# FINANCING AUCTION BIDS

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## FINANCING AUCTION BIDS ABSTRACT

In many auctions, bidders require financing to pay their cash bid. If bidders have equal access to competitive financing, then intuition suggests that the auction will be efficient. We examine numerous types of financial markets and show that any deviation from 1) perfect information about bidders, or complete information about bidder actions, 2) a precise level of competition, and 3) the need for all bidders to obtain financing, will result in an inefficient auction. Furthermore, even a seller who intervenes to provide financing cannot restore efficiency. Our work suggests that adverse selection in financial markets makes it difficult to attain efficient allocations in auctions.

## FINANCING AUCTION BIDS

The majority of auctions worldwide require cash bids. Yet in many auctions, bidders do not have cash equal to the sum they wish to bid. As a result, bidders finance part of their bids. This financing may come from the financial markets or from the seller. For example, financing was the rule in the Federal Communications Commission (FCC) bandwidth auctions, in which the government sold sections of the radio spectrum. In the FCC Class C auctions the FCC itself financed the winning bidders through an installment payment agreement. Ex-post, many of the winning bidders could not make the payments, leading to considerable litigation and finally re-auctioning of licences. In the third generation (3G) European wireless spectrum auctions the winning bidders have borrowed billions of dollars. In most real-estate sales bidders borrow from banks to pay their bid. Bankrupt firms are often sold through cash auctions with financed bids.<sup>1</sup> Many mergers and acquisitions are also auctions that involve financed bids. This is especially true of leveraged buy-outs. Privatizations also involve financed bids.<sup>2</sup> In fact, in almost any auction which sells an object of substantial value, bidders finance a portion of their bid. This paper explores the interaction between the auction and the financial markets.

Should financing have any effect on the auction? Consider an auction in which bidders have different private values and different available cash. However, bidders all have access to competitive financial markets. Intuition may suggest that the auction will behave as predicted by the seminal work in auction theory: Vickrey (1961), Harris and Raviv (1981), Myerson (1981), Riley and Samuelson (1981), Milgrom and Weber (1982), and all of the work that follows from these. As Aghion, Hart, and Moore (1992) state: "Auctions work well if raising cash for bids is easy..." (p527). Baird (1986) and Jensen (1991) imply that with competitive capital markets auctions will yield allocative efficiency. Maskin (2000) suggests that the ability to "pay for the asset out of future earnings..." (p672)<sup>3</sup> might get around the inefficiency associated with capital constraints. Hart (1995) says that, "In a world of perfect capital markets, a cash auction would (presumably) be the ideal bankruptcy procedure." Our work shows that competitive capital markets will not lead to efficient auctions.

Some may not find the lack of efficiency surprising since the bidders are asymmetric in their cash positions.<sup>4</sup> However, bidders only differ in their cash balances, and they can all borrow and lend through competitive capital markets; it would seem that this would eliminate the inefficiency.

<sup>&</sup>lt;sup>1</sup>See interesting work by Strömberg (2000) and Eckbo and Thorburn (2000). A theoretical evaluation of such bankruptcy auctions is Rhodes-Kropf and Viswanathan (2000).

<sup>&</sup>lt;sup>2</sup>See Bolton and Roland (1992).

<sup>&</sup>lt;sup>3</sup>This is the function of competitive capital markets.

 $<sup>^{4}</sup>$ Maskin and Riley (2000) and Maskin (2000) show that auctions with asymetric bidders and private values will be inefficient.

We show that capital markets *can* restore efficiency, and thus it is not the asymmetry itself that causes the inefficiency. However, we show that liquid, *competitive* financial markets cannot restore efficiency. Thus, the asymmetry in this paper is different from that usually considered because there is an active capital market that should seemingly remove the asymmetry and thus the inefficiency.

The reason that auctions with financed bids may not be efficient is because competitive capital markets cannot resolve the adverse selection problem. With financing, the bidders' values are affected by the borrowing rate in different ways depending on their type and their cash position. Thus, financing will distort the bids, altering who will win the auction and the seller's expected revenue. Since many auctions include financed bids this work has profound implications both for the theoretical design of efficient auctions and for their practical implementation.

We consider the following situation: each bidder has a different independent private expected value for the object for sale, and each bidder also has a different amount of cash. Thus, bidders differ in two dimensions.<sup>5</sup> Bidders finance the amount needed to cover their bid. Bidder financing is either debt or equity or state contingent financing, so the payment made to the financial market depends on the resulting value of the object. Furthermore, we also consider various criteria for the financing rate: financing where the rate is set before the auction with only limited information about the bidders, pre-auction financing that uses full information about the bidders, and financing that conditions the rate on the bids.

With all of these forms of financing we find that with competitive liquid financial markets the auction is unlikely to be efficient, i.e., the bidder with the highest value may not win. To get efficiency the capital market that sets rates ex-ante must make the bidder indifferent between having more or less cash. With rates conditioned on the bids, efficiency requires the capital market to make the bidder indifferent between bidding more or less, independent of his cash. However, bidders understand that their bid will influence the rate at which they borrow. Thus, bidders with a lot of cash have greater incentives to distort their bid to influence their rate than bidders with a lot of cash. A monopolist lender can set rates to counter this effect. However, a competitive capital market can only give the competitive rate. Thus, it is not the financial constraints of the bidders per se which cause the inefficiency. Instead it is the competition in the financial markets themselves and the interaction with the auction that cause inefficiencies.<sup>6</sup>

Specifically, we show that with pre-auction financing the debt financed auction will be inefficient even when the lender has perfect information about the borrower and the lending market is in a perfectly competitive equilibrium. If instead the financing depends on the bids and the level of cash,

<sup>&</sup>lt;sup>5</sup>Solving the general auction with two-dimensional bidders is an important goal. This paper is a step forward in that results are found with bidders who have different continuous values and different continuous amounts of cash.

<sup>&</sup>lt;sup>6</sup>Che and Gale (1998) also show inefficiency due to the budget constraint. Our results indicate a different kind of inefficiency that is due to the pricing of securities in the financial market.

then a competitive equilibrium in either the equity or debt market will not result in an efficient auction. In general, efficient auctions with debt or equity financed bids require the financial markets to be less than perfectly competitive. Even if the seller provides financing, the auction will not be efficient if the bidders can access competitive outside financing options. The seller-financed auction will only be efficient if the seller can mandate the rate (because they are the government) or if the seller is the only source of financing (such as in Eastern Europe or due to market imperfections).

In summary, capital markets provide rates based on different criteria in different situations. Sometimes the rate depends on the assets being purchased or the risk class of the project. Sometimes lenders try to determine who the borrower is and set different rates for different types. And sometimes the rate depends on how much is borrowed. We show that auctions with all of these forms of financing are unlikely to be efficient. Any deviation in the financial market from 1) perfect information about the bidders, or complete information about the bidders' actions, 2) a precise level of competition, and 3) the need for all bidders to obtain financing, will result in an inefficient auction. Furthermore, a seller who intervenes to provide financing cannot restore efficiency.

In an important paper, Che and Gale (1998) explore an auction in which bidders face budget constraints. These authors allow for credit. However, they do not model the financial markets. Instead they use a reduced form approach in which the marginal cost of financing is increasing in the capital borrowed. Auctions are also inefficienct in their set up. However, the question remains whether it is the lack of internal capital that leads to auction inefficiency or whether it is the poor functioning of financial markets that causes auction inefficiency. We tackle this question by examining multiple forms of financing and different levels of financial market competition. Our work allows us to understand that neither the bidder's financial constraints nor the imperfections in the capital markets are certain to cause inefficiencies. Rather, the interaction between the bidders' values and the financial markets causes the inefficient outcome.

Our work is also related to two other papers. Rhodes-Kropf and Viswanathan (2000) consider an auction model with debt where different bidders have different valuations but the same amount of an asset to pledge in the event of bankruptcy.<sup>7</sup> Zheng (2000) examines bidders who have different amounts of cash and must finance their bids, but bidders all have the same value for the object sold.<sup>8</sup> This last assumption removes the possibility that the auction is inefficient. When bidders have the same value then the object for sale is always put to its best use (awarded to the highest value bidder). However, when bidders have different values and different amounts of cash then the

<sup>&</sup>lt;sup>7</sup>In Rhodes-Kropf and Viswanathan (2000), the asset is in the form of a non-pecuinary penalty if bankruptcy occurs. It can equally be interpreted as cash held by bidders.

<sup>&</sup>lt;sup>8</sup>Further, in Zheng's (2000) model there is no distinction between debt and equity. This is because of the assumption in a two-state model that the low state is zero. Thus, whether bidders use equity or debt, the payment in the low state is zero.

auction may not be efficient.

Our paper is organized as follows. Section I considers the model with a single interest rate (and subsequently a single equity price). Section II allows the capital market to condition on information; this section considers rates based on both perfect information and rates conditioned on the bids. With each type of rate we examine first debt and then equity financing. The end of the section considers complete state contingent claims. Section III allows for seller financing while Section IV concludes.

## I The Model

The model is a two-stage game with private information. In the first stage, N potential risk-neutral bidders, with  $N = \{1, ..., n\}$  representing the set of n bidders, bid with cash for a firm whose value is not known with certainty. The seller uses a first price cash auction, and the firm is awarded to the high bidder. In order to pay the bid, the bidder may access the securities market and raise money. The financing can be thought of as occurring before the auction (if the rate does not depend on the bids) or after the auction (if the rate depends on the bids), but must occur before the final stage in which the value of the firm is revealed, and all claims are settled.

The final value of the firm for sale will be either a high value H or a low value L.<sup>9</sup> In the first stage, each bidder has private information about  $\lambda_i$ , their ability to manage the firm. If bidder *i* runs the firm then with probability  $\lambda_i$  the firm will be worth H and with probability  $(1 - \lambda_i)$  the firm will be worth L. The  $\lambda s$  of each bidder are independently and identically distributed and drawn from the distribution  $F(\lambda)$  with  $F(\underline{\lambda}) = 0$ ,  $F(\overline{\lambda}) = 1$ .  $F(\lambda)$  is strictly increasing and differentiable over the interval  $[\underline{\lambda}, \overline{\lambda}]$ .  $\underline{\lambda}$  must be greater than some lower bound, which will be defined later.<sup>10</sup> Each bidder also has private information about the amount of cash that he has,  $c_i$ . In general, we will assume that the amounts of cash held by the bidders are independently and identically distributed and drawn from the distribution  $\Psi(c)$  with  $\Psi(\underline{c}) = 0$ ,  $\Psi(\overline{c}) = 1$ .  $\Psi(c)$  is strictly increasing and differentiable over the interval  $[\underline{c}, \overline{c}]$ , and  $\underline{c} > 0$ . We will briefly consider the bidders' endogenous ability to pay dividends (adjust c).

To establish a benchmark we will first examine the case when each bidder has enough cash to cover their entire bid ( $\underline{c}$  > highest bid). We will then consider the situation when bidders use their cash and some debt, then their cash and some equity. Finally, we will consider state contingent securities.

<sup>&</sup>lt;sup>9</sup>Every result would go through if H and L had distributions.

<sup>&</sup>lt;sup>10</sup>No matter how large H is and how small c is, there exists some  $\lambda > 0$  that is so small that the assumptions we use to solve the problem (namely insolvency in the low state) are not true.

## I.1 Cash Bids, No Financing

When every bidder has enough cash to cover their entire bid, the bidder's problem is

$$\max_{b_i} [\{H\lambda_i + L(1-\lambda_i) - b_i\} Prob[b_i > \max_{\forall j \neq i} b_j]].$$
(1)

Standard techniques reveal that the optimal bid is

$$b(\lambda_i) = H\lambda_i + L(1 - \lambda_i) - (H - L) \int_{\underline{\lambda}}^{\lambda_i} \frac{F^{n-1}(s)ds}{F^{n-1}(\lambda_i)}.$$
(2)

And, the expected revenue of the seller is

$$H\overline{\lambda} + L(1-\overline{\lambda}) - n \int_{\underline{\lambda}}^{\overline{\lambda}} (H-L)F^{n-1}(\lambda)d\lambda$$
(3)

The more interesting results to follow will be compared against this benchmark.

### I.2 Cash Bids with Partial Debt Financing

If the seller requires cash bids paid in full, then bidders who do not have enough cash to pay for the object for sale require financing. In this section, the winning bidder uses debt to finance the portion of his bid that he does not currently hold in cash. Every bidder is given equal access to debt financing and no limits on the amount he can borrow. Thus, we would expect that the auction is unchanged, since it is still a cash auction. Yet, this is not the case. We will see why.

Every bidder can borrow or lend money in a perfectly competitive debt market at a rate of r.<sup>11</sup> The competition ensures that although r > 0, the lender's expected return is zero.<sup>12</sup> Thus, bidders borrow an amount b-c to make up the difference between their cash, c, and their bid, b, and owe the lender (1+r)(b-c). It may seem that bidders would like to borrow their entire bid rather than just b-c. However, we assume that the managers cannot steal. Therefore, if they borrow more than b-c then the extra cash stays in the firm and must be returned in the low state and must be returned with interest in the high state. Thus, managers only wish to borrow b-c.<sup>13</sup>

<sup>&</sup>lt;sup>11</sup>The ability to lend at the competitive rate ensures that bidders value their cash at c. If they had poor outside opportunities then retaining their cash would be a negative net present value project.

<sup>&</sup>lt;sup>12</sup>The assumption of zero systematic risk is without loss of generality. See corollary 1 for a formal definition of break even r.

<sup>&</sup>lt;sup>13</sup>Formally, suppose the bidder borrows an additional amount s. The bidder would pay an additional amount s in the low state and an additional amount (1 + r)s in the high state. Hence the expected payoff is negative and the bidder will not borrow more than he needs in equilibrium. This argument relies on the possibility of bankruptcy. If there were no bankruptcy in the low state, debt would be no different from cash.

The bidder's problem now becomes

$$\max_{b_{i}} [\{H\lambda_{i} + (1 - \lambda_{i})L - \min[b_{i}, c_{i}] - \lambda_{i}(1 + r)\max[b_{i} - c_{i}, 0] - (1 - \lambda_{i})\min[L, \max[(1 + r)(b_{i} - c_{i}), 0]]\}Prob[b_{i} > \max_{\substack{\forall i \neq i}} b_{j}]].$$
(4)

The complication from the multiple *min* and *max* functions stems from the different possible amounts of cash. The first *min* and the second and third *max* are necessary if it is possible that the bidder did not borrow. Since the interesting case is when bidders do borrow, we will assume that bidders bid more than their current cash. The second *min* checks whether the bidder earns enough in the low state to fully repay the lender, i.e. no default. Again, the interesting case is with default since debt without default is like cash.<sup>14</sup> Furthermore, without default the interest rate from a competitive market must be zero.<sup>15</sup> Therefore, we assume that bidders have cash, but must borrow enough that they cannot meet their obligations in the low state.<sup>16</sup> In this case L can be thought of as the secured part of the loan.<sup>17</sup> These assumptions require simple conditions which will be determined later. The bidder's problem becomes

$$\max_{b_i} \left[ \{ H\lambda_i - c_i - \lambda_i (1+r)(b_i - c_i) \} Prob[b_i > \max_{\forall j \neq i} b_j] \right].$$
(5)

To solve this problem we need to define the probability of winning. However, bidders now differ in two dimensions: they have different amounts of cash and different probabilities of achieving the high outcome. It looks as though two dimensional information may eliminate closed form solutions. Note, however, that we can divide the maximization by  $\lambda_i(1+r)$  and not change the answer. The bidder's problem then becomes

$$\max_{b_i} \left[ \left\{ \frac{H}{1+r} - \frac{c_i}{\lambda_i(1+r)} + c_i - b_i \right\} \operatorname{Prob}[b_i > \max_{\forall j \neq i} b_j] \right].$$
(6)

Define

$$\theta_i = \frac{H}{1+r} - \frac{c_i}{\lambda_i(1+r)} + c_i,\tag{7}$$

 $^{14}$ Che and Gale (1998) have no default. Thus, their debt is just two certain cash payments, one now and one next period. However cash payments in the future are costly.

