a firm with only straight debt outstanding (and no convertible debt). The results of the model and their implications are presented in Section 3. Section 4 summarizes our findings and concludes. Appendix 1 provides a justification for one of the simplifying assumptions of the model. Appendix 2 outlines the solution of the model in which convertible debt is converted into equity before the firm's investment option is exercised. The case of callable convertible debt is examined in Appendix 3.

2 The model

We model the effects of convertible debt on a firm's investment choices in an environment that is sufficiently simple to ensure closed-form expressions for the values of the firm's securities, yet rich enough to capture the dynamics of the firm's investment decisions under various capital structure scenarios. In the most general case, we consider a firm whose instantaneous profit is given by

$$\pi(x_t) = x_t - s_c - s_s, \tag{1}$$

where x_t is the firm's instantaneous EBIT, s_c is the instantaneous contractual coupon payment made to the holders of the firm's convertible debt with infinite maturity, and s_s is the instantaneous coupon paid to the holders of straight debt with infinite maturity. We assume that x_t follows a geometric Brownian motion:

$$\frac{dx_t}{x_t} = \mu dt + \sigma dW_t,$$

where W_t is a standard Brownian motion defined on a probability space (Ω, F, P).

We assume that if debt has the conversion option, it can be converted into a fraction η of the original equity, where $\eta = \alpha s_c$, α being a constant. This implies that if there are N shares of common stock originally outstanding, a convertible debt contract with \$1 contractual coupon payment can be converted into αN shares of equity. As mentioned in the introduction, we assume here that convertible debt contracts do not include a call provision. We analyze the case of callable convertible debt in Appendix 3 and find that callability does not affect the main conclusions of the model.

To model the firm's growth option we assume that at any time the shareholders can increase the firm's EBIT by a fraction $(a-1)$, where $a>1$ is a constant, by paying a fixed irreversible investment cost I and financing the investment by issuing new equity. The post-investment instantaneous profit is then given by

$$\pi(x_t) = ax_t - s_c - s_s. \tag{2}$$

As we discuss in detail below, the presence of convertible debt changes the shareholders' investment incentives in several ways. Some of those effects are also at work when the firm has straight debt outstanding instead of convertible debt, but some effects are unique to convertible debt and are caused by the presence of the conversion option. In order to disentangle various effects and provide meaningful comparative statics results, we consider several settings before proceeding to our ultimate goal of examining the effect of convertible debt on investment. First, we establish a benchmark model in which the firm is financed entirely with equity. Second, we model the firm that has straight debt (but no convertible debt) outstanding to study the investment distortions caused by straight debt.[4] Third, we examine the case in which the firm has convertible debt (but no straight debt) in its capital structure and analyze the unique effects on investment caused by the conversion option. Within that scenario we first examine the optimal conversion policy of a firm with no growth opportunities and then proceed to the more general case in which both convertible debt and an investment option are in place. Finally, we nest the two debt financing cases and examine the effects of a combination of convertible debt and straight debt on investment timing and the agency costs of inefficient investment. In this section we outline the solution of the model for different scenarios. We present the analysis of comparative statics in Section 3.

[4]This case is based on the analysis in Lyandres and Zhdanov (2005), who investigate various effects of straight debt on optimal investment timing under different assumptions regarding the financing of new investment.

2.1 Equity financing

The all-equity case (corresponding to zero coupon payment to straight and convertible debt holders, $s_c = 0$ and $s_s = 0$) has been extensively studied in the real options literature (see, for example, McDonald and Siegel (1986) or Dixit and Pyndick (1994)). In this case the optimization problem of the equity holders can be formalized as follows:

$$E(x_0) = \sup_{T_{x^*}>0} \mathbb{E}^{x_0} \left[\int_0^{T_{x^*}} e^{-rt} x_t dt - e^{-rT_{x^*}} I + \int_{T_{x^*}}^{\infty} e^{-rt} a x_t dt \right], \tag{3}$$

where $\mathbb{E}^{x_0}$ is the expectation operator, and T_{x^*} is a stopping time (the time at which the investment option is exercised). The standard result is that the optimal investment rule is to exercise the growth option at the first passage time of the stochastic shock to an upper threshold, x^*, given by

$$x^* = \frac{I\beta_1}{\beta_1 - 1} \frac{r - \mu}{a - 1}, \tag{4}$$

where r is the risk-free discount rate, β_1 is the positive root of the quadratic equation $\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0$,

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left[\frac{1}{2} - \frac{\mu}{\sigma^2}\right]^2 + \frac{2r}{\sigma^2}}, \tag{5}$$

and other variables are defined as above.

2.2 Equity and straight debt financing

Here we consider the case of a firm that has straight debt in its capital structure in addition to equity, but no convertible debt ($s_c = 0$ and $s_s > 0$). This case establishes a useful benchmark for analyzing the investment incentives caused by the conversion option. As before, the firm is endowed with an investment opportunity. We assume that the shareholders neither repay their debt nor change the coupon payment when they decide to exercise their investment option. Thus, the investment is financed entirely by issuing new equity.

When the firm has debt in its capital structure, the shareholders' optimal investment strategy maximizes the value of equity. Such strategy takes the form of the optimal thresholds of two types: 1) the investment threshold, x^*, and 2) two default thresholds, x_d (the pre-investment default threshold), and $x_{d,i}$ (the post-investment default threshold). The optimization problem of the shareholders can be stated as

$$E(x_0) = \sup_{T_{x^*},T_{x_d},T_{x_{d,i}}>0} \mathbb{E}^{x_0}[\int_0^{\min(T_{x^*},T_{x_d})} e^{-rt}\left[x_t - s_s\right]dt + \\ 1_{T_{x^*}<T_{x_d}}[-e^{-rT_{x^*}}I + \int_{T_{x^*}}^{T_{x_{d,i}}} e^{-rt}\left[ax_t - s_s\right]dt]], \quad (6)$$

where T_{x^*}, T_{x_d}, and $T_{x_{d,i}}$ are the stopping times upon reaching thresholds x^*, x_d, and $x_{d,i}$, respectively, $1_{T_{x^*}<T_{x_d}}$ is an indicator function that equals one if $T_{x^*} < T_{x_d}$ and equals zero otherwise. Note that in (6), the last term is positive only if $T_{x^*} < T_{x_d}$, i.e. if the optimal investment threshold is hit before the optimal default one. In the other case, when $T_{x^*} > T_{x_d}$, the value of the investment option is lost from the equity holders' perspective.

The optimal post-investment default threshold, $x_{d,i}$, is not equal to the pre-investment one, x_d. There are two reasons for that. First, instantaneous EBIT changes as a result of investment. Second, once the investment option is exercised, it does not affect the value of the option to default anymore. Both factors influence the optimal default policy of the firm.

The Bellman equation for a firm that has not exercised its investment option yet, corresponding to the optimization problem (6) has the following form:

$$rE(x_t) = x_t - s_s + \frac{1}{dt}\mathbb{E}^{x_t}(dE(x_t)). \quad (7)$$

Equation (7) states that the instantaneous rate of return on equity equals the instantaneous cash flows to equity holders plus the expected instantaneous change in the value of equity. Equation (7) is equivalent to the following ODE:

$$\frac{1}{2}x^2\sigma^2 E_{xx}(x_t) + \mu x E_x(x_t) + x_t - s_s - rE(x_t) = 0, \quad (8)$$

the solution to which is given by

$$E(x_t) = Ax_t^{\beta_1} + Bx_t^{\beta_2} + \frac{x_t}{r-\mu} - \frac{s_s}{r}, \quad (9)$$

where β_1 is given in (5), β_2 is the negative root of the quadratic equation $\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0$:

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left[\frac{1}{2} - \frac{\mu}{\sigma^2}\right]^2 + \frac{2r}{\sigma^2}}, \quad (10)$$

and A and B are constants to be determined below.

Once the investment option has been exercised, the optimal default policy is established. Using standard arguments (see, for example, Dixit and Pindyck (1994)), it is straightforward to show that the optimal post-investment default threshold is given by

$$x_{d,i} = \frac{\beta_2}{\beta_2 - 1} \frac{s_s [r - \mu]}{ar}, \tag{11}$$

and the value of existing equity at the time when the investment option is exercised is

$$E(x^*) = \frac{ax^*}{r-\mu} - \frac{s_s}{r} - \left[\frac{x^*}{x_{d,i}}\right]^{\beta_2} \left[\frac{ax_{d,i}}{r-\mu} - \frac{s_s}{r}\right] - I. \tag{12}$$

In (12), the first two terms represent the value of the perpetual entitlement to the profits of the firm net of the present value of interest payments, the third term is the value of the option to default, and the fourth one is the cost of investment.

Differential equation (8) must be solved subject to a number of boundary conditions. These conditions are as follows:

$$A [x^*]^{\beta_1} + B [x^*]^{\beta_2} = \frac{[a-1]\, x^*}{r-\mu} - \left[\frac{x^*}{x_{d,i}}\right]^{\beta_2} \left[\frac{ax_{d,i}}{r-\mu} - \frac{s_s}{r}\right] - I, \tag{13}$$

$$\beta_1 A [x^*]^{\beta_1 - 1} + \beta_2 B [x^*]^{\beta_2 - 1} = \frac{[a-1]}{r-\mu} - \frac{\beta_2 [x^*]^{\beta_2 - 1}}{[x_{d,i}]^{\beta_2}} \left[\frac{ax_{d,i}}{r-\mu} - \frac{s_s}{r}\right], \tag{14}$$

$$A [x_d]^{\beta_1} + B [x_d]^{\beta_2} + \frac{x_d}{r-\mu} - \frac{s_s}{r} = 0, \tag{15}$$

$$\beta_1 A [x_d]^{\beta_1 - 1} + \beta_2 B [x_d]^{\beta_2 - 1} + \frac{1}{r-\mu} = 0. \tag{16}$$

Substituting x^* into x_t in (9) and equalizing (9) to (12) results in (13), which is the value matching condition that ensures that the value of equity at the optimal investment threshold is equal to the new equity value net of the investment cost. On the right hand side of (13), the first term is the increase in the value of the entitlement to the perpetual cash flows of the firm due to the exercise of the growth option, and the second term is the value of the post-investment option to default. Similarly, equating (9) to zero results in (15), which is the value matching condition that requires that the value of equity at the default threshold be zero. Equations (14) and (16) are the smooth-pasting conditions that ensure the optimality of the default and investment thresholds. Equations (13)-(16) present a system of four equations in four unknown variables (A, B, x^*, and x_d; the post-investment default threshold, $x_{d,i}$, is given in (11)). This system must be solved numerically.

