Downside Consumption Risk and Expected Returns

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I derive and test empirically the implications of a consumption asset pricing model based on a first-order risk averse utility which allows for distinct aversion to downturns in consumption. The model nests the standard expected-utility-based CCAPM as a special case and is estimated using Fama-French portfolios. In the empirical specification the model is stated in a two factor form with both factors depending only on consumption growth. The model performs considerably better than the standard CCAPM and, consistent with recent studies of long-horizon consumption risk, the best performance is achieved at horizons of 4 to 8 quarters. The estimates imply that the left tail outcomes are over-weighted relative to their objective probabilities while the right tail is relatively underweighted, i.e. economic agents are particularly concerned about downturns in consumption growth. As a result, downside (or recession) risk is reflected in asset risk premia. Specifically, value and small stocks have a larger exposure to downside consumption risk than do growth and big stocks and this exposure contributes to value and size risk premia.

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Abstract

I derive and test empirically the implications of a consumption asset pricing model based on a first-order risk averse utility which allows for distinct aversion to downturns in consumption. The model nests the standard expected-utility-based CCAPM as a special case and is estimated using Fama-French portfolios. In the empirical specification the model is stated in a two factor form with both factors depending only on consumption growth. The model performs considerably better than the standard CCAPM and, consistent with recent studies of long-horizon consumption risk, the best performance is achieved at horizons of 4 to 8 quarters. The estimates imply that the left tail outcomes are over-weighted relative to their objective probabilities while the right tail is relatively underweighted, i.e. economic agents are particularly concerned about downturns in consumption growth. As a result, downside (or recession) risk is reflected in asset risk premia. Specifically, value and small stocks have a larger exposure to downside consumption risk than do growth and big stocks and this exposure contributes to value and size risk premia.

1 Introduction

The consumption capital asset pricing model (CCAPM) is one of the core models of modern finance.[1] It assumes that economic agents behave according to the expected utility theory. The expected utility (EU) model has many well known limitations and it is perhaps not surprising that poor empirical performance of the CCAPM often may be traced to preferences.[2] The EU model has been under a lot of scrutiny and a vast amount of research is accumulated developing new theories to better reflect human behavior under risk.[3] This paper integrates one of the leading models of preferences from this literature, the Rank-Dependent Expected Utility (RDEU, Quiggin (1982) and Yaari (1987)), into the standard consumption-based asset pricing framework and empirically tests the new model. The RDEU-based model performs much better than the standard CCAPM and implies that consumers are averse to downturns in consumption over and above the standard notion of risk aversion in the EU. As a result asset prices reflect a risk premium for downside consumption risk.

The main difference of RDEU from EU is that the former is using decision weights instead of objective probabilities. The weights are functions of the cumulative probabilities of ranked outcomes and may be higher or lower than the corresponding subjective probabilities.[4] Using the transformation of probabilities to weights, the RDEU allows to model "attention" to events separately from the level of utility associated with them. The decision weights provide a mechanism for risk aversion which is independent from the marginal utility of wealth.[5] This

[1]See Rubinstein (1976), Breeden (1979), Lucas (1978).

[2]Some of the early tests rejecting the model were done by Hansen and Singleton (1983). For an overview of the issues with consumption-based asset pricing see a survey by Campbell (2003). Cochrane (2001) provides a text book treatment.

[3]See Starmer (2000) for a recent survey.

[4]It is important not to confuse this with biased probability assessments or subjectively weighted utility. The RDEU decision maker is unbiased and is fully aware of the underlying subjective probability distribution. He or she chooses to direct more or less "attention" to the events depending on their rank. Because weights here depend on the event's rank they may change through the actions of the decision maker, unlike subjective weights. For example, what is bad and what is good outcome depends on whether you have short or long position in a particular asset. For the most recent axiomatization of RDEU see, for example, Abdellaoui (2002).