<sup>&</sup>lt;sup>15</sup>In debt markets the premium over the risk free rate is called the default spread. In the real world all borrowers have a probability of default and must pay a default spread.

<sup>&</sup>lt;sup>16</sup>It would be interesting to allow some bidders who do not borrow, or some bidders who can repay the loan in the bad state. However, the complication would not allow a closed form solution. We do demonstrate some results in the situation where some bidders do not need to borrow.

<sup>&</sup>lt;sup>17</sup>Unless bidders borrow more than b - c, in which case L + c is the secured part of the loan. Thus, bidders only want to borrow b - c.

and the bidder's problem becomes

$$\max_{b_i} \left[ \{\theta_i - b_i\} \operatorname{Prob}[b_i > \max_{\forall j \neq i} b_j] \right].$$
(8)

This equation is nearly identical to Equation (1), the bidder's problem in the first price cash auction.  $\theta_i$  is independently and identically distributed and drawn from the distribution  $G(\theta)$ , where  $G(\cdot)$  is the cumulative distribution of  $\theta_i$  given  $F(\lambda)$  and  $\Psi(c)$ .<sup>18</sup>  $G(\theta)$  is strictly increasing and differentiable over the interval  $[\underline{\theta}, \overline{\theta}]$ . Thus,  $\theta_i$  is the bidder's "type" or "value".

The problem can now be changed to a direct revelation mechanism by assuming that all other bidders use the equilibrium bid function  $b(\theta_j)$ . In a first price auction the high bidder wins. Therefore, bidder *i* bids in the range  $[b(\underline{\theta}), b(\overline{\theta})]$ , where  $\underline{\theta}$  is the smallest possible  $\theta$  given Equation (7) and all combinations of *c* and  $\lambda$ .<sup>19</sup> Thus,  $b_i$  can be written as b(x) where *x* is a selection from  $[\underline{\theta}, \overline{\theta}]$ . We assume, and then verify, that the equilibrium bid function is invertible in  $\theta$ . Therefore, the probability of winning can be written

$$Prob[b_i > \max_{\forall j \neq i} b_j] = Prob[b^{-1}(b_i) > \theta_j \forall j] = G^{n-1}(b^{-1}(b_i)) = G^{n-1}(x).$$
(9)

Thus, the bidder's problem is

$$\max_{x} \left[ \{ \theta_i - b(x) \} G^{n-1}(x) \right].$$
 (10)

The following theorem demonstrates the optimal bid.

**Theorem 1** If the interest rate, r, is set such that either (A)  $1 < 1 + r < \frac{1}{\underline{\lambda}} < \frac{(H-L)}{\overline{c}}$  or (B)  $\frac{1}{\underline{\lambda}} < 1 + r < \frac{H-L}{\overline{c}-\underline{c}} - \frac{\underline{c}}{(\overline{c}-\underline{c})\underline{\lambda}}$  then  $\exists$  a unique symmetric equilibrium s.t.

$$b(\lambda_i, c_i) = b(\theta_i) = \theta_i - \int_{\underline{\theta}}^{\theta_i} \frac{G^{n-1}(s)ds}{G^{n-1}(\theta_i)},$$
(11)

$$G(\theta) = G\left[\frac{H}{1+r} - \frac{c_i}{\lambda_i(1+r)} + c_i \le \theta\right]$$
$$= \int_{\underline{c}}^{\overline{c}} \Pr\left[\frac{H}{1+r} - \frac{c_i}{\lambda_i(1+r)} + c_i \le \theta \mid c_i\right] \Psi'(c_i) dc_i.$$

Rearanging yields

$$\lambda_i \le \frac{c_i}{H + (1+r)(c_i - \theta)}.$$

Therefore,

$$G(\theta) = \int_{\underline{c}}^{\overline{c}} F\left[\frac{c_i}{H + (1+r)(c_i - \theta)}\right] \Psi'(c_i) dc_i.$$

<sup>19</sup>In a first price auction, a bidder gains no probability of winning by bidding greater than the highest possible competitor's bid, and he must pay more than if he just bid the highest bid. A bidder will lose with certainty if he bids below the lowest possible bid.

with

$$b(\underline{\theta}) = \underline{\theta} = \frac{H}{1+r} - \frac{\overline{c}}{\underline{\lambda}(1+r)} + \overline{c}, \qquad (12)$$

if condition (A) holds, or with

$$b(\underline{\theta}) = \underline{\theta} = \frac{H}{1+r} - \frac{\underline{c}}{\underline{\lambda}(1+r)} + \underline{c}, \tag{13}$$

if condition (B) holds.

**Proof.** The FOC for the bidder's problem is

$$[\theta - b(x)]\frac{dG^{n-1}(x)}{dx} - b'(x)G^{n-1}(x) = 0 \text{ for } x = \theta \ \forall \theta \in [\underline{\theta}, \overline{\theta}]$$
(14)

Therefore, increasing  $\theta$  without changing x results in the FOC > 0, and decreasing  $\theta$  without changing x results in the FOC < 0. Since the bidder will never bid outside of  $[b(\underline{\theta}), b(\overline{\theta})]$  this shows pseudoconcavity. Individual rationality is easily shown since  $c_i < H\lambda_i + (1-\lambda_i)L - \lambda_i(1+r)(b_i - c_i) - (1-\lambda_i)L$ as long as  $b_i < \frac{H}{1+r} - \frac{c_i}{\lambda_i(1+r)} + c_i$ ; i.e. bidders bid less than their 'value'. Finally, integrating Equation (14) yields Equation (11).

It is also easy to verify that this bid function is strictly increasing in  $\theta$  and is, therefore, invertible as assumed. However, we must further show that bidders borrow an amount greater than they can pay back in the low state (as we assumed to start the problem). The possibility of default requires

$$(b(\theta_i) - c_i)(1+r) > L, \ \forall i.$$

$$(15)$$

A necessary and sufficient condition that ensures default is

$$(b(\underline{\theta}) - \overline{c})(1+r) > L. \tag{16}$$

If  $1 + r < 1/\underline{\lambda}$  and r > 0 (the first part of condition A), then  $\theta$  is strictly decreasing in c. In this case

$$b(\underline{\theta}) = \frac{H}{1+r} - \frac{\overline{c}}{\underline{\lambda}(1+r)} + \overline{c}, \qquad (17)$$

and the condition, (16), can be reduced to

$$(H-L)\underline{\lambda} > \overline{c},\tag{18}$$

which is the second part of condition (A).  $(H - L)\underline{\lambda} > \overline{c}$  requires that (H - L) must be relatively large compared to the most cash the worst manager could have, and  $\underline{\lambda}$  must be greater than zero. In other words, if H is large enough then even the worst manager is willing to risk bankruptcy as long as he has some decent chance of success.

If  $\underline{\lambda} > 1/(1+r)$  then  $\theta$  is strictly increasing in c. Thus,

$$b(\underline{\theta}) = \frac{H}{1+r} - \frac{\underline{c}}{\underline{\lambda}(1+r)} + \underline{c},\tag{19}$$

and the condition, (16), becomes

$$1 + r < \frac{H - L}{\overline{c} - \underline{c}} - \frac{\underline{c}}{(\overline{c} - \underline{c})\underline{\lambda}}.$$
(20)

If the interest rate is high enough then some bidders will stop borrowing since increasing their bid slightly above their cash will increase their expected payment far more than it increases their probability of winning the auction.  $\blacksquare$ 

The following corollary shows that the competitive interest rate exists and falls under condition (A) in the theorem.

**Corollary 1** If the lending market is perfectly competitive, then there exists an equilibrium competitive rate r such that  $1 < 1 + r < \frac{1}{\lambda}$ .

**Proof.** Under perfect competition the lender must expect to be paid an amount equal to what he lent. In equilibrium the lender expects to lend

$$n\int_{\underline{\lambda}}^{\overline{\lambda}}\int_{\underline{c}}^{\overline{c}}(b(\lambda,c)-c)\left[\int_{\underline{c}}^{\overline{c}}F\left(\frac{\widehat{c}}{H+(1+r)(\widehat{c}-\theta(\lambda,c))}\right)\Psi'(\widehat{c})d\widehat{c}\right]^{n-1}\Psi'(c)F'(\lambda)dcd\lambda.$$
(21)

The lender then expects to be paid

$$n\int_{\underline{\lambda}}^{\overline{\lambda}}\int_{\underline{c}}^{\overline{c}} \left[\lambda(b(\lambda,c)-c)(1+r) + (1-\lambda)L\right] \left[\int_{\underline{c}}^{\overline{c}} F\left(\frac{\widehat{c}}{H+(1+r)(\widehat{c}-\theta(\lambda,c))}\right) \Psi'(\widehat{c})d\widehat{c}\right]^{n-1} \Psi'(c)F'(\lambda)dcd\lambda$$
(22)

A competitive r is a rate such that Equation (21) = Equation (22). If r = 0 then Equation (21)  $\geq$  Equation (22) since  $\lambda \leq 1$ . If  $r = \frac{1}{\lambda} - 1$  then  $\lambda(1 + r) \geq 1$ . Therefore, Equation (21)  $\leq$  Equation (22). Thus, since everything is continuous, the competitive rate exists such that  $1 < 1 + r < \frac{1}{\lambda}$ .

Theorem 1 finds that the bid function is surprisingly simple, but why is this bid function interesting? Because it is increasing in  $\theta$ , but is not independent of c. For fixed c,  $\theta$  is increasing in  $\lambda$ , however, it also depends on c, which leads to inefficiency.

**Theorem 2** A first price auction in which bidders finance a portion of their bid with debt at an interest rate r is not efficient.

**Proof.** Efficiency requires that the bidder with the highest  $\lambda$  is sure to win the auction:  $b(\lambda_i, c_i) > b(\lambda_j, c_j) \ \forall \lambda_i > \lambda_j$  and  $\forall c_i, c_i \in [\underline{c}, \overline{c}]$ . The following Lemma shows the conditions necessary for an efficient auction.

**Lemma 1:** An efficient auction symmetric equilibrium requires bids such that  $\frac{\partial b(\lambda,c)}{\partial \lambda} > 0$ , and  $\frac{\partial b(\lambda,c)}{\partial c} = 0$ .

**Proof.** Efficiency requires a function  $b(\lambda_i, c_i)$  such that  $d\lambda > 0 \Longrightarrow db > 0$ . By definition

$$db = \frac{\partial b}{\partial \lambda} d\lambda + \frac{\partial b}{\partial c} dc.$$
(23)

Since dc may equal zero  $\frac{\partial b}{\partial \lambda}$  must be greater than zero. However, even when  $\frac{\partial b(\lambda,c)}{\partial \lambda} > 0$  there exists a small enough  $d\lambda > 0$  such that  $\frac{\partial b}{\partial \lambda} d\lambda + \frac{\partial b}{\partial c} dc < 0$  for some dc unless  $\frac{\partial b}{\partial c}$  equals zero.

Theorem 1 showed that the bid is increasing in  $\theta$ . The derivative of  $\theta$ , Equation (7), with respect to  $\lambda$  is

$$\frac{\partial\theta}{\partial\lambda} = \frac{c}{\lambda^2(1+r)}.$$
(24)

which is positive. However, a better type may also have a different amount of cash. The derivative of  $\theta$ , Equation (7) with respect to c is

$$\frac{\partial\theta}{\partial c} = 1 - \frac{1}{\lambda(1+r)}.\tag{25}$$

This derivative is only equal to zero if

$$\lambda = \frac{1}{(1+r)}.\tag{26}$$

Therefore, Lemma 1 ensures that the auction is not efficient, and a bidder *i* with  $\lambda_i > \lambda_j$  and  $c_i \neq c_j$  may have a lower  $\theta_i$  and thus, a lower bid.<sup>20</sup>

An immediate corollary to this theorem is that this is not just a feature of the first price auction.

**Corollary 2** Any form of cash auction in which the bidder willing to pay the most is certain to win, and bidders finance a portion of their bid with debt at an interest rate r, is not efficient.

**Proof.** The maximum amount that any bidder is willing to pay is

$$H\lambda_i + L(1 - \lambda_i),\tag{27}$$

his true value. If he must pay all of his cash now and borrow some part of his bid then he is willing to pay an amount, X, in the auction only if it satisfies

$$L(1-\lambda_i) + c_i + \lambda_i(1+r)(X-c_i) \le H\lambda_i + L(1-\lambda_i)$$
(28)

That is, he must pay less than he makes. Solving this equation,

$$X \le \frac{H}{1+r} - \frac{c_i}{\lambda_i(1+r)} + c_i = \theta_i.$$
<sup>(29)</sup>

<sup>20</sup>If there is no chance of bankruptcy then

$$\theta_i = \frac{H\lambda_i + L(1 - \lambda_i) - c_i}{1 + r} + c_i$$

In which case,

$$\frac{\partial \theta}{\partial c} = 1 - \frac{1}{1+r}.$$

This derivative is equal to zero if r = 0. Since there is no bankruptcy, the only efficient interest rate is zero.

In any auction the payment must not be more than  $\theta_i$ , the bidders' willingness to pay. Therefore, if bidder *i* with  $\lambda_i > \lambda_j$  and  $c_i \neq c_j$  has a lower  $\theta$ , then bidder *i* will be willing to pay less than bidder  $j^{21}$ 

Theorem 2 and Corollary 2 demonstrate that with a single interest rate for all bidders, most standard auctions are inefficient. At first glance our result may seem identical to the inefficiency results in Che and Gale (1998). However, we will show that the nature of the inefficiency is not the need for financing per se, but instead a failure of the capital markets.

When trying to understand how the bids are affected by the differing amounts of cash, the natural assumption is that since bidders must pay interest to the bank, they are willing to give less to the seller. Therefore, bidders with more cash, who borrow less and pay less interest, probably have an advantage in the auction. Or, the reader might think that since the bidder may not be able to repay the debt, the limited liability nature of debt gives the advantage to those bidders with less cash. The following corollary demonstrates that neither is correct. For some bidders more cash is better, for others less is better.

**Corollary 3** In the cash auction with debt financed bids, high types  $\lambda > 1/(1+r)$  increase their bids if they have more cash, but low types  $\lambda < 1/(1+r)$  decrease their bids if they have more cash.