2.3 Equity and convertible debt financing

2.3.1 No investment opportunity

Before analyzing the effects of convertible debt on the shareholders' investment incentives, we examine the simplest case of a firm (without an investment opportunity) that has convertible debt in its capital structure. In this case the firm's claim holders are faced with two optimization programs, which they solve simultaneously. The shareholders optimally select the (lower) default threshold, while the holders of convertible debt choose the (upper) conversion threshold. Note that the optimal conversion policy does not imply that the debt holders should convert as soon as the conversion option is in the money. By converting they make an irreversible decision and forsake the stream of coupon payments. Therefore, the value of the conversion option must be taken into account by the debt holders. The optimal conversion occurs when the value of their claim if converted to equity (i.e., the stream of dividends) becomes equal to the present value of coupon payments plus the value of the option to convert. This logic is similar to the one applied in the case of an all-equity firm endowed with an investment opportunity, discussed above. Because of the irreversibility of investment, the firm exercises its investment opportunity only when it is strictly in the money (the NPV of the investment project is strictly positive), since by making the investment the firm not only exposes itself to a fixed investment cost, but also abandons the investment option. The irreversibility of the conversion option leads to a similar intuition. The optimal conversion policy is discussed in more details in Appendix 1.

To simplify our analysis, we follow Brennan and Schwartz (1977) and assume block conversion, implying that all convertible debt holders exercise their conversion option at the same time. Incorporating partial conversion in the model would make it technically intractable. However, we show in Appendix 1 that block conversion is not an unrealistic assumption. Assuming that the conversion game is played by a number of infinitesimally small debt holders, simultaneous conversion by all of them is one of the Nash equilibria of the game.

The optimization program of the equity holders of a firm with outstanding convertible debt and no investment option reads:

$$E(x_0) = \sup_{T_{x_d}>0} \mathbb{E}^{x_0}[\int_0^{\min(T_{x_c},T_{x_d})} e^{-rt}\left[x_t - s_c\right] dt + 1_{T_{x_c}<T_{x_d}} \frac{1}{\eta+1} \int_{T_{x_c}}^{\infty} e^{-rt} x_t dt], \quad (17)$$

where T_{x_d} is the stopping time upon reaching the default threshold, x_d, and T_{x_c} is the stopping time upon reaching the conversion threshold, x_c, which is chosen by the debt holders. Their optimization problem has the following form:

$$D_c(x_0) = \sup_{T_{x_c}>0} \mathbb{E}^{x_0}[\int_0^{\min(T_{x_c},T_{x_d})} e^{-rt} s_c dt + 1_{T_{x_c}<T_{x_d}} \frac{\eta}{\eta+1} \int_{T_{x_c}}^{\infty} e^{-rt} x_t dt + 1_{T_{x_d}<T_{x_c}} \epsilon(x_d)], \quad (18)$$

where $\epsilon(x_d)$ is the abandonment value of the firm, which accrues to convertible debt holders in the event of default. We further assume that the abandonment value equals to the value of the unlevered firm net of the proportional bankruptcy cost, θ,

$$\epsilon(x_d) = [1-\theta] \frac{x_d}{r-\mu}, \quad (19)$$

where $0 \leq \theta \leq 1$.

It follows from the optimization programs of the shareholders and convertible debt holders in (17) and (18) respectively that the values of the firm's convertible debt and equity prior to default and conversion are given by

$$D_c(x_t) = A x_t^{\beta_1} + B x_t^{\beta_2} + \frac{s_c}{r} \quad (20)$$

and

$$E(x_t) = C x_t^{\beta_1} + F x_t^{\beta_2} + \frac{x_t}{r-\mu} - \frac{s_c}{r}. \quad (21)$$

Constants A, B, C, and F, together with the optimal default and conversion thresholds, x_d and x_c, have to be determined using the following set of boundary conditions:

$$A\left[x_c\right]^{\beta_1} + B\left[x_c\right]^{\beta_2} + \frac{s_c}{r} = \frac{\eta}{1+\eta} \frac{x_c}{r-\mu}, \quad (22)$$

$$\beta_1 A [x_c]^{\beta_1 - 1} + \beta_2 B [x_c]^{\beta_2 - 1} = \frac{\eta}{1+\eta} \frac{1}{r-\mu}, \tag{23}$$

$$A [x_d]^{\beta_1} + B [x_d]^{\beta_2} + \frac{s_c}{r} = [1-\theta] \frac{x_d}{r-\mu}, \tag{24}$$

$$C [x_d]^{\beta_1} + F [x_d]^{\beta_2} + \frac{x_d}{r-\mu} - \frac{s_c}{r} = 0, \tag{25}$$

$$\beta_1 C [x_d]^{\beta_1 - 1} + \beta_2 F [x_d]^{\beta_2 - 1} + \frac{1}{r-\mu} = 0, \tag{26}$$

$$C [x_c]^{\beta_1} + F [x_c]^{\beta_2} + \frac{x_c}{r-\mu} - \frac{s_c}{r} = \frac{1}{1+\eta} \frac{x_c}{r-\mu}. \tag{27}$$

Equation (22) is the value matching condition that ensures that the value of debt at the optimal conversion threshold is equal to the value of the newly issued equity. It is obtained by equating the value of convertible debt in (20) to the proportion of the unlevered firm accruing to the debt holders after conversion. In order to ensure that at the time of default the value of convertible debt equals the value of all-equity firm net of bankruptcy costs, we equate (20) to the abandonment value of the firm given in (19) and obtain (24). Equations (25) and (27) are the value matching conditions requiring that the value of equity in default be zero, while the value of equity upon reaching the conversion threshold be equal to the value of the fraction of the firm owned by the original equity holders immediately after conversion. Equations (23) and (26) are the smooth-pasting conditions that ensure the optimality of the conversion and default thresholds. Equations (22)-(27) present a system of six equations in six unknown variables (A, B, C, F, x_c, and x_d). This system must be solved numerically.

2.3.2 Investment Opportunity

We now proceed to the more general case in which the shareholders of a firm that has convertible debt (but no straight debt) on it balance sheet are endowed with an investment option. There are two possible scenarios. In the first one the equity holders exercise their investment option before the debt holders decide to convert their claims into equity. In the second scenario the bondholders exercise their conversion option first, and the firm becomes an all-equity entity, whose optimal investment strategy was established in subsection 2.1. In the second case the issuance of convertible debt does not lead to any investment distortions and results in the first-best investment policy because by the time of investment all debt is converted into equity. Here we focus on the first case, i.e. the one in which investment precedes conversion. However, it is important to analyze the second case as well, in order to ensure that the assumption of investment preceding conversion is optimal. To do that, for all combinations of the model's parameters that we examine, we calculate the convertible debt holders' value conditional on optimal conversion under each of the two scenarios and choose the scenario in which this value is higher. Since investment distortions can occur only in the case in which investment precedes conversion, we delegate the complementary case to Appendix 2.

Let x^* be the optimal investment threshold. As in the case of straight debt, we assume that the shareholders issue new equity to finance their investment opportunity. If the equity value upon reaching x^* (and immediately before the exercise of the investment option) is $E(x^*)$, and the investment in the amount I is required, then the fraction of new equity, issued to finance investment, out of total equity is $\gamma = \frac{I}{I+E(x^*)}$. Therefore, once the investment option is exercised, the holders of convertible debt are only entitled to a fraction $\eta_i = \frac{\eta}{1+\gamma}$ of the total equity. Note that the optimal conversion threshold x_c depends on γ and, therefore, on the existing equity value at the investment threshold, $E(x^*)$, while the latter depends, in turn, on the subsequent conversion policy. Therefore the optimal investment and conversion strategies must be determined jointly by examining the optimization programs of the firm's claim holders.

The optimization problem of the shareholders (conditional on conversion not occurring before the investment option is exercised) takes the following representation:

$$E(x_0) = \sup_{T_{x^*}, T_{x_d}, T_{x_{d,i}} > 0} \mathbb{E}^{x_0}[\int_0^{\min(T_{x^*}, T_{x_d})} e^{-rt}\left[x_t - s_c\right] dt + 1_{T_{x^*} < T_{x_d}}[[-e^{-rT_{x^*}} I + \int_{T_{x^*}}^{\min(T_{x_c}, T_{x_{d,i}})} e^{-rt}\left[a x_t - s_c\right] dt] + 1_{T_{x_c} < T_{x_{d,i}}} \frac{1}{\eta_i + 1} \int_{T_{x_c}}^{\infty} e^{-rt} a x_t dt]], \quad (28)$$

where T_{x^*} is the stopping time upon reaching the optimal investment threshold, x^*, T_{x_d} is the stopping time upon reaching the pre-investment default threshold, $x_d, T_{x_{d,i}}$ is the stopping time upon reaching the post-investment default threshold, $x_{d,i}$, T_{x_c} is the stopping time upon reaching the conversion threshold, x_c, (selected by the debt holders), and η_i is the fraction of total equity that accrues to convertible debt holders upon conversion,

$$\eta_i = \frac{\eta}{1 + \frac{I}{I+E(x^*)}}. \tag{29}$$

Note that the strategy of the equity holders is given by a triplet (x^*, x_d, $x_{d,i}$), consisting of two lower default thresholds, x_d and $x_{d,i}$, and one upper investment threshold, x^*. The bondholders' optimization program reads:

$$\begin{aligned} D_c(x_0) = \sup_{T_{x_c}>0} \mathbb{E}^{x_0}[\int_0^{\min(T_{x^*},T_{x_d})} e^{-rt} s_c dt + 1_{T_{x_d}<T_{x^*}} e^{-rT_{x_d}} \epsilon(x_d) + \\ 1_{T_{x^*}<T_{x_d}}[\int_{T_{x^*}}^{\min(T_{x_c},T_{x_{d,i}})} e^{-rt} s_c dt + \\ 1_{T_{x_c}<T_{x_{d,i}}} \frac{\eta_i}{\eta_i+1} \int_{T_{x_c}}^{\infty} e^{-rt} x_t dt + 1_{T_{x_{d,i}}<T_{x_c}} e^{-rT_{x_{d,i}}} \epsilon(x_{d,i})]]. \end{aligned} \tag{30}$$

