NEGATIVE FLYPE MOVE AND CYCLIC BRANCHED COVERING

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ABSTRACT. By studying negative flype moves, we show that the p-fold ($p \geq 2$) cyclic branched covering of (S^3, ξ_{std}) over transverse knots is insensitive to detecting transversally non-simple knots.

1. Introduction

Let σ_i be the standard generator of the braid group B_n satisfying

$$B_n = \{\sigma_1, \cdots, \sigma_{n-1} \mid \sigma_i\sigma_j = \sigma_j\sigma_i, \text{ for } |i-j| \geq 2, \ \sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1}\}.$$

We think that S^3 is a one-point compactification of $\mathbb{R}^3$ equipped with the cylindrical coordinates (r, θ, z). Let $\xi_{sym} = \ker(dz + r^2 d\theta)$.

A knot $T \subset (S^3, \xi_{sym})$ is called a *transversal* knot if at any point $p \in T$ the knot T transverses the contact plane ξ_p. Thanks to Bennequin [1], we can identify, up to transversal isotopy, a transverse knot in (S^3, ξ_{sym}) with a closed braid about the z-axis. The *self linking number* is a classical invariant of transverse knots measuring the obstruction to extend a non-zero vector field along T induced by the contact structure, to a Seifert surface of T. It can be combinatorially computed, namely $sl(T)$ is the algebraic crossing number (writhes) of the braid T minus the braid index of T. A topological knot type $\mathcal{K}$ is called *transversally simple* if transversal knot representatives of $\mathcal{K}$ are completely classified only by the self linking number.

There are infinitely many transversally non-simple knots due to Briman-Menasco [2], Etnyre-Honda [5], Menasco-Matsuda [9], Ng-Ozsváth-Thurston[10] and Lisca-Ozsvath-Stipsicz-Szabó [8].

As noted in [7, Example 5.10], one can observe that all the transversally non-simple examples discovered in [2, 5, 9, 10, 8] admit *negative flype* braid moves.

Definition 1.1. Let $T_1, T_2 \subset (S^3, \xi_{sym})$ be transverse knots. Under Bennequin's identification [1], we regard T_i as the braid closure of a braid word w_i. Suppose w_1, w_2 have the braid index $= n$. We say that T_1, T_2 are related to each other by an *negative flype* move if there exist $u, v \in B_{n-1}$ and $k \in \mathbb{Z}$, $k \neq 0$ such that, up to braid conjugation,

$$w_1 = u\ \sigma_{n-1}^{-1}\ v\ \sigma_{n-1}^{k}, \tag{1.1}$$

$$w_2 = u\ \sigma_{n-1}^{k}\ v\ \sigma_{n-1}^{-1}, \tag{1.2}$$

where we read braid words from the left to the right.

It is natural to ask:

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Question 1.2. *Suppose that $u \neq v$, $k \neq 1$, and T_1 and T_2 admit a negative flype. Are T_1 and T_2 transversally non-isotopic?*

Obviously, if $u = v$ then they are braid isotopic, i.e., transversally isotopic. Also, when $k = 1$, they are known to be transversally isotopic, see [3] for example. So, we have to exclude these cases.

Plamenevskaya [12] initiated study of coverings of (S^3, ξ_{std}) branched over a transversal knot intending to re-prove transverse non-simplicity of the Birman-Menasco's 3-braids [2]. She showed that the double branched covering fails to detect their transverse non-simplicity. More precisely, suppose (T_1, T_2) is a pair of 3-braids in [2], having the same topological type and $sl(T_1) = sl(T_2)$ but is known to be $T_1 \neq T_2$ as transverse knots. One would expect that the double branched covers of (S^3, ξ_{std}) over T_1, T_2 are not contactomorphic. Plamenevskaya [12] proved that the double branched covers are actually contactomorphic.

Later, many other transversally non-simple knots were discovered [5, 9, 10, 8]. In [7], it is proved that the p-fold ($p \geq 2$) cyclic branched covering cannot detect transverse non-simplicity of Birman-Menasco's pairs. Furthermore, they proved that the *double* branched cover cannot detect transverse non-simplicity of any of the examples found in [5, 9, 10].

Here is our main result extending that of [7]. Let

$$\pi : (\Sigma_p(T), \xi_p(T)) \to (S^3, \xi_{sym})$$

denote the p-fold ($p \geq 2$) cyclic branched covering of (S^3, ξ_{sym}) branched over transverse knot T. Recall that negative flype is a key to search transversally non-simple knots.