Proof.

$$\frac{\partial\theta}{\partial c} = 1 - \frac{1}{\lambda(1+r)}.\tag{30}$$

And

$$b'(\theta) > 0. \tag{31}$$

Thus, a bidder *i* with  $\lambda_i > 1/(1+r)$  and  $\lambda_i > \lambda_j$  but  $c_i < c_j$  may bid lower than bidder *j*. Or, a bidder *i* with  $\lambda_i < 1/(1+r)$  and  $\lambda_i > \lambda_j$  and  $c_i > c_j$  may bid lower than bidder *j*. This results in two regions. See figure 1. (Insert figure 1 here) For high types, Iso-bid functions are increasing in the amount of cash that they have. While for low types, Iso-bid functions are decreasing in the amount of cash that they have. Within these two regions a better type may lose to a worse type.

If bidders are allowed to pay pre-auction dividends, then low types  $(\theta_j < \theta^* = H/(1+r))$  benefit from having low cash, these bidders dump their cash until their  $\theta_j = \theta^*$ , and they pool at  $b(\theta^*) = \frac{H}{1+r}$ .

$$c_i + (1+r)(X-c_i) \le H\lambda_i + L(1-\lambda_i),$$

or

$$X \le \frac{H\lambda_i + L(1 - \lambda_i) - c_i}{1 + r} + c_i$$

Since the bidder's willingness to pay still varies with  $c_i$  the bidder willing to pay the most may not be the highest type,  $\lambda_i$ .

 $<sup>^{21}</sup>$ If there is no chance of bankruptcy then the bidder is willing to pay an amount X s.t.

The remaining bidders now have the same chance of winning the auction as they did before, but the bidders who pool at  $\theta^*$  function as a kind of reserve price. So, the high  $\theta$  bidders now increase their bid to

$$b(\theta_i) = \theta_i - \int_{\theta^*}^{\theta_i} \frac{G^{n-1}(s)ds}{G^{n-1}(\theta_i)}.$$
(32)

This result is shown in figure 6 (insert Figure 6 here). However, r must increase when the low types borrow more. A higher rate causes the best of the low types to decide not to dump their cash, but the lending market does not collapse because the high types find it better to borrow at bad rates than not to borrow at all.

Thus, a single financing rate may result in significant inefficiencies. The ability of low value bidders who have less cash to outbid higher value bidders who have more cash may explain the ex-post bankruptcy of winners in the FCC C Class auction. These winning bidders may have been low value low cash bidders since the FCC essentially offered the same financing terms to all winning bidders. Zheng (2000) suggests a similar possibility, although there is no efficiency loss in Zheng.<sup>22</sup>

Why is a competitive interest rate inefficient? Since low types like to borrow and high types do not, it seems most likely that this result is an artifact of the limited liability nature of debt. If a bidder may not have to pay his bid then he is willing to bid more. If this is the reason then the use of equity rather than debt financing should eliminate the problem.

#### I.3 Cash Bids with Partial Equity Financing

In this section the winning bidder sells equity to finance the portion of his bid that he does not currently hold in cash. Thus, the winning bidder will obtain some of the income in both the high and the low states. The key point we wish to make is that the source of the adverse selection is the financial market and not the particular security that is considered.

Every bidder now has equal access to unlimited capital from a perfectly competitive securities market. For every dollar they need to finance they sell a fraction  $\phi$  of their firm. Thus,  $\phi$  is the reciprocal of the market capitalization. Assuming that the bidder needs to finance a portion of their bid, the bidder's problem becomes

$$\max_{b_i} \left[ \{H\lambda_i + (1-\lambda_i)L - c_i - \phi(b_i - c_i)(H\lambda_i + (1-\lambda_i)L)\} Prob[b_i > \max_{\forall j \neq i} b_j] \right].$$
(33)

We can divide the maximization by  $\phi(H\lambda_i + (1 - \lambda_i)L)$  and not change the answer. The bidder's problem then becomes

$$\max_{b_i} \left[ \left\{ \frac{1}{\phi} - \frac{c_i}{\phi(H\lambda_i + (1 - \lambda_i)L)} - (b_i - c_i) \right\} Prob[b_i > \max_{\forall j \neq i} b_j] \right].$$
(34)

 $^{22}$ Our model is more general in allowing for differences in valuation and cash and in considering a richer class of securities.

Now we will define

$$\beta_i = \frac{1}{\phi} - \frac{c_i}{\phi(H\lambda_i + (1 - \lambda_i)L)} + c_i, \tag{35}$$

and the bidder's problem can be written

$$\max_{b_i} \left[ \{ \beta_i - b_i \} \operatorname{Prob}[b_i > \max_{\forall j \neq i} b_j] \right].$$
(36)

 $\beta_i$  is independently and identically distributed and drawn from the distribution  $\Omega(\beta)$ , where  $\Omega(\cdot)$  is the cumulative distribution of  $\beta_i$  given  $F(\lambda)$  and  $\Psi(c)$ .  $\Omega(\beta)$  is strictly increasing and differentiable over the interval  $[\beta, \overline{\beta}]$ .

Again, we will switch to a direct revelation mechanism. Therefore, we will assume that all other bidders use the equilibrium bid function  $b(\beta_j)$ . In this case, bidder *i* bids in the range  $[b(\underline{\beta}), b(\overline{\beta})]$ , where  $\underline{\beta}$  is the smallest possible  $\beta$  given Equation (35) and all combinations of *c* and  $\lambda$ . Thus,  $b_i$ can be written as b(x) where *x* is a selection from  $[\underline{\beta}, \overline{\beta}]$ . We will assume, and then verify, that the equilibrium bid function is invertible. Therefore, the probability of winning can be written

$$Prob[b_i > \max_{\forall j \neq i} b_j] = Prob[b^{-1}(b_i) > \beta_j \forall j] = \Omega^{n-1}(b^{-1}(b_i)) = \Omega^{n-1}(x).$$
(37)

Thus, the bidder's problem is

$$\max_{x} [\{\beta_{i} - b(x)\}\Omega^{n-1}(x)].$$
(38)

The following theorem will demonstrate the optimal bid.

**Theorem 3** If the price of equity,  $\phi$ , is set such that either (A)  $1/\phi > H\underline{\lambda} + (1 - \underline{\lambda})L > \overline{c}$  or (B)  $H\underline{\lambda} + (1 - \underline{\lambda})L > 1/\phi > \frac{(\overline{c} - \underline{c})(H\underline{\lambda} + (1 - \underline{\lambda})L)}{H\underline{\lambda} + (1 - \underline{\lambda})L - \underline{c}}$  then  $\exists$  a unique symmetric equilibrium s.t.

$$b(\lambda_i, c_i) = b(\beta_i) = \beta_i - \int_{\underline{\beta}}^{\beta_i} \frac{\Omega^{n-1}(s)ds}{\Omega^{n-1}(\beta_i)},$$
(39)

with

$$b(\underline{\beta}) = \underline{\beta} = \frac{1}{\phi} - \frac{\overline{c}}{\phi(H\underline{\lambda} + (1 - \underline{\lambda})L)} + \overline{c}, \tag{40}$$

if condition (A) holds, or with

$$b(\underline{\beta}) = \underline{\beta} = \frac{1}{\phi} - \frac{\underline{c}}{\phi(H\underline{\lambda} + (1 - \underline{\lambda})L)} + \underline{c}, \tag{41}$$

if condition (B) holds.

#### **Proof.** See Appendix

Now that we have found the equilibrium bid function with equity financing, we must determine if equity financing also causes an inefficient auction. **Theorem 4** Any auction in which the bidder willing to pay the most is certain to win, and bidders finance a portion of their bid by selling equity at a rate of  $\phi$  per dollar is not efficient.

**Proof.** This proof is virtually identical to Theorem 2 and Corollary 2 above.

Overall, we find that the auction is still inefficient with equity financed bids. Therefore, the inefficiency is not stemming from the limited liability nature of debt. What then is causing the failure? Adverse selection in capital markets.

## II Capital Markets

Previously in the paper we have assumed that the capital market is in a pooling equilibrium (possibly perfectly competitive). The pooling equilibrium seems to be the reason for the inefficiency of the auction. In this section we show that this is part, but not all of the problem. When the financial market is in a pooling equilibrium then the low types get an interest rate better than they deserve. Therefore, they benefit from borrowing money and are willing to bid more the less cash that they have. A pooling equilibrium charges high types too high a rate. Therefore, the less cash they have the less they are willing to bid. These effects cause the two regions in figure 1.

What happens if the capital market is separating and better types get lower interest rates or higher stock prices? Moving from a pooling to a separating equilibrium in the financial markets alters the equilibrium in the auction and may eliminate the two regions caused by the pooling equilibrium. However, we will show that the auction is, at best, unlikely to be efficient. Furthermore, we will show that competition in the capital markets is a source of the inefficiency in the auction.

Contracts can condition on bidder information in one of two ways. The first is that a securities market with perfect information can set different rates for each type even before the auction. The second is that the bidders' actions during the auction affect their rate. We will examine both possibilities and debt, equity and state-contingent financing to determine why competitive financial markets cause inefficient auctions.

### **II.1** Pre Auction Financing

In this section each bidder's rate will depend on their type and/or cash, but not the bid in the auction. This requires the securities market to determine the type of the bidder independent of the auction, and the market must commit not to use the information contained in the bids. It could be argued that banks specialize in determining the type of the borrower, and equity markets may aggregate information well. This seems to be the view expressed in Jensen (1991) and others who argue in favor of auctions when firms go bankrupt. Thus, the assumption of full information in the securities market is possible though extreme since the market's information is probably not complete.

Section II.2 will relax the full information assumption, and instead allow the rate to depend on the bid.

#### II.1.1 Pre Auction Debt Financing

Lenders now have the skill to determine the type  $\lambda_i$  and the cash  $c_i$  of the bidder before the auction. The lenders commit to use only  $\lambda_i$  and  $c_i$  to determine an interest rate for each bidder,  $r_i$ . Assuming the bidder still borrows and is still insolvent in the bad state, the bidder's problem is

$$\max_{b_i} \left[ \{ H\lambda_i - c_i - \lambda_i (1+r_i)(b_i - c_i) \} Prob[b_i > \max_{\forall j \neq i} b_j] \right].$$
(42)

Since the interest rate depends only on the private information of the borrower and not the bid, we can rearrange the optimization as

$$\max_{b_i} \left[ \left\{ \frac{H}{1+r_i} - \frac{c_i}{\lambda_i(1+r_i)} + c_i - b_i \right\} \operatorname{Prob}[b_i > \max_{\forall j \neq i} b_j] \right].$$

$$\tag{43}$$

Now we will define a new  $\hat{\theta}_i$ 

$$\widehat{\theta}_i = \frac{H}{1+r_i} - \frac{c_i}{\lambda_i(1+r_i)} + c_i, \tag{44}$$

and the bidder's problem becomes

$$\max_{b_i} \left[ \left\{ \widehat{\theta} - b_i \right\} Prob[b_i > \max_{\forall j \neq i} b_j] \right].$$
(45)

Given the sufficient condition,  $(b(\hat{\theta}) - \bar{c})(1 + \underline{r}) > L$ , and some bounds on  $r_i$ , the solution is, of course, the same as before, Equation (11), although the bid is a function of  $\hat{\theta}_i$  instead of  $\theta_i$ . Therefore, the bid function is an increasing function of  $\hat{\theta}_i$ . However, the question is whether this auction is inefficient. The answer depends on the interest rate.

**Theorem 5** If the lender knows  $\lambda_i$  and  $c_i$  and must set an interest rate for each bidder before the auction, then the only interest rate function which results in an efficient auction is

$$1 + r_i = \frac{1}{\lambda_i} \frac{\lambda_i H - c_i}{m_D(\lambda_i) - c_i} \tag{46}$$

where  $m_D(\cdot)$  is a monotone function subject to  $H - \frac{c_i}{\lambda_i} + c_i \ge m_D(\lambda_i) > L + c_i - \int_{\underline{\lambda}}^{\lambda_i} \frac{m'_D(x)F^{n-1}(x)dx}{F^{n-1}(\lambda_i)}$ , and where the subscript D denotes Debt.

**Proof.** Since the bid is increasing in  $\hat{\theta}_i$ , Lemma 1 shows that for the auction to be efficient  $\hat{\theta}_i$  must be increasing in  $\lambda_i$  and not change with  $c_i$ . The derivative with respect to  $c_i$  is only zero if

$$\frac{\partial \hat{\theta}_i}{\partial c_i} = 0, \text{ or } -\frac{1}{(1+r(\lambda_i, c_i))^2} [H - \frac{c_i}{\lambda_i}] \frac{\partial r_i}{\partial c_i} - \frac{1}{\lambda_i (1+r(\lambda_i, c_i))} + 1 = 0.$$
(47)

Fixing  $\lambda_i$  and integrating with respect to  $c_i$  we obtain that

$$1 + r(\lambda_i, c_i) = \frac{1}{\lambda_i} \frac{\lambda_i H - c_i}{m_D(\lambda_i) - c_i}.$$
(48)

Since the interest rate,  $r_i$ , must be positive,  $H - \frac{c_i}{\lambda_i} + c_i \ge m_D(\lambda_i)$ . Furthermore, plugging Equation (48) into  $\hat{\theta}_i$ , Equation (44), we find  $\hat{\theta}_i = m_D(\lambda_i)$ . Therefore,  $\frac{db}{d\hat{\theta}}\frac{d\hat{\theta}}{d\lambda} > 0$  requires  $m'_D(\cdot) > 0.^{23}$  Finally, we need to ensure that every bidder borrows and is possibly insolvent,  $b(\lambda_i) - c_i > L$ . It is easy to show that the bid function is

$$b(\lambda_i) = m_D(\lambda_i) - \int_{\underline{\lambda}}^{\lambda_i} \frac{m'_D(x)F^{n-1}(x)dx}{F^{n-1}(\lambda_i)}.$$
(49)

Thus, if  $m_D(\lambda)$  is too small, then the interest rate will be too large and some bidders will not borrow enough. Substituting the bid function into  $b(\lambda_i) - c_i > L$  yields  $m_D(\lambda_i) > L + c_i - \int_{\underline{\lambda}}^{\lambda_i} \frac{m'_D(x)F^{n-1}(x)dx}{F^{n-1}(\lambda_i)}$ .

If we set  $m_D(\lambda_i) = \lambda_i H$ , then we obtain  $(1 + r_i) = 1/\lambda_i$  as a special case where the interest rate does not depend on  $c_i$ . Furthermore, if we set  $m_D(\lambda_i) = \lambda_i H + (1 - \lambda_i)L$  then the auction bids will be same as the standard cash auction.

This theorem shows that it is not the bidder's need for financing but the financial market's failure to set interest rates appropriately that can cause inefficiency in the auction. If the financial market were to set interest rates according to Theorem 5, efficient auctions would be obtained. Figure 2 (insert figure 2 here) shows the iso-bid functions for the efficient separating equilibrium. Since the bids no longer depend on  $c_i$  the iso-bid functions are flat. The two regions in Figure 1 have disappeared. The two regions existed because the lending market was in a pooling equilibrium. Those bidders with high types found the interest rate too high and bid higher if they had more cash, while those with low types thought the interest rate quite low and bid higher if they needed to borrow.

If instead, the debt market is in a separating equilibrium where the interest rates are not given by Theorem 5, then it is possible that every bidder faces too high or too low an interest rate. In this case there is only one region; see Figure 3 (insert figure 3 here). However, the auction is still inefficient because the bids still change with c. It is also possible that many, but not all bidders face the efficient rate, resulting in many regions. Figure 4 (insert figure 4 here) shows this possibility. Since  $r_i$  needs to be integrable for our solution to be valid, the regions are not caused by an r that jumps around. Instead, the regions result as the rate moves above and below Equation (48) for different  $\lambda_i$ .