To jointly solve the shareholders' and convertible debt holders' problems in (28) and (30), we start from the moment at which the investment option is exercised and work backwards to determine the optimal investment threshold. Similar to (21), the post-investment value of debt equity is given by

$$E(x_t) = Cx_t^{\beta_1} + Fx_t^{\beta_2} + \frac{ax_t}{r-\mu} - \frac{s_c}{r}, \tag{31}$$

while the value of convertible debt is given in (20):

$$D_c(x_t) = Ax_t^{\beta_1} + Bx_t^{\beta_2} + \frac{s_c}{r}.$$

After the investment has been made, the equity holders optimally select the default threshold, $x_{d,i}$, while the debt holders choose the optimal timing of conversion by selecting the conversion threshold, x_c. The set of boundary conditions is as follows:

$$A[x_c]^{\beta_1} + B[x_c]^{\beta_2} + \frac{s}{r} = \frac{\eta_i}{1+\eta_i} \frac{ax_c}{r-\mu}, \tag{32}$$

$$\beta_1 A[x_c]^{\beta_1-1} + \beta_2 B[x_c]^{\beta_2-1} = \frac{\eta_i}{1+\eta_i} \frac{a}{r-\mu}, \tag{33}$$

$$A[x_{d,i}]^{\beta_1} + B[x_{d,i}]^{\beta_2} + \frac{s_c}{r} = [1-\theta] \frac{ax_{d,i}}{r-\mu}, \tag{34}$$

$$C[x_{d,i}]^{\beta_1} + F[x_{d,i}]^{\beta_2} + \frac{ax_{d,i}}{r-\mu} - \frac{s_c}{r} = 0, \tag{35}$$

$$\beta_1 C\left[x_{d,i}\right]^{\beta_1 - 1} + \beta_2 F\left[x_{d,i}\right]^{\beta_2 - 1} + \frac{a}{r-\mu} = 0, \tag{36}$$

$$C\left[x_c\right]^{\beta_1} + F\left[x_c\right]^{\beta_2} + \frac{a x_c}{r-\mu} - \frac{s}{r} = \frac{1}{1+\eta_i}\frac{a x_c}{r-\mu}, \tag{37}$$

where η_i is given in (29) and $E(x^*)$ is the value of old equity at the investment threshold (that depends on the subsequent conversion policy of the debt holders):

$$E(x^*) = C\left[x^*\right]^{\beta_1} + F\left[x^*\right]^{\beta_2} + \frac{a x^*}{r-\mu} - \frac{s_c}{r} - I. \tag{38}$$

Equation (32) is the value matching condition that ensures that the value of convertible debt at the optimal conversion threshold is equal to the value of the proportion of the firm owned by convertible debt holders after conversion. Similarly, (34) is the value matching condition that ensures that the value of debt at the post-investment default threshold is equal to the abandonment value of the firm net of the proportional bankruptcy cost. Equations (35) and (37) are the value matching conditions postulating that the value of equity in default is zero, while the value of equity upon reaching the conversion threshold is equal to the value of the fraction of the firm owned by the original equity holders after conversion. Equations (33) and (36) are the smooth-pasting conditions that ensure the optimality of the conversion and default thresholds. Equations (32)-(37), along with (29) and (38) present a system of eight equations in eight unknown variables (A, B, C, F, x_c, $x_{d,i}$, η_i, and $E(x^*)$). Note that the investment threshold, x^*, is treated as pre-determined at this stage and not as a decision variable.

There is an important difference between the current problem and the one corresponding to the case with no investment opportunity, represented by the system of equations (22)-(27). Now there is a "feedback loop" in the system, represented by (29) and (38). The value of equity at the investment threshold determines the conversion ratio, which, in turn, affects the optimal conversion policy, and, therefore, influences the equity value at the time of investment.

We now turn to the original problem of the equity holders. Their objective is to maximize the value of equity by optimally choosing the pre-investment default threshold, x_d, and the investment threshold, x^*. Once the investment option is exercised (i.e., if the pre-investment default threshold is not reached before the investment threshold), the problem transforms to the one discussed above.

Similar to (21), the value of equity before investment is given by

$$E(x_t) = Gx_t^{\beta_1} + Hx_t^{\beta_2} + \frac{x_t}{r-\mu} - \frac{s_c}{r}, \tag{39}$$

where G and H are some constants. The appropriate value-matching conditions are:

$$G\,[x^*]^{\beta_1} + H\,[x^*]^{\beta_2} + \frac{x^*}{r-\mu} - \frac{s_c}{r} = E(x^*), \tag{40}$$

$$G\,[x_d]^{\beta_1} + H\,[x_d]^{\beta_2} + \frac{x_d}{r-\mu} - \frac{s_c}{r} = 0, \tag{41}$$

postulating that the value of equity at the investment threshold equals to the share of the post-investment equity accruing to old shareholders, and that the value of equity in default is zero.

Solving (40) and (41) for G and H, plugging the resulting expressions into (39) and rearranging gives the following pre-investment value of equity:

$$E(x_0) = \max_{x^*,x_d}[\frac{x_0}{r-\mu} - \frac{s_c}{r} + p_{i\ before\ d}[E(x^*) - \frac{x^*}{r-\mu} + \frac{s_c}{r}] - p_{d\ before\ i}[\frac{x_d}{r-\mu} - \frac{s_c}{r}]], \tag{42}$$

where

$$p_{i\ before\ d} = \frac{[x_d]^{\beta_2}\,[x_0]^{\beta_1} - [x_d]^{\beta_1}\,[x_0]^{\beta_2}}{[x_d]^{\beta_2}\,[x^*]^{\beta_1} - [x_d]^{\beta_1}\,[x^*]^{\beta_2}}, \tag{43}$$

is the present value of one dollar to be received at the first passage time of the stochastic shock x to x^*, conditional on x staying above x_d, $\min x > x_d$, and

$$p_{d\ before\ i} = \frac{[x^*]^{\beta_1}\,[x_0]^{\beta_2} - [x^*]^{\beta_2}\,[x_0]^{\beta_1}}{[x_d]^{\beta_2}\,[x^*]^{\beta_1} - [x_d]^{\beta_1}\,[x^*]^{\beta_2}}, \tag{44}$$

is the present value of one dollar to be received at the first passage time of the stochastic shock x to x_d, conditional on x staying below x^*, $\max x < x^*$.

2.4 Equity, straight debt, and convertible debt financing

In this subsection we focus on a general case in which a firm has both straight and convertible debt outstanding. For simplicity reasons, we assume here that straight and convertible debt have the same priority. Thus, in the event of default, the holders of straight and convertible debt are entitled to payoffs of $\frac{s_s}{s_s+s_c}\epsilon(x)$ and $\frac{s_c}{s_s+s_c}\epsilon(x)$ respectively, where $\epsilon(x)$ is the value of the firm at bankruptcy. This assumption is not critical for the subsequent analysis, and other types of seniority can also be incorporated and would lead to qualitatively similar results.

The optimization programs of the equity holders and the holders of convertible debt admit similar representations to (28) and (30), with an adjustment for the coupon that has to be paid to the holders of straight debt. We, therefore, proceed directly to formulating the values of convertible debt and equity. As in the previous subsection, we focus on the case in which conversion happens after investment. The case in which conversion precedes investment is delegated to Appendix 2. Similar to (20)-(21), the value of equity is given by

$$E(x_t) = Cx_t^{\beta_1} + Fx_t^{\beta_2} + \frac{ax_t}{r-\mu} - \frac{s_s + s_c}{r}, \tag{45}$$

while the value of convertible debt is given in (20):

$$D_c(x_t) = Ax_t^{\beta_1} + Bx_t^{\beta_2} + \frac{s_c}{r},$$

where A, B, C, and F are constants to be determined jointly with the optimal conversion, investment, and default thresholds. Note that the equity holders commit to making a total instantaneous payment of $s_c + s_s$ (in the numerator of the last term on the right-hand side in (45)). Once the investment option has been exercised, the equity holders optimally select the post-investment default threshold, $x_{d,i}$, while the convertible debt holders choose the optimal exercise timing of their conversion option by selecting the conversion threshold, x_c, (conditional on not having converted before).

The first boundary condition, corresponding to (32) in the case of no straight debt, is the value matching condition that equates the value of convertible debt at the conversion threshold to the value of the fraction of equity accruing to convertible debt holders:

$$A\left[x_c\right]^{\beta_1} + B\left[x_c\right]^{\beta_2} + \frac{s_c}{r} = \frac{\eta_i}{1+\eta_i}E(x_c), \tag{46}$$

where η_i is the fraction of equity accruing to the convertible debt holders upon conversion, given in (29):

$$\eta_i = \frac{\eta}{1 + \frac{I}{I+E(x^*)}},$$

and $E(x^*)$ is the value of equity immediately before the exercise of the investment option, which in the presence of straight debt equals

$$E(x^*) = C\,[x^*]^{\beta_1} + F\,[x^*]^{\beta_2} + \frac{ax^*}{r-\mu} - \frac{s_s + s_c}{r} - I. \tag{47}$$

$E(x_c)$ is similar to (12) and equals the value of equity of a firm with only straight debt outstanding:

$$E(x_c) = \frac{ax_c}{r-\mu} - \frac{s_s}{r} - \left[\frac{x_c}{x_{d,i}}\right]^{\beta_2} \left[\frac{ax_{d,i}}{r-\mu} - \frac{s_s}{r}\right], \tag{48}$$

where $x_{d,i}$ is the optimal post-conversion (and post-investment) default threshold, given in (11).

The second value matching condition equates the value of convertible debt at the default threshold with the corresponding fraction of the abandonment value:

$$A\,[x_{d,i}]^{\beta_1} + B\,[x_{d,i}]^{\beta_2} + \frac{s_c}{r} = [1-\theta]\,\frac{s_c}{s_s + s_c}\frac{ax_{d,i}}{r-\mu}. \tag{49}$$

The following smooth-pasting condition ensures the optimality of the conversion threshold, x_c :

$$\beta_1 A\,[x_c]^{\beta_1 - 1} + \beta_2 B\,[x_c]^{\beta_2 - 1} = \frac{\eta_i}{1+\eta_i}\left[\frac{a}{r-\mu} - \frac{\beta_2\,[x_c]^{\beta_2 - 1}}{[x_{d,i}]^{\beta_2}}\left[\frac{ax_{d,i}}{r-\mu} - \frac{s_s}{r}\right]\right]. \tag{50}$$

The other three conditions (two value matching and one smooth-pasting) correspond to the value of equity and were discussed above:

$$C\,[x_{d,i}]^{\beta_1} + F\,[x_{d,i}]^{\beta_2} + \frac{ax_{d,i}}{r-\mu} - \frac{s_s + s_c}{r} = 0 \tag{51}$$

$$\beta_1 C\,[x_{d,i}]^{\beta_1 - 1} + \beta_2 F\,[x_{d,i}]^{\beta_2 - 1} + \frac{a}{r-\mu} = 0 \tag{52}$$

$$C\,[x_c]^{\beta_1} + F\,[x_c]^{\beta_2} + \frac{ax_c}{r-\mu} - \frac{s_s + s_c}{r} = \frac{1}{1+\eta_i}E(x_c) \tag{53}$$

As before, the equity holders choose the optimal investment threshold, x^*, together with the pre-investment default threshold, x_d, with the objective to maximize the value of equity. The pre-investment value of equity can be obtained along the lines of the solution of the model in the previous subsection and is given by the following expression:

$$E(x_0) = \max_{x^*, x_d}[\frac{x_0}{r-\mu} - \frac{s_s + s_c}{r} + p_{i\ before\ d}[E(x^*) - \frac{x^*}{r-\mu} + \frac{s_s + s_c}{r}] - p_{d\ before\ i}[\frac{x_d}{r-\mu} - \frac{s_s + s_c}{r}]], \quad (54)$$

where $p_{i\ before\ d}$ and $p_{d\ before\ i}$ are given in (43) and (44).