[5]In fact, in Yaari's (1987) model marginal utility is constant (linear utility is assumed) and the risk averse behavior is generated only through the decision weights. In contrast, in the EU the only way to generate risk averse behavior is to assume a concave utility function.

turns out to be empirically important feature of the model for asset pricing implications.

I use RDEU in the otherwise standard consumption-based setting and derive joint restrictions on consumption, weighting function and returns, similar to the Euler equation in the CCAPM. The closest theoretical counterpart of this model is the model in Epstein and Zin (1991). They consider a two-state endowment economy with a recursive utility and RDEU certainty equivalent and use it to study the effects of the first-order risk aversion on asset prices.[6] I suppress the recursivity here to focus on the risk aversion stemming from the weighting function. The Euler equation is obtained using theorems from Carlier and Dana (2003) and Ai (2005) which show the differentiability of the RDEU functional with respect to continuously distributed random variables.

For estimation, the model is converted into a two-factor linear form with both factors depending only on consumption.[7] The model is estimated using GMM on the cross section of 25 Fama and French portfolio sorted on market capitalization (size) and ratios of book equity to market equity (book-to-market). I use various horizons to estimate the model and find that the best performance is achieved between 4 and 6 quarters when, depending on specification, the model explains between 70% and 50% of the cross sectional variation in expected returns. The estimated decision weighting functions imply that the outcomes with lower consumption growth are over-weighted relative to their probabilities and the outcomes with higher consumption growth are underweighted. This corresponds to what I refer in this paper as aversion to downside consumption risk: consumers “pay more attention” to events in recessions than they do in normal or in good times. This main result of the model is robust to various changes in specification and sample periods used for estimation. On the other hand, the linearity of the decision weights (as in expected utility) is almost always rejected.

The stochastic discount factor associated with the RDEU-based model exhibits a pro-

[6]They use the same structure as in Mehra and Prescott (1985). Since their model has only two states for aggregate consumption, it does not take advantage of the properties of the RDEU beyond the first order risk aversion (a kink in the indifference curves around riskless allocations).

[7]Strictly speaking the model is always nonlinear due to decision weighting function. However, for a given parameterization of the weighting function it is a linear two-factor model.

nounced counter-cyclical pattern: it is high in recessions and low in normal or high growth periods. The main difference with the CCAPM appears to be in the ability of the RDEU model to separate recession risk from the standard consumption fluctuations captured by both models. I also use the model to derive a decomposition of risk premium into standard consumption risk premium and a downside risk premium. Examining these components across the 25 Fama-French portfolios I find that value stocks and small stocks have a larger exposure to the downside risk than do growth stocks and big stocks.[8] Thus, the value and size premia are explained in part by the consumers' aversion to downside risk.

The behavior of the stochastic discount factor (SDF) in the RDEU-based model bears important similarities with external habit formation model in Campbell and Cochrane (1999). The estimated model implies that decision weight part of the SDF rises sharply during bad times acting similarly to the consumption surplus multiplier in the habit model. Habit utility gives rise to time varying risk aversion so that risk aversion is higher in recessions than in normal times. This in turn results in counter-cyclical variation of risk premia which is consistent with stylized facts. The time varying risk aversion has rather appealing economic intuition and works quite well in theoretical models, but linking empirically risk premia with consumer's risk attitude proved hard because the external habit process is unobservable. Empirical studies of conditional CAPM and CCAPM (Jagannathan and Wang (1996), Lettau and Ludvigson (2000)) confirm that time variation in the coefficients of the SDF is very important for explaining expected returns. However, these studies constitute only indirect evidence for the idea of time-varying risk aversion because the conditioning variables are not explicitly connected with risk preferences. In the context of the present paper the estimated decision weights function is theoretically tied to risk aversion. The RDEU-based model therefore provides a direct connection between the dynamics of asset risk premia and

[8] The readers unfamiliar with this strand of finance literature may require a clarification of the terminology originating from the sorting procedure introduced by Fama and French (1993). Value stocks are the ones with higher book-to-market ratios and growth stocks have lower book-to-market ratios. Big and small refers to market capitalization. The value premium is the difference in returns on a portfolio of value stocks and a portfolio of growth stocks. Similarly size premium is the difference of returns on a portfolio of small stocks and a portfolio of large stocks.

consumer's risk attitude.