Therefore, to generate efficient bid functions, the lender must both know the borrower's type, and choose to set the interest rates according to Theorem 5. The following corollary shows that the

<sup>&</sup>lt;sup>23</sup>Looking at the bid function, Equation (49), it would seem that  $m'(\cdot)$  could be negative for large  $\lambda_i$ . However, this bid function is derived using  $\frac{db}{d\theta} > 0$ . Therefore, since  $\hat{\theta}_i = m(\lambda_i), m'(\cdot)$  must be positive.

competitive equilibrium in the lending market is not one of the equilibria in Theorem 5, i.e. the competitive equilibrium cannot be efficient.

**Corollary 4** If the debt market sets rates without conditioning on the bid and is perfectly competitive then the auction is inefficient.

**Proof.** If the debt market is perfectly competitive and the lender knows both  $\lambda_i$  and  $c_i$ , then each bidder must pay an interest rate that depends on their type and their cash, while the lender earns nothing.<sup>24</sup> In order for the lender to break even the interest rate,  $r_i$ , must be set such that

$$\lambda_i(b(\lambda_i) - c_i)(1 + r(\lambda_i, c_i)) + (1 - \lambda_i)L = b(\lambda_i) - c_i,$$
(50)

where  $b(\cdot)$  is the equilibrium bid function (which the rational lender can ascertain). Therefore,  $r_i$  does not change if the bidder chooses to bid out of equilibrium. This can be rewritten as

$$1 + r(\lambda_i, c_i) = \left(1 - \frac{(1 - \lambda_i)L}{b(\lambda_i) - c_i}\right) \frac{1}{\lambda_i}.$$
(51)

For the competitive auction to be efficient, this interest rate must equal Equation (48), or

$$\frac{\lambda_i H - c_i}{m_D(\lambda_i) - c_i} = \frac{b(\lambda_i) - c_i - (1 - \lambda_i)L}{b(\lambda_i) - c_i}.$$
(52)

This equality must hold for all  $c_i$ . Substituting for  $b(\lambda_i)$  from equation (49) and rearranging, we find the equality implies that

$$b(\lambda_i) = \lambda_i H + (1 - \lambda_i) L - \left(1 - \frac{(1 - \lambda_i)}{m_D(\lambda_i) - \int_{\underline{\lambda}}^{\lambda_i} \frac{m'_D(x)F^{n-1}(x)dx}{F^{n-1}(\lambda_i)} - c_i}\right) \int_{\underline{\lambda}}^{\lambda_i} \frac{m'_D(x)F^{n-1}(x)dx}{F^{n-1}(\lambda_i)}.$$
 (53)

Which requires that the bid is a function of  $c_i$ , and thus, not efficient; a contradiction.<sup>25,26</sup>

#### **II.1.2** Pre Auction Equity Financing

The equity market now has the ability to aggregate information and determine the  $\lambda_i$  and  $c_i$  of the bidder before the auction. The equity market commits to use only  $\lambda_i$  and  $c_i$  to determine the market

 $<sup>^{24}</sup>$ If the lender does not know  $c_i$  then the following proof will entail expectations over  $c_i$ , but the end result will not change.

<sup>&</sup>lt;sup>25</sup>This does not mean that  $m_i(\lambda_i) = \lambda_i H + (1 - \lambda_i)L$  is not a valid function in determining the efficient rate. It is. However, the rate in such a case is not the competitive interest rate.

<sup>&</sup>lt;sup>26</sup>It should be noted that the assumption that the financial market can only contract on the type and cash is important here. If the market could contract on the type, bid and cash, then competitive debt could yield efficiency and implement the cash auction (See Section II.2.4). However, the informational requirements are very strong. In contrast, we can implement efficiency and competitive equilibrium with equity while only contracting on the type.

capitalization,  $1/\phi_i$ . Assuming that the bidder needs to finance a portion of their bid, the bidder's problem becomes

$$\max_{b_i} \left[ \{H\lambda_i + (1-\lambda_i)L - c_i - \phi_i(b_i - c_i)(H\lambda_i + (1-\lambda_i)L)\} Prob[b_i > \max_{\forall j \neq i} b_j] \right].$$
(54)

Define  $\hat{\beta}_i$ 

$$\widehat{\beta}_i = \frac{1}{\phi_i} - \frac{c_i}{\phi_i (H\lambda_i + (1 - \lambda_i)L)} + c_i, \tag{55}$$

and the bidder's problem becomes

$$\max_{b_i} \left[ \left\{ \widehat{\beta}_i - b_i \right\} Prob[b_i > \max_{\forall j \neq i} b_j] \right].$$
(56)

With the necessary and sufficient condition,  $b(\hat{\beta}) - \bar{c} > 0$ , the resulting bid function is the same as above with  $\hat{\beta}_i$  replacing  $\beta_i$ . Whether this auction is efficient depends on the stock price.

**Theorem 6** If the lender knows  $\lambda_i$  and  $c_i$  and must set a market capitalization for each bidder before the auction, then the only market capitalization which results in an efficient auction is

$$1/\phi_i = [H\lambda_i + (1 - \lambda_i)L] \frac{m_E(\lambda_i) - c_i}{H\lambda_i + (1 - \lambda_i)L - c_i},$$
(57)

with  $m_E(\lambda_i) > c_i$  and  $m'_E(\lambda_i) > 0$ , where the subscript E denotes Equity.

**Proof.** Since the bids increase with  $\hat{\beta}_i$ , Lemma 1 shows that for the auction to be efficient  $\hat{\beta}_i$  must be increasing in  $\lambda_i$  and not change with  $c_i$ . Therefore,  $\phi_i$  can be set such that

$$\frac{1}{\phi_i} - \frac{c_i}{\phi_i(H\lambda_i + (1 - \lambda_i)L)} + c_i = m_E(\lambda_i),$$
(58)

where  $m'_E(\lambda_i) > 0$ . Rearranging yields Equation (57). Since  $\phi_i$  must be positive,  $m_E(\lambda_i) > c_i$ .

If we set  $m_E(\lambda_i) = \lambda_i H + (1 - \lambda_i)L$ , then we obtain  $1/\phi_i = H\lambda_i + (1 - \lambda_i)L$  as a special case where the price does not depend on  $c_i$ . We further note that setting  $1/\phi_i = H\lambda_i + (1 - \lambda_i)L$  results in  $\hat{\beta}_i = H\lambda_i + (1 - \lambda_i)L$  and the same bid as the cash bid in a standard cash auction.

**Corollary 5** If the equity market sets the rate without conditioning on the bid and is perfectly competitive then the auction is efficient.

**Proof.** If the equity market is perfectly competitive and the equity market knows both  $\lambda_i$  and  $c_i$ , then each bidder must sell equity at a rate that depends on their type, while the equity market earns nothing. In order for the equity market to break even the rate,  $\phi_i$ , must be set such that

$$\phi_i(H\lambda_i + (1 - \lambda_i)L) = 1. \tag{59}$$

Which is identical to one of the rates that results in an efficient auction.

In this case the perfectly competitive equilibrium also results in an efficient auction and the outcome of the perfectly competitive equilibrium is identical to that of the cash auction. Understanding why the equity financed auction is efficient if the securities market is perfectly competitive, while perfect competition in the debt market results in an inefficient market, explains why only one pre-auction financing function will result in an efficient auction. In both markets financing distorts the auction unless each bidder is indifferent between using his own cash and getting financing. If the financing rate is anything but the indifference point then the bidder either benefits from having cash or wishes that he needed to borrow more. This causes his bid to change with the amount of cash that he has, and a bidder with a better type may lose to a bidder with a worse type who has a different amount of cash.

In the debt market the point of indifference was not the competitive equilibrium because the view of the bidder and lender was different. The bidder was not affected by L since he paid it independent of the interest rate. The lender, however, was willing to lower the rate because he knew he would receive at least L. However, in the equity auction the seller and the buyer face the same trade-offs, so the bidder's indifference rate is the same as the equity market's break-even point. The secured part of the loan (L) changes the perspective of the lender but does not alter the marginal decision of the bidder. Thus, it is the secured part of the loan that results in the inefficiency in the auction.

Overall, the need for financing and the difference between the debt and equity auction may or may not affect the seller, it depends on financial markets. If the financing rates are set such that  $m_D(\lambda_i) = m_E(\lambda_i) \ \forall \lambda_i$ , then the bids in an auction with either debt or equity financing or BOTH types of financing are identical. In fact if  $m_D(\lambda_i) = m_E(\lambda_i) = \lambda_i H + (1 - \lambda_i)L$ , then even if some bidders do not need to borrow, the auction will be efficient and the bids will be the same as the standard cash auction. Thus, the seller would be unaffected by the cash shortfall. However, note that the debt market must be in an equilibrium other than a competitive equilibrium! If the debt market is competitive then the auction will not be efficient.

Lenders set the interest rate independently of the auction in many circumstances (such as asset purchases). Often lenders set the interest rate based on the risk of the project and do not even consider different abilities. We have shown this leads to an inefficient auction, as efficiency requires a different rate for each type. It is extreme to assume that the lender can determine the type of every borrower. If the lender incorrectly determines the bidder's type pre auction, then the auction will be inefficient. Even if the lender has perfect information then with all but one choice of financing rate function the auction will be inefficient, and with debt the efficient rate is not the competitive equilibrium. However, we must consider the possibility that the lender can condition the interest rate on the auction. The next section allows the lender to use the information from the auction to determine the interest rate.

## **II.2** Post Auction Financing

After the auction the securities market learns the bid of the winning bidder and may be able to determine the bidder's cash (by auditing). Thus, the securities market could use the information from the auction to determine the rate. The rate could be a function of  $b_i$  and/or  $c_i$ ; with debt  $r(b_i, c_i)$  or with equity  $\phi(b_i, c_i)$ . With this general functional form for the rate, we will not be able to determine a closed form solution for the bid function. However, we will be able to determine some general conclusions.

In the above section we demonstrated that the only pre-auction interest rate which resulted in an efficient auction required the lender to know the bidder's type. In reality, one could argue that the bidder's type is unverifiable and thus, banks are unlikely to be able to do this. Furthermore, if the type is easily determined then the seller should not need to use an auction. In this section, the lender can use the bidder's actions in the auction to help determine the bidder's type. So, the important questions are whether the lender is able to use this information to set an interest rate function that results in an efficient auction, and furthermore, is this rate the competitive equilibrium.

#### **II.2.1** Post Auction Debt Financing

Assuming the bidder still borrows and is still insolvent in the bad state, the bidder's problem is

$$\max_{b_i} \left[ \{ H\lambda_i - c_i - \lambda_i (1 + r(b_i, c_i))(b_i - c_i) \} Prob[b_i > \max_{\forall j \neq i} b_j] \right].$$
(60)

Because the interest rate is a function of the bid, we cannot rearrange this maximization in the way we did above. For a general  $r(b_i, c_i)$  there is no closed form solution. However, as the following theorem shows, we do not need a general solution to determine if there exists an interest rate function that results in an efficient auction.

**Theorem 7** If the lender does not know  $\lambda_i$  but can set an interest rate after seeing the bid, then the only interest rate which results in an efficient auction is

$$1 + r(b_i, c_i) = \left[H - \frac{c_i}{b^{-1}(b_i)} - \int_{\underline{\lambda}}^{b^{-1}(b_i)} \frac{c_i F^{n-1}(x)}{x^2 F^{n-1}(b^{-1}(b_i))} dx\right] \frac{1}{M_D(b^{-1}(b_i)) - c_i},\tag{61}$$

where  $M_D(b^{-1}(b_i))$  is any function of the inverted bid which satisfies,  $M_D(b^{-1}(b_i)) \ge c_i, M'_D(b^{-1}(b_i)) > 0.^{27}$ 

$$M_D(b^{-1}(b_i)) \le H - \frac{c_i}{b^{-1}(b_i)} + c_i - \int_{\underline{\lambda}}^{b^{-1}(b_i)} \frac{c_i F^{n-1}(x)}{x^2 F^{n-1}(b^{-1}(b_i))} dx.$$

<sup>&</sup>lt;sup>27</sup>Also,  $1 + r(b_i, c_i) < 1$  is not economically reasonable. Therefore,

**Proof.** To prove this theorem we will assume that there is an interest rate which depends on the bid and results in an efficient auction. Then we will determine what interest rate function is required, and show that with that interest rate the auction is efficient. To begin we will use the direct revelation mechanism and write  $b_i$  as b(x) where x is a selection from  $[\underline{\lambda}, \overline{\lambda}]$ . Since we have assumed the auction is efficient (which we will later verify) the probability of winning the auction is  $F^{n-1}(x)$ . Furthermore,  $1 + r(b_i, c_i)$  can be written as  $1 + r(b(x), c_i)$  or just  $1 + r(x, c_i)$ . Therefore, the bidder's problem is

$$\max_{x} \left[ \{ H\lambda_i - c_i - \lambda_i (1 + r(x, c_i))(b(x) - c_i) \} F^{n-1}(x) \right].$$
(62)

**Lemma 2** If the interest rate is set as in Equation (61) and  $(H-L)\underline{\lambda} > \overline{c}$  then  $\exists$  a unique symmetric equilibrium s.t.

$$b(\lambda_i) = \frac{H}{1 + r(\lambda_i, c_i)} - \frac{c_i}{\lambda_i (1 + r(\lambda_i, c_i))} + c_i - \int_{\underline{\lambda}}^{\lambda_i} \frac{c_i F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i) (1 + r(\lambda_i, c_i))} dx = M_D(\lambda_i).$$
(63)

**Proof.** Divide the bidder's problem by  $\lambda_i$ , differentiate w.r.t. x, and the resulting FOC is

$$\{H - \frac{c_i}{\lambda_i}\}\frac{dF^{n-1}(x)}{dx} = \frac{d\{(1 + r(x, c_i))(b(x) - c_i)F^{n-1}(x)\}}{dx} at \ x = \lambda_i.$$
(64)

Since this must hold for all  $\lambda_i$ , we can set  $\lambda_i = x$  then integrate x from  $\underline{\lambda}$  to  $\lambda_i$ , and solve for Equation (63).

To ensure that all bidders borrow and go bankrupt in the low state,  $(1 + r(\lambda_i, c_i))(b(\lambda_i) - c_i) > L \forall \lambda_i, c_i$ . With equation (63) this reduces to the sufficient and necessary condition  $(H - L)\underline{\lambda} > \overline{c}$ .

