Securities Trading when Liquidity Providers are Informed

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Comments are Most Welcome

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Abstract

We study securities trading when liquidity is provided by informed agents—either because dealers have superior access to market information or because informed traders exploit strategies involving limit orders. In the case of informed dealers, we show that dealers and informed traders profit more at the expense of uninformed liquidity traders when markets are transparent than when markets are opaque. This is because transparency serves as a coordination device that reduces competition among informed traders. When the informed are allowed to choose whether to trade via market orders or price contingent (limit) orders, informed traders gravitate toward limit orders. The endogenous allocation of traders to order types maximizes competition among the informed thereby minimizing expected losses to liquidity traders. This extends Glosten's (1994) conclusion on the robustness of limit order markets to a setting where liquidity is provided by informed traders and competition among liquidity providers is imperfect in equilibrium. The predictions of our model are consistent with recent empirical evidence that the price impact of limit order arrivals is greater than that of market order executions. This suggests that the usual approach in empirical microstructure research of measuring adverse selection as the trade-correlated permanent component of price changes significantly understates the true importance of adverse selection in securities markets.

Introduction

Models of securities trading with asymmetric information typically examine settings in which informed traders submit market orders to liquidity providers who are uninformed and perfectly competitive. For example, the dealers in Glosten and Milgrom (1985), Kyle (1985) and Easley and O'Hara (1987) have no private information and set prices to break even in expectation. Similarly, in Glosten (1994) liquidity is provided by a book of limit orders submitted by perfectly competitive uninformed traders. A few models abandon the assumption of perfect competition. Glosten (1989) examines the price schedule posted by a single monopolist specialist in a setting where market orders are submitted by a trader with information and a liquidity need. Dennert (1993), Biais, Martimort and Rochet (2000) and Bondarenko (2001), consider settings where each of a collection of profit maximizing dealers posts a separate price schedule. A single informed trader and noise traders then choose how to allocate their market orders across dealers. Though dealers are imperfectly competitive in those models, they are still uninformed.

This paper relaxes the assumption that liquidity providers are uninformed. We also relax the assumption that they are perfectly competitive, albeit in a manner different from Dennert (1993), Biais, et.al. (2000), and Bondarenko (2001).

Two ideas motivate our approach. First, in security markets with designated dealers, liquidity providers might have information by virtue of their role as dealers that is not available to others. Depending on the rules of the market, they observe who wishes to trade, the size of desired trades and order executions earlier or with greater precision than non-dealers. These observations convey information to dealers about the direction of future price changes and might also give dealers an advantage at interpreting public information about fundamentals. This reasoning extends beyond markets with designated dealers as well. If public limit orders are exposed to the market, those with private information have the option to trade as providers of liquidity. If they exercise this option, liquidity is provided by agents with private information.

The second idea is that in many public securities markets, dealer quotes and depths are consolidated into best bid and offer demand and supply schedules against which market orders are crossed. That is, dealers' price contingent orders are pooled centrally, and demand for liquidity is then allocated to dealers based on price priority. Our model incorporates this feature. This departure from the approach taken in Dennert (1993), Biais, et.al. (2000) and Bondarenko (2001) is merely aesthetic when dealers are uninformed. However, it is important when dealers have private information as studied here, because the strategies that dealers and traders adopt depend on how the best bid and offer schedules aggregate dealers' information. The model is also easier to solve when dealers' orders are consolidated. Demanders of liquidity need not optimize over dealers because the market's aggregation into best bid and offer schedules takes care of this.

Another difference between our model and those cited above is that we allow for *multiple* informed traders. In some cases, our model's predictions depend on the intensity of competition among informed traders. The dependence of these predictions on the competitive environment is lost by focusing on a single informed trader.

In our model, informed dealers submit supply schedules to a centralized market that respects price priority. Informed and liquidity traders submit market orders that execute against the consolidated schedule of orders submitted by dealers. Market order traders do not observe the supply schedules submitted by dealers, though they are aware of the strategies dealers follow in formulating those schedules. Thus, market order traders are uncertain as to the execution price of their orders, and liquidity providers are uncertain as to the quantities they will trade.

We study two settings. In the first, the number of dealers is exogenous. This corresponds to a situation where dealers are either designated by the market or precommit to playing that role. Dealers have information in common that they are assumed to possess by virtue of their position as dealers. If this information is unavailable to non-dealers, the market is said to be opaque. We then study the impact of instituting a rule that promotes transparency with respect to the information that dealers possess.

We show that dealers earn nothing from the information they have in common, even in an opaque market. Instead, they profit from price discrimination against informed traders according to how intensely they expect the informed to trade. Promoting transparency leads traders' strategies to depend on information that includes the market information that dealers observe. This, in turn, enables dealers to better forecast the trades of the informed and extract more from them via price discrimination. The informed react to this by reducing the aggressiveness with which they trade, thereby competing less aggressively with each other. The net effect is that expected profit to both dealers and informed traders increases, provided that the number of dealers and the number of informed traders are not too small (i.e., neither has excessive monopoly power to begin with).

This result suggests that if dealers and informed traders determine a market's rules, a commitment to transparency will arise endogenously as a market matures. Such rules can be *detrimental* to uninformed liquidity traders, however. Transparency acts as a coordination device that limits competition among informed traders, which increases expected profit to dealers and informed traders at the expense of uninformed liquidity traders.

A practical implication of this result is that regulators need not be too concerned about the lack of transparency in trading in so-called dark pools of liquidity. These are crossing networks or call auctions in which traders do not see the state of liquidity supply at the time they submit their orders. Thus, in dark pools, traders are prevented from observing value relevant information that liquidity suppliers possess. Our analysis suggests that with sufficient competition among the informed, the uninformed actually *benefit* from the lack of transparency by suffering smaller losses to the informed. The *Wall Street Journal* and *Financial Times* report that volume traded using these mechanisms has tripled since 2004 to between 5% and 10% of total equities trading in the United States.[1] The ascendance of these trading protocols is consistent with how our model would view competition between markets—if the incumbent market is transparent, it is vulnerable to competition from another that is opaque.[2]

In the second setting of the model, distinctions between dealers and traders are not imposed exogenously. All informed agents are identically situated ex-ante—their private signals are independent and identically distributed, and agents *choose* whether to trade as liquidity providers or liquidity demanders. We also examine the possibility that liquidity provision is not fully anonymous; so by trading as a liquidity provider, an informed trader sacrifices a portion of his informational advantage.

If liquidity provision is fully anonymous, we show that the resulting (endogenous) market structure has the informed trading exclusively as liquidity providers, and that this allocation of traders to order types minimizes losses suffered by uninformed liquidity traders. This is reminiscent of Glosten's (1994) results on the competitiveness of a limit order market, but in a very different setting. In Glosten's model, liquidity providers are uninformed and the informed are constrained to trade via market order. In our model, the allocation of traders between liquidity provision and liquidity demand is endogenous because the informed are unconstrained as to order type. Our result is that in a fully anonymous market, competition among the informed in terms of order *type* in addition to order quantity, leads to an outcome that minimizes harm to liquidity traders.

When liquidity provision is less than fully anonymous, informed traders do not choose exclusively to supply liquidity. Some demand liquidity by trading via market orders to preserve their full informational advantage. Thus, a lack of anonymity acts as an impediment to competition among liquidity providers, similar to the effect of transparency in the setting with designated dealers.

[1] See *Wall Street Journal,* January 9, 2007, p. C1 and *Financial Times* January 31, 2007, p. 23.

[2] Apart from transparency, regulators are also concerned with the possible effects of fragmentation and the ability of these networks to attract a disproportionate share of informationless trades (i.e., cream skimming). Pirrong (2002) analyzes this issue.

An important feature of the equilibrium is that private information flows into prices through *both* supply schedule submissions *and* market order executions. This prediction is consistent with empirical evidence. Kaniel and Liu (2006) and Rourke (2006) compare the price impacts of limit order submissions and market order executions in intra-day data. Both studies conclude that limit order submissions convey more information to equity markets than market order executions. Interpreting their findings in light of our model suggests that the usual approach of measuring adverse selection as the *trade*-correlated permanent component of price changes misses the information content of limit order submissions. Therefore, extant estimates of adverse selection in securities trading may be very significantly understated. Our model also predicts that the relative importance of limit versus market orders in conveying information into prices depends on the degree of anonymity afforded to liquidity suppliers. The more anonymous the market, the more important will be limit order submissions. To our knowledge, this hypothesis has not been tested empirically.

From a methodological perspective, the analytical structure of our model combines the strategic informed market order traders of Kyle (1985) with informed who trade via supply schedules as in Kyle (1989). Our general setting where market orders *and* supply schedules are used by the informed combines the settings considered in Kyle's models into a single framework, yet it retains a very simple structure. This demonstrates that the seemingly distinct approaches to modeling trading strategies in Kyle's models are quite compatible and can be combined to yield a tractable model of securities trading with informed traders providing *and* demanding liquidity.

Another perspective on how our model relates to the existing literature is based on the observation that price contingent supply schedules are bundles of limit orders. Liquidity suppliers in our model can be interpreted as limit order traders. Though many studies model trading in limit order markets, most assume that informational asymmetries are absent and instead model heterogeneity that generates trading as idiosyncratic liquidity needs or private values of the asset [see, for example, Seppi (1997), Parlour (1998), Parlour and Seppi (2003), Rosu (2003), Foucault, Kadan and Kandel (2005) and Goettler, Parlour and Rajan (2005), Hollifield, Miller and Sandas (2004)]. Traders' order strategies do not convey information about security values to others, which simplifies things enough that the dynamics of the limit order book can be analyzed. There are a few studies of limit order markets besides Glosten (1994) in which asymmetric information does play a central role. These models are static like ours, but differ from our model in some important ways.

Kumar and Seppi (1993) focus on commonalities in order submissions between the informed and liquidity traders that arise endogenously from strategic behavior. Liquidity traders choose market and limit orders to optimize over price improvement and the cost of falling short of their target share holdings. To camouflage his trading, an informed trader must mimic the strategy of liquidity traders. Kumar and Seppi show that equilibrium market and limit order submissions are therefore driven by the exogenous factors that determine liquidity traders' order strategies. Their model focuses on the interaction between strategic liquidity traders and a single informed trader. Our results explore the implications of different allocations of information across agents and competition among the informed. To focus clearly on these issues, we suppress strategic behavior of liquidity traders and assume the distribution of their orders is exogenous.

Chakravarty and Holden (1995) consider an informed trader who can submit a market order and a single limit order. They show that the best strategic use of the limit order is often to place it on the opposite side of the market as the market order. Doing so bounds the price paid or received for the market order by the limit price. In our model, traders are allowed to submit full supply schedules (multiple limit orders). Our traders have enough control over execution prices that they need not pair limit and market orders to bound the range of execution prices. Kaniel and Liu (2006) show how informed traders can submit limit orders that undercut the quotes of an uninformed competitive market maker. However, they abstract from competition among the informed, which plays an important role in our model. In their setting, only one limit order exists at a time, so a given limit order submission does not face competition from limit orders of other traders. Our informed traders compete with each other in supplying and demanding liquidity, and the intensity of competition among them is important to explaining our results.

Though we capture the essence of limit orders by allowing individual informed traders to submit price contingent orders, our model does not incorporate an operational feature of real limit order markets that can affect the characterization of equilibrium. Specifically, we assume that trades clear at a uniform price rather than by market orders "marching" up or down the limit order book (discriminatory pricing). Baruch (2005) also makes this assumption in his comparison of open versus closed limit order books. This assumption enables us to obtain closed form solutions and facilitates comparisons with the existing models of strategic informed trading that are most familiar [i.e., Kyle (1985, 1989), Dennert (1993), etc.].

Whether there is a one-to-one mapping between the two pricing environments for a given model depends on other elements of the model. Viswanathan and Wang (2002) study a static model with imperfect competition among risk-averse uninformed liquidity providers. In their model, equilibrium allocations depend on the pricing convention employed by the market, so the two pricing environments are not equivalent. In contrast, Back and Baruch (2004) study a dynamic model with perfectly competitive uninformed risk neutral liquidity providers. In their setting, the two pricing conventions yield identical allocations because traders capture the full surplus to adjusting their strategies in response to the pricing convention. Since liquidity provision in our model is imperfectly competitive, our results might not characterize an otherwise identical market with discriminatory pricing. Nevertheless, the intuition behind our results is general enough that the assumption of uniform pricing does not distort the qualitative conclusions.

The next section sets out the details of the model and considers the case where dealers are uninformed as in Dennert (1993). Section 2 considers informed dealers where the numbers of dealers and informed market order traders are exogenous, and analyzes rules that promote market transparency. Section 3 allows the informed to choose between market and limit orders and examines the endogenous provision of liquidity. Section 4 concludes with a review of our main results and a discussion of possible extensions to the model. Proofs of propositions are collected in the Appendix.

1. Model

Our model builds on those of Kyle (1985) and Dennert (1993). The departure from Kyle that our model and Dennert's shares is that dealers are assumed to be less than perfectly competitive. The differences between our model and Dennert's are that (i) we allow dealers to be informed, (ii) dealer quotes are consolidated in a central market against which all market orders clear, and (iii) we allow for multiple informed agents to trade via market orders. This is important when dealers are informed because trading gains depend on the relative intensity of competition within dealer and (non-dealer) trader groups. Initially, we fix the number of informed dealers and informed market order traders exogenously. Later, we allow the informed to choose their order type and those numbers are endogenized.

There is a single security in zero net supply whose terminal payoff is $\tilde{v} \sim N(0, \sigma_v^2)$. We assume there are J risk-neutral dealers who submit supply schedules to the centralized market. The information possessed by dealer j is denoted by s_j, and his price-contingent schedule of orders is denoted by $y_j(s_j, p)$ where $y > 0$ indicates a quantity sold. We often refer to the collection of such schedules submitted by the J dealers as the limit order book. There are N informed traders who submit market orders. The information possessed by trader i is denoted by s_i, and his market order is denoted by $x_i(s_i)$ where $x > 0$ indicates a quantity purchased. The distinction between dealers and traders is simply that the former supply liquidity by submitting supply schedules and the latter demand it by placing market orders. Dealers and informed traders are assumed to maximize expected profit. The specification of the signals s, and whether J and N are exogenous or endogenous, vary across settings of the model that we analyze. Finally, we assume there are uninformed liquidity traders who submit net market orders of $\tilde{u} \sim N(0, \sigma_u^2)$, with $\sigma_u > 0$.

We focus on symmetric Nash equilibria. Thus we seek a pair of order strategies $\{y(\cdot), x(\cdot)\}$ that maximizes the expected profit of dealer j and informed trader i, respectively, given that (i) other dealers and informed traders are conjectured to follow these strategies and (ii) markets clear. The market clearing condition evaluated at the equilibrium orders implicitly defines the equilibrium price. The equilibrium price is a random variable in the dealers' and traders' optimization problems. Using a Nash equilibrium concept means that market order traders do not observe the consolidated supply schedule before submitting their orders. This situation is commonly referred to as a closed limit order book.[3]

In the equilibria that we study, limit and market order strategy functions are both linear. Consequently, the market clearing condition can be inverted to obtain a closed-form expression for the equilibrium price schedule implied by the limit order book as a function of the net market orders submitted, ω. We write the equilibrium price schedule as $\tilde{P} = \tilde{P}(\tilde{\omega})$, where the tilde on the functional $P(\cdot)$ emphasizes that the price schedule is random for reasons other than randomness of $\tilde{\omega}$ when dealers are informed. This is an important difference associated with modeling *informed* dealers. The price contingent orders that dealers submit are based on their private information. This implies that the consolidated price schedule will depend on dealers' information, and traders must take this into account when choosing their strategies.