Theorem 1.3. *Let T_1 and $T_2 \subset (S^3, \xi_{sym})$ be closed braids related to each other by an negative flype move. Then for any $p \geq 2$, the p-fold cyclic branched coverings $(\Sigma_p(T_1), \xi_p(T_1))$ and $(\Sigma_p(T_2), \xi_p(T_2))$ are contactomorphic.*

Assuming that the answer to Question 1.2 is "Yes", Theorem 1.3 terminates our project to detect transversal non-simplicity via cyclic branched covers.

To prove Theorem 1.3, we propose a recipe for a contact surgery diagram for $(\Sigma_p(w), \xi_p(w))$ where braid word w represents transversal knot T. (In [7] it requires that w starts with "unknotting head' $\sigma_1 \sigma_2 \cdots \sigma_{n-1}$, which is not assumed in this paper.) By a nature of the recipe, we obtain an upper bound of the *support genus*, $g(\xi_p(T))$, defined in [6], the minimal possible genus for a page of an open book that supports the contact structure $(\Sigma_p(T), \xi_p(T))$. Let $n_T = \min\{$ braid index of $w \mid w$ is a braid representative of T $\}$ and let $c_{p,T}$ be the number of the link components of the (n_T, p)-torus link. We have

$$g(\xi_p(T)) \leq \frac{1}{2}(1 - c_{p,T} + (n_T - 1)(p - 1)).$$

It is interesting to find a family $\{T_i\}$ of transversal knots which have $(\Sigma_p(T), \xi_p(T)) = (\Sigma_p(T_i), \xi_p(T))$, hoping to obtain a sharper bound for $g(\xi_p(T))$.

In fact, in Remark 2.6, we see that many transversal knots of distinct topological types give a same p-fold cyclic branched cover up to contactomorphism.

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2. Construction of contact surgery diagrams

Let $b \in B_n$ be a braid word. In the following, we inductively construct a contact surgery diagram (a framed Legendrian link) $\Omega(b) \subset (S^3, \xi_{std})$ for $(\Sigma_p(T), \xi_p(T))$, where $\xi_{std} = \ker(dz - ydx)$. This construction is essentially the same as the one introduced in [7]. However this one is more convenient since our braid word b does not have to have the "unknotting head" (i.e., $b = \sigma_1 \cdots \sigma_{n-1} b'$) as required in [7].

Let $D = \{(r, 0, z) | \ 0 < r, \ \theta = 0, \ z \in \mathbb{R}\}$ be a disk with $\partial D = z$-axis. Let $x_1, \cdots, x_n \in D$ be n points on which braid word $b \in B_n$ acts: Namely, $\sigma_i \in B_n$ exchanges points x_i and x_{i+1} as in Figure 1. Let $\phi_b \in \mathrm{Diffeo}(D, \partial D)$ be a monodromy map of D associated to

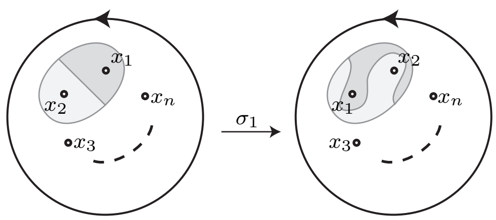

Figure 1. The map $\phi_{\sigma_1} : D \to D$ acts on the shaded neighborhood of x_1 and x_2.

the braid word b. Since the Alexander's trick states that the mapping class group of D is trivial, the open book decomposition (D, ϕ_b) supports the contact manifold (S^3, ξ_{sym}). The p-fold cyclic branched covering map $\pi : (\Sigma_p(T), \xi_p(T)) \to (S^3, \xi_{sym})$ induces an open book decomposition $(\tilde{D}, \tilde{\phi}_b) = (\pi^{-1}(D), \pi^*(\phi_b))$ supporting $(\Sigma_p(b), \xi_p(b))$. If $b \in B_n$ then $\tilde{D}$ is the Seifert surface of the (n, p)-torus knot (link) sketched in Figure 2.

Let $\alpha^i_j \subset S^3$ $(i = 1, 2, \cdots, n-1, \ j = 0, 1, \cdots, p-1)$ be an unknot sketched in Figure 2 and its Legendrian realization in (S^3, ξ_{std}) is sketched in Figure 3.