To show single crossing, we will write the objective function, Equation (62), as

$$[H\lambda_i - c_i - \lambda_i(1 + r(x, c_i))(b(x) - c_i)]Q$$
(65)

where  $Q = F^{n-1}(x)$ . Therefore, the indifference curves in (Q, x) space have slope given by

$$\frac{dQ}{dx} = \frac{Q\lambda_i \left[\frac{\partial r(x,c_i)}{\partial x}(b(x) - c_i) + (1 + r(x,c_i))\frac{\partial b(x)}{\partial x}\right]}{H\lambda_i - c_i - \lambda_i (1 + r(x,c_i))(b(x) - c_i)},\tag{66}$$

and the budget constraint has slope given by

$$\frac{dQ}{dx} = (n-1)F^{n-2}(x)F'(x).$$
(67)

The FOC ensures that these two equations are equal at  $x = \lambda_i$ . Single crossing with efficiency requires that

$$\frac{\partial}{\partial \lambda_i} \frac{dQ}{dx} < 0, \tag{68}$$

at any x and Q. Let  $S(x) = (1+r(x,c_i))(b(x)-c_i)$ , thus,  $S_x(x) = \frac{\partial r(x,c_i)}{\partial x}(b(x)-c_i) + (1+r(x,c_i))\frac{\partial b(x)}{\partial x}$ . Therefore,

$$\frac{\partial}{\partial\lambda_i}\frac{dQ}{dx} = \frac{QS_x(x)(H\lambda_i - c_i - \lambda_i S(x)) - Q\lambda_i S_x(x)(H - S(x))}{[H\lambda_i - c_i - \lambda_i S(x)]^2}$$
$$= \frac{-c_i}{[H\lambda_i - c_i - \lambda_i (1 + r(x, c_i))(b(x) - c_i)]^2}.$$
(69)

Since  $c_i$  is positive  $\frac{\partial}{\partial \lambda_i} \left( \frac{dQ}{dx} \right) < 0.$ 

In order for the auction to be efficient, the tangency in Figure 5 (insert Figure 5 here) must occur at the same point regardless of  $c_i$ , or  $\frac{\partial}{\partial c_i} \left(\frac{dQ}{dx}\right)|_{x=\lambda_i}=0$ . Therefore,

$$\frac{\partial}{\partial c_i} \left( \frac{dQ}{dx} \right) = Q\lambda_i \frac{\frac{\partial S_x(x)}{\partial c} (H\lambda_i - c_i - \lambda_i S(x)) + S_x(x)(1 + \lambda_i \frac{\partial S(x)}{\partial c})}{[H\lambda_i - c_i - \lambda_i S(x)]^2} \Big|_{x = \lambda_i} = 0,$$
(70)

or,

$$\frac{\partial S_x(x)}{\partial c} (H\lambda_i - c_i - \lambda_i R(x)) + S_x(x)(1 + \lambda_i \frac{\partial S(x)}{\partial c}) \mid_{x = \lambda_i} = 0$$
(71)

Substituting in the bid function, Equation (63) yields

$$\lambda_{i}c_{i}(n-1)F'(\lambda_{i})\int_{\underline{\lambda}}^{\lambda_{i}} \frac{F^{n-1}(x)}{x^{2}F^{2n-2}(\lambda_{i})} dx \int_{\underline{\lambda}}^{\lambda_{i}} \frac{F^{n-1}(x)}{x^{2}F^{1n-1}(\lambda_{i})} dx -\lambda_{i}c_{i}(n-1)F'(\lambda_{i})\int_{\underline{\lambda}}^{\lambda_{i}} \frac{F^{n-1}(x)}{x^{2}F^{2n-2}(\lambda_{i})} dx \int_{\underline{\lambda}}^{\lambda_{i}} \frac{F^{n-1}(x)}{x^{2}F^{1n-1}(\lambda_{i})} dx = 0,$$
(72)

which is obviously true. However, Equation (63) can only be used if the auction is efficient, since it was derived under the assumption of efficiency. Therefore, Lemma 1 tells us we must check that the bid function is increasing in  $\lambda_i$ ,  $b'(\lambda_i) > 0$ , and does not change with  $c_i$ ,  $\frac{\partial b(\lambda_i)}{\partial c_i} = 0$ . Substituting the interest rate function from Equation (61) into the bid function, Equation (63), we find that  $b(x) = M_D(x)$ , which is increasing in x and does not change with  $c_i$ .

This proves lemma 2, but to prove the theorem we must show that Equation (61) is the only interest rate that results in efficiency. Using lemmas 1 and 2, efficiency requires

$$\frac{\partial b(\lambda_i)}{\partial c_i} = 1 - \frac{1}{\lambda_i (1 + r(\lambda_i, c_i))} - \frac{(H\lambda_i - c_i) r_{c_i}(\lambda_i, c_i)}{\lambda_i (1 + r(\lambda_i, c_i))^2} - \int_{\underline{\lambda}}^{\lambda_i} \frac{F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i) (1 + r(\lambda_i, c_i))} dx - \int_{\underline{\lambda}}^{\lambda_i} \frac{c_i F^{n-1}(x) r_{c_i}(\lambda_i, c_i)}{x^2 F^{n-1}(\lambda_i) (1 + r(\lambda_i, c_i))^2} dx = 0,$$
(73)

which can be reduced to

$$(1+r(\lambda_i,c_i))^2 - \left[\frac{1}{\lambda_i} + \int_{\underline{\lambda}}^{\lambda_i} \frac{F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i)} dx\right] (1+r(\lambda_i,c_i))$$
$$= \left[H - \frac{c_i}{\lambda_i} - \int_{\underline{\lambda}}^{\lambda_i} \frac{c_i F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i)} dx\right] r_{c_i}(\lambda_i,c_i).$$
(74)

The solution to this ODE is Equation (61).<sup>28</sup> However, we must check that with this interest rate function the bid function is increasing in  $\lambda_i$ .

$$b'(\lambda_i) = -\left[H - \frac{c_i}{\lambda_i} - \int_{\underline{\lambda}}^{\lambda_i} \frac{c_i F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i)} dx\right] \frac{r_{\lambda_i}(\lambda_i, c_i)}{(1 + r(\lambda_i, c_i))^2} + \int_{\underline{\lambda}}^{\lambda_i} \frac{c_i F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i)(1 + r(\lambda_i, c_i))} dx \left[\frac{(n-1)F'(\lambda_i)}{F(\lambda_i)}\right] > 0.$$
(75)

Substituting for  $r(\lambda_i, c_i)$  and  $r_{\lambda_i}(\lambda_i, c_i)$  this reduces to the condition  $M'_D(b^{-1}(b_i)) > 0$  if  $M_D(b^{-1}(b_i)) > c_i$  or  $M'_D(b^{-1}(b_i)) < 0$  if  $M_D(b^{-1}(b_i)) < c_i$ . However, if  $M_D(b^{-1}(b_i)) < c_i$  then the interest rate is negative. Therefore, it is only economically reasonable for  $M_D(b^{-1}(b_i)) > c_i$ .

This theorem demonstrates that the lender must choose a specific form of interest rate function if the auction is to be efficient. However, an infinite number of interest rate functions have this form. The following two corollaries consider how the different players are affected by the choice of efficient rate, and whether the lender must know the cash of the bidder to set an efficient rate. Together they will help provide the intuition as to how an interest rate can allow an auction with two dimensional bidders to be efficient. Then, theorem 8 will bring us back to the larger question of whether the efficient rate is the competitive rate.

**Corollary 6** In an efficient auction with debt financing that is conditioned on the bids, the bidders are indifferent to the choice of interest rate function, but having greater cash improves their payoff. However, the seller's payoff increases if the interest rate is set lower. And, although the lender's receipts are not affected by the interest rate, their rate of return is.

**Proof.** In equilibrium, the lender receives

1

$$\lambda_i(b(\lambda_i) - c_i)(1 + r(b(\lambda_i), c_i)) + L(1 - \lambda_i).$$
(76)

And the bidder pays this plus  $c_i$ . Substituting in for  $r(\lambda_i, c_i)$  and  $b(\lambda_i)$  from Equations (61) and (63) yields

$$\frac{H\lambda_i - c_i - \lambda_i}{\Delta} \int_{\underline{\lambda}}^{\lambda_i} \frac{c_i F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i)} dx + L(1-\lambda_i)$$
(77)

 $^{28}$ Let

$$q(\lambda_i) = \frac{1}{\lambda_i} + \int_{\underline{\lambda}}^{\lambda_i} \frac{F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i)} dx$$
$$+ r(\lambda_i, c_i) = z(H - c_i q(\lambda_i)).$$

Therefore, the ODE can be rewritten

$$z^{2}(H - c_{i}q(\lambda_{i})) = \frac{\partial z(H - c_{i}q(\lambda_{i}))}{\partial c_{i}}$$

and the bidder pays this plus  $c_i$ . Therefore, the bidder's payment and the lender's receipts are unaffected by the interest rate. However, the total payment made by the bidder is decreasing in  $c_i$ and thus, having higher cash yields higher utility to the bidders.

The seller receives the bid, Equation (63) with Equation (61) substituted in for the interest rate. Thus, the seller gets  $M_D(b^{-1}(b_i))$ . If  $M_D(b^{-1}(b_i))$  is set lower, then the interest rate is higher, the bidder bids less and seller does worse. If the bidder bids less, then the lender must lend less. Since Equation (77) shows that the lender's total receipts are unaffected, his rate of return must increase.

An interest rate can result in an efficient auction by ensuring that bidders with more cash make lower expected payments. This works because bidders understand that their bid will affect their interest rate. Therefore, bidders with less cash must borrow more and thus, have a greater desire to influence the rate. This suggests that for low cash bidders the marginal benefit of raising their bid is higher.<sup>29</sup> However, the other benefit from raising the bid is an increased probability of winning. Therefore, if the interest rate can be set such that the payment of those with more cash is lower, then the marginal benefit of raising the bid for bidders with any amount of cash can be made to equate. Since the marginal cost (a larger expected payment) is the same for all bidders, if the marginal benefits are the same then bidders with the same type,  $\lambda$ , bid the same, and the auction is efficient.<sup>30</sup>

This discussion indicates that one way to get efficiency is to reduce the ability of the low cash bidders to affect their interest rate. This is accomplished by setting the slope of the interest rate function with respect to the bid, low for low cash bidders. This makes it difficult for low cash bidders to lower their interest rate by raising their bid, and leaves them with higher payments to the lender. This argument would suggest that any efficient interest rate function must depend on  $c_i$ . Examining the interest rate function, Equation (61), we see that its derivative with respect to  $b_i$  is lower when  $c_i$  is smaller. However, the following corollary shows that there is another possibility.

**Corollary 7** If the lender's only information is the winning bidder's bid and he does not know the bidder's cash then the lender can set the interest rate to

$$1 + r(b_i) = \frac{1}{b^{-1}(b_i)} + \int_{\underline{\lambda}}^{b^{-1}(b_i)} \frac{F^{n-1}(x)}{x^2 F^{n-1}(b^{-1}(b_i))} dx,$$
(78)

and the auction will be efficient.

**Proof.** Set

$$M_D(b^{-1}(b_i)) = \frac{H}{\frac{1}{b^{-1}(b_i)} + \int_{\underline{\lambda}}^{b^{-1}(b_i)} \frac{F^{n-1}(x)}{x^2 F^{n-1}(b^{-1}(b_i))} dx}$$
(79)

<sup>&</sup>lt;sup>29</sup>The derivative of Equation (78) with respect to  $b_i$  is negative. Therefore, higher bidders get better interest rates.

<sup>&</sup>lt;sup>30</sup>Corollary 6 shows that the bidder's expected payment decreases if their cash increases. Therefore, cash is valuable and bidders will not pay pre-auction dividends. Instead, bidders may attempt to build up cash before an auction.

in Equation (61), and the result is Equation (78).  $\blacksquare$ 

It is surprising that the lender is able to set an interest rate that results in efficiency, but depends only on the bid. It would seem that if bidders with different amounts of cash face the same interest rate function, then the marginal benefit of raising the bid would be greater for the low cash bidder. However, the other marginal benefit of raising the bid is a higher probability of winning. To respond to the effect of the interest rate on the low cash bidders, the ex-post efficient interest rate can be set high. With a high interest rate, bidders with less cash who in equilibrium bid the same as those with high cash (and the same type) are worse off: bidders with different cash make the same bid, but the high interest rate makes the low cash bidders pay more to the lender. Therefore, the marginal benefit of a higher probability of winning is lower for the low cash bidders. Thus, the interest rate can be set such that the marginal benefit of raising the bid is the same for every bidder with the same type regardless of their cash.

Mathematically, if the interest rate only depends on the bid, then the bidder's expected payoff from the auction with a bid of  $b_i$  is

$$(H\lambda_i - (1 + r(b_i))(b_i - c_i)\lambda_i - c_i)F^{n-1}(b^{-1}(b_i)).$$
(80)

The marginal cost of raising the bid is the increased payment, or

$$MC = -(1 + r(b_i))\lambda_i F^{n-1}(b^{-1}(b_i)),$$
(81)

which does not depend on  $c_i$ . The marginal benefit of raising the bid is the better interest rate and higher probability of winning, or

$$MB = -\frac{d(1+r(b_i))}{db_i}(b_i - c_i)\lambda_i F^{n-1}(b^{-1}(b_i)) + (H\lambda_i - (1+r(b_i))(b_i - c_i)\lambda_i - c_i)\frac{dF^{n-1}(b^{-1}(b_i))}{db_i}.$$
 (82)

The derivative of the interest rate, Equation (78) with respect to  $b_i$  is

$$\frac{d(1+r(b_i))}{db_i} = -\int_{\underline{\lambda}}^{b^{-1}(b_i)} \frac{F^{n-1}(x)}{x^2 F^{2n-2}(b^{-1}(b_i))} dx \frac{dF^{n-1}(b^{-1}(b_i))}{db_i}.$$
(83)

Substituting into the MB yields

$$MB = (H\lambda_i - b(\lambda_i))\frac{dF^{n-1}(\lambda_i)}{d\lambda_i},$$
(84)

at the equilibrium bid. Thus, at the equilibrium, neither the marginal benefit nor the marginal cost depend on  $c_i$ .

Interestingly, if the lender sets  $M_D(\lambda_i) = b(\lambda_i)$ , where  $b(\lambda_i)$  is the cash bid (and therefore, does not depend on c), then the bid, Equation (63), reduces to the cash bid. Thus, it is possible to obtain exactly the same bids as in a cash auction. Clearly the standard cash bids do not depend on  $c_i$ , so an efficient interest rate does not need to depend on  $c_i$ . Theorem 7 and the resulting corollaries show that a lender whose only desire is to set an interest rate that results in an efficient auction is able to do so. The auction will be efficient if the interest rate function can counter the low cash bidder's stronger desire to influence the interest rate. However, the next theorem shows that no efficient interest rate is the result of a competitive equilibrium in the lending market.

**Theorem 8** The ex-post interest rate which results in an efficient auction is not the result of perfect competition in the lending market.

**Proof.** The perfectly competitive interest rate is the rate such that

$$\lambda_i (b_i - c_i) (1 + r(b_i, c_i) + (1 - \lambda_i)L = b_i - c_i,$$
(85)

or

$$1 + r(b_i, c_i) = \left(1 - \frac{(1 - \lambda_i)L}{b_i - c_i}\right) \frac{1}{\lambda_i}.$$
(86)

Substituting in for  $r(b_i, c_i)$  with Equation (61) and noting  $b_i = M_D(b^{-1}(b_i))$  in equilibrium, Equation (86) requires that

$$M_D(\lambda_i) = H\lambda_i + (1 - \lambda_i)L - \int_{\underline{\lambda}}^{\lambda_i} \frac{c_i F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i)} dx.$$
(87)

However, this depends on  $c_i$ . Thus, there is no  $M_D(\cdot)$  which satisfies Equation (86) that does not depend on  $c_i$ . Since the  $M_D(\cdot)$  that results in the efficient outcome does not depend on  $c_i$ , the competitive equilibrium is not efficient.