It is useful to illustrate how our model works in the informational setting considered by Dennert (1993), but allowing for more than one informed trader $N \geq 1$. In Dennert's model, the informed trader observes the realization of the security's payoff in advance of trading, $s_i = v$. However, dealers are *uninformed* in Dennert's model, so s_j is null for all $j = 1, \ldots, J$. Thus we write each dealer's order as a function of the single argument P. Dealer j determines his optimal supply schedule $y_j(p)$ by solving this set of problems

$$\max_{y_j} E\left[(\tilde{P} - \tilde{v})y_j | \tilde{P} = p\right] \qquad \text{for every possible } p,$$

[3]In modeling an open limit order book, market order traders would observe the consolidated demand and supply schedules before choosing their quantities. A Stackelberg equilibrium concept is needed for this setting because dealers must anticipate how their orders affect market order traders' beliefs and strategies through the contents of the book. This is a very difficult problem, and perhaps explains why there are no existing models of an open limit order book where informed traders supply liquidity. For example, though Glosten (1994) models an open limit order book in which asymmetric information is a key feature, only uninformed traders supply liquidity. Baruch (2005) compares open versus closed limit order books, also in an environment where liquidity providers are uninformed. In his model, an open book communicates information about market depth but not about the security's value. We discuss the likely impact of opening the book in our model later in the paper.

taking as given the strategies of all other agents, $\{y_k(\cdot)$ for all $k \neq j$ and $x_i(\cdot)$ for all $i\}$, and market clearing. Informed trader i determines his optimal market order $x_i(v)$ by solving

$$\max_{x_i} E\left[(\tilde{v} - \tilde{P})x_i | \tilde{v} = v\right],$$

taking as given his information, $s_i = v$, the strategies of all other agents, $\{y_j(\cdot)$ for all j and $x_k(\cdot)$ for all $k \neq i\}$, and market clearing. The market clearing condition is

$$\sum_{j=1}^{J} y_j(\tilde{P}) = \sum_{i=1}^{N} x_i(\tilde{v}) + \tilde{u} \qquad \text{whatever are the realizations of } \tilde{v} \text{ and } \tilde{u}.$$

A pair of functions $\{y(\cdot), x(\cdot)\}$ is a symmetric Nash equilibrium if (i) for all realizations of $\tilde{v}$ and $\tilde{u}$, these strategies are optimal for dealer j and trader i when they conjecture that these strategies are followed by the other dealers and informed traders, and (ii) markets clear. The following result generalizes Dennert's model to the case where there can be more than a single informed trader (i.e., $N \geq 1$).

<u>Proposition 1</u>. *If $J > 2$, there exists a symmetric Nash equilibrium in which dealer and informed trader strategies are linear. Moreover, this is the unique symmetric equilibrium in which strategies are linear. Explicit expressions for the strategies are*

$$y(P) = \delta P \qquad \text{and} \qquad x(v) = \beta v$$

where

$$\beta = \frac{J\delta}{N+1} \qquad \text{and} \qquad J\delta = \frac{\sigma_u}{\sigma_v}\left(\frac{1}{N} + 1\right)\sqrt{\frac{J-2}{\frac{1}{N}(J-1)+1}}.$$

Dealer and trader ex-ante expected profits, denoted by π_j and π_i respectively, satisfy

$$J\pi_j + N\pi_i = \frac{\sigma_u^2}{J\delta} \qquad \text{and} \qquad \pi_i = \frac{\beta\sigma_v^2}{N+1}.$$

The condition $J > 2$ is necessary because, when $J \leq 2$, there is insufficient competition among dealers to produce a consolidated price schedule whose slope is finite. This is explained in Kyle (1989). Bernhardt and Hughson (1997) study the two-dealer case in detail and show that an equilibrium can exist with two dealers if liquidity trading is price elastic.

Substituting the equilibrium dealer orders into the market clearing condition yields

$$j\delta P = \omega$$

where $\omega \equiv \sum_{i=1}^{N} x_i + u$ is the flow of market orders. Thus, the equilibrium price schedule is

$$P(\omega) = \left(\frac{1}{J\delta}\right)\omega.$$

Since dealers are uninformed, randomness in the price is related only to ω. In the equilibria analyzed later in the paper, the price also depends on *dealers'* private information.

Kyle (1985) considers the case of a single informed trader facing a perfectly competitive market maker. Note that when $N = 1$, $J\delta \to \dfrac{2\sigma_u}{\sigma_v}$ as $J \to \infty$. Thus, when $N = 1$, the equilibrium in Proposition 1 converges to that of Kyle (1985) as competition among dealers intensifies. When J is finite, the slope of the price schedule is steeper than in Kyle's model, and this extra slope is the source of dealer profit.

When $N = 1$, the equilibrium strategies in Proposition 1 match those studied by Dennert (1993), though in our model, market orders are executed against the *aggregated* collection of limit orders. In Dennert's model, dealers post separate price schedules. Informed traders optimize over quantities traded, and over the distribution of market orders routed to different dealers. Liquidity traders do not optimize over quantity, but do optimize over dealers. In our model, consolidation of dealer orders into a centralized limit order book removes the need for market order traders to optimize over separate price schedules posted by dealers.

An important difference between our model and those based on Kyle (1985) with competitive market makers relates to what happens as $N \to \infty$. In Kyle's model, the informed are the only source of losses to liquidity traders because market making is perfectly competitive. So in Kyle-style models with multiple informed traders, losses to liquidity traders tend to zero as competition among informed traders intensifies. In our model, both dealers and informed traders are sources of losses to liquidity traders. Profit to informed traders tends to zero as $N \to \infty$, but losses to liquidity traders do not even when dealers are uninformed. We show later that liquidity trader losses tend to zero only if *both* $N \to \infty$ and $J \to \infty$ in our model.

We now examine a setting where dealers are informed, potentially as a consequence of superior access to market information. To highlight that this is a common source of information, dealers are assumed to observe identical signals. The case of uncorrelated signals is considered in section 3.

2. Informed Dealers and Market Transparency

By virtue of their position as liquidity providers, dealers may have access to information that can be used to forecast a security's future value. We model this by assuming that the security's payoff has two components, $\tilde{v} = \tilde{v}_1 + \tilde{v}_2$, where all dealers observe the realization of $\tilde{v}_1$ before submitting their supply schedules (i.e., $s_j = v_1$ for all j). One can think of v_1 as incomplete information about the security's payoff that dealers are able to glean from their presence in the trading crowd and what they observe during prior rounds of trading.

We define the market to be *opaque* if non-dealers are prevented from learning what dealers observe either because past market statistics such as prices and trade volumes are not reported publicly, or because non-dealers cannot observe the composition of the trading crowd. Blocking traders out of drawing inferences from these items would impair their ability to forecast the security's payoff. In this case, $s_i = v_2$. We contrast this with a *transparent* market where informed traders are not barred from learning what dealers observe from the trading process and $s_i = v$.[4]

We assume that $\tilde{v}_1$ and $\tilde{v}_2$ are independent normal random variables with zero means and variances $\sigma_{v_1}^2$ and $\sigma_{v_2}^2$, respectively. In this section, J is exogenous, meaning that there are designated dealers who provide liquidity and the N non-dealers demand liquidity. We examine the opaque market first then compare it to a transparent market.

<u>PROPOSITION 2 - OPAQUE MARKET</u>. *Assume that $s_j = v_1$ for all $j = 1, \ldots, J$ and $s_i = v_2$ for all $i = 1, \ldots, N$. If $J > 2$, there exists a symmetric Nash equilibrium in which dealer and informed trader strategies are linear. Moreover, this is the unique symmetric equilibrium in which strategies are linear. Explicit expressions for the strategies are*

$$y(v_1, p) = \gamma v_1 + \delta p$$
$$x(v_2) = \beta v_2$$

where

$$\gamma = -\delta \qquad \beta = \frac{J\delta}{N+1} \qquad \text{and} \qquad J\delta = \frac{\sigma_u}{\sigma_{v_2}} \left(\frac{1}{N} + 1 \right) \sqrt{\frac{J-2}{\frac{1}{N}(J-1)+1}}.$$

Dealer and trader ex-ante expected profits, denoted by π_j and π_i respectively, satisfy

$$J\pi_j + N\pi_i = \frac{\sigma_u^2}{J\delta} \qquad \text{and} \qquad \pi_i = \frac{\beta \sigma_{v_2}^2}{N+1}.$$

Total expected profit extracted from liquidity traders is $\sigma_u^2 / J\delta$. Using the formula for π_i and substituting for β and $J\delta$ yields explicit expressions for trader and dealer expected profit in terms

[4] Our perspective is that the difference between transparency and opacity lies in the ability of informed traders to forecast the realization of $\tilde{v}$ precisely, and not in their ability to forecast both $\tilde{v}$ and the dealers' beliefs. In a transparent market, informed traders are not impeded from forecasting the security payoff. However, they are not enabled to forecast the component of the payoff that the dealers predict. This strikes us as more realistic than assuming that transparent markets allow traders to actually unbundle the components of $\tilde{v}$ to figure out what dealers believe as well. In other words, the market is transparent, but dealers' beliefs are not.

of exogenous variables:

$$\pi_i = \left(\frac{\sigma_{v_2}\sigma_u}{N+1}\right)\sqrt{\frac{J-2}{N(J+N-1)}}$$
$$\pi_j = \left(\frac{\sigma_{v_2}\sigma_u}{J}\right)\sqrt{\frac{N}{(J-2)(J+N-1)}}.$$

These expressions verify some straightforward conclusions. First, informed traders compete among themselves (π_i is decreasing in N) as do dealers (π_j is decreasing in J). As competition among informed traders (dealers) intensifies, informed trader (dealer) profit tends to zero ($\lim_{N\to\infty}\pi_i = \lim_{J\to\infty}\pi_j = 0$). Second, agents in one group benefit from competition between members of the other group—π_i is increasing in J and π_j is increasing in N. Since informed traders benefit from intense competition among dealers, π_i converges to a positive constant as $J \to \infty$. This limit is the perfectly competitive market making environment of Kyle (1985) with multiple insiders. Similarly, dealers benefit from intense competition among the informed, and π_j therefore converges to a positive constant as $N \to \infty$. These observations imply our earlier assertion that liquidity traders suffer expected losses unless competition is intense among *both* informed traders *and* dealers.

A somewhat surprising asymmetry exists between traders and dealers. Though traders' expected profit depends on the quality of their private information, σ_{v_2}, dealers' expected profit is independent of σ_{v_1}. Dealers do not profit from their informational advantage over non-dealers. This is an indication of how much more intense is competition among traders when price contingent strategies are allowed than when they are not.[5] Despite being strategic, and having the option not to condition on price by setting $\delta = 0$, dealers choose to submit price contingent orders and $J > 2$ dealers are enough to compete away all rents to the information they hold in common. This can be seen in the consolidated price schedule by substituting dealers' equilibrium strategy into the market clearing condition and inverting for the consolidated price schedule. Noting that $\gamma = -\delta$, this yields

$$P(\omega) = v_1 + \frac{1}{J\delta}\omega = E\left[\tilde{v}|v_1\right] + \frac{1}{J\delta}(\omega - E\left[\tilde{\omega}|v_1\right]), \tag{1}$$

where $\omega \equiv \sum_{i=1}^{N} x_i + u$ is the net flow of market orders. Since $E\left[\tilde{\omega}|v_1\right] = 0$, the dependence of the price schedule on v_1 is through the intercept. Dealers' private information, v_1, anchors the price schedule (1) in the same way the *unconditional* expected security value anchors the price schedule of Kyle's (1985) perfectly competitive market maker. Thus, dealers' private information affects prices as though it were public information.

[5] This observation is explored more fully in the next section.

When there is a single informed trader ($N = 1$) the slope of the price schedule above converges to Kyle's (1985) as $J \to \infty$. However, for finite J, the slope is greater here than in Kyle's model. Dealer profit derives from this "extra" slope, which enables dealers to *share* in the rents that the informed earn from liquidity traders. Interestingly, expected dealer profit behaves like a tax on informed traders for access to the market. When there are no informed traders, dealers' expected profit is zero (i.e., $\pi_j \to 0$ as $N \to 0$). For a given number of informed traders, $N > 0$, the higher the quality of their information, or the more liquidity trading there is, the larger is $\sigma_{v_2}\sigma_u$ and the greater are informed trader *and* dealer profits. Finally, the more competition there is among the informed, the more intensively they trade for given values of σ_{v_2} and σ_u. Thus, dealer profit is greater the greater is N.

The greater is J, the greater is competition among the dealers and the lesser is their aggregate expected profit. In fact, how the losses of liquidity traders are shared between dealers and informed traders is determined entirely by the relative competitiveness of their groups, and not at all by the relative qualities of their information:

$$\frac{N\pi_i}{J\pi_j} = \frac{J-2}{N+1}.$$

Since dealers do not profit directly from their private information, equilibrium in this case is the same as if dealers are totally uninformed and informed traders know everything (i.e., $\sigma_{v_1} = 0$ and therefore $\sigma_v = \sigma_{v_2}$), the situation depicted in Proposition 1. We noted earlier that the equilibrium in Kyle's (1985) single-period model obtains as the limit of the equilibrium in Proposition 1 when dealers who are uninformed increase in number. The same is true with informed dealers in an opaque market. Key to this is *independence* between the information possessed by dealers and traders that occurs when non-dealers are prevented from observing what dealers observe. Dealer and trader strategies are more subtle in the transparent market considered next because dealers' and traders' information is correlated. The transparent market does not converge to Kyle's as $J \to \infty$.

<u>Proposition 3 - Transparent Market</u>. *Assume that $s_j = v_1$ for all $j = 1, \ldots, J$ and $s_i = v$ for all $i = 1, \ldots, N$. If $J > 2$, there exists a symmetric Nash equilibrium in which dealer and informed trader strategies are linear. Moreover, this is the unique symmetric equilibrium in which strategies are linear. Explicit expressions for the strategies are*

$$y(v_1, p) = \gamma v_1 + \delta p$$

$$x(v) = \beta v$$

where

$$\gamma = -\delta + \left(\frac{J-2}{J(J-1)}\right) N\beta \qquad \beta = J\delta\Psi \qquad \text{and} \qquad J\delta = \frac{\sigma_u}{\sigma_{v_2}}(\Phi+1)\sqrt{\frac{J-2}{\Phi(J-1)+1}}.$$

The symbols Ψ and Φ depend on exogenous variables only and are defined as

$$\Psi = \frac{1}{N+1}\left\{\frac{(\sigma_{v_2}^2/\sigma_{v_1}^2)}{1+(\sigma_{v_2}^2/\sigma_{v_1}^2) - \left(\frac{J-2}{J-1}\right)\left(\frac{N}{N+1}\right)}\right\} \qquad \Phi = \frac{1}{N}\left\{1 + \left(\frac{\sigma_{v_1}^2}{\sigma_{v_2}^2}\right)\frac{N+J-1}{(J-1)}\right\}.$$

Dealer and trader ex-ante expected profits, denoted by π_j and π_i respectively, satisfy

$$J\pi_j + N\pi_i = \frac{\sigma_u^2}{J\delta} \qquad \text{and} \qquad \pi_i = \beta\sigma_{v_2}^2\left\{1 - N\Psi - \frac{N\Psi}{J-1}\frac{\sigma_{v_1}^2}{\sigma_{v_2}^2}\right\}.$$

We know from the previous case that dealers do not profit directly from knowing v_1, but instead by extracting a share of the profit of *informed* traders. The fact that informed traders base their strategy on $v_1 + v_2$ in a transparent market means that dealers can use v_1 to forecast how the informed will trade. This sharpens dealers' ability to extract profit from the informed in a way that is not possible when the two information sets are disjoint as in an opaque market. This impacts the dealers' strategy in two ways.