As already mentioned, the performance of the model is not uniform across the decision horizons chosen for estimation. Using horizons from 1 to 10 quarters I find that pricing errors and cross-sectional R^2's are hump-shaped with the best performance concentrated mainly between 4 and 6 quarters. The "horizon effect" is consistent with CCAPM's better performance at longer time intervals found by Jagannathan and Wang (2005) when testing standard CCAPM on non-overlapping annual data. Unlike their paper, I use all available observations with a given horizon. Similarly Parker and Julliard (2005) look at different horizons but their "best" horizons are different (10-12 quarters), because of the different specification. I must use the stochastic discount factor (SDF) which is contemporaneous with returns due to state-nonseparability of RDEU. Parker and Julliard instead use the SDF which leads returns by substituting future Euler equations into the current one.

The RDEU-based model does share a shortcoming common to almost all consumption-based models: it can not fully explain the observed level of risk premia.[9] While the RDEU-based model does better than the standard CCAPM on this dimension, it does not eliminate the problem completely. Many consumption-based models which use only non-durable consumption seem to share this fate despite providing a good fit of the cross-section of returns, for example Lettau and Ludvigson (2000), Parker Julliard (2005) and Jagannathan and Wang (2005). For the RDEU specification of utility this result is consistent with the conclusion made by Epstein and Zin (1991) from their theoretical model that the first-order risk aversion in the RDEU can not fully resolve the equity premium puzzle of Mehra and Prescott (1985).

The rest of this paper is organized as follows. Section 2 develops CCAPM with RDEU preferences. Sections 3 and 4 describe estimation methodology and data. Main estimation results are presented in section 5 and their economic intuition is discussed in section 6. Section 7 investigates robustness of results to model specification and sample choice and section 8 concludes.

[9]The only exception to this drawback is the model in Yogo (2006) which is based on a nonlinear SDF with durable and non-durable consumption.

2 CCAPM with aversion to downside risk

The generalization of CCAPM considered in the present paper is based on the Rank-Dependent Expected Utility. RDEU was originally proposed by Quiggin (1982) and independently discovered by Yaari (1987) in a special case with linear outcome utility. The central idea of the RDEU is that utility is weighted by the decision weights which are transformations of the cumulative objective probabilities of ranked events (from worst to best). The value function may over- or under-weigh the events relative to their objective probabilities based on how desirable the event is to a decision-maker. The idea of rank-dependent weights is incorporated into other models of preferences such as Cumulative Prospect Theory of Tversky and Kahneman (1991) and Recursive Utilities of Epstein and Zin (1989). Epstein and Zin (1991) integrate the RDEU-based certainty equivalent into a dynamic framework suitable for asset pricing. Their model of preferences, but without recursive feature, is used in the present paper to derive the implications of CCAPM with aversion to downside risk.

I begin by describing RDEU in a static discrete-state setting to provide some intuition. Let $i = 1, ..., N$ index possible outcomes (wealth or consumption) ordered from lowest to highest. Every outcome is assigned a decision weight w_i. The RDEU function is given by:

$$V^{RDEU} = \sum_{i=1}^{N} u(c_i)w_i,$$

where $u(\cdot)$ is the outcome utility defined on consumption c_i in state i. The decision weights w_i are constructed using a strictly increasing and differentiable function $Q(\cdot) : [0, 1] \rightarrow [0, 1]$, s.t. $Q(0) = 0$ and $Q(1) = 1$, defined on the cumulative probability of outcomes P_i as follows:

$$\begin{aligned} w_1 &= Q(P_1) \\ w_i &= Q(P_i) - Q(P_{i-1}) \text{ for } i = 2, \ldots, N-1 \\ w_N &= 1 - Q(P_{N-1}). \end{aligned}$$