The intuition for the result is as follows. The efficient auction requires bidders with the same type but different cash to make the same bid but actually pay different expected payments. It is this feature that it makes it inconsistent with a competitive financial market. In a competitive financial market bidders who make the same bid will make the same total expected payment (this is a consequence of the competition). Thus, our results demonstrate that debt financing from a competitive lending market will not yield efficient auctions. We have now seen this result with all three types of lending: a set interest rate for every bidder, a pre auction interest rate that depends on the lender's perfect information, and a post auction interest rate that depends on the bid in the auction.

#### **II.2.2** Post Auction Equity Financing

Although competitive debt financing is always inefficient, Section II.1.2 demonstrated that equity financing could result in an efficient auction if the equity market knew the underlying valuations of the bidders. As we have argued, this may not be verifiable and thus, a more realistic assumption is that the equity market must learn the type of the bidder from the bidder's actions in the auction. In this section we examine an equity financed cash auction where the stock price is set after the bids are revealed. This approach follows our argument with debt.

Assuming that the bidder still needs to finance a portion of their bid, the bidder's problem is

$$\max_{b_i} \left[ \{H\lambda_i + (1-\lambda_i)L - c_i - \phi(b_i, c_i)(b_i - c_i)(H\lambda_i + (1-\lambda_i)L)\} Prob[b_i > \max_{\forall j \neq i} b_j] \right].$$
(88)

As with ex-post debt, we cannot solve for the general solution. We can, however, still determine if a stock price exists that will result in an efficient auction.

**Theorem 9** If the equity market does not know  $\lambda_i$  but can determine the market capitalization after seeing the bid, then the only  $\phi$  which results in an efficient auction is  $\phi(b_i, c_i) =$ 

$$\left[1 - \frac{c_i}{Hb^{-1}(b_i) + (1 - b^{-1}(b_i))L} - \int_{\underline{\lambda}}^{b^{-1}(b_i)} \frac{c_i(H - L)F^{n-1}(x)}{(Hx + (1 - x)L)^2F^{n-1}(b^{-1}(b_i))}dx\right] \frac{1}{M_E(b^{-1}(b_i)) - c_i},$$
(89)

where  $M_E(b^{-1}(b_i))$  is any function of the inverted bid which satisfies,  $M_E(b^{-1}(b_i)) \ge c_i$ ,  $M'_E(b^{-1}(b_i)) > 0.^{31}$ 

#### **Proof.** See Appendix. ■

Thus, again with equity financing it is possible that the auction is efficient. Although, once again only one form of rate function will result in efficiency. Furthermore, as with debt, the bidder pays the same absolute amount with any choice of  $M_E(\cdot)$ , but the markets rate of return increases with  $\phi$ , and the seller does worse. Finally, as the following corollary demonstrates, the market need not know  $c_i$  in order to get efficiency.

**Corollary 8** If the equity market's only information is the winning bid and the market does not know the winner's cash, then the market can set

$$\phi(b_i) = \frac{1}{Hb^{-1}(b_i) + (1 - b^{-1}(b_i))L} + \int_{\underline{\lambda}}^{b^{-1}(b_i)} \frac{(H - L)F^{n-1}(x)}{(Hx + (1 - x)L)^2 F^{n-1}(b^{-1}(b_i))} dx, \tag{90}$$

and the auction will be efficient.

**Proof.** Set

$$M_E(b^{-1}(b_i)) = \frac{1}{\frac{1}{Hb^{-1}(b_i) + (1-b^{-1}(b_i))L} + \int_{\underline{\lambda}}^{b^{-1}(b_i)} \frac{(H-L)F^{n-1}(x)}{(Hx + (1-x)L)^2F^{n-1}(b^{-1}(b_i))}dx}$$
(91)

<sup>31</sup>Also,  $\phi(b_i, c_i) > 1$  is not economically reasonable. Therefore,

$$M_E(b^{-1}(b_i)) \ge 1 - \frac{c_i}{Hb^{-1}(b_i) + (1 - b^{-1}(b_i))L} + c_i - \int_{\underline{\lambda}}^{b^{-1}(b_i)} \frac{c_i(H - L)F^{n-1}(x)}{(Hx + (1 - x)L)^2F^{n-1}(b^{-1}(b_i))} dx.$$

in Equation (89). ■

Once again the equity market must set a high rate to ensure that at the equilibrium bid, bidders with low cash have the same marginal benefit from increasing their bid as a high cash bidder.

We saw earlier with pre auction equity financing that the competitive rate was also the efficient rate. However, the next theorem demonstrates that with ex-post financing the efficient rate is not the result of a competitive equilibrium in the equity market.

**Theorem 10** The ex-post stock price which results in an efficient auction is not the result of perfect competition in the equity market.

**Proof.** The perfectly competitive stock price is the rate such that

$$(b_i - c_i)\phi(b_i, c_i)(H\lambda_i + (1 - \lambda_i)L) = b_i - c_i,$$
(92)

or

$$\phi_i = \frac{1}{H\lambda_i + (1 - \lambda_i)L}.$$
(93)

Which does not equal Equation (89). Therefore, the competitive equilibrium is not efficient.

Again the result hinges on the idea that in an efficient auction bidders with the same type but different cash make the same bid but different expected payments. This is inconsistent with a competitive financial market where bidders who make the same cash bid must make the same expected payment.

Why is the post auction competitive equity market not efficient even though the pre-auction perfect information competitive equity market was efficient? The main problem is the desire of the bidders to convince the market that they are a better type so that they get a better rate. In particular, the bidders with less cash have a greater desire to convince the market they are a good type (because they need more money), so they are willing to increase their bid by more than if they had a lot of cash.<sup>32</sup> To counter this effect, the expected payment of types with different levels of cash is different. This can only occur with a rate function that is not competitive.

Our general conclusion is that with standard securities like debt and equity, perfectly competitive financial markets cannot produce the efficient outcome in the auction unless bidders use only equity financing and the equity market has ex-ante perfect information. Therefore, efficiency is an unlikely result.

 $<sup>^{32}</sup>$ Engelbrecht-Wiggans and Kahn (1991) and Rothkopf et al (1990) show that auction outcomes can have affects outside the auction. Katzman and Rhodes-Kropf (2000) generalize how bid information can affect secondary games such as obtaining financing.

#### **II.2.3** Post Auction State-Contingent Financing

When bidders obtain competitive financing with either debt or equity, and the financing depends on their bids (or the amount that they borrow), we have shown that the auction will not be efficient. However, it is more realistic to assume that the bidders can use both debt and equity financing. Can a combination of competitive debt and equity financing result in an efficient auction? In other words, does security design matter? We allow for contingent claims and find that if every bidder is cash constrained then a proper capital structure can yield an efficient auction. However if we allow the possibility that some bidders do not need to get financing, then the auction will not be efficient.

Since there are only two states, we consider state contingent payment functions, where  $h(b_i, c_i) \leq H$  is the payment in the high state and  $l(b_i, c_i) \leq L$  is the payment in the low state. We do not impose the condition that  $h(b_i, c_i) \geq 0$  and  $l(b_i, c_i) \geq 0$ . Thus, at this stage, we allow for the possibility that the lender may pay the bidder in some states of the world. The bidder's problem becomes

$$\max_{b_i} \left[ \{H\lambda_i + (1-\lambda_i)L - c_i - \lambda_i h(b_i, c_i) - (1-\lambda_i)l(b_i, c_i)\} Prob[b_i > \max_{\forall j \neq i} b_j] \right]$$
(94)

Remember that the seller is not directly concerned with how the bidders raise money; the seller just awards the firm to the bidder who pays him the most (has the highest bid). Although we will be unable to solve for a general closed from solution, we can prove the following theorem.

**Theorem 11** If  $H\underline{\lambda} + (1 - \underline{\lambda})L > \overline{c}$  (all types borrow), then there exist competitive state-contingent financing schemes that result in an efficient auction, such that  $h(b_i, c_i) = w(b_i) - c_i \leq H$ ,  $l(b_i, c_i) = z(b_i) - c_i \leq L$ , with the restriction that  $h_x(x, c_i) - l_x(x, c_i) \leq 0$  and  $H - L - [h(x, c_i) - l(x, c_i)] \geq 0$ , at least one inequality strict,  $\forall c_i$ .

**Proof.** We will once again use the direct revelation mechanism and write  $b_i$  as b(x) where x is a selection from  $[\underline{\lambda}, \overline{\lambda}]$ . Next we will assume that the auction is efficient, which we will later verify. If the auction is efficient the probability of winning the auction is  $F^{n-1}(x)$ . Furthermore,  $h(b_i, c_i)$ and  $l(b_i, c_i)$  can be written as  $h(b(x), c_i)$  and  $l(b(x), c_i)$  or just  $h(x, c_i)$  and  $l(x, c_i)$ . We will show that  $H\underline{\lambda} + (1 - \underline{\lambda})L > \overline{c}$  ensures that every bidder needs to borrow, therefore, the bidder's problem is

$$\max_{x} \left[ \{ H\lambda_i + (1 - \lambda_i)L - c_i - \lambda_i h(x, c_i) - (1 - \lambda_i)l(x, c_i) \} F^{n-1}(x) \right].$$
(95)

Maximizing, the FOC is

$$\{-\lambda_{i}h_{x}(x,c_{i}) - (1-\lambda_{i})l_{x}(x,c_{i})\}F^{n-1}(x) + \{H\lambda_{i} + (1-\lambda_{i})L - c_{i} - \lambda_{i}h(x,c_{i}) - (1-\lambda_{i})l(x,c_{i})\}\frac{\partial F^{n-1}(x)}{\partial x} = 0.$$
(96)

Pseudo-concavity holds if the derivative of the FOC w.r.t  $\lambda_i$  is positive for all x. This requires

$$\{H - L - h(x, c_i) + l(x, c_i)\}\frac{\partial F^{n-1}(x)}{\partial x} - \{h_x(x, c_i) - l_x(x, c_i)\}F^{n-1}(x) > 0.$$
(97)

The sufficient condition is therefore,

$$h_x(x,c_i) - l_x(x,c_i) \le 0,$$
(98)

and

$$H - L - h(\underline{\lambda}, c_i) + l(\underline{\lambda}, c_i) \ge 0, \ \forall c_i,$$
(99)

with one of the two inequalities strict. We also need the efficiency requirement that  $\frac{\partial F^{n-1}(x)}{\partial x} \ge 0$ .

Setting  $\lambda_i = x$  and integrating from  $\underline{\lambda}$  to  $\lambda_i$  yields

$$\lambda_{i}h(\lambda_{i},c_{i}) + (1-\lambda_{i})l(\lambda_{i},c_{i}) + c_{i} = H\lambda_{i} + (1-\lambda_{i})L - \int_{\underline{\lambda}}^{\lambda_{i}} \frac{[H-L-[h(x,c_{i})-l(x,c_{i})]]F^{n-1}(x)dx}{F^{n-1}(\lambda_{i})}.$$
(100)

We cannot in general solve for a closed form solution. However, if the financing is competitive then

$$b(\lambda_i) - c_i = \lambda_i h(\lambda_i, c_i) + (1 - \lambda_i) l(\lambda_i, c_i).$$
(101)

Therefore, the bid function is

$$b(\lambda_i) = H\lambda_i + (1 - \lambda_i)L - \int_{\underline{\lambda}}^{\lambda_i} \frac{[H - L - [h(x, c_i) - l(x, c_i)]]F^{n-1}(x)dx}{F^{n-1}(\lambda_i)}.$$
 (102)

Since the bids are increasing in  $\lambda_i$ , the lowest bid is  $H\underline{\lambda} + (1 - \underline{\lambda})L$  which must be greater than  $\overline{c}$  to ensure that every bidder needs to borrow.

We now have two descriptions of the bid function, Equations (101) and (102), however, we still have three unknown functions,  $b(\cdot)$ ,  $h(\cdot)$  and  $l(\cdot)$ . Thus, we cannot determine a closed form solution. However, we can determine the properties of  $h(\cdot)$  and  $l(\cdot)$  which result in an efficient auction. Lemma 1 notes that efficient bids must not change with  $c_i$ . Therefore, the derivative of the bid, Equations (101) and (102), must equal zero for all  $\lambda_i$ . The derivative of Equation (102) w.r.t.  $c_i$  is

$$\int_{\underline{\lambda}}^{\lambda_i} \left[ \frac{\partial h(x, c_i)}{\partial c_i} - \frac{\partial l(x, c_i)}{\partial c_i} \right] F^{n-1}(x) dx = 0.$$
(103)

Thus,

$$\frac{\partial h(x,c_i)}{\partial c_i} = \frac{\partial l(x,c_i)}{\partial c_i},\tag{104}$$

almost everywhere. Differentiating Equation (101) w.r.t.  $c_i$  tells us that

$$\lambda_i \left[ \frac{\partial h(x, c_i)}{\partial c_i} - \frac{\partial l(x, c_i)}{\partial c_i} \right] + \frac{\partial l(x, c_i)}{\partial c_i} = -1.$$
(105)

Thus,

$$\frac{\partial l(x,c_i)}{\partial c_i} = -1 = \frac{\partial h(x,c_i)}{\partial c_i}.$$
(106)

Therefore, the only solution that allows for an efficient auction with competitive state contingent financing is  $h(\lambda_i, c_i) = w(\lambda_i) - c_i$  and  $l(\lambda_i, c_i) = z(\lambda_i) - c_i$ .<sup>33</sup>

We note that the above scheme may require payments from the lender to the bidder in some states. In particular, the maximum payment in the low state is L. Since  $l(\lambda_i, \underline{c}) = z(\lambda_i) - \underline{c} \leq L$ , we have that  $l(\lambda_i, \overline{c}) = l(\lambda_i, \underline{c}) + (\underline{c} - \overline{c}) \leq L + (\underline{c} - \overline{c}) \leq 0$  if  $L \leq (\overline{c} - \underline{c})$ . This is certainly true if L = 0. In this case, all such schemes require subsidies in the low state to some bidders. If subsidies are not allowed, then no competitive state contingent schemes exist.

While the above theorem provides the requirements for a contingent scheme that is consistent with competitive equilibrium, no scheme is actually shown. Given the conditions of the theorem, there exist many competitive contingent schemes. One example is the following. Let  $h(\lambda_i, c_i) = H + \delta(\lambda_i - \underline{\lambda}) - c_i$ and  $l(\lambda_i, c_i) = L + \nu(\lambda_i - \underline{\lambda}) - c_i$  where  $\delta < 0$  and  $\nu > 0$ . Also, the parameters satisfy  $\nu(\overline{\lambda} - \underline{\lambda}) < \underline{c}$ . This ensures that the total payment in the low state is less than L. The scheme reduces the payment in the high state for higher types and increases payment in the low state. This makes it incentive compatible. Thus, there exists many contingent payment schemes that satisfy the conditions of Theorem (11).