3 Results and discussion

In this section we present solutions to the variants of the model of the previous section and discuss various effects of convertible debt on the shareholders' investment incentives.

3.1 Convertible debt without straight debt

Figure 1 plots the optimal investment threshold as a function of instantaneous coupon payment for the case in which the firm has convertible debt (but no straight debt) outstanding and for the case in which it has straight debt (but no convertible debt). The following set of parameter values was used to produce Figure 1: $\alpha = 2.5$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $a = 2$, $I = 5$.[5] The solid line depicts the optimal investment threshold for the case of convertible debt, while the dashed line represents the investment threshold for the case of straight debt. The dotted line is the (first-best) investment trigger of an all-equity firm. For each value of the coupon payment we check that the debt holders do not have an incentive to convert prior to the exercise of the investment option, whose timing is selected by the equity holders. (The derivation of the values of the firm's securities in the case when conversion precedes investment is presented in Appendix A.2.)

Insert Figure 1 here

Figure 1 reveals that there is a non-monotonic relation between the optimal investment threshold, x^*, and the contractual coupon payment on convertible debt,

[5] Here and below, the results are insensitive to the choice of parameter values, unless explicitly stated otherwise.

s_c. For moderate values of the coupon payment, investment is accelerated relative to the value-maximizing all-equity case. Thus, for the set of parameter values used to produce Figure 1, while $s < 0.34$ the optimal investment threshold is below the all-equity one and the equity holders have an incentive to speed up the exercise of the investment opportunity. We call the range $0 < s_c < 0.34$ the "overinvestment" region. For values of s_c exceeding 0.34 the shareholders' investment incentives reverse. In that region the optimal investment threshold is above its value in the all-equity case, and the investment opportunity is delayed relative to the first-best, leading to "underinvestment". In figure 1 $s_c > 0.34$ is the "underinvestment" region.

Unlike convertible debt, straight debt always leads to underinvestment, as long as the exercise of the investment opportunity is financed with equity. The dashed line in Figure 1 clearly shows that straight debt increases the optimal investment threshold and, therefore, delays the exercise of the investment option. Also, as is evident from Figure 1, even in the region where both straight and convertible debt lead to underinvestment, the incentives to underinvest caused by convertible debt are smaller in magnitude than those caused by straight debt. For all values of the contractual coupon payment the optimal investment threshold corresponding to convertible debt always lies below the one resulting from the issuance of straight debt.

Several effects lead to the results presented in Figure 1. Some are common to straight and convertible debt, while others arise because of the presence of the conversion option. Specifically, there are four different effects at work. The first one is the underinvestment effect of Myers (1977). When the firm's shareholders exercise their investment option, they increase the firm's operating cash flow, and, therefore, reduce the probability of default and increase the value of debt (regardless of whether it is straight or convertible). Thus, a fraction of the NPV of the investment project is captured by the firm's debt holders. In other words, the investment facilitates a wealth transfer from the firm's equity holders to the debt holders. This wealth transfer reduces the attractiveness of the investment opportunity from the perspective of the shareholders and delays the exercise of the growth option.

The second effect is the "overinvestment" effect of Lyandres and Zhdanov (2005). In short, this effect is caused by the fact that the equity holders of a firm in default lose their investment opportunity. Thus, the presence of debt (straight or convertible) makes the option to wait less valuable. This tilts the trade-off between investing and starting receiving cash flows from investment, and waiting to invest when the investment opportunity reaches a higher value, in the direction of exercising the investment option sooner, and forces the equity holders to speed up investment.[6] In a similar setting Lyandres and Zhdanov (2005) show, however, that when the investment is financed entirely with equity (and when all outstanding debt is straight debt), the underinvestment effect dominates the overinvestment effect, and the resulting relation between the investment threshold and the coupon payment is positive, as demonstrated by the dashed line in Figure 1.

The two remaining effects are pertinent to convertible debt only. The first one comes from the dilution of the claims of convertible debt holders occurring because of the issuance of new equity, required to finance the investment opportunity. The cost of investment, I, is fixed. The lower the value of the stochastic shock at which the growth option is exercised (the lower the investment threshold), the lower the value of equity at the investment threshold and the higher the number of new shares issued to finance investment. Higher number of new shares leads to lower η_i, the fraction of the value of the total firm that accrues to convertible debt holders upon the exercise of their conversion option. (It follows directly from (29) that η_i is an decreasing function of $E(x^*)$, and, therefore, a decreasing function of x^*.) By investing earlier, the equity holders are able to expropriate wealth from convertible debt holders by ensuring that when (and if) the latter finally convert, they will get a lower fraction of the total equity value. This leads to an incentive to speed up investment.

Another difference between straight debt and convertible debt is that the conversion option reduces the magnitude of both Myers' (1977) underinvestment effect and Lyandres and Zhdanov's (2005) overinvestment effect. Recall that both effects are caused by the possibility of bankruptcy. In the case of convertible debt, it is possible that the stochastic shock is going to reach the conversion threshold (at which the firm becomes an all-equity entity) before reaching the default threshold. This reduces the probability of reaching the default threshold while still having debt outstanding, reducing the magnitudes of both the underinvestment and the overinvestment effects.

The four effects discussed above depend differently on the magnitude of the con-

[6] For a detailed discussion see Lyandres and Zhdanov (2005).

tractual coupon payment. Both the underinvestment effect of Myers (1977) and the overinvestment effect of Lyandres and Zhdanov (2005) are increasing in the probability of default. Both effects disappear when debt is riskless. When the coupon payment (either s_c or s_s) is low, debt is relatively safe and the probability of default is negligible, so these two effects are second-order in magnitude. For small values of coupon payment the "dilution" effect of convertible debt that leads to accelerated investment dominates. Therefore, for moderate values of coupon payment, the net effect of convertible debt on investment is positive, and the presence of convertible debt leads to accelerated investment or overinvestment, in contrast with the effect of straight debt on investment. However, when the coupon payment becomes large, the underinvestment effect becomes more substantial because of the higher probability of default. When convertible debt coupon is sufficiently high, the net underinvestment effect is strong enough to dominate the "dilution" effect that leads to accelerated investment. In that region ($s_c > 0.34$ in Figure 1) convertible debt leads to underinvestment, though the underinvestment incentives associated with convertible debt are not as strong as those associated with straight debt.

One of the important parameters of a convertible debt contract is the conversion ratio, η. We now proceed to examine the effects of η on the shareholders' investment incentives. The magnitude of the "dilution" effect, as well as that of Myers' (1977) underinvestment (net of accelerated investment) effect depend on the conversion ratio. The higher the conversion ratio, the more equity-like the convertible debt, and, therefore, the stronger the dilution effect. In addition, as argued above, the higher the probability of conversion, the lower the probability of reaching the default threshold while still having debt outstanding, and the weaker the net underinvestment effect. To illustrate this, Figure 2 depicts the relation between the investment threshold, coupon payment to convertible debt holders for the set of input parameters used in Figure 2 and three different values of the conversion ratio: $\alpha = 1.25$ (solid line), $\alpha = 1.8$ (dash-dotted line), and $\alpha = 2.5$ (dotted line).

Insert Figure 2 here

As expected, the magnitude of the accelerated investment effect is decreasing in the conversion ratio. In addition, conversion ratio is positively related to the optimal level of convertible debt (i.e., the one that implies the first-best investment policy).

When $\alpha = 1.25$ the dilution effect exactly offsets the underinvestment incentives for $s_c = 0.22$ (compared with $s_c = 0.29$ for $\alpha = 1.8$ and $s_c = 0.34$ for $\alpha = 2.5$). This suggests that there is an additional degree of freedom available to the managers of the firm: different values of the conversion ratio lead to different optimal levels of convertible debt (from the investment perspective). This finding is certainly useful if there are non-investment-related considerations that influence capital structure decisions (such as the tax benefits of debt or its ability to alleviate the free cash flow problem). By setting the leverage ratio of their firm at the target and varying the conversion ratio, financial managers are able to commit to the optimal investment policy and, therefore, preserve the total value of the firm.

The analysis above has interesting implications. First, as discussed above, there is a certain level of convertible debt (corresponding to the coupon payment of 0.34 for the set of parameter values used in Figure 1) that leads to first-best investment strategy. If the convertible debt coupon is set at the optimal level, the net investment distortion is zero. (This is not the case with straight debt. There is no positive level of straight debt that leads to first-best investment.) By issuing convertible debt, the equity holders can exploit the benefits of debt (e.g. tax shields on interest payments prior to conversion), while committing to value-maximizing investment policy.

Second, even in the case in which convertible debt leads to underinvestment (which happens when the convertible debt coupon is high enough), the underinvestment incentives caused by it are not as strong as those caused by straight debt, reducing the overall agency costs of debt.

Finally, since for relatively low values of s_c convertible debt leads to accelerated investment, while straight debt leads to underinvestment (as long as the investment is financed entirely by issuing equity), there has to be an optimal portfolio of straight and convertible debt, so that the two opposite effects exactly offset each other. By issuing a certain combination of convertible and straight debt, the equity holders would be able to commit to first-best investment strategy while choosing the level of total debt (straight and convertible) that maximizes the firm value with respect to parameters exogenous to our model (e.g., tax benefits versus expected bankruptcy costs). We explore this implication in the next subsection.