The first effect is that dealers restrict supply somewhat to earn rents on their information about how the informed will trade. This can be seen by substituting dealer orders into the market clearing condition and inverting for the equilibrium price schedule.

$$\begin{aligned} P(\omega) &= v_1 - \frac{1}{J\delta}\left(\frac{J-2}{J-1}\right)N\beta v_1 + \frac{1}{J\delta}\omega \\ &= v_1 + \frac{1}{J\delta}\left(\frac{1}{J-1}\right)N\beta v_1 + \frac{1}{J\delta}(\omega - N\beta v_1) \end{aligned}$$

where $\omega \equiv \sum_{i=1}^N x_i + u$ is the net flow of market orders. Noting that $N\beta v_1$ is the dealers' expectation of ω conditional on their information v_1, the price schedule can be written as

$$P(\omega) = E\left[\tilde{v}|v_1\right] + \frac{1}{J\delta}\left(\frac{1}{J-1}\right)E\left[\tilde{\omega}|v_1\right] + \frac{1}{J\delta}(\omega - E\left[\tilde{\omega}|v_1\right]).$$

Comparing this expression to the price schedule associated with the opaque market (equation (1)) indicates that dealers' strategies increase the intercept when dealers expect the informed to be buying and decrease the intercept when they expect the informed to be selling. This price discrimination makes the shares more expensive to informed traders and is clearly a source of expected profit to dealers that does not exist in an opaque market. Of course, the informed anticipate this and scale back their trading as evidenced by the $\Psi < 1$ multiplier in the expression for β.

Second, the slope, δ, of the dealers' supply schedule now depends on $\Phi > \frac{1}{N}$ rather than $\frac{1}{N}$ as in Proposition 2. It is only through Φ that N enters $J\delta$. Thus, the shift from $1/N$ to $\Phi > 1/N$ that occurs as a result of moving to a transparent market has the same effect on traders' and dealers' strategies as if N were to decrease in the opaque market. This means that transparency has the effect of reducing the intensity of competition among informed traders.

For all but the smallest values of N to begin with, restraining competition among the informed leads to greater losses for liquidity traders. So a move to transparency acts as a coordination device that prevents the informed from trading so intensively that they aggressively compete away the rents to their information. Since dealers skim their profit from the informed, this generally leads to greater profit for *both* dealers *and* the informed. This will happen as long as N is not very small (i.e., when competition is low to begin with), and also as long as J is not so small (or $\sigma^2_{v_1}/\sigma^2_{v_2}$ not so large) that the dealers extract so much profit from the informed that the level of competition between the informed has only a small effect on dealers' profit.

Analytic proofs of these assertions are available, but they are illustrated more clearly in graphical form. In Figures 1 and 2 we plot, for various choices of J and N, the percentage *change* in expected losses to liquidity traders, and profit to informed traders, associated with moving from an opaque to a transparent market. In percentages, the figure for dealers is identical to that of the informed, so a dealer figure is omitted. In both pictures, the black curve denotes a zero change, and progressively lighter shading corresponds to positive changes of greater magnitudes. Note that the move to transparency *increases* the losses to liquidity traders, and profit to *both* dealers and informed traders, unless N or J are small.

This discussion suggests that when either information or liquidity provision (dealership) are concentrated (small N or small J), liquidity traders benefit from market transparency at the expense of dealers and informed traders. Alternatively, as a market "matures" and both dealership and private information become more widely disbursed, liquidity traders lose less if the market is opaque than if it is transparent. The reason is because transparency restrains competition among the informed, and competition among the informed benefits liquidity traders. It is common for security markets to be organized as membership organizations where the members who set policies are dealers and active traders. Our model predicts that as a market grows from concentrated to diffuse dealership and possession of information about fundamentals, the market's policies will evolve from opacity to transparency. This evolutionary path might appear to favor the interests of uninformed liquidity traders. However, our analysis indicates that exactly the opposite can be true. This also suggests that if large transparent markets possess a dominant share of trading activity, such markets would be vulnerable to competition from opaque rivals, provided that the rivals can achieve a sufficient degree of "maturity."

Briefly summarizing, when dealers observe information that is common among them, they do not profit from it directly. The source of their profit is extracting a portion of the profit that informed traders earn at the expense of liquidity traders. Transparency enables dealers to better forecast how the informed will trade. The strategic response of the informed is to reduce the intensity of their trading. This reduces the intensity of competition among the informed. As long as informed traders and dealers do not already exercise a great deal of monopoly power, the reduction in competition associated with transparency increases the profit both dealers and the informed earn at the expense of liquidity traders.

3. Ex-Ante Identical Traders and Choice of Order Type

The analysis in the previous section exogenously specifies traders as providers or demanders of liquidity. Traders' information sets are also determined by their role in the market, and those with a particular role are homogeneously informed. In this section, we consider the situation where informed agents are identical ex-ante, except that their information signals are distinct. We then examine the endogenous provision of liquidity by allowing agents to choose whether to submit supply schedules or market orders. We show that this gives rise to the usual prisoners dilemma inherent in imperfect competition. However, in addition to applying to order quantities, it applies to order *types*. It is individually rational for informed agents to trade as liquidity providers to capture rents to liquidity provision, but doing so intensifies competition among them. Intense competition benefits liquidity traders, whose losses are minimized when the informed all trade as liquidity suppliers.

This conclusion is reminiscent of Glosten's (1994) results on the robustness of a market in which liquidity is provided by perfectly competitive uninformed agents in an open limit order book with discriminatory pricing. Our results illustrate that the spirit of Glosten's conclusion extends to a setting where *informed* traders provide liquidity and *imperfect* competition among liquidity providers exists in an equilibrium with uniform pricing.

We assume that there are M informed agents. Each agent observes $s_i = v_i$. The $\tilde{v}_i$s are iid $N(0, \sigma_{vi}^2)$, where $\sigma_{vi} > 0$ is the same for all i. The security pays off $\tilde{v} = \tilde{v}_1 + \tilde{v}_2 + \cdots + \tilde{v}_M$. Since we denote the variance of $\tilde{v}$ by σ_v^2, we have $\sigma_v^2 = M\sigma_{vi}^2$. Of the M agents, $J > 2$ trade as dealers by submitting price contingent supply schedules denoted $y_j(v_j, p)$, where $y > 0$ indicates a sale. The remaining $M - J$ traders submit market orders denoted $x_i(v_i)$, where $x > 0$ indicates a market purchase. An equilibrium mixture of agents between dealing and trading is defined as the value J_* so that when J_* agents act as dealers by submitting supply schedules, neither dealers nor market order traders have an incentive to switch order types. As before, net market order liquidity trading is denoted by $\tilde{u} \sim N(0, \sigma_u^2)$, with $\sigma_u > 0$.

Since we wish to draw comparisons with Glosten (1994), we begin with the case in which liquidity provision is frictionless. There are no direct costs or loss of informational advantage associated with supplying liquidity. This is a maintained hypothesis in Propositions 4 - 7. Later, we introduce the possibility that liquidity provision is not totally anonymous and, consequently, informed traders sacrifice a portion of their private information if they trade as liquidity providers.

The next result characterizes equilibrium when J is exogenous. We then use traders' expected profit functions to endogenize J by solving for J_*.

<u>Proposition 4 - Traders Identical Ex-Ante</u>. *Assume that $v = \tilde{v}_1 + \cdots + \tilde{v}_M$, $s_i = v_i$ for all $i = 1, \ldots, M$, and that $J > 2$ traders provide liquidity by trading as dealers and $M - J$ demand liquidity by utilizing market orders. There exists a symmetric Nash equilibrium in which order strategies are linear. Moreover, this is the unique symmetric equilibrium in which strategies are linear. Explicit expressions for the strategies are*

$$y_j(v_j, p) = \gamma v_j + \delta p \qquad \text{for } j = 1, \ldots, J$$
$$x_i(v_i) = \beta v_i \qquad \text{for } i = J+1, \ldots, M$$

where

$$\gamma = -\beta \qquad \beta = \frac{J\delta}{2} \qquad \text{and} \qquad J\delta = \frac{2\sigma_u}{\sigma_{vi}}\sqrt{\frac{(J-2)}{J(M-1)}}.$$

Dealer and trader ex-ante expected profits, denoted by π_j and π_i respectively, satisfy

$$J\pi_j + (M-J)\pi_i = \frac{\sigma_u^2}{J\delta} \qquad \text{and} \qquad \pi_i = \frac{\beta\sigma_{vi}^2}{2}.$$

Dealer strategies are linear and separable in the signal and price as in the earlier sections. Supply schedules are upward sloping because $\delta > 0$. This slope and the responsiveness of orders to signals are jointly determined in equilibrium. This means, for example, that the β and γ parameters are not the same as they would be in a market with perfectly competitive liquidity provision such as Kyle (1985).

To identify the equilibrium number of liquidity providers, let $\pi_j(J)$ and $\pi_i(J)$ denote dealer and trader profit given that J agents act as dealers by trading via supply schedules. It turns out that dealer profit is greater than trader profit over the entire feasible range of J, $2 < J \leq M$. This means that $J_* = M$. In equilibrium, all M informed agents trade via schedules. In addition, liquidity trader losses, $\frac{\sigma_u^2}{J\delta}$, are *decreasing* in J over $2 < J \leq M$. This means that the most favorable pricing possible for liquidity traders is achieved when $J = M$. Together, these facts imply that competition on the basis of order *type* (in addition to the usual competition in quantities given order type) gives rise to an equilibrium that minimizes expected losses sustained by liquidity traders. These assertions, proved in the Appendix, are summarized in the following proposition.

<u>Proposition 5 - Endogenous Liquidity Provision</u>. *In equilibrium, the number of liquidity providers is $J_* = M$: all informed traders submit supply schedules and the equilibrium flow of market orders is composed entirely of liquidity trades. This outcome minimizes expected losses sustained by liquidity traders across all possible distributions of informed traders to order types.*

We interpret this as an extension of Glosten's (1994) result to an environment where informed traders participate in liquidity provision and competition is less than perfect in equilibrium. In Glosten's model, uninformed traders place zero-profit limit orders and informed traders place market orders *by assumption.* Proposition 5 says that without these assumptions the *opposite* allocation of informed traders to order type occurs endogenously. Nevertheless, that allocation produces an outcome that is maximally competitive.

Abandoning the assumption that liquidity provision is necessarily perfectly competitive, draws informed agents into trading as liquidity providers to capture a share of the rents to supplying liquidity. Since there are rents to providing liquidity even when all the informed do so, $J = M$, the resulting equilibrium is not perfectly competitive (unless, of course, $M \to \infty$). Consequently, the price schedule in the limit order book is steeper, and transaction prices more volatile, than in a market with perfect competition among liquidity providers. The implications of this are summarized in the next result.

<u>Proposition 6 - Liquidity Trader Losses</u>. *In the equilibrium of Proposition 5, the expected losses of liquidity traders are given by $\frac{\sigma_u \sigma_v}{2}\sqrt{\frac{(M-1)}{(M-2)}}$, which is greater than the expected losses of liquidity traders in a market where each of the M informed traders submit market orders to a perfectly competitive, uninformed, risk-neutral market maker: $\frac{\sigma_u \sigma_v}{2}$. In addition, the bid-ask spread is wider, and transaction prices more volatile, in the equilibrium of Proposition 5 than with a perfectly competitive market maker. These differences disappear as the number of informed*

traders gets large ($M \to \infty$).

When the informed provide liquidity, some of their private information is incorporated into the consolidated price schedule in the limit order book. This contrasts with the case where liquidity providers are uninformed and information becomes incorporated only through market order executions. To illustrate, consider the following decomposition:

$$\tilde{v} - E[\tilde{v}] = \left\{\tilde{P}(0) - E[\tilde{v}]\right\} + \left\{\tilde{P}(\tilde{\omega}) - \tilde{P}(0)\right\} + \left\{\tilde{v} - \tilde{P}(\tilde{\omega})\right\}. \tag{3}$$

Before trading occurs, the unconditional expectation is the market's consensus estimate of the security's value. Sometime later, the security's price comes to reflect the security's true value $\tilde{v}$. The difference $\tilde{v} - E[\tilde{v}]$ is the information that the market must eventually absorb. This decomposition enables us to trace the steps by which information flows into the market.

In the first component, $\tilde{P}(0)$ denotes the midquote of the price schedule in the limit order book, so $\tilde{P}(0) - E[\tilde{v}]$ reflects information incorporated into prices by the submission of supply schedules. The second term, $\tilde{P}(\tilde{\omega}) - \tilde{P}(0)$, captures information conveyed to the market by the execution of market orders, $\tilde{\omega}$, beyond that conveyed by the arrival of limit orders. This component also reflects whatever premium liquidity providers earn from market order traders. The first two components correspond to what an econometrician would measure as the price impacts of limit and market order arrivals, respectively. The last term reflects incorporation of whatever private information remains, and a reversal of the temporary effect of the liquidity premium embedded in the transaction price.

Equation (3) implicitly subdivides the single period in our model into stages. In the first, strategies $\{y_j\}_{j=1}^{J_*}$ are submitted to the central market and consolidated into a price schedule. In the second stage, market orders $\omega \equiv \sum_{i=J_*+1}^{M} x_j + u$ are cleared against that price schedule. This interprets the signal-contingent component of y_j as the intercept of a supply schedule that is submitted in the first stage, rather than as a market order that is "held back" for submission in the second stage. So far in the paper, we have not made any assumptions about whether the components of y_j remain bundled. This is a matter of indifference to traders because all orders are cleared simultaneously in a Nash equilibrium. However, to clarify the model's empirical implications, a hypothesis in the next result is that liquidity provider strategies remain bundled. This interpretation is consistent with the existence of a (small) cost to separating order submissions.