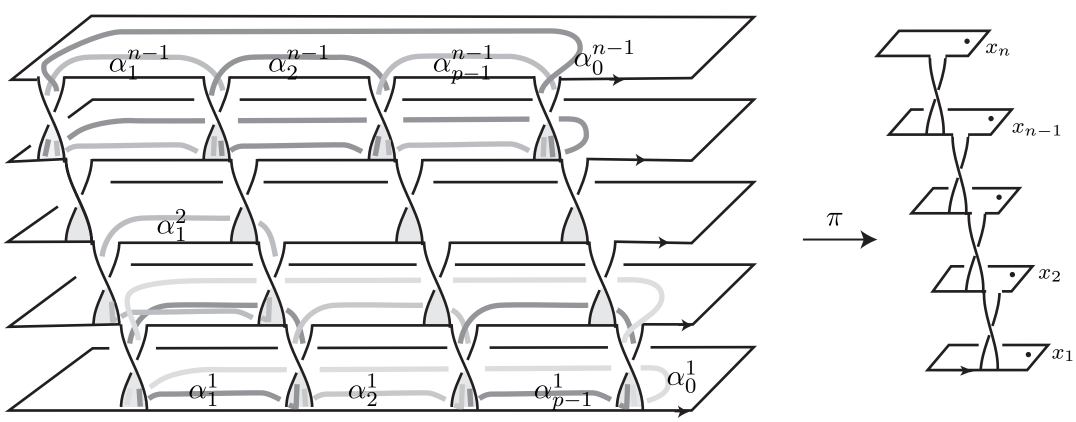

Figure 2. The left sketch is a page $\tilde{D}$ of the open book decomposition of $(\Sigma_p(T), \xi_p(T))$ where $p = 4$ and $n = 5$. The right sketch is the disk D, a page of the open book decomposition of (S^3, ξ_{sym}).

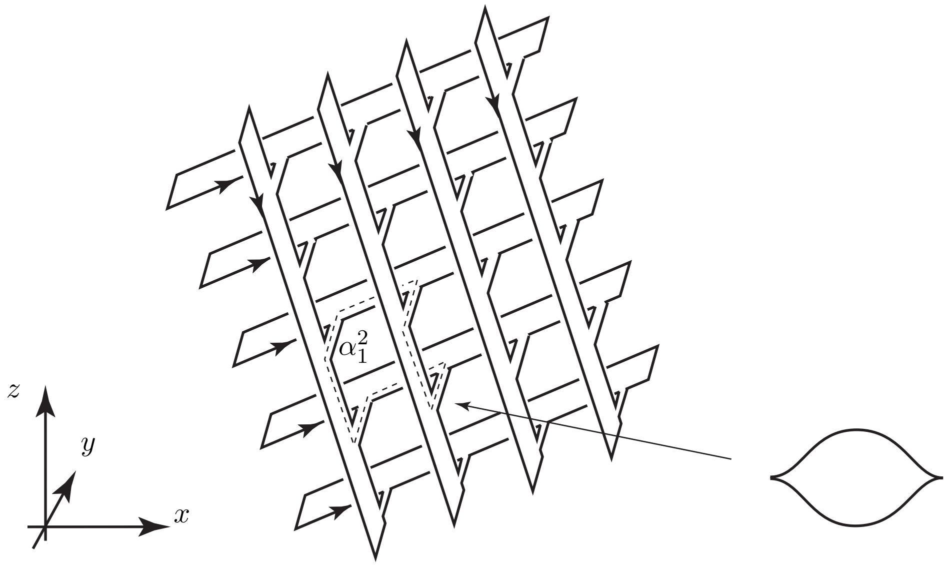

FIGURE 3. Legendrian unknot α_j^i.

Lemma 2.1. [7, Lemma 3.1] *Let $\mathcal{D}_j^i$ be the right-handed Dehn twist about α_j^i specified in Figure 2. The lift $\tilde{\sigma}_i \in \mathrm{Diffeo}(\tilde{\mathrm{D}})$ of σ_i is $\mathcal{D}_1^i \circ \mathcal{D}_2^i \circ \cdots \circ \mathcal{D}_{p-1}^i$ (composition of Dehn twists read from the right to the left).*

Lemma 2.2. *By symmetry of $\tilde{D}$, we also have $\tilde{\sigma}_i = \mathcal{D}_0^i \circ \mathcal{D}_1^i \circ \cdots \circ \mathcal{D}_{p-2}^i$.*

In the following we explain how to construct a contact surgery diagram (framed Legendrian link) $\Omega(b) \subset (S^3, \xi_{std})$ for contact manifold $(\Sigma_p(T), \xi_p(T))$.

• Let $b_0 \in B_1$ be the one-strand braid representative of the unknot and let

$$\Omega(b_0) = empty. \tag{2.1}$$

• Suppose $b = b'\sigma_{n-1} \in B_n$ is a positive braid stabilization of $b' \in B_{n-1}$. Then we define

$$\Omega(b) = \Omega(b'). \tag{2.2}$$

In the contact manifold level, we are taking the connect sum

$$\left(\overset{p-1}{\#}(S^3, \xi_+)\right) \ \# \ (\Sigma_p(b'), \xi_p(b'))$$

where (S^3, ξ_+) is the tight contact structure compatible with the open book decomposition of S^3 induced by the positive Hopf link. Therefore, a page of the open book decomposition for $(\Sigma_p(b), \xi_p(b))$ is obtained by plumbing (or Murasugi sum of) $p-1$ positive Hopf bands and the page for $(\Sigma_p(b'), \xi_p(b'))$ sketched in Figure 2.