3.2 Convertible debt with straight debt

Here we analyze the case in which the firm's capital structure includes both convertible and straight debt, the solution to which is outlined in subsection 2.4. In our setting, moderate amounts of convertible debt lead to overinvestment because of the shareholders' incentive to invest earlier and dilute the value of the claims of convertible debt holders by doing so. On the other hand, as long as new investment is entirely financed by equity, straight debt always leads to underinvestment and delays the optimal investment decision. Both effects are costly for the firm as they lead to inefficient investment decisions and reduce the value of the investment option. This reduction in value is incorporated in the price of debt, so as typical for most agency conflicts, the shareholders bear the (ex-ante) cost of sub-optimal investment policy. It is therefore in their interest to commit to efficient investment and preserve the value of their claims. Our argument is that they can commit to the optimal exercise of the investment option by issuing certain proportions of convertible and straight debt, such that the accelerated investment effect of convertible debt is exactly offset by the underinvestment effect of straight debt. This offers a potential explanation of why many firms choose to issue a combination of convertible and straight bonds. By itself each of those securities leads to a deviation from the optimal investment policy, while if taken together in certain proportions they result in the first-best investment policy. The equity holders are, therefore, able to enjoy the benefits of debt (i.e. the tax shields) while reducing or even totally eliminating the associated agency costs.

Figure 3a plots the optimal investment thresholds as functions of total (convertible and straight) coupon, for firms with different levels of convertible debt. The dashed line in Figure 3a represents the investment threshold of a firm with straight debt only. The solid line refers to a firm having a convertible coupon of 0.10, while the dashed-dotted line depicts the threshold of a firm with a convertible coupon of 0.20. The dotted line represents the (first-best) investment threshold of an all-equity firm. The following set of input parameters was used to produce Figure 3a: $\alpha = 2.5$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $a = 2$, $I = 5$.

Insert Figure 3a here

There is one point on both the solid and the dashed-dotted lines in Figure 3a at which the underinvestment and dilution (or accelerated investment) effects neutralize each other, and the combination of straight debt and convertible debt results in the first-best investment policy. For different amounts of convertible debt (corresponding to coupon payments of 0.10 and 0.20) there are different amounts of total debt that lead to the first-best investment. Thus, for a firm with $s_c = 0.10$, the optimal total coupon payment to the holders of straight debt is $s_s = 0.255$, while for a firm with $s_c = 0.2$ it is 0.309. Note that in both cases it is possible to ensure the first-best policy by appropriately choosing the weights of straight and convertible debt. However, it may not be possible for certain values of the coupon payment to convertible debt holders. As Figure 1 suggests, when $s_c = 0.34$, the optimal amount of straight debt is zero. For any value of s_c exceeding 0.34, the presence of convertible debt leads to underinvestment. Since in our setting straight debt always leads to underinvestment, issuing more straight debt can only aggravate the underinvestment problem. Likewise, there is a maximum amount of straight debt that can be "offset" by issuing convertible debt. If the amount of straight debt is too high, issuing convertible debt may not fully solve the underinvestment problem. It may, however, considerably mitigate it by providing the equity holders with the offsetting accelerated investment incentives.

Figure 3b presents the optimal proportion of straight debt out of total debt that results in the first-best investment policy as a function of the coupon payment to convertible debt holders. The set of input parameters is the same as in Figure 3a.

Insert Figure 3b here

As expected, the optimal fraction of straight debt decreases with convertible debt coupon. This happens because the underinvestment incentives of convertible debt increase as its value increases. (In other words, the marginal net accelerated investment effect is decreasing with each additional unit of convertible debt. This is evident from the convex shape of the solid line in Figure 1.) On the other hand, the marginal net underinvestment incentives of straight debt increase with each additional unit of straight debt (as evident from the increasing and convex dashed line in Figure 1). When the coupon payment to convertible debt holders reaches the value of 0.34, convertible debt alone leads to the first-best investment, so the optimal amount of straight debt zero. For $s_c > 0.34$ it is impossible to achieve the first-best policy by combining straight and convertible debt, but having 100% convertible debt minimizes the agency costs of debt.

It is important to emphasize that our model does not predict a certain fixed optimal proportion of convertible and straight debt. As suggested in Figure 3, different combinations of coupon payments, s_c and s_s, may lead to the first-best investment policy. The managers of the firm, therefore, have some flexibility in choosing the weights of convertible and straight debt[7]. Other factors, such as the tax advantages of debt could play a role in determining the optimal portfolio of the firm's securities. Our message is that by including convertible debt in this portfolio, the managers are able to substantially mitigate and in some cases completely eliminate the agency costs of debt associated with inefficient investment.

4 Conclusions

We propose and examine a new explanation for why firms issue convertible debt. In our model convertible debt helps to alleviate or even totally eliminate the underinvestment problem of Myers (1977), arising from the issuance of straight debt. The reason is that while the debt-like features of a convertible debt contract result in underinvestment incentives, just like in the case of straight debt, the presence of the conversion option gives rise to an opposing accelerated investment effect. In a dynamic setting, the possibility of conversion provides the equity holders with an incentive to speed up the exercise of an investment opportunity, or "overinvest". By investing earlier, when the value of equity is lower, the equity holders are able to dilute the value accruing to the holders of convertible debt once they convert their claims into equity.

We discuss various effects of convertible debt on investment and show that for a certain level of convertible debt these effects completely offset each other, resulting in the shareholders choosing the first-best investment policy. In a scenario in which a firm issues both straight and convertible debt, for a wide range of total debt levels there exists a combination of the two types of debt that leads to first-best investment choices. This finding can potentially explain why many firms choose to issue both straight and convertible debt. Straight debt may have stronger advantages (i.e., tax benefits), while the right amount of convertible debt may offset the agency costs of

[7] As discussed above, the managers have an additional degree of freedom to manipulate investment incentives - the conversion ratio.

straight debt (arising because of an inefficient investment policy), while still providing certain benefits (i.e., tax benefits until the conversion option is exercised). Our arguments hold also in the case when convertible debt is callable. We show that while call provisions reduce the accelerated investment incentives of convertible debt, they can lead to a situation in which all debt is called before the investment option is exercised, resulting in the first-best investment policy.

A Appendix

A.1 Why block conversion is an equilibrium?

In this section we show that block conversion, assumed in the analysis of Sections 2 and 3, is one of the Nash equilibria of a game in which infinitesimally small bondholders decide whether to convert or to wait. We start by solving for the block conversion threshold for different levels of convertible debt coupon. Figure 4 plots the optimal conversion thresholds as functions of the coupon payment, s_c, for the following set of parameter values: $\alpha = 2.5$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $a = 2$, $I = 5$, $s_s = 0$.

Insert Figure 4 here

There are two different conversion thresholds presented in Figure 4. The block conversion threshold, represented by the solid line, corresponds to the case in which all convertible debt holders exercise their options to convert simultaneously, resulting in block conversion. The other threshold, represented by the dashed line, is the optimal conversion threshold of the marginal bondholder. It corresponds to the case in which all bondholders but the last one have converted their claims into equity, and provides the optimal conversion threshold resulting from the optimization program of that marginal bondholder.

As Figure 4 demonstrates, optimal conversion thresholds are increasing functions of the coupon payment. The reason is simple. A higher debt level leads to a higher probability of default, and makes the conversion option less valuable, since the debt holders have a higher priority claim on the assets of the firm in default. Convertible debt holders are willing to wait longer before converting their claims into equity and, therefore, abandoning their rights to the assets of the firm in default.[8]

The fact that the marginal conversion threshold lies below the block conversion threshold is important for our analysis and supports the existence of an equilibrium

[8] The optimal conversion does not occur immediately when the conversion option moves into the money, i.e., at a point where the present value of dividends to be received if the claim is converted into equity equals the present value of coupon payments. Since the conversion decision is irreversible, it is always optimal to convert when that option is strictly in the money. The option to wait is valuable and is taken into account just like the option to wait in the similar case of investment under uncertainty. (See McDonald and Siegel (1986).)

when all bondholders convert simultaneously. Assume that all small convertible debt holders decide to convert their debt at the block conversion threshold, depicted by the solid line in Figure 4. Once all bondholders except for one convert, the conversion threshold of the remaining bondholder is the marginal threshold, represented by the dashed line in Figure 4, which is lower than the block conversion threshold for any $s_c > 0$. Thus, no bondholder would rationally want to deviate from an equilibrium in which everybody converts at the block conversion threshold. Therefore, block conversion constitutes a Nash equilibrium, albeit possibly not a unique one. This supports the validity of our assumption of block conversion.

A.2 Conversion preceding investment

In this section we derive the values of the firm's securities for the case in which the holders of convertible debt exercise their conversion option before the investment is undertaken. We focus on two different cases considered in Section 2: the case without straight debt, and the case in which both straight and convertible debt are present.

A.2.1 Convertible debt only

If the bondholders convert before investment, then the post-conversion investment problem transforms into the all-equity case, analyzed in subsection 2.1. Therefore, the value of equity after conversion is

$$E(x_t) = \frac{x_t}{r-\mu} + \left[\frac{x_t}{x_{eq}^*}\right]^{\beta_1} \left[\frac{[a-1]\, x_{eq}^*}{r-\mu} - I\right], \tag{A.1}$$

where the optimal all-equity investment threshold, x_{eq}^*, is given in (4). The values of convertible debt and equity before the conversion option is exercised are given in (20)-(21):

$$E(x_t) = Cx_t^{\beta_1} + Fx_t^{\beta_2} + \frac{x_t}{r-\mu} - \frac{s_c}{r},$$

and

$$D_c(x_t) = Ax_t^{\beta_1} + Bx_t^{\beta_2} + \frac{s_c}{r}.$$

The set of the appropriate value matching and smooth-pasting conditions is slightly different from that in subsection 2.3, in which the assumption was that conversion occurs after investment:

$$A\left[x_c\right]^{\beta_1}+B\left[x_c\right]^{\beta_2}+\frac{s_c}{r}=\frac{\eta}{1+\eta}\left[\frac{x_c}{r-\mu}+\left[\frac{x_c}{x_{eq}^*}\right]^{\beta_1}\left[\frac{(a-1)x_{eq}^*}{r-\mu}-I\right]\right], \tag{A.2}$$

$$\beta_1 A\left[x_c\right]^{\beta_1-1}+\beta_2 B\left[x_c\right]^{\beta_2-1}=\frac{\eta}{1+\eta}\left[\frac{1}{r-\mu}+\frac{\beta_1\left[x_c\right]^{\beta_1-1}}{\left[x_{eq}^*\right]^{*\beta_1}}\left[\frac{\left[a-1\right]x_{eq}^*}{r-\mu}-I\right]\right], \tag{A.3}$$

$$A\left[x_d\right]^{\beta_1}+B\left[x_d\right]^{\beta_2}+\frac{s_c}{r}=\left[1-\theta\right]\frac{x_d}{r-\mu}, \tag{A.4}$$

$$C\left[x_d\right]^{\beta_1}+F\left[x_d\right]^{\beta_2}+\frac{x_d}{r-\mu}-\frac{s_c}{r}=0, \tag{A.5}$$

$$\beta_1 C\left[x_d\right]^{\beta_1-1}+\beta_2 F\left[x_d\right]^{\beta_2-1}+\frac{1}{r-\mu}=0, \tag{A.6}$$

$$C\left[x_c\right]^{\beta_1}+F\left[x_c\right]^{\beta_2}+\frac{x_c}{r-\mu}-\frac{s_c}{r}=\frac{1}{1+\eta}\left[\frac{x_c}{r-\mu}+\left[\frac{x_c}{x_{eq}^*}\right]^{\beta_1}\left[\frac{\left[a-1\right]x_{eq}^*}{r-\mu}-I\right]\right]. \tag{A.7}$$

The intuition behind (A.2)-(A.7) is similar to that behind (32)-(37). Equations (A.2) - (A.7) jointly determine the values of the optimal conversion threshold, x_c, and the optimal default threshold, x_d, together with the four unknowns, A, B, C, and F.