<u>Proposition 7 - Decompositions</u>. *If liquidity providers do not unbundle their strategies, the*

equilibrium in Proposition 5 has these components:

$$\tilde{P}(0) - E[\tilde{v}] = \frac{1}{2}\sum_{i=1}^{M}\tilde{v}_i$$

$$\tilde{P}(\omega) - \tilde{P}(0) = \left(\frac{\sigma_v}{2\sigma_u}\sqrt{\frac{M-1}{M-2}}\right)\tilde{u}$$

$$\tilde{v} - \tilde{P}(\tilde{\omega}) = \frac{1}{2}\sum_{i=1}^{M}\tilde{v}_i - \left(\frac{\sigma_v}{2\sigma_u}\sqrt{\frac{M-1}{M-2}}\right)\tilde{u}.$$

If liquidity is provided by a perfectly competitive, uninformed, risk-neutral market maker (denoted by "hats"), the components are:

$$\hat{P}(0) - E[\tilde{v}] = 0$$

$$\hat{P}(\hat{\omega}) - \hat{P}(0) = \frac{1}{2}\sum_{i=1}^{M}\tilde{v}_i + \left(\frac{\sigma_v}{2\sigma_u}\right)\tilde{u}$$

$$\tilde{v} - \hat{P}(\hat{\omega}) = \frac{1}{2}\sum_{i=1}^{M}\tilde{v}_i - \left(\frac{\sigma_v}{2\sigma_u}\right)\tilde{u}.$$

In particular, in the equilibrium of Proposition 5, the midpoint of the bid-ask spread has less mean-square error as a predictor of the security's true value than does the midpoint of the bid-ask spread if liquidity is provided by a perfectly competitive, uninformed, risk-neutral market maker.

The private information that flows into prices before $\tilde{v}$ is revealed is the same in both markets and is equal to $\frac{1}{2}\sum_{i=1}^{M}\tilde{v}_i$. In the equilibrium of Proposition 5, this occurs with the arrival of *limit* orders. Market orders are informationless because all informed traders gravitate to supplying liquidity. However, when liquidity providers are uninformed as in the second market, midquotes are informationless and the arrival of *market* orders is how prices come to reflect private information. Transaction prices in both markets contain liquidity premiums. The premium is greater in the former market because liquidity provision is imperfectly competitive. In both markets, the liquidity premium reverses and additional information is revealed when $\tilde{v}$ becomes public knowledge.

The perspective that *private* information is incorporated into prices without order executions is vastly different from the assumption underlying almost all existing empirical work measuring adverse-selection in security markets [e.g., George, Kaul, Nimalendran (1991), Hasbrouck (1991a, 1991b), Huang and Stoll (1997), Madhavan, Richardson and Roomans (1997), and George and Hwang (2001) and many others]. In those studies the impact of private information is defined as the component of permanent price changes that is correlated with order executions, which necessarily means market orders or marketable limit orders. The remainder of permanent price changes are attributed to *public* information arrivals. In the equilibrium of Proposition 5, private information is incorporated into prices without order executions, suggesting that private information is misclassified as public news in existing studies. Thus, existing studies are likely to understate the importance of private information on security price dynamics because limit order arrivals are ignored as a means by which private information is impounded into prices.

Two exceptions to the approach taken in the studies cited above are Kaniel and Liu (2006) and Rourke (2006). They examine the price impacts of limit and market orders separately for evidence that either type of order is motivated by private information. Both studies conclude that limit orders contain more private information than market orders for equities. Their findings are consistent with Proposition 7, which predicts that the price impact of both limit and market order arrivals is non-zero.[6]

We now turn to the case in which supplying liquidity is not frictionless. The extreme prediction of Proposition 5, that informed traders utilize limit orders *exclusively*, depends critically on the absence of market frictions. If there are costs or barriers to liquidity provision, informed trading will be split between market and limit orders. Such frictions exist in real markets and could be modeled as exogenous monetary costs to limit order submission (e.g., the cost of monitoring limit order positions). We believe that a more interesting possibility is the loss of informational advantage associated with a trader revealing his willingness to provide liquidity. The cost associated with this friction is endogenous, and depends on the intensity of competition among informed traders. We now consider how this alters traders' choices of order types.

In a non-anonymous market, the fact that a trader shows up willing to supply liquidity could convey information to market participants. For example, the appearance of an owner of a large coffee grove to deal in coffee futures might convey an indication of future spot market conditions to traders who observe his presence and trading pattern. Here, an informed trader's election to trade as a dealer conveys information to markets and dissipates part of the informational advantage the agent would possess if he were to trade anonymously via market order. If providing liquidity causes traders to sacrifice enough of their informational advantage, it is no longer true that limit orders uniformly dominate market orders.

We model this loss of informational advantage by assuming that if trader i elects to trade via

[6]Distinguishing empirically between limit orders that are motivated by private information versus those that are placed to reposition quotes to reflect public news is not entirely straightforward. However, Rourke (2006) finds a significant relation between price impacts and the *size* of limit order submissions. If limit order traders react to public news by repositioning their quotes, there is no obvious reason why limit orders of larger sizes would be placed in reaction to bigger news of which everyone is already aware. His finding suggests that a significant fraction of limit order trading is motivated by private information.

market order, his signal is drawn from $N(0, \sigma_I^2)$. However, if he elects to submit a supply schedule, his presence as a liquidity provider reduces his informational advantage and his signal is drawn from $N(0, \sigma_D^2)$, where $\sigma_D < \sigma_I$. Traders that choose to provide liquidity convey information to the market as a whole and therefore reduce uncertainty associated with the security's payoff. Since $\tilde{v} = \tilde{v}_1 + \cdots + \tilde{v}_M$, total payoff uncertainty, $\sigma_v^2 = J\sigma_D^2 + (M - J)\sigma_I^2$, is smaller, the more traders elect to be dealers.

<u>Proposition 8 - Interior vs. Boundary Equilibrium</u>. *Assume that $M > 2$, that those who choose to trade as dealers observe signals that are iid draws from $N(0, \sigma_D^2)$, and non-dealers' signals are iid draws from $N(0, \sigma_I^2)$. If $\dfrac{\sigma_D^2}{\sigma_I^2} < \dfrac{M-2}{M-1}$, then $J_* < M$; otherwise $J_* = M$.*

Proposition 8 indicates that the informed use *both* market and limit orders in markets where their decision to provide liquidity conveys sufficient information to other traders. The degree to which σ_D^2/σ_I^2 is less than one measures the extent of the informational advantage sacrificed by those who choose to supply liquidity. The more informed traders there are, the smaller is the sacrifice that leads potential liquidity providers to switch to market orders.

This result also indicates that the level of competition that emerges endogenously among liquidity providers is more intense (i.e., J_* is larger), the greater is the level of anonymity provided by the market's structure (i.e., the closer is σ_D^2/σ_I^2 to one). This mirrors the conclusion in section 2—anonymity here, like opacity before, promotes more intense competition. Here, competition becomes more intense among liquidity providers.

The empirical implication of Proposition 8 is that when $J_* < M$, both limit *and* market order arrivals cause information to be impounded into prices. This softens the prediction in Proposition 7. In reference to the expressions in Proposition 7, the average signal of J_* liquidity providers arrives with the submission of orders into the limit order book, and the average signal of the remaining $M - J_*$ traders arrives with the execution of market orders. Thus, there is an informational role for both limit and market orders as documented by Kaniel and Liu (2006) and Rourke (2006). A further implication is that, for a given M, the relative importance of limit versus market orders in conveying information to markets depends on the degree to which traders' anonymity is protected. The more anonymous is the market, the more information will flow into prices via limit orders. To our knowledge, this prediction has not been empirically tested.

This result also sheds light on how equilibrium might differ in a model of an open versus closed limit order book when liquidity providers are informed. As noted earlier, a proper analysis of an open book with informed liquidity providers requires that liquidity providers explicity account for how their actions (as reflected in the contents of the book) affect the actions of market order traders. This can be complicated. For example, if incentives to mislead market order traders are sufficiently strong, an equilibrium might not exist. However, if an equilibrium does exist, the essence of the difference is that an open book conveys information to the market about the security's value before traders submit market orders. This dissipates the informational advantage of informed liquidity providers, as in Proposition 8. Thus, Proposition 8 suggests how the situation differs in an open book environment. The clearer is the book as a signal of liquidity providers' information, the closer an open book replicates a large sacrifice of informational advantage by those who provide liquidity.

An interesting case to consider is the extreme at which an open book causes liquidity providers to lose their informational advantage entirely ($\sigma_D^2 \to 0$). Though this minimizes the number of liquidity providers, even in this case markets do not break down. Liquidity will be provided because the rents to liquidity provision are sufficient to compensate enough liquidity providers for their loss of private information that an equilibrium still exists.

<u>Proposition 9</u>. *Under the same assumptions as in Proposition 8, even if* $\sigma_D^2 = 0$, $J_* > 2$.

In this case, liquidity providers have no informational advantage and private information is possessed only by market order traders. This result can be interpreted as a justification for the assumptions made in many of the models described in the introduction that liquidity is provided by uninformed traders and the informed trade exclusively via market orders. In particular, if liquidity provision is sufficiently transparent that liquidity providers forego their entire informational advantage, a market will still exist with uninformed traders providing liquidity and the informed trading via market orders. Liquidity provision will not be perfectly competitive, however.

Before concluding, we note that the analytical structure of our model combines elements of Kyle (1985) and Kyle (1989). We observed earlier that with a single informed trader ($N = 1$), our model converges to Kyle's (1985) as the number of dealers grows without bound ($J \to \infty$). Another special case is the boundary equilibrium of Proposition 5, where the informed submit only limit orders and market orders derive entirely from liquidity traders. This case coincides with Kyle (1989) as risk aversion coefficients tend to zero. In his model, informed traders with constant absolute risk aversion submit demand schedules to a centralized market against which market orders submitted by liquidity traders clear.[7] In our model, when the equilibrium is interior ($J_* < M$),

[7]A sketch of the proof is as follows. Kyle's (1989) N is our J. By his equation (37), his γ_I is our $-\delta$. These definitions and his equation (41) imply that his $1/\lambda_I$ equals our $-(J-1)\delta$. His equation (46) with $\rho \to 0$ (risk aversion disappears) is the same as our equation (2.3). So traders' strategies as functions of their conditional expectations are the same in our and his models. (The actual strategies are a bit different because his agents each observe a noisy

informed market order traders of Kyle (1985) coexist with informed liquidity providers who trade via schedules as in Kyle (1989). The tractability of our model shows that the key elements of Kyle's models are compatible when combined into a single framework.

4. Conclusion

Existing models of security trading with asymmetric information focus on uninformed liquidity providers who are either perfectly competitive, or compete in decentralized markets with each posting his own price schedule. This paper develops a tractable model of securities trading in centralized markets that allows private information to be possessed by both suppliers and demanders of liquidity, and where competitiveness on both the liquidity supply and demand sides of the market is endogenous. We analyze two settings of this model. In the first, the designation of agents as market order traders versus dealers (liquidity suppliers) is exogenous, and we examine the impact of market transparency on the various classes of agents. In an opaque market, dealers possess private information about the security's value that is hidden from informed traders. In a transparent market, such information is not hidden.

In this setting, dealers do not profit from private information that is hidden from informed traders. This is because the source of dealers' profit lies in their ability to forecast how the informed will trade and to price discriminate across states of varying intensity of informed trading. In transparent markets, traders condition on the information observed by dealers. Consequently, transparency enhances dealer profit by increasing the dealers' ability to forecast informed trading. This also enhances the profit of informed traders because the price discrimination they face by dealers deters them from trading as aggressively on their information as they otherwise would. Since transparency increases the profit to both dealers and informed traders, it increases losses to uninformed liquidity traders. This seemingly counterintuitive result arises because transparency serves as a coordination device to limit competition among informed traders. We conclude that if dealers and informed traders are responsible for setting a market's rules, then as a market matures, both dealers and the informed favor transparency as a way to limit competition that reduces their profit. However, this also suggests that existing transparent markets are vulnerable to competition from opaque markets.

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signal of $\tilde{v}$ whereas ours observe a component of $\tilde{v}$. However, if the structure of signals were the same in both models, the strategies would be identical.) When there are no uninformed speculators in Kyle's model (i.e., his parameter $M{=}0$), his market clearing condition (38) is the same as ours. The strategies and market clearing condition define the equilibrium. Since they are the same in both models, then under identical information structures the equilibria are the same also.

In the second setting of the model, liquidity supply and demand are endogenous. Informed agents are ex-ante identical and *choose* whether they wish to trade via market or limit order. In this setting, liquidity provision is endogenous, and we consider two cases. In the first, liquidity provision is totally anonymous in the sense that the informed can trade as liquidity providers without sacrificing any of their informational advantage. In this case, *all* informed agents choose limit rather than market orders. We also show that pricing is maximally competitive in the sense that liquidity trader losses are minimized when the informed all choose limit orders. This extends Glosten's (1994) conclusions about the robustness of a limit order book to a situation where the informed are *unconstrained* as to order type and *imperfect* competition exists among liquidity providers in equilibrium.

The second case we consider allows for the possibility that the market is less than fully anonymous. In this case, informed traders utilize both limit and market orders. Quotes and prices behave differently in our model from models where the informed are *assumed* to submit only market orders. In our model, private information flows into prices through limit and market order arrivals, rather than market orders alone. This prediction is consistent with the empirical results of Kaniel and Liu (2006) and Rourke (2006).

Our model endogenizes the order choices of informed traders and, in equilibrium, the model can accommodate some informed traders submitting market orders and others supply schedules. Moreover, the equilibrium can be solved in closed form. The model is static and therefore invites an extension to a dynamic setting to capture the extent to which impatience to trade before information is revealed through prices affects agents' strategies. The simplicity of the model suggests that it might be tractable in a dynamic setting. The dynamic tradeoffs involved in selecting dynamic *market order* strategies have been analyzed by Foster and Viswanathan (1996), Back, Cao and Willard (2000) and Boulatov and Livdan (2006) all under the assumption that liquidity provision is perfectly competitive. To our knowledge, there are no dynamic models of asymmetric information where traders are permitted to submit price schedules or limit orders.

APPENDIX

Proof of Proposition 1: This result is a corollary to Proposition 2 in the case where $\sigma_{v_1} = 0$ and hence $\sigma_{v_2} = \sigma_v$.