• Suppose $b = b'\sigma_k \in B_n$ where b' contains σ_k or σ_k^{-1}. Let P_θ, $\theta \in [0, 2\pi)$ denote the pages of the open book decomposition of $(\Sigma_p(T), \xi_p(T))$. Topologically, $P_\theta = \tilde{D}$. Assume that $\Omega(b') \subset \bigcup_{0<\theta<\theta_0} P_\theta \subset S^3$ for some $\theta_0 < 2\pi$ and pick $\theta_0 < \theta_1 < \theta_2 < \cdots < \theta_{p-1} < 2\pi$. Denote the copy of α_j^i in the page P_θ by $\alpha_j^{i,\theta}$. Define diagram u_k^+ as in Figure 4 and

$$\Omega(b) = \Omega(b') \cup u_k^+. \tag{2.3}$$

Notice that u_k^+ may link with $\Omega(b')$.

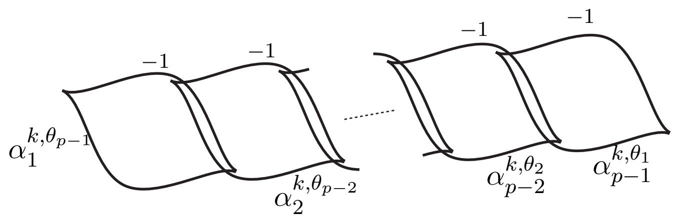

FIGURE 4. Framed Legendrian link u_k^+ (front projection).

• Suppose $b = b'\sigma_{n-1}^{-1} \in B_n$ is a negative braid stabilization of $b' \in B_{n-1}$. Assume that $\Omega(b') \subset \bigcup_{0<\theta<\theta_0} P_\theta$ for some $\theta_0 < 2\pi$. Pick $\theta_1, \cdots, \theta_{2p-2}$ so that $\theta_0 < \theta_1 < \theta_2 < \cdots < \theta_{2p-2} < 2\pi$. Let u_k^{ot} be as sketched in Figure 5. We define

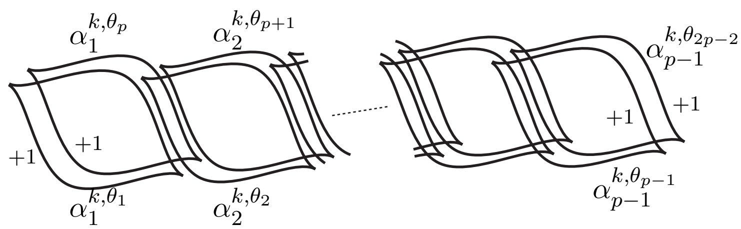

FIGURE 5. Framed Legendrian link u_k^{ot}.

$$\Omega(b) = \Omega(b') \sqcup u_{n-1}^{ot}, \tag{2.4}$$

where '$\sqcup$' implies that u_{n-1}^{ot} is not linked with $\Omega(b')$.

• Suppose $b = b'\sigma_k^{-1} \in B_n$ where b' contains σ_k or σ_k^{-1}. Let $\theta_0 < \cdots < \theta_{p-1}$ be as above. Let u_k^- as in Figure 6 and define

$$\Omega(b) = \Omega(b') \cup u_k^-. \tag{2.5}$$

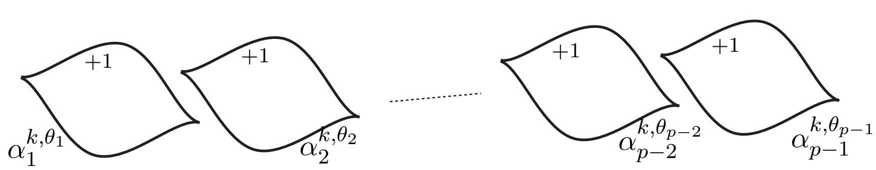

FIGURE 6. Framed Legendrian link u_k^-.