Thus, the decision weight of each outcome depends both on the ranking of the outcome and on the transformation of its cumulative probability by the function Q. A special case $Q(P) = P$ coincides with expected utility. The form of $Q(\cdot)$ determines how the decision maker transforms objective probabilities into subjective decision weights. In particular, concave Q corresponds to a form of risk aversion when consumer over-weighs bad outcomes and under-weighs good outcomes. Note that this form of risk aversion does not rely on the curvature in the utility function u, in fact in Yaari's model u is linear to underscore this point.[10] The continuous-state version of the RDEU functional is given by:

$$V^{RDEU} = \int u(c) dQ(P_c),$$

where P_c is the cdf of consumption. If the outcomes have a continuous cdf then the integration can be done using the original probability P_c. Denoting $Z(P) \equiv Q'(P) \geq 0$ we can write value function as follows:

$$V^{RDEU} = \int u(c) Q'(P_c) dP_c \equiv \int u(c(w)) Z(P_c) dP_c$$

The outcomes with $Z > 1$ are weighted heavier than their objective probability and vice versa. Note that decision weights integrate to 1 so that $EZ = 1$.

Using the RDEU certainty equivalent Epstein and Zin (1991) study the effects of first-order risk aversion on asset prices. They use recursive utility which separates the effect of the intertemporal substitution from the curvature of the utility u. While this is a flexible and generally desirable feature of preferences, the empirical implementation of the recursive model requires the (unobservable) return on total wealth of the consumer. Empirical studies therefore must use a proxy for this variable, typically a stock market index. To keep the model parsimonious and focus on the role of the aversion to downside risk, I suppress the recursivity here. Note that aggregation of preferences is similar to the expected utility and requires homogeneous utility functions and complete markets.[11]

[10] This separation of risk aversion from the marginal utility of wealth is one of the main contributions of the RDEU. Concavity of Q is a sufficient condition for risk aversion under linear or concave utility u, in general risk aversion only requires $Q(P) \geq P$, see Quiggin (1993).

[11] See Epstein and Zin (1989) for the derivations of representative agent under recursive preferences with general certainty equivalent, including RDEU. Homogeneity of utilities is not required for the existence of the representative agent. However, similarly to the expected utility, in order for the representative agent preferences to have the same structure as the individual preferences, it is necessary to assume homogeneous preferences, see Chapman and Polkovnichenko (2006) for an analysis of static economies with RDEU agents.

Consider an infinitely-lived representative consumer who receives endowment e_t and has access to N securities with returns vector R_t. Denote w_t the value of the securities held by the consumer at the beginning of period t. Every period the consumer chooses consumption $c_t > 0$ and the value of portfolio holdings $\theta_t \in R^N$ subject to the budget constraints:

$$c_t + \theta_t' \mathbf{1} \leq e_t + w_t, \text{ and } \quad w_{t+1} = \theta_t' R_{t+1}$$

The value function of the consumer is given by:

$$V_t^{RDEU} = \max_{c_t, \theta_t} \left\{ u(c_t) + \delta E_t \{ V_{t+1}^{RDEU} (c_{t+1}, \theta_t) \, Z(P_{V_{t+1}^{RDEU}}) \} \right\}$$

Assuming that value function V_t is monotone in optimal consumption, the ranking of possible outcomes may be equivalently done by observing consumption choice:

$$V_t^{RDEU} = \max_{c_t, \theta_t} \left\{ u(c_t) + \delta E_t \{ V_{t+1}^{RDEU} (c_{t+1}, \theta_t) \, Z(P_{c_{t+1}}) \} \right\} \tag{1}$$