Proof of Proposition 2: Consider informed traders first. Suppose that informed trader i believes that (i) dealers $j \in \{1, \ldots, J\}$ follow symmetric strategies of the form $y_j = \gamma v_1 + \delta P$, and (ii) other informed traders $k \neq i$ follow symmetric strategies of the form $x_k = \beta v_2$. Then trader i perceives the market clearing condition as

$$J\gamma\tilde{v}_1 + J\delta\tilde{P} = \tilde{\omega} \qquad \text{where} \qquad \tilde{\omega} = x_i + (N-1)\beta v_2 + \tilde{u},$$

or equivalently that the market clearing price is the realization of the random variable

$$\tilde{P} = \tilde{P}(\tilde{\omega}) = \frac{-\gamma}{\delta}\tilde{v}_1 + \frac{1}{J\delta}\tilde{\omega}.$$

Trader i then chooses x_i to maximize expected profit conditional on his information $s_i = v_2$:

$$\max_{x_i} E\left[\left(\tilde{v}_1 + v_2 + \frac{\gamma}{\delta}\tilde{v}_1 - \frac{1}{J\delta}\left[x_i + (N-1)\beta v_2) + \tilde{u}\right]\right) x_i \big| v_2\right].$$

Since $E\left[\tilde{v}_1 | v_2\right] = 0$, the first-order condition implies

$$x_i = \frac{1}{2}\left[J\delta - (N-1)\beta\right] v_2.$$

The second-order condition is $\delta > 0$, which we later show is satisfied by choosing the positive root of the quadratic that defines δ. This verifies that if any trader i believes the others follow symmetric linear strategies, he also follows a strategy that is linear in $s_i = v_2$. Symmetry among the traders' strategies implies that any trader i's strategy coefficient must be equal to the coefficient he believes defines the strategies of the other informed traders:

$$\beta = \frac{1}{2}\left[J\delta - (N-1)\beta\right] \qquad \Longleftrightarrow \qquad \beta = \frac{J\delta}{N+1}. \tag{2.1}$$

Now consider dealers. Suppose dealer j believes that (i) other dealers $k \neq j$ follow symmetric strategies of the form $y_k = \gamma v_1 + \delta P$ and (ii) informed traders $i \in \{1, \ldots, N\}$ follow symmetric strategies of the form $x_i = \beta v_2$. Then dealer j perceives the market clearing condition as

$$(J-1)\left(\gamma v_1 + \delta\tilde{P}\right) + y_j = \tilde{\omega} \qquad \text{where} \qquad \tilde{\omega} = N\beta\tilde{v}_2 + \tilde{u},$$

or equivalently that the market clearing price is a realization of the random variable

$$\tilde{P} = \tilde{P}(\tilde{\omega}) = \frac{-\gamma}{\delta}v_1 + \frac{1}{(J-1)\delta}\left(\tilde{\omega} - y_j\right). \tag{2.2}$$

Dealer j then chooses a price-contingent supply schedule $y_j(s_j, p)$ to maximize expected profit conditional on his information $s_j = v_1$ and on the realized price $\tilde{P} = p$:

$$\max_{y_j} E\left[\left(\frac{-\gamma}{\delta}v_1 + \frac{1}{(J-1)\delta}\left(N\beta\tilde{v}_2 + \tilde{u} - y_j\right) - \tilde{v}\right) y_j \big| v_1, \tilde{P} = p\right].$$

The first-order condition is

$$E\left[\left(\frac{-\gamma}{\delta}v_1 + \frac{1}{(J-1)\delta}\left(N\beta\tilde{v}_2 + \tilde{u} - 2y_j\right) - \tilde{v}\right) \big| v_1, \tilde{P} = p\right] = 0.$$

The second-order condition is $\delta > 0$. By (2.2), $\tilde{P} = \frac{-\gamma}{\delta}v_1 + \frac{1}{(J-1)\delta}\left(N\beta\tilde{v}_2 + \tilde{u} - y_j\right)$ so the first-order condition is equivalent to

$$E\left[\tilde{P} - \tilde{v} - \frac{1}{(J-1)\delta}y_j \big| v_1, \tilde{P} = p\right] = 0$$

or

$$y_j = (J-1)\delta\left(p - E\left[\tilde{v}|v_1, \tilde{P} = p\right]\right). \tag{2.3}$$

From the perspective of dealer j, all variates are jointly normal, so

$$\begin{aligned} E\left[\tilde{v}|v_1, \tilde{P} = p\right] &= v_1 + E\left[\tilde{v}_2|v_1, \tilde{P} = p\right] \\ &= v_1 + \frac{\mathcal{C}}{\mathcal{V}}\left(p - E\left[\tilde{P}|v_1\right]\right) \\ &= v_1 + \frac{\mathcal{C}}{\mathcal{V}}\left(p + \frac{\gamma}{\delta}v_1 + \frac{1}{(J-1)\delta}y_j\right) \end{aligned} \tag{2.4}$$

where

$$\mathcal{C} \equiv \mathrm{Cov}\left[\tilde{v}_2\,, \tilde{P}|v_1\right] = \frac{N\beta}{(J-1)\delta}\sigma^2_{v_2} \tag{2.5}$$

$$\mathcal{V} \equiv \mathrm{Var}\left[\tilde{P}|v_1\right] = \frac{(N\beta)^2\sigma^2_{v_2} + \sigma^2_u}{(J-1)^2\delta^2}. \tag{2.6}$$

Substituting (2.4) into (2.3) and solving for y_j yields

$$y_j(v_1, p) = (J-1)\delta\left\{\left(\frac{1 - \frac{\mathcal{C}}{\mathcal{V}}}{1 + \frac{\mathcal{C}}{\mathcal{V}}}\right)p - \left(\frac{1 + \frac{\mathcal{C}}{\mathcal{V}}\frac{\gamma}{\delta}}{1 + \frac{\mathcal{C}}{\mathcal{V}}}\right)v_1\right\}.$$

This verifies that if any dealer j believes the others follow symmetric linear strategies, he also follows a strategy that is linear in $s_j = v_1$ and the realized price p. Symmetry among the dealers' strategies implies that any dealer j's strategy coefficients must be equal to the coefficients he believes define other dealers' strategies:

$$\delta = (J-1)\delta\left(\frac{1 - \frac{\mathcal{C}}{\mathcal{V}}}{1 + \frac{\mathcal{C}}{\mathcal{V}}}\right) \qquad \Longleftrightarrow \qquad \frac{\mathcal{C}}{\mathcal{V}} = \frac{J-2}{J} \tag{2.7}$$

and

$$\gamma = -(J-1)\delta\left(\frac{1+\frac{\mathcal{C}}{\mathcal{V}}\frac{\gamma}{\delta}}{1+\frac{\mathcal{C}}{\mathcal{V}}}\right) \qquad \text{using (2.7)}$$

$$= -(J-1)\left(\frac{1+\frac{J-2}{J}\frac{\gamma}{\delta}}{1+\frac{J-2}{J}}\right) \qquad \Longleftrightarrow \qquad \frac{\gamma}{\delta} = -1. \tag{2.8}$$

To identify separately γ and δ, substitute (2.5) and (2.6) into (2.7)

$$\frac{N\beta}{(J-1)\delta}\sigma_{v_2}^2 = \frac{J-2}{J}\left\{\frac{(N\beta)^2\sigma_{v_2}^2+\sigma_u^2}{(J-1)^2\delta^2}\right\}.$$

Substituting for β from (2.1) gives an expression in terms of δ alone. Solving that expression for δ and selecting the positive root so second-order conditions are satisfied yields

$$\delta = \frac{N+1}{J}\frac{\sigma_u}{\sigma_{v_2}}\sqrt{\frac{J-2}{N(J+N-1)}}. \tag{2.9}$$

Equations (2.1), (2.8) and (2.9) define the parameter values that characterize a symmetric equilibrium that is unique in the linear class. Note that the assumption in the proposition that $J > 2$ ensures that δ is real valued.

Unconditional expected profit of dealer j in equilibrium is

$$\pi_j \equiv E\left[\left(\tilde{P}_* - \tilde{v}\right)\tilde{y}_{j*}\right]$$

where y_{j*} is the dealer's optimal strategy and P_* is the equilibrium price schedule (i.e., evaluated at dealers' and traders' optimal strategies). Thus,

$$\pi_j = E\left[\left(\tilde{v}_1 + \frac{\tilde{\omega}_*}{J\delta} - \tilde{v}_1 - \tilde{v}_2\right)\delta(\tilde{P}_* - \tilde{v}_1)\right]$$

$$= \delta E\left[\left(\frac{\tilde{\omega}_*}{J\delta} - \tilde{v}_2\right)\frac{\tilde{\omega}_*}{J\delta}\right] \qquad \text{where} \qquad \tilde{\omega}_* = \frac{NJ\delta}{N+1}\tilde{v}_2 + \tilde{u}.$$

Substituting for $\tilde{\omega}_*$ and taking expectations yields

$$\pi_j = \delta\left\{\frac{N}{N+1}\left(\frac{N}{N+1}-1\right)\sigma_{v_2}^2 + \left(\frac{1}{J\delta}\right)^2\sigma_u^2\right\}. \tag{2.10}$$

Substituting from (2.9) for δ and simplifying,

$$\pi_j = \frac{\sigma_u\sigma_{v_2}}{J}\sqrt{\frac{N}{(J-2)(J+N-1)}}. \tag{2.11}$$

From this, it is easy to show that

$$\frac{\partial \pi_j}{\partial \sigma_{v_2}} > 0 \qquad \frac{\partial \pi_j}{\partial \sigma_u} > 0 \qquad \frac{\partial \pi_j}{\partial J} < 0 \qquad \text{and} \qquad \frac{\partial \pi_j}{\partial N} > 0.$$

Unconditional expected profit of informed trader i in equilibrium is

$$\begin{aligned}
\pi_i &= E\left[\left(\tilde{v} - \tilde{P}_*\right)\tilde{x}_{i*}\right] \\
&= E\left[\left(\tilde{v}_1 + \tilde{v}_2 - \tilde{v}_1 - \frac{\tilde{\omega}_*}{J\delta}\right)\beta\tilde{v}_2\right] \\
&= \beta E\left[\tilde{v}_2^2 - \frac{1}{J\delta}(N\beta\tilde{v}_2 + \tilde{u})\tilde{v}_2\right] \\
&= \beta\left(1 - \frac{N\beta}{J\delta}\right)\sigma_{v_2}^2.
\end{aligned}$$

Substituting for β from (2.1) and simplifying

$$\pi_i = \frac{J\delta}{(N+1)^2}\sigma_{v_2}^2. \tag{2.12}$$

Substituting for δ from (2.9) yields

$$\pi_i = \frac{\sigma_{v_2}\sigma_u}{N+1}\sqrt{\frac{J-2}{N(J+N-1)}}. \tag{2.13}$$

From this, it is easy to show that

$$\frac{\partial \pi_i}{\partial \sigma_{v_2}} > 0 \qquad \frac{\partial \pi_i}{\partial \sigma_u} > 0 \qquad \frac{\partial \pi_i}{\partial N} < 0 \qquad \text{and} \qquad \frac{\partial \pi_i}{\partial J} > 0.$$

Finally, note that (2.10) is equivalent to

$$\pi_j = -N\delta\frac{\sigma_{v_2}^2}{(N+1)^2} + \frac{1}{J^2\delta}\sigma_u^2.$$

Using (2.12) to substitute for the first term

$$\pi_j = -\frac{N}{J}\pi_i + \frac{\sigma_u^2}{J^2\delta}$$

or

$$J\pi_j + N\pi_i = \frac{\sigma_u^2}{J\delta}.$$

This completes the proof of Proposition 2. ‖

Proof of Proposition 3: The differences between this proof and that of Proposition 2 are that dealers can forecast order flow and traders can forecast the intercept of the price schedule posted by dealers. This complicates the calculations, but the steps are identical to those of Proposition 2.

Consider informed traders first. Suppose that informed trader i believes that (i) dealers $j \in \{1, \ldots, J\}$ follow symmetric strategies of the form $y_j = \gamma v_1 + \delta P$, and (ii) other informed traders $k \neq i$ follow symmetric strategies of the form $x_k = \beta v$. Then trader i perceives the market clearing condition as

$$J\gamma\tilde{v}_1 + J\delta\tilde{P} = \tilde{\omega} \qquad \text{where} \qquad \tilde{\omega} = x_i + (N-1)\beta v + \tilde{u}$$

or equivalently that the market clearing price is the realization of the random variable

$$\tilde{P} = \tilde{P}(\tilde{\omega}) = \frac{-\gamma}{\delta}\tilde{v}_1 + \frac{1}{J\delta}\tilde{\omega}.$$

Trader i then chooses x_i to maximize expected profit conditional on his information $s_i = v$:

$$\max_{x_i} E\left[\left(v + \frac{\gamma}{\delta}\tilde{v}_1 - \frac{1}{J\delta}\left[x_i + (N-1)\beta v + \tilde{u}\right]\right) x_i \middle| v\right].$$

The first-order condition is equivalent to

$$x_i = \frac{1}{2}\left\{J\delta\left(v + \frac{\gamma}{\delta}E\left[v_1|v\right]\right) - (N-1)\beta v\right\}. \tag{3.1}$$

The second order condition is $\delta > 0$, which we later show is satisfied. Since

$$E\left[v_1|v\right] = \left(\frac{\sigma_{v_1}^2}{\sigma_{v_1}^2 + \sigma_{v_2}^2}\right)v,$$

equation (3.1) can be written as

$$x_i = \frac{1}{2}\left\{J\delta + J\gamma\left(\frac{\sigma_{v_1}^2}{\sigma_{v_1}^2 + \sigma_{v_2}^2}\right) - (N-1)\beta\right\}v.$$

This verifies that if any trader i believes the others follow symmetric linear strategies, he also follows a strategy that is linear in $s_i = v$. Symmetry among the traders' strategies implies that any trader i's strategy coefficient must be equal to the coefficient he believes defines the strategies of the other informed traders:

$$\begin{aligned}\beta &= \frac{1}{2}\left\{J\delta + J\gamma\left(\frac{\sigma_{v_1}^2}{\sigma_{v_1}^2 + \sigma_{v_2}^2}\right) - (N-1)\beta\right\} \\ &= \frac{J}{N+1}\left\{\delta + \gamma\left(\frac{\sigma_{v_1}^2}{\sigma_{v_1}^2 + \sigma_{v_2}^2}\right)\right\} \qquad (3.2)\end{aligned}$$

Now consider dealers. Suppose dealer j believes that (i) other dealers $k \neq j$ follow symmetric strategies of the form $y_k = \gamma v_1 + \delta P$ and (ii) informed traders $i \in \{1, \ldots, N\}$ follow symmetric strategies of the form $x_i = \beta v$. Then dealer j perceives the market clearing condition as

$$(J-1)\gamma v_1 + (J-1)\delta\tilde{P} + y_j = \tilde{\omega} \qquad \text{where} \qquad \tilde{\omega} = N\beta\tilde{v} + \tilde{u}$$

or equivalently that the market clearing price is a realization of the random variable

$$\tilde{P} = \tilde{P}(\tilde{\omega}) = \frac{-\gamma}{\delta} v_1 + \frac{1}{(J-1)\delta} (\tilde{\omega} - y_j) . \tag{3.3}$$

Dealer j then chooses a price-contingent supply schedule $y_j(s_j, p)$ to maximize expected profit conditional on his information $s_j = v_1$ and on the realized price $\tilde{P} = p$:

$$\max_{y_j} E\left[\left(\frac{-\gamma}{\delta} v_1 + \frac{1}{(J-1)\delta} (N\beta\tilde{v} + \tilde{u} - y_j) - v\right) y_j \middle| v_1, \tilde{P} = p\right].$$

The first-order condition is

$$E\left[\left(\frac{-\gamma}{\delta} v_1 + \frac{1}{(J-1)\delta} (N\beta\tilde{v} + \tilde{u} - 2y_j) - \tilde{v}\right) | v_1, \tilde{P} = p\right]$$

and the second-order condition is $\delta > 0$. By equation (3.3), $\tilde{P} = \frac{-\gamma}{\delta} v_1 + \frac{1}{(J-1)\delta} (N\beta\tilde{v} + \tilde{u} - y_j)$ so the first-order condition is equivalent to

$$E\left[\tilde{P} - \frac{1}{(J-1)\delta} y_j - v \middle| v_1, \tilde{P} = p\right] = 0$$

or

$$y_j = (J-1)\delta \left\{p - E\left[\tilde{v}|v_1, \tilde{P} = p\right]\right\}. \tag{3.4}$$