By [11] we know that the surface framing (induced by the page $\tilde{D}$) of an α-curve is the same as its contact framing induced by ξ_{std}. Therefore, by Lemma 2.1 we have the following:

Proposition 2.3. *(cf.* [7, Theorem 3.4]*) Let b be a braid word of a transverse knot $T \subset (S^3, \xi_{sym})$. The contact surgery description $\Omega(b)$ gives the p-fold cyclic branched cover $(\Sigma_p(T), \xi_p(T))$.*

Remark 2.4. Definition (2.4) is consistent with (2.5) and (2.2) in the following sense. Braid $b = b'\sigma_{n-1}^{-1}$ is braid conjugate to $b'\sigma_{n-1}\sigma_{n-1}^{-2}$, thus, as contact manifold, we have $(\Sigma_p(b), \xi_p(b)) = (\Sigma_p(b'\sigma_{n-1}\sigma_{n-1}^{-2}), \xi_p(b'\sigma_{n-1}\sigma_{n-1}^{-2}))$ and

$$\begin{aligned}
\Omega(b'\sigma_{n-1}\sigma_{n-1}^{-2}) &\stackrel{(2.5)}{=} \Omega(b'\sigma_{n-1}\sigma_{n-1}^{-1}) \cup u_{n-1}^{-} \\
&\stackrel{(2.5)}{=} \Omega(b'\sigma_{n-1}) \cup u_{n-1}^{-} \cup u_{n-1}^{-} \\
&\stackrel{(2.2)}{=} \Omega(b') \sqcup (u_{n-1}^{-} \cup u_{n-1}^{-}) \\
&= \Omega(b') \sqcup u_{n-1}^{ot}.
\end{aligned}$$

Since the positive push-off $(\alpha_j^{n-1})^+$of α_j^{n-1} and $\alpha_{j'}^{n-2}$ do not link to each other, u_{n-1}^{ot} is *not* linked with $\Omega(b')$ and we use symbol '$\sqcup$'.

Remark 2.5. Equality (2.2) allows many topologically distinct braids give the same contact surgery diagram, i.e., contactomorphic branched coveing of (S^3, ξ_{sym}). For example, let $b = \sigma_1\sigma_2\sigma_3\sigma_4\sigma_3\sigma_3\sigma_4$, $b' = \sigma_1\sigma_2\sigma_3\sigma_3\sigma_4\sigma_3\sigma_4$, and $b'' = \sigma_2\sigma_1\sigma_3\sigma_3\sigma_3\sigma_4\sigma_4$. Then branched covers are $(\Sigma_p(b), \xi_p(b)) = (\Sigma_p(b'), \xi_p(b')) = (\Sigma_p(b''), \xi_p(b''))$.

Remark 2.6. Let $b \in B_n$ be a braid word which contains σ_k or σ_k^{-1}. The *Cancellation Lemma* of Ding-Geiges [4, Proposition 8] allows us to identify $\Omega(b\sigma_k\sigma_k^{-1})$ with $\Omega(b)$, since

$$\Omega(b\sigma_k\sigma_k^{-1}) \stackrel{(2.5)}{=} \Omega(b\sigma_k) \cup u_k^{-} \stackrel{(2.3)}{=} \Omega(b) \cup u_k^{+} \cup u_k^{-} \xleftarrow{\text{cancelation lemma}} - - - \longrightarrow \Omega(b).$$

The case when $b \in B_{n-1}$ and $k = n-1$ is discussed in (2.7).

Proposition 2.7. *Suppose $b = b_1b_2 \in B_n$ consists of two unlinked components b_1, b_2 where b_1 is written in braid generators $\sigma_1, \cdots, \sigma_{k-1}$ and b_2 is written in $\sigma_{k+1}, \cdots, \sigma_{n-1}$. Then*

$$\Omega(b) := \Omega(b_1) \sqcup u_k^{-} \sqcup \Omega(b_2). \tag{2.6}$$

In particular, if b_2 is the unknot of braid index 1 (i.e., $k = n-1$) and unlinked with b_1 then

$$\Omega(b) = \Omega(b_1) \sqcup u_{n-1}^{-}. \tag{2.7}$$

Proof of Proposition 2.7. Applying (2.2) and (2.5) and get $\Omega(b_1\sigma_k\sigma_k^{-1}) = \Omega(b_1) \sqcup u_k^{-}$ which is (2.7). Diagram $\Omega(b_2)$ consists of $\alpha_j^{k+1}, \alpha_j^{k+2}, \cdots, \alpha_j^{n-1}$'s $(j = 0, 1, \cdots, p-1)$. Since push off $(\alpha_j^{k+1})^+$ of α_j^{k+1} and $\alpha_{j'}^{k}$ do not link to each other, we have (2.6). □