A.2.2 Both straight and convertible debt

Here we consider the case in which both straight and convertible debt are outstanding and the conversion decision precedes the investment one. Once the conversion option is exercised, the problem converts to the case of a firm with an investment option and straight debt. This case was considered in subsection 2.2, and the value of equity is given by the solution to (6). Denote the value of equity in this case as E^*.

Before conversion (and investment), two different optimization problems are solved jointly – the equity holders choose the optimal default threshold, x_d, while the holders of convertible debt select the optimal conversion threshold, x_c. In equilibrium it must be optimal for the equity holders to default at x_d if the debt holders convert at x_c and vice versa. Formally, the optimization problems of the equity holders and the holders of convertible debt read:

$$E(x_0)=\max_{x_d}\Big[\frac{x_0}{r-\mu}-\frac{s_c}{r}+p_{c\ before\ d}\Big[\frac{1}{1+\eta}E^*-\frac{x_c}{r-\mu}+\frac{s_c}{r}\Big] \\ -p_{d\ before\ c}\Big[\frac{x_d}{r-\mu}-\frac{s_c}{r}\Big]\Big] \tag{A.8}$$

$$D_c(x_0) = \max_{x_c}[\frac{s_c}{r} + p_{c\ before\ d}[\frac{\eta}{1+\eta}E^*(s_s) - \frac{s_c}{r}] + p_{d\ before\ c}[\epsilon(x_d) - \frac{s_c}{r}]], \quad (A.9)$$

where

$$p_{c\ before\ d} = \frac{[x_d]^{\beta_2}[x_0]^{\beta_1} - [x_d]^{\beta_1}[x_0]^{\beta_2}}{[x_d]^{\beta_2}[x_c]^{\beta_1} - [x_d]^{\beta_1}[x_c]^{\beta_2}}, \quad (A.10)$$

is the present value of one dollar to be received at the first passage time of the stochastic shock x to x_c, conditional on x staying above x_d, $\min x > x_d$, and

$$p_{d\ before\ c} = \frac{[x_c]^{\beta_1}[x_0]^{\beta_2} - [x_c]^{\beta_2}[x_0]^{\beta_1}}{[x_d]^{\beta_2}[x_c]^{\beta_1} - [x_d]^{\beta_1}[x_c]^{\beta_2}}, \quad (A.11)$$

is the present value of one dollar to be received at the first passage time of the stochastic shock x to x_d, conditional on x staying below x_c, $\max x < x_c$.

A.3 Callable convertible debt

Empirical evidence suggests that the majority of convertible debt contracts include call provisions. In the body of the paper we examined the case of convertible debt without a call provision. In this section we examine the other extreme – convertible debt with a call provision and no call protection (i.e., the shareholders can call the debt anytime for a pre-specified call price). We show that the major intuition of our results still holds. Convertible debt still leads to accelerated investment, it can still neutralize the underinvestment effect of Myers (1977), and it always results in lower agency costs than those of straight debt. On top of that, for relatively high conversion ratios, the equity holders would call convertible debt prior to the exercise of the investment option. In that case by the time of investment, the firm would be an all-equity entity (if there is no straight debt outstanding) and, therefore, would adopt the first-best investment strategy. As in Section 2, we start with a simple case of a firm without an investment opportunity and then extend our analysis to incorporate the investment option.

A.3.1 No investment opportunity

The optimization problem of the shareholders of a firm without the investment opportunity is to optimally choose the (lower) default threshold, x_d, and the (upper) call threshold, x_l, while the debt holders' problem is to choose the optimal conversion threshold, x_c. Debt can be either called or converted, and the call option can only be exercised as long as $x_l < x_c$. If the $x_l \geq x_c$ then the problem becomes identical to the one of non-callable convertible debt considered above. However, as discussed below, it is never optimal to call convertible debt when the conversion option is in the money, so $x_l < x_c$ always holds.

In the analysis below we assume that convertible debt can be called at a price exceeding its par value.[9] In what follows, we assume that the call price is given by $\gamma D_c(x_0, x_l, x_d)$, where $D_c(x_0, x_l, x_d)$ is the par value of convertible debt (i.e. its market price at the time of issuance), $\gamma > 1$ is a constant that shows by how much the call price is marked up relative to par[10], x_l is the call threshold, and x_d is the default threshold.

The value of equity is given in (39):

$$E(x_t) = Gx_t^{\beta_1} + Hx_t^{\beta_2} + \frac{x_t}{r-\mu} - \frac{s_c}{r},$$

where G and H are constants. The first value matching condition states that the value of equity at the call threshold equals the post-conversion value of equity minus the call price or to the post-conversion value accruing to the debt holders, whichever is higher:

$$G\,[x_l]^{\beta_1} + H\,[x_l]^{\beta_2} + \frac{x_l}{r-\mu} - \frac{s_c}{r} = \frac{x_l}{r-\mu} - \max(\gamma D_c(x_0, x_c, x_d), \frac{\eta}{1+\eta}\frac{x_l}{r-\mu}). \quad \text{(A.12)}$$

The second value matching condition postulates that the value of equity in default is zero:

$$G\,[x_d]^{\beta_1} + H\,[x_d]^{\beta_2} + \frac{x_d}{r-\mu} - \frac{s_c}{r} = 0. \quad \text{(A.13)}$$

Solving (A.12)-(A.13) for G and H and rearranging results in the following value of

[9] In practice, call price is usually set at a premium relative to par (see, for example, Tuckman (1996)).

[10] We assume constant γ in order to preserve analytical tractability. In reality, call premiums tend to be functions of the time to maturity.

equity:

$$E(x_0) = \max_{x_l, x_d}[\frac{x_0}{r-\mu} - \frac{s_c}{r} + p_{l\ before\ d}[\frac{s_c}{r} - \max(\gamma D_c(x_0, x_l, x_d), \frac{\eta}{1+\eta}\frac{x_l}{r-\mu})] - p_{d\ before\ l}[\frac{x_d}{r-\mu} - \frac{s_c}{r}]], \quad \text{(A.14)}$$

where

$$p_{l\ before\ d} = \frac{[x_d]^{\beta_2}[x_0]^{\beta_1} - [x_d]^{\beta_1}[x_0]^{\beta_2}}{[x_d]^{\beta_2}[x_l]^{\beta_1} - [x_d]^{\beta_1}[x_l]^{\beta_2}}, \quad \text{(A.15)}$$

is the present value of one dollar to be received at the first passage time of the stochastic shock x to x_l, conditional on x staying above x_d, $\min x > x_d$,

$$p_{d\ before\ l} = \frac{[x_l]^{\beta_1}[x_0]^{\beta_2} - [x_l]^{\beta_2}[x_0]^{\beta_1}}{[x_d]^{\beta_2}[x_l]^{\beta_1} - [x_d]^{\beta_1}[x_l]^{\beta_2}}, \quad \text{(A.16)}$$

is the present value of one dollar to be received at the first passage time of the stochastic shock x to x_d, conditional on x staying below x_l, $\max x < x_l$, and $D_c(x_0, x_l, x_d)$ is the par value of convertible debt, given by

$$D_c(x_0, x_l, x_d) = \frac{s_c}{r} + p_{l\ before\ d}\left[-\frac{s_c}{r} + \max(\gamma D_c(x_0, x_l, x_d), \frac{\eta}{1+\eta}\frac{x_c}{r-\mu})\right] + p_{d\ before\ l}\left[\epsilon(x_d) - \frac{s_c}{r}\right]. \quad \text{(A.17)}$$

Figure 5 provides the optimal call threshold as a function of the contractual coupon payment to convertible debt holders, s_c, for the following set of input parameters: $\alpha = 2.5$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$. In addition, γ is set to 1.1, so the call price is marked up by 10% relative to par.

Insert Figure 5 here

The decision to call the debt is based on several considerations. If the call price $\gamma D_c(x_0, x_l, x_d)$ is lower than the value of a riskless bond without the conversion option, $\frac{s_c}{r}$, then the equity holders always lose by not exercising the call option immediately, since they effectively pay interest to convertible debt holders at a rate higher than r. (i.e. they can buy back the asset for $\gamma D_c(x_0, x_l, x_d)$, so their effective interest rate is $\frac{s}{\gamma D_c(x_0, x_l, x_d)} > r$). On the other hand, calling convertible debt immediately would eliminate the valuable default option. Thus, as long as the conversion option is out of the money (i.e., if convertible debt holders would prefer not to convert but rather receive cash in the amount of $\gamma D_c(x_0, x_l, x_d)$ upon debt being called), the optimal call decision is determined by the relative magnitudes of the two opposite effects.

Once the conversion option moves into the money, there appears an additional cost of waiting. By waiting until a higher value of x is reached, not only the equity holders would have to pay an above-market effective interest to bondholders, but they also improve the payoff to bondholders if the call provision is exercised. In this region, convertible debt would be converted into equity, and higher values of x lead to a higher value of equity. The only benefit of waiting is preserving the option to default. This option, however, is less valuable for higher values of x, due to the lower probability of default. It is therefore likely that the optimal call decision is to call convertible debt when conversion option is exactly at the money. Indeed, this is the case for the set of parameters used in Figure 5. This is also consistent with the findings of Brennan and Schwartz (1977). Note, however, that it does not always have to be the case. For example, when $\gamma = 1.05$ it becomes optimal to call when conversion option is out of the money. Lower γ reduces the price at which debt can be called and, therefore, increases the cost of waiting. On the other hand, consistent with the intuition above, calling debt when the conversion option is strictly in the money does not constitute an optimal strategy for any γ.