From the perspective of dealer j, all variates are jointly normal, so

$$\begin{aligned} E\left[\tilde{v}|v_1, \tilde{P} = p\right] &= v_1 + E\left[v_2|v_1, \tilde{P} = p\right] \\ &= v_1 + \frac{\mathcal{C}}{\mathcal{V}}\left(p - E\left[\tilde{P}|v_1\right]\right) \\ &= v_1 + \frac{\mathcal{C}}{\mathcal{V}}\left(p + \frac{\gamma}{\delta} v_1 - \frac{N\beta v_1 - y_j}{(J-1)\delta}\right) \end{aligned} \tag{3.5}$$

where

$$\mathcal{C} \equiv \mathrm{Cov}\left[\tilde{v}_2\,, \tilde{P}|v_1\right] = \frac{N\beta\sigma_{v_2}^2}{(J-1)\delta} \tag{3.6}$$

$$\mathcal{V} \equiv \mathrm{Var}\left[\tilde{P}|v_1\right] = \frac{(N\beta)^2\sigma_{v_2}^2 + \sigma_u^2}{(J-1)^2\delta^2}. \tag{3.7}$$

Substituting (3.5) into (3.6) and solving for y_j yields

$$y_j(v_1, p) = (J-1)\delta \left\{ \left(\frac{1 - \frac{\mathcal{C}}{\mathcal{V}}}{1 + \frac{\mathcal{C}}{\mathcal{V}}}\right) p - \left(\frac{1 + \frac{\mathcal{C}}{\mathcal{V}}\left[\frac{\gamma}{\delta} - \frac{N\beta}{(J-1)\delta}\right]}{1 + \frac{\mathcal{C}}{\mathcal{V}}}\right) v_1 \right\}.$$

This verifies that if any dealer j believes the others follow symmetric linear strategies, he also follows a strategy that is linear in $s_j = v_1$ and the realized price p. Symmetry among the dealers' strategies implies that any dealer j's strategy coefficients must be equal to the coefficients he believes define the other dealers' strategies:

$$\delta = (J-1)\delta\left(\frac{1-\frac{\mathcal{C}}{\mathcal{V}}}{1+\frac{\mathcal{C}}{\mathcal{V}}}\right) \qquad \Longleftrightarrow \qquad \frac{\mathcal{C}}{\mathcal{V}} = \frac{J-2}{J} \tag{3.8}$$

and

$$\begin{aligned}
\gamma &= -(J-1)\delta\left(\frac{1+\frac{\mathcal{C}}{\mathcal{V}}\left[\frac{\gamma}{\delta}-\frac{N\beta}{(J-1)\delta}\right]}{1+\frac{\mathcal{C}}{\mathcal{V}}}\right) \qquad \text{using (3.8)}\\
&= -(J-1)\delta\left(\frac{1+\frac{J-2}{J}\left[\frac{\gamma}{\delta}-\frac{N\beta}{(J-1)\delta}\right]}{1+\frac{J-2}{J}}\right)\\
&= \left(\frac{J-2}{J-1}\right)\frac{N\beta}{J} - \delta
\end{aligned} \tag{3.9}$$

the expression in the statement of the proposition. Substituting for β from (3.2), this is equivalent to

$$\frac{\gamma}{\delta} = \frac{-\left(1-\frac{J-2}{J-1}\frac{N}{N+1}\right)}{\left(1-\frac{J-2}{J-1}\frac{N}{N+1}\frac{\sigma_{v_1}^2}{\sigma_{v_1}^2+\sigma_{v_2}^2}\right)} \equiv \mathcal{R} \tag{3.10}$$

where $\mathcal{R}$ depends on exogenous variables only. To separately identify γ and δ, substitute (3.6) and (3.7) into (3.8)

$$J(J-1)\delta N\beta\sigma_{v_2}^2 = (J-2)(N\beta)^2\sigma_{v_2}^2 + (J-2)\sigma_u^2$$

then substitute for β from (3.2)

$$J(J-1)\delta N\left[\frac{J}{N+1}(\delta+\mathcal{R}\Sigma)\right]\sigma_{v_2}^2 = (J-2)N^2\left[\frac{J}{N+1}(\delta+\mathcal{R}\Sigma)\right]^2\sigma_{v_2}^2 + (J-2)\sigma_u^2$$

where $\Sigma \equiv \sigma_{v_1}^2/(\sigma_{v_1}^2+\sigma_{v_2}^2)$. Note that this expression depends on δ but not on γ. Collecting like terms

$$\frac{N(1+\mathcal{R}\Sigma)}{N+1}\left\{\left(\frac{J-1}{J-2}\right) - \frac{N(1+\mathcal{R}\Sigma)}{N+1}\right\}\delta^2 = \frac{\sigma_u^2}{\sigma_{v_2}^2}\frac{1}{J^2}. \tag{3.11}$$

A somewhat tedious calculation shows that

$$\frac{N(1+\mathcal{R}\Sigma)}{N+1} = \frac{1}{1+\Phi} \qquad \text{where} \qquad \Phi = \frac{\sigma_{v_1}^2}{\sigma_{v_2}^2}\left(\frac{N+J-1}{N(J-1)}\right) + \frac{1}{N}.$$

Substituting this into (3.11) and solving for δ^2 yields

$$\delta^2 = \frac{\sigma_u^2}{\sigma_{v_2}^2}\frac{1}{J^2}\left[\frac{(1+\Phi)^2(J-2)}{(J-1)(1+\Phi)-(J-2)}\right].$$

The assumption in the proposition that $J > 2$ guarantees real solutions for δ. Therefore,

$$J\delta = \frac{\sigma_u}{\sigma_{v_2}} (1 + \Phi) \sqrt{\frac{J-2}{\Phi(J-1)+1}} \tag{3.12}$$

the expression in the proposition, where the positive square root is selected to satisfy the second order conditions ($\delta > 0$).

To derive the expression for β in terms of δ and exogenous parameters, begin with (3.2) then substitute for γ using $\gamma = \mathcal{R}\delta$ (equation (3.10)):

$$\beta = \frac{J}{N+1} \left\{ \delta + \left(\frac{\sigma_{v_1}^2}{\sigma_{v_1}^2 + \sigma_{v_2}^2} \right) \mathcal{R}\delta \right\}$$

factoring out δ and simplifying yields

$$\beta = J\delta\Psi \qquad \text{where} \qquad \Psi = \frac{1}{N+1} \left\{ \frac{\frac{\sigma_{v_2}^2}{\sigma_{v_1}^2}}{1 + \frac{\sigma_{v_2}^2}{\sigma_{v_1}^2} - \frac{J-2}{J-1}\frac{N}{N+1}} \right\}. \tag{3.13}$$

Equations (3.9), (3.12) and (3.13) define the parameter values that characterize a symmetric equilibrium that is unique in the linear class.

Unconditional expected profit of informed trader i in equilibrium is

$$\pi_i \equiv E\left[\left(\tilde{v} - \tilde{P}_* \right) \tilde{x}_{i*} \right]$$

where x_{i*} is informed trader i's optimal strategy and P_* is the equilibrium price schedule (i.e., evaluated at dealers' and traders' optimal strategies). Thus,

$$\begin{aligned}
\pi_i &= E\left[\left(\tilde{v} + \frac{\gamma}{\delta}\tilde{v}_1 - \frac{1}{J\delta}(N\beta\tilde{v} + \tilde{u}) \right) \beta\tilde{v} \right] \\
&= \beta E\left[\tilde{v}^2 + \frac{\gamma}{\delta}\tilde{v}\tilde{v}_1 - \frac{N\beta}{J\delta}\tilde{v}^2 \right] \\
&= \beta \left\{ (\sigma_{v_1}^2 + \sigma_{v_2}^2) + \frac{\gamma}{\delta}\sigma_{v_1}^2 - \frac{N\beta}{J\delta}(\sigma_{v_1}^2 + \sigma_{v_2}^2) \right\}.
\end{aligned}$$

Using (3.13),

$$\pi_i = \beta \left\{ (1 - N\Psi)(\sigma_{v_1}^2 + \sigma_{v_2}^2) + \frac{\gamma}{\delta}\sigma_{v_1}^2 \right\}.$$

By (3.9),

$$\frac{\gamma}{\delta} = \left(\frac{J-2}{J-1} \right) \frac{N\beta}{J\delta} - 1 = \left(\frac{J-2}{J-1} \right) N\Psi - 1$$

so

$$
\begin{aligned}
\pi_i &= \beta \left\{ (1 - N\Psi)(\sigma_{v_1}^2 + \sigma_{v_2}^2) + \left[\left(\frac{J-2}{J-1} \right) N\Psi - 1 \right] \sigma_{v_1}^2 \right\} \\
&= \beta \left\{ (1 - N\Psi)\sigma_{v_2}^2 - \frac{N\Psi}{J-1} \sigma_{v_1}^2 \right\} \\
&= \beta \sigma_{v_2}^2 \left\{ 1 - N\Psi - \frac{N\Psi}{J-1} \frac{\sigma_{v_1}^2}{\sigma_{v_2}^2} \right\} \qquad (3.14)
\end{aligned}
$$

the expression in the proposition.

Dealer j's unconditional expected profit in equilibrium is

$$
\begin{aligned}
\pi_j &= E\left[\left(\tilde{P}_* - \tilde{v} \right) \tilde{y}_{j*} \right] \\
&= E\left[\gamma \tilde{P}_* \tilde{v}_1 - \gamma \tilde{v} \tilde{v}_1 + \delta \tilde{P}_*^2 - \delta \tilde{P}_* \tilde{v} \right],
\end{aligned}
$$

Now, $\tilde{P}_* = -\frac{\gamma}{\delta} \tilde{v}_1 + \frac{1}{J\delta} \left(N\beta(\tilde{v}_1 + \tilde{v}_2) + \tilde{u} \right)$, which implies that

$$
\begin{aligned}
E\left[\tilde{P}_* \tilde{v}_1 \right] &= \frac{-\gamma}{\delta} \sigma_{v_1}^2 + \frac{1}{J\delta} N\beta \sigma_{v_1}^2 = \frac{1}{\delta} \left[\frac{N\beta}{J} - \gamma \right] \sigma_{v_1}^2 \\
E\left[\tilde{P}_*^2 \right] &= \left(\frac{N\beta}{J\delta} - \frac{\gamma}{\delta} \right)^2 \sigma_{v_1}^2 + \left(\frac{N\beta}{J\delta} \right)^2 \sigma_{v_2}^2 + \frac{\sigma_u^2}{(J\delta)^2} \\
E\left[\tilde{P}_* \tilde{v} \right] &= \left(\frac{N\beta}{J\delta} - \frac{\gamma}{\delta} \right) \sigma_{v_1}^2 + \frac{N\beta}{J\delta} \sigma_{v_2}^2.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\pi_j = (\gamma - \delta) \left(\frac{N\beta}{J\delta} - \frac{\gamma}{\delta} \right) \sigma_{v_1}^2 - \gamma \sigma_{v_1}^2 + \delta \left(\frac{N\beta}{J\delta} - \frac{\gamma}{\delta} \right)^2 \sigma_{v_1}^2 \\
+ \delta \left(\frac{N\beta}{J\delta} \right)^2 \sigma_{v_2}^2 + \frac{1}{J^2\delta} \sigma_u^2 - \delta \frac{N\beta}{J\delta} \sigma_{v_2}^2.
\end{aligned}
$$

Noting that $\frac{N\beta}{J\delta} - \frac{\gamma}{\delta} = N\Psi - \frac{J-2}{J-1} N\Psi + 1 = \frac{N\Psi}{J-1} + 1$,

$$
\begin{aligned}
\pi_j &= (\gamma - \delta) \left(1 + \frac{N\Psi}{J-1} \right) \sigma_{v_1}^2 - \gamma \sigma_{v_1}^2 + \delta \left(1 + \frac{N\Psi}{J-1} \right)^2 \sigma_{v_1}^2 \\
&\quad + \delta N^2 \Psi^2 \sigma_{v_2}^2 + \frac{1}{J^2\delta} \sigma_u^2 - \delta N\Psi \sigma_{v_2}^2 \\
&= \left\{ (\gamma - \delta) \left(1 + \frac{N\Psi}{J-1} \right) - \gamma + \delta \left(1 + \frac{N\Psi}{J-1} \right)^2 \right\} \sigma_{v_1}^2 \\
&\quad + \delta N\Psi (N\Psi - 1) \sigma_{v_2}^2 + \frac{\sigma_u^2}{J^2\delta}
\end{aligned}
$$

Using $\dfrac{\gamma}{\delta} = \dfrac{J-2}{J-1}N\Psi - 1$ and $\gamma - \delta = \delta\left[\dfrac{J-2}{J-1}N\Psi - 2\right]$, we have

$$\begin{aligned}
\pi_j &= \delta\left\{\left(\frac{J-2}{J-1}N\Psi - 2\right)\left(1 + \frac{N\Psi}{J-1}\right) - \left(\frac{J-2}{J-1}N\Psi - 1\right) + \left(1 + \frac{N\Psi}{J-1}\right)^2\right\}\sigma_{v_1}^2 \\
&\qquad + \delta N\Psi(N\Psi - 1)\sigma_{v_2}^2 + \frac{\sigma_u^2}{J^2\delta} \\
&= \delta\sigma_{v_1}^2\left(\frac{N\Psi^2}{J-1}\right) + \delta\sigma_{v_2}^2 N\Psi(N\Psi - 1) + \frac{\sigma_u^2}{J^2\delta} \\
&= N\Psi\delta\sigma_{v_2}^2\left\{N\Psi - 1 + \frac{N\Psi}{J-1}\frac{\sigma_{v_1}^2}{\sigma_{v_2}^2}\right\} + \frac{\sigma_u^2}{J^2\delta} \\
&= \frac{\sigma_u^2}{J^2\delta} - \frac{\beta\sigma_{v_2}^2}{J}\left\{1 - N\Psi - \frac{N\Psi}{J-1}\frac{\sigma_{v_1}^2}{\sigma_{v_2}^2}\right\}.
\end{aligned} \tag{3.15}$$

Combining (3.14) and (3.15)

$$J\pi_j + N\pi_i = \frac{\sigma_u^2}{J\delta}$$

as stated in the proposition. ‖

Proof of Proposition 4: The difference between this and the proof of proposition 2 is that agents are heterogeneously informed. However, the strategy of the proof is the same. In anticipation of Proposition 8, we maintain different notation for the variances of the signals of those who elect to trade as dealers and those who trade via market orders. In particular, $\sigma_{vi} = \sigma_D$ if $i \in \{1, \ldots, J\}$ and $\sigma_{vi} = \sigma_I$ if $i \in \{J+1, \ldots, M\}$.