So we have found that with state contingent payoffs, a form of competitive financing results in an efficient auction. The intuition for this result is straightforward: with any form of contingent auction bidders have the incentive to try to convince the 'lender' that they are a better type so that they receive better financing. With either just debt or just equity financing those bidders with low cash have a greater incentive to influence the 'lender', so with competitive financing low cash bidders out bid higher cash bidders. However, with state contingent financing the amount of cash a bidder has can be removed from his decision; in every state, the amount of cash he paid to the seller is refunded, then he is charged an amount that depends only on his type.<sup>34</sup> We note that such contingent schemes are very different from standard financing where the contingent payment does not involve rebating of the cash paid.

It seems then that with an optimal capital structure for the bid, then the auction will be efficient. However, the intuition above makes it clear that the above result relied on a particular assumption that will not in general be true: all bidders needed to obtain financing. If a bidder does not obtain financing then his cash is not refunded and he does not make a contingent payment and his incentives to affect the lender disappear. In most of the paper we have focused on the case when all bidders need financing, and we have found that the auction is rarely efficient. Now, we can show in the most general case that the auction will not be efficient if some bidders do not require financing!

<sup>&</sup>lt;sup>33</sup>The bidder clearly has no incentive to lie about or reduce  $c_i$ , since his payment just goes up by the amount he reduces  $c_i$ . And the bidder cannot increase  $c_i$  since he is cash constrained.

<sup>&</sup>lt;sup>34</sup>It should be noted that this requires the lender to be able to determine and contract on  $c_i$ .

**Theorem 12** If some bidders require financing and other bidders do not, and some of the bidders who require financing bid such that  $b(\overline{\lambda}) > L + \underline{c}$  (lending entails risk), then competitive financing will not result in an efficient auction.

**Proof.** If the auction is efficient then the probability that a particular bidder with type x wins is  $F^{n-1}(x)$ , regardless of whether he needs to borrow. Therefore, the bidder's problem of a bidder who needs financing is Equation (95) and his resulting bid is Equation (102). Simultaneously, a bidder that does not need to borrow faces

$$\max_{x} \left[ \{ H\lambda_i + (1 - \lambda_i)L - b(x) \} F^{n-1}(x) \right].$$
(107)

Remember, this is true assuming the auction is efficient. Thus, a bidder who does not need to borrow bids

$$b(\lambda_i) = H\lambda_i + (1 - \lambda_i)L - \int_{\underline{\lambda}}^{\lambda_i} \frac{[H - L]F^{n-1}(x)dx}{F^{n-1}(\lambda_i)}.$$
(108)

This bid function is only equal to Equation (102) if  $h(x, c_i) = l(x, c_i)$ , bidders make the same payment in both states of the world. If bidders make the same payment in both states then this payment must be less than L plus c or the bidders cannot pay. This payment must also be larger than or equal to what they 'borrowed', or else the 'lender' does not break even. Therefore, the largest amount borrowed must be less than or equal to  $L, b(\overline{\lambda}) - \underline{c} \leq L$ . In other words, the financing must not entail any risk. If instead, the financing requires risk, as assumed by  $b(\overline{\lambda}) > L + \underline{c}$ , then the auction will not be efficient.

In order to get complete efficiency, the amount financed would need to be so low that the 'borrower' would never default. Since this is not an accurate picture of the financial markets the theorem follows. Thus, auctions with bidders who obtain competitive financing are not efficient.

#### **II.2.4** Perfect and Complete Information

To make it clear that the inefficiency in the auction stems from the adverse selection problem in the financial markets, we briefly consider the situation when the financial market has both perfect ex-ante information and complete information about the actions of the bidders. In this case it is intuitively obvious that if the 'lender' can set the rate using the true type,  $\lambda_i$ , the cash,  $c_i$ , and the bid,  $b_i$ , then the competitive rate in the financial market will result in an efficient auction. Furthermore, the result of this auction will be the same as the cash auction. Therefore, the auction will be efficient even if some bidders do not need to borrow. To see an example of this simply plug the competitive interest rate,

$$1 + r(\lambda_i, c_i) = \left(1 - \frac{(1 - \lambda_i)L}{b_i - c_i}\right) \frac{1}{\lambda_i},\tag{109}$$

into the bidder's problem,

$$\max_{b_i} \left[ \{ H\lambda_i - c_i - \lambda_i (1+r_i)(b_i - c_i) \} Prob[b_i > \max_{\forall j \neq i} b_j] \right].$$
(110)

The result is the same bidder's problem as the standard cash auction, Equation (1).

We have shown that even when the financial market is competitive and efficient the auction will likely not be efficient. However, if the financial market has an extraordinary amount of information then the inefficiency is removed. This demonstrates conclusively that it is the information asymmetries and adverse selection problems in the financial market (and not the financing per se) which result in an inefficient auction.

## III Efficient Seller Financing

When governments hold auctions they are often very interested in efficiency. Our discussion in the previous sections suggests that a cash auction is unlikely to be efficient when bidders finance their bids in competitive financial markets. To obtain efficiency, the government might offer financing. We analyze this under two different assumptions. First, the seller is the only source of financing. Second, a competitive financial market exists (the outside option).

In some parts of the world (such as Eastern Europe) or for some objects, financing is difficult to acquire. In these situations the seller may be the only avenue for financing. Aghion, Hart and Moore (1992) and others have suggested that when bidders cannot get financing that the seller allow bids in the form of securities such as debt. Rhodes-Kropf and Viswanathan (2000) examine this idea for the case when bidders have identical amounts of cash. This section analyses efficiency and revenue for the seller when the seller is the only source of financing for bidders. In doing so, we extend the revenue results of Rhodes-Kropf and Viswanathan (2000) to bidders with unequal cash.

It is uninteresting to consider a seller who can provide pre-auction efficient financing, because a seller with this much information about the bidders would not need an auction. More reasonably, the seller provides financing after seeing the bid. In this case the seller prefers equity financing.

**Theorem 13** There exists an efficient auction even if some bidders do not require financing. The seller expects to earn more providing equity than debt if he sets the rate after seeing the bids and if the resulting auction is efficient.

**Proof.** First we note that Theorems (7) and (9) provide for efficient auctions with debt and equity. Further by setting  $M_D(\lambda_i) = b(\lambda_i)$ , in Theorem (7) (where  $b(\lambda_i)$  is the cash bid), we can mimic the cash auction. Thus, there is an efficient auction even when some bidders do not require

financing. A similar argument holds for the equity auction. See Appendix for the proof that the equity financing results in greater seller revenue.  $\blacksquare$ 

Therefore, if the government has the appropriate information about the distribution of bidder types and they can require bidders to use the seller financing (or the seller is the only source of financing), then they can ensure efficiency and increase revenue using equity financing. Of course the requirement that the seller is the only source of financing is a strong assumption. If the auction is efficient then the bidder could use the bid and his contractible cash to go to a competitive financial market. Under such circumstances, seller financing will only be used if it offers terms better than a competitive financial market. If there is one bidder who does not require financing, the seller cannot attain efficiency. Under the conditions of Theorem (12), seller financing cannot improve upon the competitive financial market (in terms of achieving efficiency).

**Theorem 14** Suppose some bidders require financing and other bidders do not, and some of the bidders who require financing bid such that  $b(\underline{\lambda}) > L + \underline{c}$  (there is risk in lending to the lowest type with the lowest cash). Further suppose the seller is constrained to offer financing at equal or better terms than a competitive financial market (the outside option), then there is no efficient auction.

**Proof.** Assume the auction is efficient. Since there is one bidder who is not financially constrained, Theorem 12 demonstrates that all bidders must make the same bid as their respective cash auction bid. With competitive outside financing the bidders who borrow from the seller should expect to payback no more than what they borrowed. Thus, in total, the bidders should expect to pay no more than the cash bid. Hence, with seller financing the expected payment made by any bidder must be less than or equal to the cash auction (otherwise this bidder will obtain financing on better terms in the competitive financial market, the outside option). Therefore, the cash bid, Equation (108), minus the contingent auction expected payment, Equation (100), must be less than or equal to zero, or

$$\int_{\underline{\lambda}}^{\lambda_i} \frac{[h(x,c_i) - l(x,c_i)] F^{n-1}(x) dx}{F^{n-1}(\lambda_i)} \le 0.$$
(111)

However, from Equation (100) we know that for the lowest type,  $\underline{\lambda}h(\underline{\lambda}, c_i) + (1 - \underline{\lambda})l(\underline{\lambda}, c_i) + c_i = H\underline{\lambda} + (1 - \underline{\lambda})L$ . Rearranging this equality yields  $\underline{\lambda}[h(\underline{\lambda}, c_i) - l(\underline{\lambda}, c_i)] = H\underline{\lambda} + (1 - \underline{\lambda})L - [l(\underline{\lambda}, c_i) + c_i] > H\underline{\lambda} + (1 - \underline{\lambda})L - [L + c_i] > 0$ . By assumption (that there is risk in lending) the last equation is true at least for the lowest type with the lowest cash. Thus,  $h(\underline{\lambda}, c_i) - l(\underline{\lambda}, c_i) > 0$  and hence we cannot satisfy the integral inequality in Equation (111) locally around  $\underline{\lambda}$  (we are using the continuity of the bid function here). Hence the financing provided by the seller must be more expensive than competitive financing and thus, no efficient equilibrium exists.

The intuition for Theorem (14) is as follows. Since financing involves contingent payments, locally around the lowest type, the expected payment increases with type faster than the cash bid payment.

Since the bidder only bids the cash bid (a consequence of the presence of a bidder who does not need financing), this implies that the expected payment exceeds the bid. Under such circumstances, the bidder would go to the competitive market for financing and thus, the seller is unable to implement the efficient auction. This suggests that even when financing schemes exist that yield the efficient auction, the presence of a competitive financial market imposes constraints on the seller that make it impossible to attain efficiency.

## IV Conclusion

Cash auctions are used worldwide to sell assets of significant unknown value. Since the bidder with the best use for the object for sale is often not the bidder with the most cash, financing is a regular occurrence. For example, in the FCC bandwidth auctions, in which the government sold sections of the radio spectrum, many bidders obtained financing. For a segment of the auction (the C Block) the government actually provided the financing. Many winners of the European 3G bandwidth auctions financed their bids. Bidders who buy property often acquire asset based financing. Firms sold in bankruptcy regularly have financed bids, as the current management team is often a bidder. In privatization auctions, bidders attempt to acquire financing. And mergers and acquisitions of all types have financed bidders.

Standard intuition is that as long as bidders have cash in the auction, the auction will be efficient; the bidder with the highest value will win. Or, at least if there are competitive liquid capital markets, then the auction will be efficient. Baird (1986) and Jackson (1986) have argued that a cash auction is the efficient procedure for selling a bankrupt firm. Aghion, Hart, and Moore (1992) agree with this view but argue that the issue of seller financing (allowing non-cash bids) is important if capital markets are not efficient. Work by Bolton and Roland (1992) and Rhodes-Kropf and Viswanathan (2000) focuses on non-cash auctions for privatizations and Eastern European bankruptcies because of the lack of access to cash. Our paper suggests that while market imperfections may be a source of auction inefficiencies, financed auctions of all types will be inefficient, even with competitive liquid capital markets.

We consider bidders who have private values and different amounts of available cash. However, they all have access to capital. Capital markets can be reasonably modeled as having either no information about individual bidders, full information, or information about the actions of the bidders. We consider each possibility to show that it is unlikely that the auction will be efficient.

Generally, the inefficiency arises because the bidders' values change in different ways when the borrowing rate is changed depending on their cash position. To get efficiency the capital market that sets rates ex-ante must make the bidder indifferent between having more or less cash. With rates set after the auction, efficiency requires the capital market to make the bidder indifferent between bidding more or less, independent of his cash. However, bidders understand that their bids will influence the rate at which they borrow. Thus, bidders with little cash have greater incentives to distort their bids to influence their rate than bidders with a lot of cash. A monopolist lender can set rates to counter this effect. However, a competitive capital market can only give the competitive rate. Seen from this point of view it seems unlikely that financing will not distort the auctions.

Even if the seller desires efficiency and is able to set the financing rate, they need to do so with particular skill in order to get efficiency. However, if they accomplish efficiency, we show that the seller will generate greater revenue by providing equity financing rather than debt financing. In the FCC bandwidth auction the government provided a form of debt financing for a particular section of the auction called the C block. Our work suggests that the government could have increased their revenue if they had provided equity financing. At the very least the government would not be having the default problems that are occurring as the debt financed bidders are unable to pay. However, in the presence of a competitive financial market, our results indicate that a seller cannot create an efficient auction by offering rates which a bidder would accept.

Auctions around the world have bidders who attain financing based on project risk, or by demonstrating they are a good type, or after showing the amount they need to borrow. We have shown that the efficiency of the auction design cannot be considered separately from the way bidders finance their bids even if the financial markets are liquid and competitive. The adverse selection in the securities market affects the valuation of the object in the auction through the terms of financing. We also emphasize that it is this distortion in the valuation of the object that causes problems for efficiency and changing the auction from first price to second price to Vickrey does not change the underlying rationale for this problem which stems from the financial market. Competitive liquid financial markets cannot provide the incentives which make bids independent of the bidders' cash positions. Thus, an auction with financed bids will be inefficient.

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#### Appendix

#### **Proof of Theorem 3 :** The FOC for the bidder's problem is

$$[\beta - b(x)]\frac{d\Omega^{n-1}(x)}{dx} - b'(x)\Omega^{n-1}(x) = 0 \text{ for } x = \beta \ \forall \beta \in [\underline{\beta}, \overline{\beta}] \tag{A1}$$

Therefore, increasing  $\beta$  without changing x results in the FOC >0 and decreasing  $\beta$  without change x results in the FOC <0. Since the bidder will never bid outside of  $[b(\underline{\theta}), b(\overline{\theta})]$  this shows pseudo-concavity.

The bid function is strictly increasing in  $\beta$  and is, therefore, invertible as we assumed. However, we need to ensure that bidders must finance a portion of their bid,  $b(\beta(\lambda_i, c_i) > c_i \forall i)$ . Since the bids increase with  $\beta$ ,

$$(b(\beta) - \overline{c}) > 0, \tag{A2}$$

is a necessary and sufficient condition that ensures all bidders use financing.  $\beta$  is strictly decreasing in c if  $(H\underline{\lambda} + (1 - \underline{\lambda})L)$  is less than  $1/\phi$ . In this case, condition (A2) can be reduced to

$$H\underline{\lambda} + (1 - \underline{\lambda})L > \overline{c}. \tag{A3}$$

This condition requires that  $H\underline{\lambda} + (1 - \underline{\lambda})L$  must be larger than the most cash the worst manager could have. If  $(H\underline{\lambda} + (1 - \underline{\lambda})L) > 1/\phi$  then  $\beta$  is strictly increasing in c. Thus,

$$b(\underline{\beta}) = \frac{1}{\phi} - \frac{\underline{c}}{\phi(H\underline{\lambda} + (1 - \underline{\lambda})L)} + \underline{c}, \tag{A4}$$

and condition (A2), becomes

$$\frac{1}{\phi} > \frac{(\overline{c} - \underline{c})(H\underline{\lambda} + (1 - \underline{\lambda})L)}{H\underline{\lambda} + (1 - \underline{\lambda})L - \underline{c}}.$$
(A5)

If the equity rate is high enough then bidders will stop financing since increasing their bid slightly above their cash will increase their expected payment far more than their probability of winning.