3. Proof of Theorem 1.3

We deform the braid word w_1, w_2 specified in (1.1) and (1.2) as follows:

$$\begin{array}{rcll} w_1 &=& u\ \sigma_{n-1}^{-1}\ v\ \sigma_{n-1}^{k} & \text{by (1.1)} \\ &=& u\ \sigma_{n-1}^{-1}\ v\ (\sigma_{n-1}\sigma_n\sigma_{n-1}^{-1})\ \sigma_{n-1}^{k} & \text{positive stabilization} \\ &=& u\ \sigma_{n-1}^{-1}\ \sigma_n^{-1}\ \sigma_{n-1}\ v\ \sigma_{n-1}\ \sigma_{n-1}^{-1}\ \sigma_n^{k}\ \sigma_{n-1} & \text{conjugation} \\ &=& u\ \sigma_{n-1}^{-1}\ \sigma_n^{-1}\ \sigma_{n-1}\ \sigma_n^{k}\ v\ \sigma_{n-1} & \text{conjugation} \\ &=& \sigma_n^{-1}\ \sigma_{n-1}\ \sigma_n^{k}\ \tau & \text{putting } \tau := v\ \sigma_{n-1}\ u\ \sigma_{n-1}^{-1} \\ w_2 &=& u\ \sigma_{n-1}^{k}\ v\ \sigma_{n-1}^{-1} = \sigma_n^{k}\ \sigma_{n-1}\ \sigma_n^{-1}\ \tau & \end{array}$$

Here the equality means transversal isotopic. Since positive braid stabilization preserves transversal knot class, these deformations do not change the transversal knot types.

Proof. Let w_i be a braid word for the transversal knot T_i. Based on the observation above, we may assume that:

$$w_1 = \sigma_{n-1}^{-1}\ \sigma_{n-2}\ \sigma_{n-1}^{k}\ w = (\sigma_{n-2}\ \sigma_{n-1})\ \sigma_{n-1}^{k-1}\ w\ \sigma_{n-1}^{-1} \tag{3.1}$$

$$w_2 = \sigma_{n-1}^{k}\ \sigma_{n-2}\ \sigma_{n-1}^{-1}\ w = (\sigma_{n-1}\ \sigma_{n-2})\ \sigma_{n-1}^{-1}\ w\ \sigma_{n-1}^{k-1} \tag{3.2}$$

for some braid word w in $\sigma_1, \cdots, \sigma_{n-2}$ and $k \in \mathbb{Z}$, $k \neq 0$.

When $k = 1$, T_1 and T_2 are related by a so called exchange move, which has been known to preserve transversal knot type, the p-fold cyclic branched coverings for T_1, T_2 are contactomorphic.

When k is negative, by [7, Proposition 4.2, Remark 4.3], the p-fold cyclic branched covers are overtwisted and contactomorphic.

Therefore, in the following we consider the case when $k \geq 2$.

By plumbing the positive Hopf bands in a different manner as in Figure 7, we can take a different set of simple closed curves $\{\beta_j^i\}_{j=0,1,\cdots,p}^{i=1,2,\cdots,n-1}$ defined by $\beta_j^i = \alpha_j^i$ for $i \neq n-2$, and β_j^{n-2} as sketched in Figure 8.

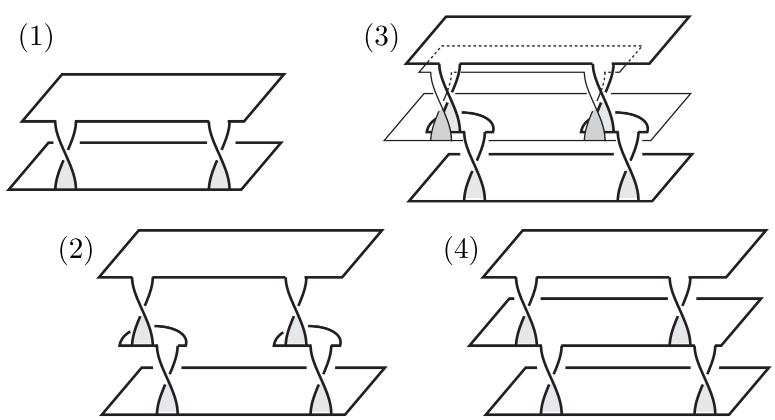

Figure 7. Plumbing of Hopf-bands. (1) = (2) $\overset{\text{plumbing}}{\longrightarrow}$ (3) = (4).