Consider informed traders first. Suppose that informed trader i believes that (i) dealers $j \in \mathcal{J} \equiv \{1, \ldots, J\}$ follow symmetric strategies of the form $y_j = \gamma v_j + \delta P$, and (ii) other informed traders $k \neq i$ follow symmetric strategies of the form $x_k = \beta v_k$. Then trader i perceives the market clearing condition as

$$\gamma \sum_{j\in\mathcal{J}} \tilde{v}_j + J\delta\tilde{P} = \tilde{\omega} \qquad \text{where} \qquad \tilde{\omega} = x_i + \beta \sum_{k\in\mathcal{I}\sim i} \tilde{v}_k + \tilde{u},$$

$\mathcal{I} \equiv \{J+1, \ldots, M\}$, and $\mathcal{I} \sim i$ is the set of integers in $\mathcal{I}$ with the exception of i. Equivalently, trader i perceives the market clearing price is the realization of the random variable

$$\tilde{P} = \tilde{P}(\tilde{\omega}) = -\frac{\gamma}{\delta}\overline{v}_{\mathcal{J}} + \frac{1}{J\delta}\tilde{\omega} \qquad \text{where} \qquad \overline{v}_{\mathcal{J}} = \frac{1}{J}\sum_{j\in\mathcal{J}} \tilde{v}_j.$$

Trader i then chooses x_i to maximize expected profit conditional on his information $s_i = v_i$:

$$\max_{x_i} E\left[\left(\sum_{h=1}^{M} \tilde{v}_h + \frac{\gamma}{\delta}\overline{v}_{\mathcal{J}} - \frac{1}{J\delta}\left(x_i + \beta\sum_{k\in\mathcal{I}\sim i}\tilde{v}_k + \tilde{u}\right)\right)x_i \middle| v_i\right].$$

Since $E[\tilde{v}_k|v_i] = 0$ for $k \neq i$, the first-order condition implies

$$x_i = \frac{J\delta}{2} v_i.$$

The second-order condition is that $\delta > 0$, which we later show is satisfied. This verifies that if any trader i believes the others follow symmetric linear strategies, he also follows a strategy that is linear in $s_i = v_i$. Symmetry among the traders' strategies implies that any trader i's strategy coefficient must be equal to the coefficient he believes defines other informed traders' strategies:

$$\beta = \frac{J\delta}{2}. \tag{4.1}$$

Now consider dealers. Suppose dealer j believes that (i) other dealers $k \neq j$ follow symmetric strategies of the form $y_k = \gamma v_k + \delta P$ and (ii) informed traders $i \in \mathcal{I}$ follow symmetric strategies of the form $x_i = \beta v_i$. Then dealer j perceives the market clearing condition as

$$\gamma \sum_{k \in \mathcal{J} \sim j} \tilde{v}_k + (J-1)\delta \tilde{P} + y_j = \tilde{\omega} \qquad \text{where} \qquad \tilde{\omega} = \beta \sum_{i \in \mathcal{I}} \tilde{v}_i + \tilde{u},$$

and $\mathcal{J} \sim j$ is the set of integers in $\mathcal{J}$ with the exception of j. Equivalently, dealer j perceives the market clearing price as a realization of the random variable

$$\tilde{P} = \tilde{P}(\tilde{\omega}) = \frac{-\gamma}{\delta} \overline{v}_{\mathcal{J} \sim j} + \frac{1}{(J-1)\delta} (\tilde{\omega} - y_j) \qquad \text{where} \qquad \overline{v}_{J \sim j} = \frac{1}{J-1} \sum_{k \in \mathcal{J} \sim j} \tilde{v}_k. \tag{4.2}$$

Dealer j then chooses a price-contingent supply schedule $y_j(s_j, p)$ to maximize expected profit conditional on his information $s_j = v_j$ and on the realized price $\tilde{P} = p$:

$$\max_{y_j} E\left[\left(\frac{-\gamma}{\delta} \overline{v}_{\mathcal{J} \sim j} + \frac{1}{(J-1)\delta} [\beta(M-J)\overline{v}_{\mathcal{I}} + \tilde{u} - y_j]) - \tilde{v}\right) y_j \middle| v_j, \tilde{P} = p\right]$$

where $\overline{v}_{\mathcal{I}} = \dfrac{1}{M-J} \sum_{i \in \mathcal{I}} \tilde{v}_i$. The first-order condition is

$$E\left[\frac{-\gamma}{\delta} \overline{v}_{\mathcal{J} \sim j} + \frac{1}{(J-1)\delta} [\beta(M-J)\overline{v}_{\mathcal{I}} + \tilde{u} - 2y_j] - \tilde{v} \middle| v_j, \tilde{P} = p\right] = 0$$

and the second-order condition is $\delta > 0$. By equation (4.2),

$$\tilde{P} = \frac{-\gamma}{\delta} \overline{v}_{\mathcal{J} \sim j} + \frac{1}{(J-1)\delta} [\beta(M-J)\overline{v}_{\mathcal{I}} + \tilde{u} - y_j] \tag{4.3}$$

so the first-order condition is equivalent to

$$E\left[\tilde{P} - \tilde{v} - \frac{1}{(J-1)\delta} y_j \middle| v_j, \tilde{P} = p\right] = 0$$

or

$$y_j = (J-1)\delta \left\{ p - E\left[\tilde{v} | v_j, \tilde{P} = p \right] \right\}. \tag{4.4}$$

From the perspective of dealer j, all variates are jointly normal so

$$\begin{aligned} E\left[\tilde{v}|v_j, \tilde{P}=p\right] &= E\left[\tilde{v}_1 + \cdots + \tilde{v}_J + \tilde{v}_{J+1} + \cdots + \tilde{v}_M | v_j, \tilde{P}=p\right] \\ &= v_j + (J-1)E\left[\tilde{v}_{\hat{k}}|v_j, \tilde{P}=p\right] + (M-J)E\left[\tilde{v}_i|v_j, \tilde{P}=p\right] \end{aligned}$$

where $\hat{k} \in \mathcal{J} \sim j$ and $i \in \mathcal{I}$. Using the expression in (4.3) for $\tilde{P}$, we have

$$E\left[\tilde{v}_{\hat{k}}|v_j, \tilde{P}=p\right] = \frac{\mathcal{C}_1}{\mathcal{V}}\left(p - E\left[\tilde{P}|v_j\right]\right) = \frac{\mathcal{C}_1}{\mathcal{V}}\left(p + \frac{1}{(J-1)\delta}y_j\right) \tag{4.5}$$

$$E\left[\tilde{v}_i|v_j, \tilde{P}=p\right] = \frac{\mathcal{C}_2}{\mathcal{V}}\left(p - E\left[\tilde{P}|v_j\right]\right) = \frac{\mathcal{C}_2}{\mathcal{V}}\left(p + \frac{1}{(J-1)\delta}y_j\right) \tag{4.6}$$

where

$$\begin{aligned} \mathcal{C}_1 &\equiv \mathrm{Cov}\left[\tilde{v}_{\hat{k}}\,, \tilde{P}|v_j\right] = \frac{-\gamma}{(J-1)\delta}\sigma_D^2 \qquad \text{for } \hat{k} \in \mathcal{J} \sim j \\ \mathcal{C}_2 &\equiv \mathrm{Cov}\left[\tilde{v}_i\,, \tilde{P}|v_j\right] = \frac{\beta}{(J-1)\delta}\sigma_I^2 \qquad \text{for } i \text{ in } \mathcal{I}, \text{ and} \\ \mathcal{V} &\equiv \mathrm{Var}\left[\tilde{P}|v_j\right] = \left(\frac{\gamma}{\delta}\right)^2 \frac{\sigma_D^2}{J-1} + \left(\frac{1}{(J-1)\delta}\right)^2 \left\{(M-J)\beta^2\sigma_I^2 + \sigma_u^2\right\}. \end{aligned}$$

Therefore, (4.5) and (4.6) imply that

$$E\left[\tilde{v}|v_j, \tilde{P}=p\right] = v_j + \frac{(J-1)\mathcal{C}_1 + (M-J)\mathcal{C}_2}{\mathcal{V}}\left[p + \frac{1}{(J-1)\delta}y_j\right].$$

Substituting this into (4.4) and solving for y_j yields

$$y_j(v_j, p) = (J-1)\delta\left\{\left(\frac{1-\phi}{1+\phi}\right)p - \left(\frac{1}{1+\phi}\right)v_j\right\}$$

where $\phi \equiv \dfrac{(J-1)\mathcal{C}_1 + (M-J)\mathcal{C}_2}{\mathcal{V}}$. This verifies that if any dealer j believes the others follow symmetric linear strategies, he also follows a strategy that is linear in $s_j = v_j$ and the realized price p. Symmetry among the dealers' strategies implies that any dealer j's strategy coefficients must be equal to the coefficients he believes define the strategies of the other dealers:

$$\delta = \frac{(J-1)\delta(1-\phi)}{1+\phi} \qquad \Longleftrightarrow \qquad \phi = \frac{J-2}{J} \tag{4.7}$$

and

$$\begin{aligned} \gamma &= \frac{-(J-1)\delta}{1+\phi} \qquad \text{using (4.7)} \\ &= \frac{-J\delta}{2} \qquad \Longleftrightarrow \qquad \frac{\gamma}{\delta} = \frac{-J}{2}. \end{aligned} \tag{4.8}$$

To separately identify γ and δ write $\phi = \dfrac{J-2}{J}$ explicitly

$$(J-1)\mathcal{C}_1 + (M-J)\mathcal{C}_2 = \left(\frac{J-2}{J}\right)\mathcal{V}$$

then substitute for $\mathcal{C}_1$, $\mathcal{C}_2$, β/δ and γ/δ

$$(J-1)\left(\frac{J}{2}\right)\sigma_D^2 + (M-J)\left(\frac{J}{2}\right)\sigma_I^2 = \frac{(J-1)(J-2)}{J}\times \left[\left(\frac{-J}{2}\right)^2\frac{\sigma_D^2}{J-1} + \left(\frac{1}{(J-1)\delta}\right)^2\left\{(M-J)\beta^2\sigma_I^2 + \sigma_u^2\right\}\right].$$

Substituting for the remaining β/δ and collecting like terms:

$$\frac{J}{2}\left[(J-1) - \frac{J-2}{2}\right]\sigma_D^2 + \frac{J}{2}\left[(M-J) - \frac{(J-2)(M-J)}{2(J-1)}\right]\sigma_I^2 = \frac{(J-2)}{J(J-1)}\frac{\sigma_u^2}{\delta^2}$$

$$(J\delta)^2 = 4\sigma_u^2\left(\frac{J-2}{J}\right)\left\{(J-1)\sigma_D^2 + (M-J)\sigma_I^2\right\}^{-1}$$

$$J\delta = \frac{2\sigma_u}{\sigma_I}\sqrt{\frac{J-2}{J[M-(1-\alpha)J-\alpha]}} \tag{4.9}$$

where $\alpha \equiv \dfrac{\sigma_D^2}{\sigma_I^2} \in (0,1]$, and the positive root is selected to ensure the second-order conditions are satisfied. Thus, equations (4.1), (4.8) and (4.9) define the parameter values that characterize a symmetric equilibrium that is unique in the linear class. Setting $\alpha = 1$ in (4.9) and noting that $\sigma_{vi} = \sigma_I$ yields the expressions in Proposition 4.

Unconditional expected profit of informed trader i in equilibrium is

$$\pi_i \equiv E\left[\left(\tilde{v} - \tilde{P}_*\right)\tilde{x}_{i*}\right]$$

where $\tilde{x}_{i*}$ is the dealer's optimal strategy and $\tilde{P}_*$ is the equilibrium price schedule (i.e., evaluated at dealers' and traders' optimal strategies). Thus,

$$\begin{aligned}\pi_i &= E\left[\left(\tilde{v} + \frac{\gamma}{\delta}\overline{v}_{\mathcal{J}} - \frac{1}{J\delta}\left(\beta\sum_{i\in\mathcal{I}}\tilde{v}_i + \tilde{u}\right)\right)\beta v_i\right]\\ &= E\left[\beta\tilde{v}_i^2 - \frac{1}{J\delta}\left(\beta^2\tilde{v}_i^2\right)\right]\end{aligned}$$

substituting for β from (4.1) and noting that $\sigma_I^2 = E\left[\tilde{v}_i^2\right]$,

$$\pi_i = \frac{J\delta\sigma_I^2}{4} = \frac{\beta\sigma_I^2}{2}.$$

Dealers' unconditional expected profit in equilibrium is

$$\begin{aligned}
\pi_j &\equiv E\left[\left(\tilde{P}_* - \tilde{v}\right)\tilde{y}_{j*}\right] \\
&= E\left[\gamma\left(\tilde{P}_* - \tilde{v}\right)\tilde{v}_j + \delta\left(\tilde{P}_* - \tilde{v}\right)\tilde{P}_*\right] \\
&= \gamma E\left[\left\{\frac{-\gamma}{\delta}\overline{v}_{\mathcal{J}} + \frac{1}{J\delta}\left(\beta\sum_{i\in\mathcal{I}}\tilde{v}_i + \tilde{u}\right) - \tilde{v}\right\}\tilde{v}_j\right] + \delta E\left[\tilde{P}_*^2\right] - \delta E\left[\tilde{v}\tilde{P}_*\right] \\
&= \gamma E\left[\frac{-\gamma}{\delta}\frac{1}{J}\tilde{v}_j^2 - \tilde{v}_j^2\right] + \delta E\left[\tilde{P}_*^2\right] - \delta E\left[\frac{-\gamma}{\delta}\overline{v}_{\mathcal{J}}\tilde{v} + \frac{\beta}{J\delta}\left(\sum_{i\in\mathcal{I}}\tilde{v}_i\right)\tilde{v}\right] \\
&= \gamma\left\{\frac{-\gamma}{\delta}\frac{1}{J}\sigma_D^2 - \sigma_D^2\right\} + \delta E\left[\tilde{P}_*^2\right] - \delta\left\{\frac{-\gamma}{\delta}\frac{1}{J}(J\sigma_D^2)\right\} - \frac{\beta}{J}(M-J)\sigma_I^2.
\end{aligned}$$

Now,

$$E\left[\tilde{P}_*^2\right] = \left(\frac{\gamma}{\delta}\right)^2\frac{\sigma_D^2}{J} + \frac{1}{J^2\delta^2}\left[\beta^2(M-J)\sigma_I^2 + \sigma_u^2\right]$$

so upon substituting, and after cancelation,

$$\pi_j = \frac{1}{J^2\delta}\left[\beta^2(M-J)\sigma_I^2 + \sigma_u^2\right] - \frac{\beta}{J}(M-J)\sigma_I^2.$$

Substituting for β from (4.1) and simplifying

$$J\pi_j = \frac{\sigma_u^2}{J\delta} - \frac{J\delta}{4}(M-J)\sigma_I^2 = \frac{\sigma_u^2}{J\delta} - (M-J)\pi_i$$

as stated in the proposition. ‖

Proof of Propositions 5 and 8: An *interior* equilibrium mixture of agents between trading via limit orders and trading via market order is $\hat{J} \in \{2, \ldots, M\}$ such that:

(i) Agents i trading via market order have no incentive to switch to limit orders

$$\pi_i(\hat{J}) \geq \pi_j(\hat{J}+1), \quad \text{and}$$

(ii) agents j trading via limit orders have no incentive to switch to market orders

$$\pi_j(\hat{J}) \geq \pi_i(\hat{J}-1).$$

Ignoring the integer constraint, $\hat{J}$ is a point at which π_i and π_j cross when both are regarded as functions of J. Substituting from the expected profit expressions in Proposition 6, $\pi_j(\hat{J}) = \pi_i(\hat{J})$ is equivalent to

$$\begin{aligned}
\hat{J}\pi_j(\hat{J}) = \frac{\sigma_u^2}{\hat{J}\delta} - (M-\hat{J})\pi_i(\hat{J}) &= J\pi_i(\hat{J}) \\
\frac{\sigma_u^2}{\hat{J}\delta} &= M\frac{\hat{J}\delta}{4}\sigma_I^2 \\
(\hat{J}\delta)^2 &= \frac{4}{M}\frac{\sigma_u^2}{\sigma_I^2}.
\end{aligned}$$