**Proof of Theorem 9:** To prove this theorem we will assume that there is a stock price which depends on the bid and results in an efficient auction. Then we will determine what stock price function is required, and show that with that stock price the auction is efficient. To begin we will use the direct revelation mechanism and write  $b_i$  as b(x) where x is a selection from  $[\underline{\lambda}, \overline{\lambda}]$ . Since we have assumed the auction is efficient (which we will later verify) the probability of winning the auction is  $F^{n-1}(x)$ . Furthermore,  $\phi(b_i, c_i)$  can be written as  $\phi(b(x), c_i)$  or just  $\phi(x, c_i)$ . Therefore, the bidder's problem is

$$\max_{x} \left[ \{H\lambda_i + (1-\lambda_i)L - c_i - \phi(x, c_i)(b(x) - c_i)(H\lambda_i + (1-\lambda_i)L)\}F^{n-1}(x) \right].$$
(A6)

Lemma 2: If  $\phi$  is set as in equation (89) and  $H\underline{\lambda} + (1 - \underline{\lambda})L > \overline{c}$  then  $\exists$  a unique symmetric equilibrium s.t.

$$b(\lambda_i) = \frac{1}{\phi(\lambda_i, c_i)} - \frac{c_i}{\phi(\lambda_i, c_i)(H\lambda_i + (1 - \lambda_i)L)} + c_i - \int_{\underline{\lambda}}^{\lambda_i} \frac{c_i(H - L)F^{n-1}(x)}{(Hx + (1 - x)L)^2F^{n-1}(\lambda_i)\phi(\lambda_i, c_i)} dx.$$
(A7)

**Proof.** Divide the bidder's problem by  $H\lambda_i + (1 - \lambda_i)L$  and differentiate,

$$\left\{1 - \frac{c_i}{H\lambda_i + (1 - \lambda_i)L}\right\} \frac{dF^{n-1}(x)}{dx} = \frac{d\{\phi(x, c_i)(b(x) - c_i)F^{n-1}(x)\}}{dx}.$$
 (A8)

Since this must hold for all  $\lambda_i$  we can integrate both sides and solve for the bid. To ensure bidders borrow as assumed,  $(b(\beta_i) - c_i) > 0 \ \forall \beta_i, c_i$ . Using the bid function, this reduces to the necessary and sufficient condition  $H\underline{\lambda} + (1 - \underline{\lambda})L > \overline{c}$ . Single crossing can be shown to hold if  $\phi$  is set as in equation (89) in a very similar manner to Theorem 7.

In order for the auction to be efficient, this bid function must be invertible, or  $b'(\lambda_i) > 0$  and  $\frac{\partial b(\lambda_i)}{\partial c_i} = 0$ .

$$\frac{\partial b(\lambda_i)}{\partial c_i} = 1 - \frac{1}{\phi(\lambda_i, c_i)(H\lambda_i + (1 - \lambda_i)L)} - \int_{\underline{\lambda}}^{\lambda_i} \frac{(H - L)F^{n-1}(x)}{(Hx + (1 - x)L)^2 F^{n-1}(\lambda_i)\phi(\lambda_i, c_i)} dx - \frac{\phi_{c_i}(\lambda_i, c_i)}{\phi^2(\lambda_i, c_i)} \left[ 1 - \frac{c_i}{H\lambda_i + (1 - \lambda_i)L} - \int_{\underline{\lambda}}^{\lambda_i} \frac{(H - L)F^{n-1}(x)}{(Hx + (1 - x)L)^2 F^{n-1}(\lambda_i)} dx \right] = 0.$$
(A9)

The solution to this is Equation (89). It is easy to show that with this stock price the bid is increasing in  $\lambda_i$ as long as  $M_E(b^{-1}(b_i)) \ge c_i$ ,  $M'_E(b^{-1}(b_i)) > 0$ .

**Proof of Theorem 13:** The winner of the debt financed auction expects to pay

$$\lambda_i(1+r_i)(b(\lambda_i)-c_i)+c_i+(1-\lambda_i)L.$$
(A10)

If the auction is efficient his expected payment is

$$H\lambda_i + (1 - \lambda_i)L - \int_{\underline{\lambda}}^{\lambda_i} \frac{\lambda_i c_i F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i)} dx.$$
 (A11)

The winner of the equity financed auction expects to pay

$$\phi_i(b(\lambda_i) - c_i)(H\lambda_i + (1 - \lambda_i)L) + c_i \tag{A12}$$

If the auction is efficient his expected payment is

$$H\lambda_i + L(1-\lambda_i) \left[ 1 - \int_{\underline{\lambda}}^{\lambda_i} \frac{(H-L)c_i F^{n-1}(x)}{(Hx+L(1-x))^2 F^{n-1}(\lambda_i)} dx \right].$$
 (A13)

The expected payment in the equity financed auction is greater than the debt financed auction if

$$\int_{\underline{\lambda}}^{\lambda_i} \frac{\lambda_i c_i F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i)} dx > \int_{\underline{\lambda}}^{\lambda_i} \frac{(H-L)c_i F^{n-1}(x)(H\lambda_i + L(1-\lambda_i))}{(Hx + L(1-x))^2 F^{n-1}(\lambda_i)} dx$$
(A14)

$$\Leftrightarrow \int_{\underline{\lambda}}^{\lambda_i} F^{n-1}(x) \left[ \frac{\lambda_i}{x^2} - \frac{(H-L)(H\lambda_i + L(1-\lambda_i))}{(Hx + L(1-x))^2} \right] dx > 0$$
(A15)

Therefore, if

$$\frac{\lambda_i}{x^2} > \frac{(H-L)(H\lambda_i + L(1-\lambda_i))}{(Hx + L(1-x))^2} \quad \forall x < \lambda_i$$
(A16)

then the equity auction expected payment is greater. If L = 0 then the two sides are equal. Therefore, if

$$\frac{d}{dL}\frac{(H-L)(H\lambda_i + L(1-\lambda_i))}{(Hx + L(1-x))^2} < 0$$
(A17)

then the theorem is proved. This derivative is negative if

$$(Hx + L(1 - x))^{2} [(H - L)(1 - \lambda_{i}) - (H\lambda_{i} + L(1 - \lambda_{i}))]$$
  
< 2(H - L)(H\lambda\_{i} + L(1 - \lambda\_{i}))(Hx + L(1 - x))(1 - x) (A18)

 $\operatorname{or}$ 

$$(Hx + L(1 - x)) [(H - L)(1 - \lambda_i) - (H\lambda_i + L(1 - \lambda_i))] < 2(H - L)(H\lambda_i + L(1 - \lambda_i))(1 - x).$$
(A19)

It is sufficient to show that

$$(Hx + L(1 - x))(H - L)(1 - \lambda_i) < 2(H - L)(H\lambda_i + L(1 - \lambda_i))(1 - x),$$
(A20)

since the other term on the left hand side is negative. Since  $1 - \lambda_i < 1 - x$  and  $Hx + L(1 - x) < H\lambda_i + L(1 - \lambda_i)$ , the theorem is proved for the case where all bidders require financing.

## Notes

<sup>1</sup>See interesting work by Strömberg (2000) and Eckbo and Thorburn (2000). A theoretical evaluation of such bankruptcy auctions is Rhodes-Kropf and Viswanathan (2000).

<sup>2</sup>See Bolton and Roland (1992).

<sup>3</sup>This is the function of competitive capital markets.

<sup>4</sup>Maskin and Riley (2000) and Maskin (2000) show that auctions with asymetric bidders and private values will be inefficient.

<sup>5</sup>Solving the general auction with two-dimensional bidders is an important goal. This paper is a step forward in that results are found with bidders who have different continuous values and different continuous amounts of cash.

<sup>6</sup>Che and Gale (1998) also show inefficiency due to the budget constraint. Our results indicate a different kind of inefficiency that is due to the pricing of securities in the financial market.

<sup>7</sup>In Rhodes-Kropf and Viswanathan (2000), the asset is in the form of a non-pecuinary penalty if bankruptcy occurs. It can equally be interpreted as cash held by bidders.

<sup>8</sup>Further, in Zheng's (2000) model there is no distinction between debt and equity. This is because of the assumption in a two-state model that the low state is zero. Thus, whether bidders use equity or debt, the payment in the low state is zero.

 $^9\mathrm{Every}$  result would go through if H and L had distributions.

<sup>10</sup>No matter how large H is and how small c is, there exists some  $\lambda > 0$  that is so small that the assumptions we use to solve the problem (namely insolvency in the low state) are not true.

<sup>11</sup>The ability to lend at the competitive rate ensures that bidders value their cash at c. If they had poor outside opportunities then retaining their cash would be a negative net present value project.

 $^{12}{\rm The}$  assumption of zero systematic risk is without loss of generality. See corollary 1 for a formal definition of break even r.

<sup>13</sup>Formally, suppose the bidder borrows an additional amount s. The bidder would pay an additional amount s in the low state and an additional amount (1 + r)s in the high state. Hence the expected payoff is negative and the bidder will not borrow than he needs in equilibrium. The argument relies on their being bankruptcy. If there were no bankruptcy in the low state, debt is no different from cash.

<sup>14</sup>Che and Gale (1998) have no default. Thus, their debt is just two certain cash payments, one now and one next period. However cash payments in the future are costly.

<sup>15</sup>In debt markets the premium over the risk free rate is called the default spread. In the real world all borrowers have a probability of default and must pay a default spread.

<sup>16</sup>It would be interesting to allow some bidders who do not borrow, or some bidders who can repay the loan in the bad state. However, the complication would not allow a closed form solution. We do demonstrate some results in the situation where some bidders do not need to borrow.

<sup>17</sup>Unless bidders borrow more than b-c, in which case L+c is the secured part of the loan. Thus, bidders only want to borrow b-c.

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$$G(\theta) = G\left[\frac{H}{1+r} - \frac{c_i}{\lambda_i(1+r)} + c_i \le \theta\right]$$
$$= \int_{\underline{c}}^{\overline{c}} \Pr\left[\frac{H}{1+r} - \frac{c_i}{\lambda_i(1+r)} + c_i \le \theta \mid c_i\right] \Psi'(c_i) dc_i.$$

Rearanging yields

$$\lambda_i \le \frac{c_i}{H + (1+r)(c_i - \theta)}.$$

Therefore,

$$G(\theta) = \int_{\underline{c}}^{\overline{c}} F\left[\frac{c_i}{H + (1+r)(c_i - \theta)}\right] \Psi'(c_i) dc_i.$$

<sup>19</sup>In a first price auction, a bidder gains no probability of winning by bidding greater than the highest possible competitor's bid, and he must pay more than if he just bid the highest bid. A bidder will lose with certainty if he bids below the lowest possible bid.

 $^{20}\mathrm{If}$  there is no chance of bankruptcy then

$$\theta_i = \frac{H\lambda_i + L(1 - \lambda_i) - c_i}{1 + r} + c_i.$$

In which case,

$$\frac{\partial \theta}{\partial c} = 1 - \frac{1}{1+r}.$$

This derivative is equal to zero if r = 0. Since there is no bankruptcy, the only efficient interest rate is zero.

<sup>21</sup>If there is no chance of bankruptcy then the bidder is willing to pay an amount X s.t.

$$c_i + (1+r)(X-c_i) \le H\lambda_i + L(1-\lambda_i),$$

or

$$X \le \frac{H\lambda_i + L(1 - \lambda_i) - c_i}{1 + r} + c_i.$$

Since the bidder's willingness to pay still varies with  $c_i$  the bidder willing to pay the most may not be the highest type,  $\lambda_i$ .

<sup>22</sup>Our model is more general in allowing for differences in valuation and cash and in considering a richer class of securities.

<sup>23</sup>Looking at the bid function, Equation (49), it would seem that  $m'(\cdot)$  could be negative for large  $\lambda_i$ . However, this bid function is derived using  $\frac{db}{d\hat{\theta}} > 0$ . Therefore, since  $\hat{\theta}_i = m(\lambda_i), m'(\cdot)$  must be positive.

<sup>24</sup>If the lender does not know  $c_i$  then the following proof will entail expectations over  $c_i$ , but the end result will not change.

<sup>25</sup>This does not mean that  $m_i(\lambda_i) = \lambda_i H + (1 - \lambda_i)L$  is not a valid function in determining the efficient rate. It is. However, the rate in such a case is not the competitive interest rate.

<sup>26</sup>It should be noted that the assumption that the financial market can only contract on the type and cash is important here. If the market could contract on the type, bid and cash, then competitive debt could yield efficiency and implement the cash auction (See Section II.2.4). However, the informational requirements are very strong. In contrast, we can implement efficiency and competitive equilibrium with equity while only contracting on the type.

<sup>27</sup>Also,  $1 + r(b_i, c_i) < 1$  is not economically reasonable. Therefore,

$$M_D(b^{-1}(b_i)) \le H - \frac{c_i}{b^{-1}(b_i)} + c_i - \int_{\underline{\lambda}}^{b^{-1}(b_i)} \frac{c_i F^{n-1}(x)}{x^2 F^{n-1}(b^{-1}(b_i))} dx.$$

 $^{28}\mathrm{Let}$ 

$$q(\lambda_i) = \frac{1}{\lambda_i} + \int_{\underline{\lambda}}^{\lambda_i} \frac{F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i)} dx$$
$$1 + r(\lambda_i, c_i) = z(H - c_i q(\lambda_i)).$$

Therefore, the ODE can be rewritten

$$z^{2}(H - c_{i}q(\lambda_{i})) = rac{\partial z(H - c_{i}q(\lambda_{i}))}{\partial c_{i}}$$

<sup>29</sup>The derivative of Equation (78) with respect to  $b_i$  is negative. Therefore, higher bidders get better interest rates.

<sup>30</sup>Corollary 6 shows that the bidder's expected payment decreases if their cash increases. Therefore, cash is valuable and bidders will not pay pre-auction dividends. Instead, bidders may attempt to build up cash before an auction.

<sup>31</sup>Also,  $\phi(b_i, c_i) > 1$  is not economically reasonable. Therefore,

$$M_E(b^{-1}(b_i)) \ge 1 - \frac{c_i}{Hb^{-1}(b_i) + (1 - b^{-1}(b_i))L} + c_i - \int_{\underline{\lambda}}^{b^{-1}(b_i)} \frac{c_i(H - L)F^{n-1}(x)}{(Hx + (1 - x)L)^2F^{n-1}(b^{-1}(b_i))} dx.$$

<sup>32</sup>Engelbrecht-Wiggans and Kahn (1991) and Rothkopf et al (1990) show that auction outcomes can have affects outside the auction. Katzman and Rhodes-Kropf (2000) generalize how bid information can affect secondary games such as obtaining financing.

<sup>33</sup>The bidder clearly has no incentive to lie about or reduce  $c_i$ , since his payment just goes up by the amount he reduces  $c_i$ . And the bidder cannot increase  $c_i$  since he is cash constrained.

 $^{34}$ It should be noted that this requires the lender to be able to determine and contract on  $c_i$ .

Figure 1 Iso-Bid functions with interest rate r.



 $Figure \ 2$  Iso-Bid functions with  $\lambda_i = 1/(1+r_i)$  for each bidder.



 $Figure \ 3$  Iso-Bid functions with  $\lambda_i < 1/(1+r_i)$  for each bidder.





Iso-Bid functions with some interest rates above  $1/\lambda_i$  and some below.





## Figure 6

Iso-bid functions with endogenous dividends and interest rate r.



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