Following the construction of contact surgery diagram from (2.1) through (2.5) introduced in Section 2, we obtain contact surgery diagram $\Omega(w_1)$ by using $\{\alpha_j^i\}$'s and $\Omega(w_2)$

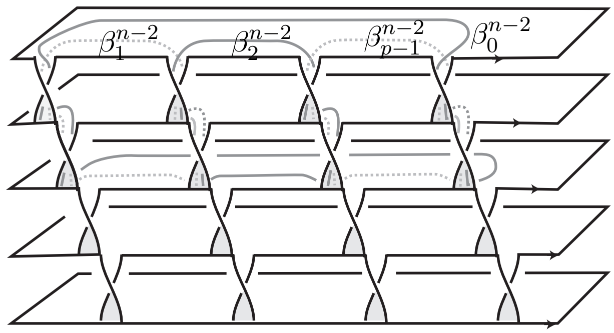

FIGURE 8. Unknot β_j^{n-2}.

by $\{\beta_j^i\}$'s sketched in Figure 9-(1) and (2-a) respectively. There, due to construction (2.1) and (2.2), there is no diagram corresponding to $(\sigma_{n-2}\,\sigma_{n-1})$ in (3.1) and $(\sigma_{n-1}\,\sigma_{n-2})$ in (3.2). By (2.4) the solid gray components correspond to σ_{n-1}^{-1}. By (2.3) the solid black ones represent σ_{n-1}^k. The dashed represent w and they consist of α_j^i or β_j^i with $i \neq n-1$ since w is a word in $\sigma_1, \cdots, \sigma_{n-2}$.

Claim 3.1. *In Figure 9, the solid gray components do not link with the dashed components.*

Proof. In (3.1), word w is "before" σ_{n-1}^{-1}. Since $\mathrm{lk}(\alpha_j^i, (\alpha_{j'}^{n-1})^+) = 0$ for $i = 1, \cdots, n-2$ and $j, j' = 1, \cdots, p-1$, where $(\alpha_j^i)^+$ is a positive transversal push off of α_j^i. On the other hand, in (3.2), word w is "after" σ_{n-1}^{-1}. By definition of β curves we have $\mathrm{lk}(\beta_{j'}^{n-1}, (\beta_j^i)^+) = 0$ for all $i = 1, \cdots, n-2$ and $j, j' = 1, \cdots, p-1$. These observations imply that the dashed and the solid gray do not link to each other in both diagrams. □

Claim 3.2. *Framed links (2-a) and (2-b) in Figure 9 are Legendrian isotopic.*

Proof. This follows from Claim 3.1 and Legendrian Reidemeister moves described in Figure 10. □

Claim 3.1 also implies that diagrams (2-b) and (2-c) are Legendrian isotopic. Notice that each gray or dashed component labeled β_j^i in (2-b) is identified with α_{j-1}^i in (2-c) where $j = 1, \cdots, p-1$.

Claim 3.3. *Contact surgery descriptions (1) and (2-c) of Figure 9 give contactomorphic manifolds.*

Proof. Recall that the black link in Figure 9 has contact surgery coefficient -1 and it corresponds to the monodromy $(\mathcal{D}_1^{n-1} \circ \cdots \circ \mathcal{D}_{p-1}^{n-1})^{k-1}$ of the open book decomposition. Due to Lemma 2.2, it is equivalent to $(\mathcal{D}_0^{n-1} \circ \cdots \circ \mathcal{D}_{p-2}^{n-1})^{k-1}$. This means that we can replace each component labeled $\beta_j^{n-1} = \alpha_j^{n-1}$ of the black link in (2-a) by α_{j-1}^{n-1} for $j = 1, \cdots, p-1$. This replacement gives exactly diagram (1).