Substituting the expression for $J\delta$ from equation (4.9) in the proof of Proposition 4,

$$\left(\frac{4\sigma_u^2}{\sigma_I^2}\right)\left(\frac{\hat{J}-2}{\hat{J}\left[(M-\hat{J})+\alpha(\hat{J}-1)\right]}\right)=\frac{4}{M}\frac{\sigma_u^2}{\sigma_I^2}$$
$$(1-\alpha)\hat{J}^2+\alpha\hat{J}-2M=0.$$

This quadratic has two real solutions. The negative solution is meaningless since $J>2$ is assumed. The positive solution is

$$\hat{J}=\frac{-\alpha+\sqrt{\alpha^2+8(1-\alpha)M}}{2(1-\alpha)}. \tag{5.1}$$

Thus, there is a *single* crossing point of π_i and π_j in the domain $J\in(2,\infty)$. Furthermore, π_j crosses π_i from above because

$$\lim_{J\to 2}\pi_i(J)=0 \qquad \text{and} \qquad \lim_{J\to 2}\pi_j(J)=\infty.$$

Therefore, when the crossing point $\hat{J}\geq M$, all informed traders elect to trade via limit order and equilibrium J_* is at the *boundary,* $J_*=M$. When $\hat{J}<M$, some informed traders elect to trade via market order and the rest trade via limit order. In this case, equilibrium J_* is interior, $J_*=\hat{J}$. To identify conditions under which these cases obtain, note from (5.1) that $\hat{J}<M$ if and only if

$$\alpha^2+8(1-\alpha)M<\left[2(1-\alpha)M+\alpha\right]^2$$
$$\alpha<\frac{M-2}{M-1}.$$

Thus, J_* is interior if $\frac{\sigma_D^2}{\sigma_I^2}<\frac{M-2}{M-1}$; otherwise $J_*=M$. $\|$

Proof of Propositions 6 and 7: Substituting $J=M$ from Proposition 5 into the expression for liquidity trader expected losses in Proposition 4 yields

$$\frac{\sigma_u^2}{M\delta}=\frac{\sigma_u\sigma_{vi}}{2}\sqrt{\frac{M(M-1)}{M-2}}=\frac{\sigma_u\sigma_v}{2}\sqrt{\frac{M-1}{M-2}},$$

where the last equality follows from $\sigma_v^2=M\sigma_{vi}^2$. This is the first expression in the statement of Proposition 6. To obtain the second expression, note that if liquidity is provided by a perfectly competitive, uninformed, risk-neutral market maker, the price schedule is $\hat{P}(\hat{\omega})=E\left[\tilde{v}|\hat{\omega}\right]$, where $\hat{\omega}=\sum_{i=1}^{M}\hat{x}_i+u$ because informed traders submit market orders only. The derivation of a linear equilibrium follows the steps in Kyle (1985) with minor modifications, so we do not reproduce them here. The solution is:

$$\hat{x}_i=\hat{\beta}v_i \qquad \text{for } i=1,\ldots,M$$
$$\hat{P}(\hat{\omega})=\hat{\lambda}\hat{\omega}$$

where

$$\hat{\beta} = \frac{\sigma_u}{\sigma_v} \qquad \text{and} \qquad \hat{\lambda} = \frac{\sigma_v}{2\sigma_u}.$$

Expected profit of informed trader i is

$$\begin{aligned}
\hat{\pi}_i &= E\left[(\tilde{v} - \hat{P}(\hat{\omega}))\hat{x}_i\right] = E\left[(\tilde{v} - \hat{\lambda}\hat{x}_i)\hat{x}_i\right] \\
&= E\left[v_i\hat{\beta}v_i - \hat{\lambda}\hat{\beta}^2 v_i^2\right] = \sigma_{vi}^2\hat{\beta}(1 - \hat{\lambda}\hat{\beta}) \\
&= \sigma_{vi}^2 \frac{\sigma_u}{\sigma_v}\left(1 - \frac{1}{2}\right) = \frac{\sigma_u \sigma_v}{2}
\end{aligned}$$

where the last equality follows from $\sigma_v = M\sigma_{vi}^2$. This verifies the second expression in the statement of Proposition 6. Next we prove Proposition 7, then return to the statements in Proposition 6 relating to the bid-ask spread and price volatility.

The equilibrium price schedule of Proposition 5 is easily shown to be

$$P_M(\omega) = \frac{1}{2}\sum_{i=1}^{M} v_i + \left(\frac{\sigma_v}{2\sigma_u}\sqrt{\frac{M-1}{M-2}}\right)\omega \tag{7.1}$$

where $\omega = u$ because the informed do not use market orders in equilibrium. A standard calculation shows that the price function of a perfectly competitive, uninformed, risk-neutral market maker in our informational environment is given by

$$\hat{P}_M(\hat{\omega}) = 0 + \left(\frac{\sigma_v}{2\sigma_u}\right)\hat{\omega}. \tag{7.2}$$

where $\hat{\omega} = \sum_{i=1}^{M} \hat{\beta}v_i + u$, $\hat{\beta} = \frac{\sigma_u}{\sigma_v}$, and the "hats" denote the competitive market maker setting. Substituting these into (7.2),

$$\hat{P}_M(\hat{\omega}) = \frac{1}{2}\sum_{i=1}^{M} v_i + \left(\frac{\sigma_v}{2\sigma_u}\right)u. \tag{7.3}$$

The decompositions in the proposition follow from equations (7.1) and (7.3).

Using (7.1) and (7.3), the midpoints of the bid and ask prices are

$$P_M(0) = \frac{1}{2}\sum_{i=1}^{M} v_i \qquad \text{and} \qquad \hat{P}_M(0) = 0.$$

Thus,

$$E\left[(\tilde{v} - \tilde{P}_M(0))^2\right] < E\left[(\tilde{v} - \hat{P}_M(0))^2\right].$$

This completes the proof of Proposition 7.

Returning to the final statements in Proposition 6, bid-ask spreads are differences between prices quoted for unit sized market buy and sell orders. Thus,

$$\begin{aligned} s_M &\equiv P_M(1) - P_M(-1) = \frac{\sigma_v}{\sigma_u}\sqrt{\frac{M-1}{M-2}} \\ \hat{s}_M &\equiv \hat{P}_M(1) - \hat{P}_M(-1) = \frac{\sigma_v}{\sigma_u}, \end{aligned}$$

so $s_M > \hat{s}_M$. Finally, the difference in the variance of prices is evident from equations (7.1) and (7.3):

$$\mathrm{Var}\left[\tilde{P}_M(\tilde{u})\right] - \mathrm{Var}\left[\hat{P}_M(\tilde{u})\right] = \left\{\sqrt{\frac{M-1}{M-2}} - 1\right\}\sigma_u^2.$$

It is clear that the differences in spreads and variances disappear as $M \to \infty$.$\|$

Proof of Proposition 9: . If $\sigma_D^2 = 0$, then $\alpha = 0$ in equation (5.1). This implies that $\hat{J} = \sqrt{2M} > 2$ provided that $M > 2$. Since $J_* = \min\{\hat{J}, M\}$, $J_* > 2$. $\|$

Figure 1
Percentage Change in Informed Traders' Expected Profit

Percentage change in informed traders' expected profit associated with moving from an opaque to a transparent market as a function of the number of informed traders *N* and informed dealers *J*. The black line indicates zero change. The dark grey region in the lower left corresponds to negative values where transparency reduces informed traders' expected profit. Regions above and to the right of zero indicate an increase in informed traders' expected profit. The grey region corresponds to the positive changes between 0 and 100%, light grey corresponds to the positive changes between 100% and 200%, and white corresponds to positive changes above 200%. In this figure, $\sigma_{v1}^2 = 6$ and $\sigma_{v2}^2 = 1$.

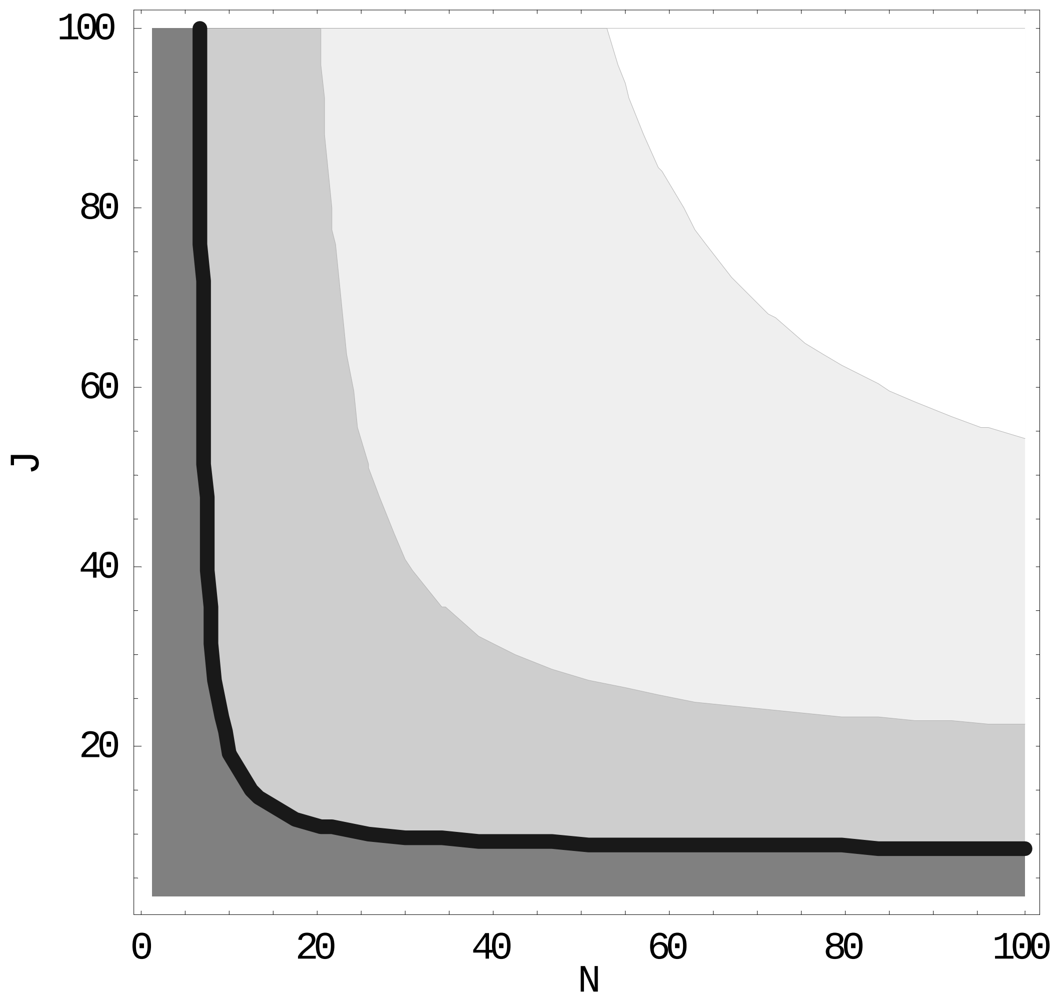
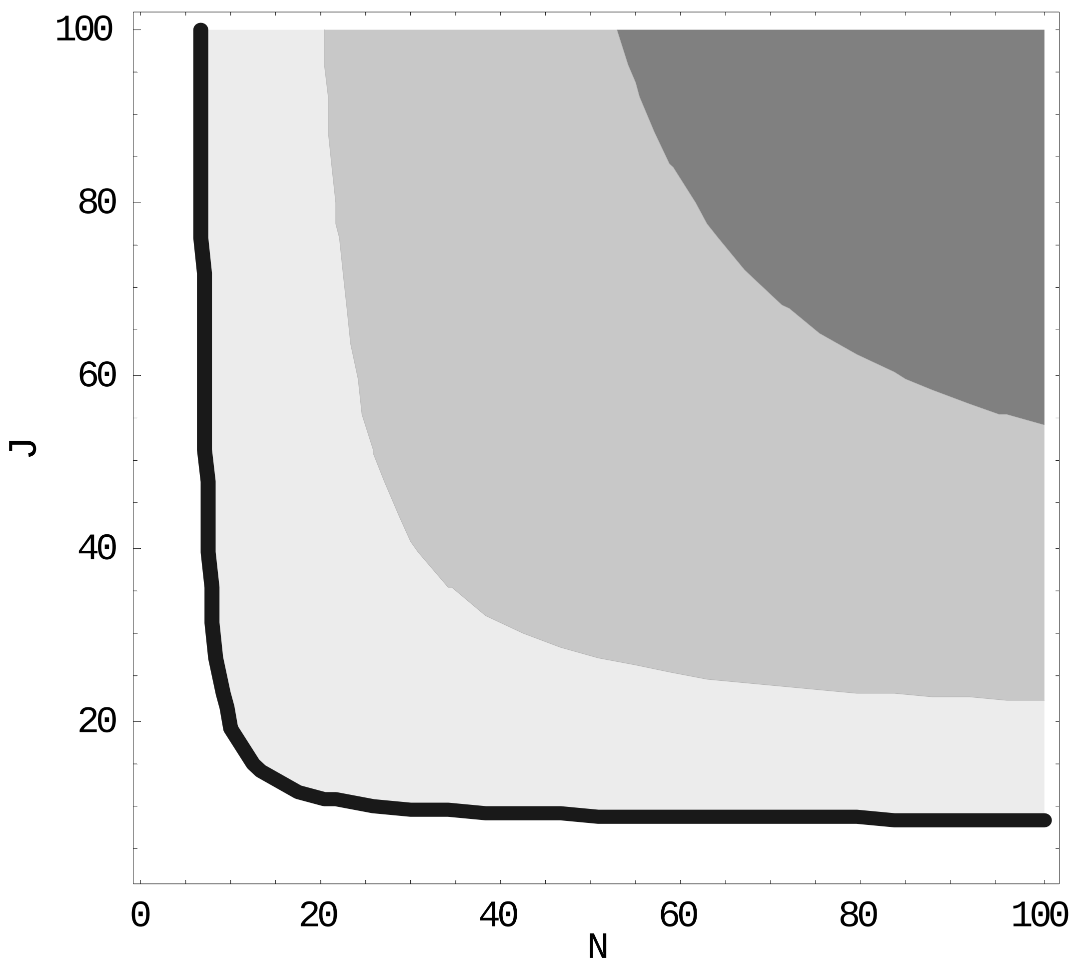


Figure 2
Percentage Change in Liquidity Traders' Expected Losses

Percentage change in liquidity traders' expected losses associated with moving from an opaque to a transparent market as a function of the number of informed traders *N* and informed dealers *J*. The black line indicates zero change. The white region in the lower left corresponds to negative values where transparency reduces liquidity traders' expected losses. Regions above and to the right of zero indicate an increase in liquidity traders' expected losses. The light grey region corresponds to an increase in losses between 0 and 100%, medium grey corresponds to 100% to 200%, and dark grey corresponds to increases in losses above 200%. In this figure, $\sigma_{v1}^2 = 6$ and $\sigma_{v2}^2 = 1$.



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