A.3.2 Investment opportunity

We now proceed to the more interesting case in which convertible debt has an unprotected call provision, while the firm is endowed with an investment option. Similar to subsection 2.2, two different scenarios are possible. The equity holders may optimally choose to call debt either before or after the exercise of the investment option. We focus first on the case in which investment precedes the exercise of the call option.

Investment preceding call

Let x^* be the investment threshold (chosen by the equity holders). Similar to (A.14), the value of equity immediately before investment is given by

$$E(x^*) = \max_{x_l, x_{d,i}} [\frac{ax^*}{r-\mu} - \frac{s_c}{r} + p_{l\ before\ d,i}[\frac{s_c}{r} - \max(\gamma D_c(x_0, x_l, x_d, x_{d,i}), \frac{\eta_i}{1+\eta_i}\frac{ax_l}{r-\mu})] - p_{d,i\ before\ l}[\frac{ax_{d,i}}{r-\mu} - \frac{s_c}{r}] - I], \quad (A.18)$$

where

$$p_{l\ before\ d,i} = \frac{[x_{d,i}]\,[x^*]^{\beta_1} - [x_{d,i}]^{\beta_1}\,[x^*]^{\beta_2}}{[x_{d,i}]^{\beta_2}\,[x_l]^{\beta_1} - [x_{d,i}]^{\beta_1}\,[x_l]^{\beta_2}}, \tag{A.19}$$

is the present value of one dollar to be received at the first passage time of the stochastic shock x to x_l, conditional on x staying above $x_{d,i}$, $\min x > x_{d,i}$, and

$$p_{d,i\ before\ l} = \frac{[x_l]^{\beta_1}\,[x^*]^{\beta_2} - [x_l]^{\beta_2}\,[x^*]^{\beta_1}}{[x_{d,i}]^{\beta_2}\,[x_l]^{\beta_1} - [x_{d,i}]^{\beta_1}\,[x_l]^{\beta_2}}, \tag{A.20}$$

is the present value of one dollar to be received at the first passage time of the stochastic shock x to $x_{d,i}$, conditional on x staying below x_l, $\max x < x_l$.

Note that the issuance of new equity leads to the dilution of convertible debt holders' claims, just as in the case of non-callable convertible debt, considered in Section 2. The fraction of the total firm value accruing to the holders of convertible debt is $\frac{\eta_i}{1+\eta_i}$, not $\frac{\eta}{1+\eta}$ as in the case without the investment opportunity. As before, the fraction η_i is given in (29). Therefore, after the exercise of the investment option, the equity holders maximize the value of equity given by (A.18) by optimally choosing the (post-investment) default threshold, $x_{d,i}$, and the call threshold, x_l, subject to (29).

Before investment, the value of equity is given by

$$E(x_0) = \max_{x^*,x_d}[\frac{x_0}{r-\mu} - \frac{s_c}{r} + p_{i\ before\ d}[E(x^*) - \frac{x^*}{r-\mu} + \frac{s_c}{r}] - p_{d\ before\ i}[\frac{x_d}{r-\mu} - \frac{s}{r}]], \tag{A.21}$$

where

$$p_{i\ before\ d} = \frac{[x_d]^{\beta_2}\,[x_0]^{\beta_1} - [x_d]^{\beta_1}\,[x_0]^{\beta_2}}{[x_d]^{\beta_2}\,[x^*]^{\beta_1} - [x_d]^{\beta_1}\,[x^*]^{\beta_2}}, \tag{A.22}$$

is the present value of one dollar to be received at the first passage time of the stochastic shock x to x^*, conditional on x staying above x_d, $\min x > x_d$, and

$$p_{d\ before\ i} = \frac{[x^*]^{\beta_1}\,[x_0]^{\beta_2} - [x^*]^{\beta_2}\,[x_0]^{\beta_1}}{[x_d]^{\beta_2}\,[x^*]^{\beta_1} - [x_d]^{\beta_1}\,[x^*]^{\beta_2}}, \tag{A.23}$$

is the present value of one dollar to be received at the first passage time of the stochastic shock x to x_d, conditional on x staying below x^*, $\max x < x^*$. $E(x^*)$ is the solution to the maximization program in (A.18). The value of convertible debt is given by

$$D_c(x_0, x_l, x_d, x_{d,i}) = \frac{s_c}{r} + p_{i\ before\ d}[-\frac{s_c}{r} + D_c(x^*, x_l, x_{d,i})] + p_{d\ before\ i}[\epsilon(x_d) - \frac{s_c}{r}], \quad (A.24)$$

where $D_c(x^*, x_c, x_{d,i})$ is the value of debt right after the investment:

$$D_c(x^*, x_l, x_{d,i}) = \frac{s_c}{r} + p_{l\ before\ d,i}[-\frac{s}{r} + \max(\gamma D_c(x_0, x_l, x_d, x_{d,i}), \frac{\eta_i}{1+\eta_i}\frac{ax_c}{r-\mu})] + p_{d,i\ before\ l}[\epsilon(x_{d,i}) - \frac{s_c}{r}]. \quad (A.25)$$

The equity holders maximize (A.21) with respect to their optimal pre-investment default threshold, x_d, and the investment threshold, x^*, given first-stage maximization in (A.18) with respect to the post-investment default threshold, $x_{d,i}$, and the call threshold, x_l, subject to (A.24), (A.25), and (29).

Call preceding investment

Calling convertible debt before exercising the investment option may be optimal from the shareholders' perspective if the conversion ratio, α, is relatively high, and the conversion option moves into the money before the optimal investment threshold is reached. Note that once the conversion option is exercised, the firm becomes an all-equity entity and would, therefore, follow first-best investment policy.

Assume that convertible debt is called at a stopping time upon reaching the call threshold, x_l, and the investment option is still open. In this case the optimal investment policy is to invest at the first passage time upon reaching the investment threshold corresponding to the all-equity case x^*_{eq}, given in (4), or immediately upon calling debt, if $x_l > x^*_{eq}$:

$$x^* = \max(x_l, x^*_{eq}).$$

It is straightforward to show that the value of equity immediately after convertible debt is called is given by

$$E^* = \max_{x^*} \left[\frac{x_l}{r-\mu} + \left[\frac{x_l}{x^*}\right]^{\beta_1} \left[\frac{[a-1]\, x^*}{r-\mu} - I \right]\right]. \quad (A.26)$$

Therefore, the value of equity before convertible debt is called is given by

$$E(x_0) = \max_{x_l, x_d}[\frac{x_0}{r-\mu} - \frac{s_c}{r} + p_{l\ before\ d}[\frac{s_c}{r} - \max(\gamma D_c(x_0, x_c, x_d), \frac{\eta}{1+\eta}E^*)] - p_{d\ before\ l}[\frac{x_d}{r-\mu} - \frac{s_c}{r}]], \quad \text{(A.27)}$$

where $p_{l\ before\ d}$ and $p_{d\ before\ l}$ are given in (A.15) and (A.16), and $D_c(x_0, x_l, x_d)$ is the par value of convertible debt. As before, when convertible debt is issued, its par value must equal the present value of the cash flows received by the holders of convertible debt:

$$D_c(x_0, x_l, x_d) = \frac{s_c}{r} + p_{l\ before\ d}[-\frac{s_c}{r} + \max(\gamma D_c(x_0, x_l, x_d), \frac{\eta}{1+\eta}E^*)] + p_{d\ before\ l}[\epsilon(x_d) - \frac{s}{r}]. \quad \text{(A.28)}$$

Note that if debt is called while the investment option is still open, there is no dilution effect, and, therefore, the fraction of total equity to be received by convertible debt holders upon conversion, in (A.27) and (A.28), is given by $\frac{\eta}{1+\eta}$, and not by $\frac{\eta_i}{1+\eta_i}$, as in the optimization problem (A.18).

Figure 6 presents the investment threshold of a firm that has non-protected callable convertible debt in its capital structure. The values of input parameters used to produce Figure 6 are as follows: $\alpha = 1.25$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $a = 2$, $I = 5$, $\gamma = 1.1$. The solid line represents the investment threshold of a firm with callable convertible debt as a function of coupon rate. The dashed line depicts the investment threshold of a firm with non-callable convertible debt (and the same conversion ratio, $\alpha = 1.25$), while the dashed-dotted line represents the investment threshold of a firm with straight debt (but no convertible debt). The dotted line depicts the investment threshold of an all-equity firm.

Insert Figure 6 here

Figure 6 reveals that callable convertible debt can still lead to accelerated investment incentives and mitigate the underinvestment effect of straight debt. However, the accelerated investment incentives of callable debt are weaker than those of debt without a call provision. The intuition is simple: the call provision erodes the value of the conversion option. As discussed above, the equity holders always find it optimal to call before the optimal conversion threshold is reached. Lower value of the option to convert reduces the magnitude of the dilution effect and tilts the overinvestment-underinvestment balance in favor of underinvestment.

For the set of parameter values used in Figure 6, by the time of the exercise of the investment option, the conversion option is far out of the money, and it is not optimal for the equity holders to call the debt before investment. Therefore, the equity holders prefer to invest first and only later call convertible debt (conditional on staying solvent). The value of equity given by the solution to the optimization problem (A.18) and (A.21) exceeds the one given by the solution to (A.27).

Note, however, that for some values of the conversion ratio, α, it may be optimal for the equity holders to call convertible debt before proceeding with investment. This is the case for the base set of input parameters and $\alpha = 2.5$. When the call threshold is reached, the debt is converted into equity, and the investment threshold of the resulting all-equity firm turns out to be lower than the call threshold. Thus, calling the debt for conversion results in immediate investment thereafter. This moves the investment strategy closer to the first-best strategy (represented by the investment threshold of an all-equity firm), and increases the ex-ante value of the equity holders' claims. Thus, call provision may provide additional benefits as it may be optimal for the equity holders to call in all debt before making investment, thus mitigating the underinvestment problem (and completely eliminating it when $x_l \leq x^*_{eq}$).

To summarize, both callable and non-callable convertible debt contracts are able to alleviate and, in some cases, completely eliminate the underinvestment problem of Myers (1977). Non-callable convertible debt provides the shareholders with accelerated investment incentives, which can neutralize the underinvestment effect of debt. By appropriately choosing the amount of convertible debt, the equity holders are able to ensure that the first-best investment policy is pursued. This is true both when all outstanding debt is convertible and when part of the debt is straight. On the other hand, while the call provision reduces the accelerated investment incentives of convertible debt, it can lead to a situation in which all debt is called before the investment option is exercised, resulting in the first-best investment policy.

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Figure 1. Optimal investment threshold and coupon rate – one type of debt

This figure presents the optimal investment thresholds for the case of convertible debt without straight debt (solid line), and for the case of straight debt without convertible debt (dashed line) as a functions of the coupon payment. The dotted line represents the investment threshold of an all-equity firm. The following set of parameter values was used to produce this figure: $\alpha = 2.5$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $a = 2$, $I = 5$.