□

This complets the proof of Theorem 1.3. □

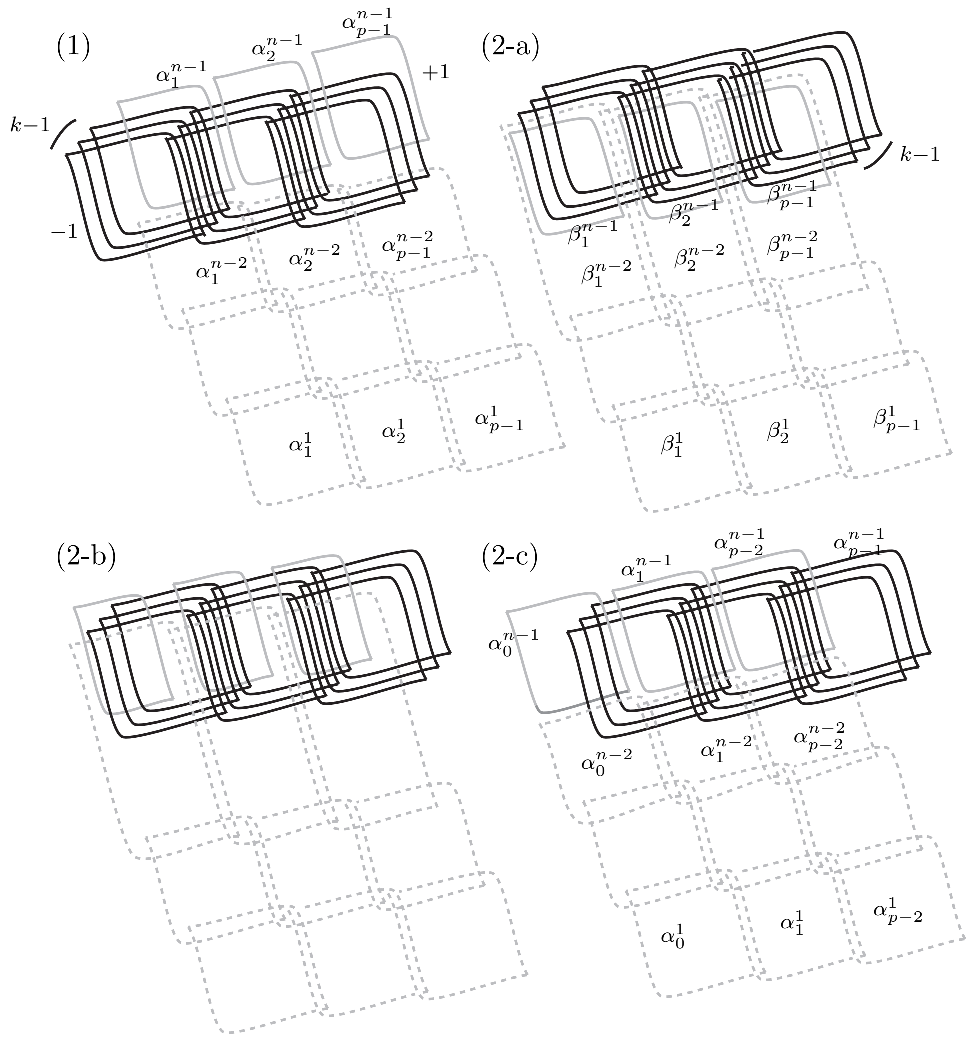

FIGURE 9. (1) Diagram $\Omega(w_1)$. (2-a) Diagram $\Omega(w_2)$. The black components correspond to σ_{n-1}^{k-1} ($k \geq 2$) and have contact surgery coefficient -1. The solid gray ones correspond to σ_{n-1}^{-1} having contact surgery coefficient $+1$. Linking and surgery coefficients of the dashed components vary depending on the braid word w.

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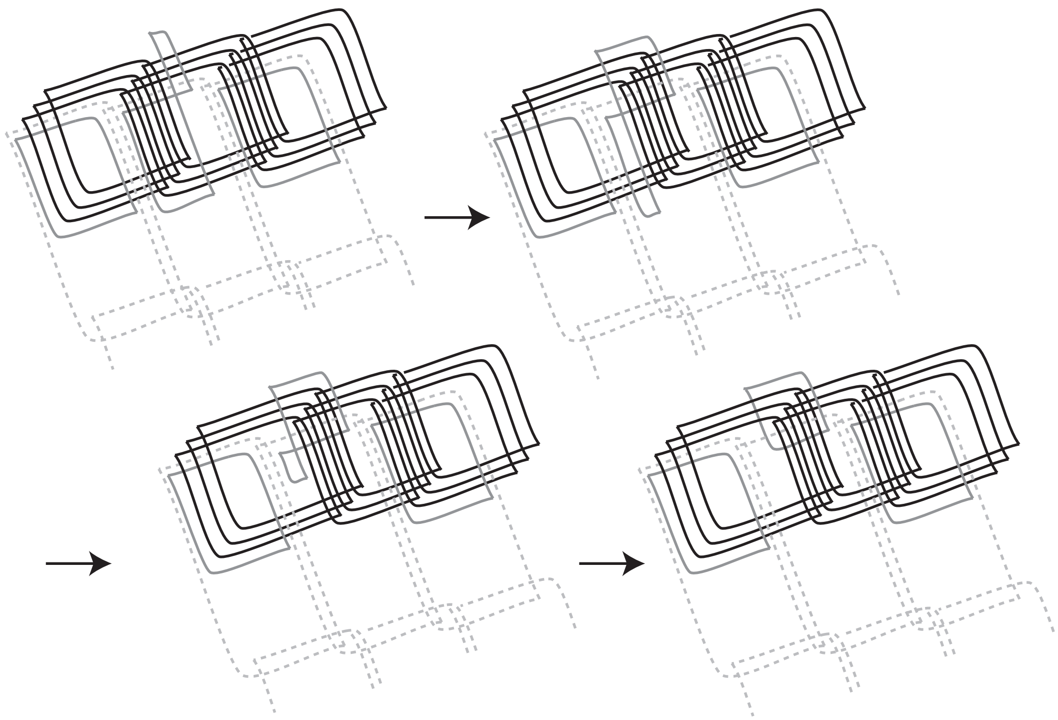

Figure 10. Legendrian Reidemister moves

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