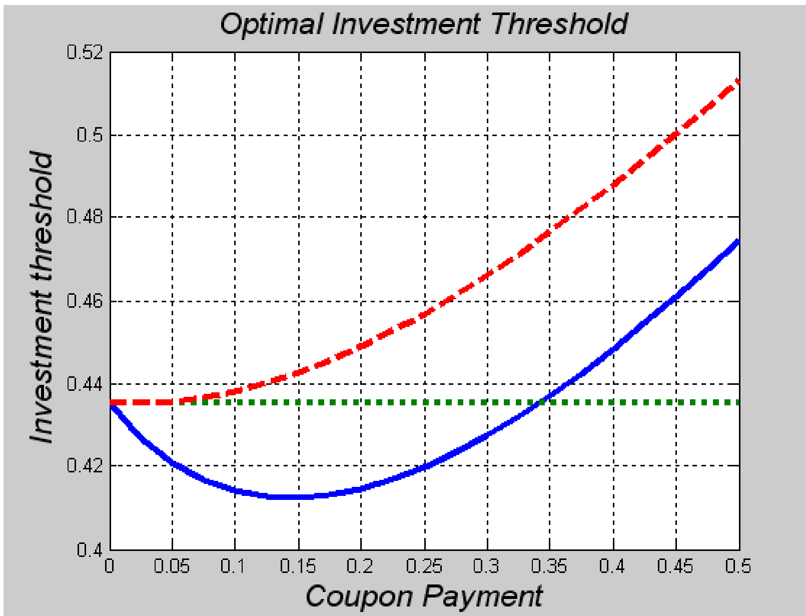

Figure 2. Optimal investment threshold and conversion ratio

This figure presents the optimal investment thresholds of a firm with outstanding convertible debt for the set of input parameters used in figure 2 and three different conversion ratios: $\alpha = 1.25$ (solid line), $\alpha = 1.8$ (dashed-dotted line), and $\alpha = 2.5$ (dotted line). The all-equity investment threshold is given by the dashed line.

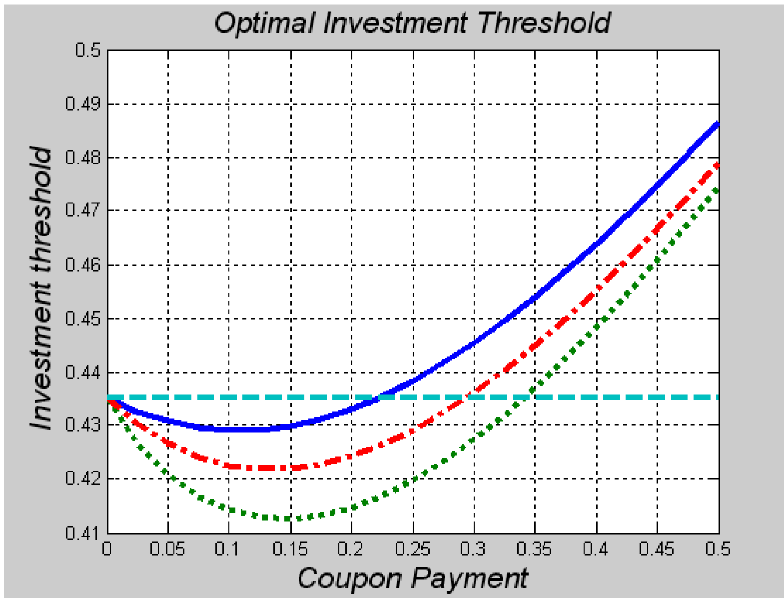

Figure 3. Optimal investment threshold – two types of debt

Figure 3a presents the optimal investment thresholds as functions of total (convertible and straight) coupon, for firms with different levels of convertible debt. The dashed line represents the investment threshold of a firm with straight debt only. The solid line refers to a firm having a convertible coupon of 0.10, and the dashed-dotted line depicts the threshold of a firm with a convertible coupon of 0.20. The dotted line represents the (first-best) investment threshold of an all-equity firm. Figure 3b presents the optimal proportion of straight debt out of total debt that results in the first-best investment policy as a function of the coupon payment to convertible debt holders. The following set of input parameters was used to produce Figures 3a and 3b: $\alpha = 2.5$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $a = 2$, $I = 5$.

Figure 3a

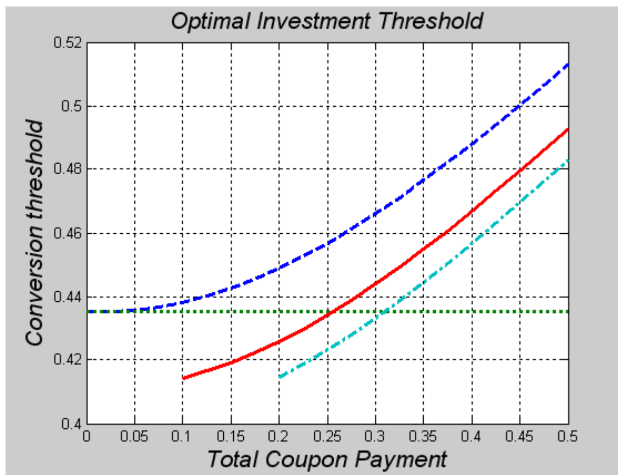

Figure 3b

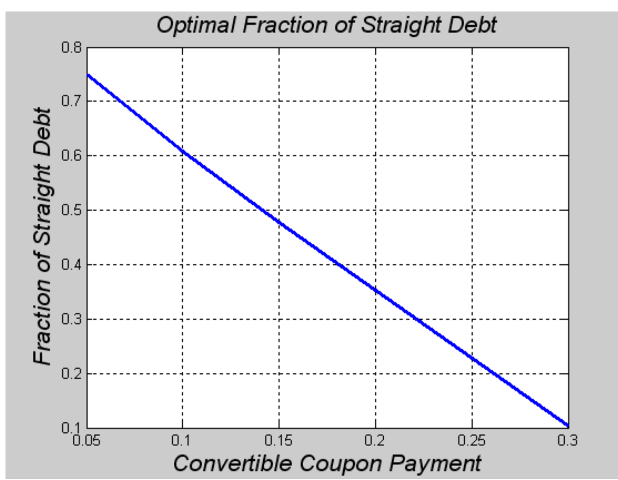

Figure 4. Optimal conversion thresholds and coupon rate

This figure presents the optimal conversion thresholds as functions of the convertible debt coupon. The solid line provides the "block conversion" threshold, while the dashed line provides the "marginal conversion" threshold (see Appendix for details). The following set of parameter values was used to produce this figure: $\alpha = 2.5$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $a = 2$, $I = 5$, $s_s = 0$.

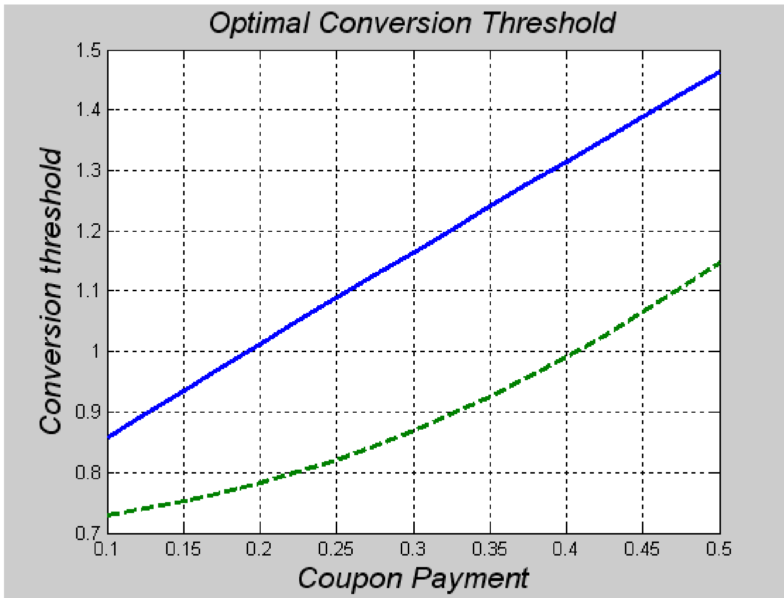

Figure 5. Optimal call threshold

This figure presents the optimal call threshold for a firm with no investment opportunity and unprotected callable convertible debt, for the following set of input parameters: $\alpha = 2.5$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $\gamma = 1.1$

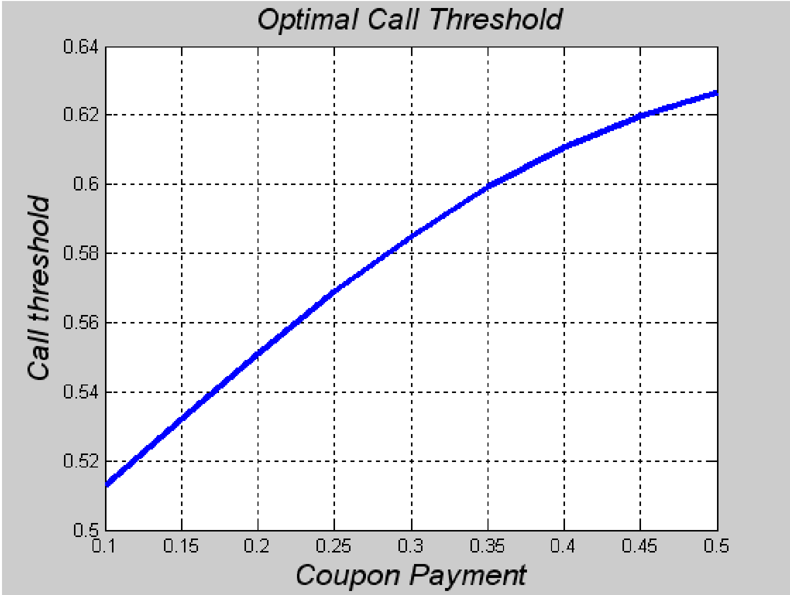

Figure 6. Optimal investment threshold – callable convertible debt

This figure presents the optimal investment threshold of a firm with unprotected callable convertible debt outstanding (represented by the solid line). The following set of parameter values was used to produce this figure: $\alpha = 1.25$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $a = 2$, $I = 5$. The dotted line provides the first-best investment threshold of an all-equity firm. The dashed line gives the investment threshold of a firm with non-callable convertible debt (and the same conversion ratio), while the dash-dotted line represents the investment threshold of a firm with straight debt.