To obtain the first order optimality conditions for the value function in (1) it is necessary to differentiate RDEU functional with respect to continuous random variables. Two recent papers, Carlier and Dana (2003) and Ai (2005), provide the solution. Consider adding a random variable αR to consumption c. Assuming c and R are random variables with continuous densities, the derivative of the RDEU value functional is given by:

$$\frac{\partial V^{RDEU}}{\partial \alpha} (c + \alpha R) \left|_{\alpha=0} \right. = \int u'(c) R Z(P_c) dP_c$$

To obtain the Euler equation, differentiate (1) with respect to $\theta_{i,t}$, take unconditional expectation and note that ranking on consumption level coincides with ranking on consumption growth:

$$\delta E \left\{ \frac{u'(c_{t+1})}{u'(c_t)} Z(P_{c_{t+1}/c_t}) R_{i,t+1} \right\} = 1,$$

where P_{c_{t+1}/c_t} is the cdf of consumption growth. We can rewrite the Euler equation for an arbitrary excess return $R_t^e = R_{i,t} - R_{j,t}$ as follows:

$$E \left\{ \frac{u'(c_{t+1})}{u'(c_t)} Z(P_{c_{t+1}/c_t}) R_{t+1}^e \right\} = 0 \tag{2}$$

To estimate the model, more structure has to be imposed on the functions Q and u. I assume that $Q(P) = P^{\phi}$ and therefore $Z(P) = Q'(P) = \phi P^{\phi-1}$. This allows for over-weighting of outcomes either in the left tail ($\phi < 1$) or in the right tail ($\phi > 1$) of the distribution. The special case $\phi = 1$ coincides with the standard CCAPM. Therefore CCAPM is nested in a general model specification and its restriction can be formally tested. The case $\phi < 1$ corresponds to "aversion to downside risk", i.e. this is risk aversion beyond that captured by the declining marginal utility. I also assume that u' is homogeneous and log-linearize the Euler equation in consumption growth:

$$E\left\{\left(b_0 + b_1 \log\left(\frac{c_{t+1}}{c_t}\right)\right) Z(P_{c_{t+1}/c_t}) R^e_{t+1}\right\} = 0 \tag{3}$$

The linearization is preferred for the estimation because it does not impose much structure on utility and makes the restricted specification with $\phi = 1$ comparable to the literature on linear factor models. The equation (3) is the basis for empirical investigation. Note that for a given parameter ϕ the model is linear with two factors Z and $Z \times \log\left(\frac{c_{t+1}}{c_t}\right)$ which collapses into a single factor model for $\phi = 1$. This simplifies the estimation since non-linearity is confined to the decision weights function.

3 Empirical method

To estimate the model in (3) I use Generalized Method of Moments (GMM). As the test assets I use 25 Fama-French portfolios sorted on size and book-to-market ratios. The model is estimated using different frequencies, denoted as h. Recent work by Parker and Julliard (2005) and Jagannathan and Wang (2005) suggests that CCAPM performance is better at longer horizons because consumption may be responding slowly to returns. Motivated by their evidence I experiment with the time period over which consumer makes decisions. In my specification the returns and the SDF are always contemporaneous because the general RDEU model is not state-separable.[12]

[12] Parker and Julliard (2005) use a consumption-based SDF which leads returns. This is possible because we can substitute future consumption growth into the SDF under the expected utility assumption. In RDEU this substitution is not possible.

When the planning horizon h is longer than the data intervals we have to make a choice of how to compute returns and consumption growth. I use two approaches to make sure the estimation is robust to this choice. First is to use the "averaging" method. Second is to use growth and return from the last quarter of the current planning period to the last quarter of the next planning period.[13] In the averaging method I compute average consumption over the planning interval and compute the growth of this average as follows:

$$\frac{C^h_{t+h}}{C^h_t} = \frac{\frac{1}{h}\sum_{k=1}^{h} c_{t+k}}{\frac{1}{h}\sum_{k=0}^{h-1} c_{t-k}}$$

The returns are first converted from monthly to quarterly to match consumption and then the average returns are computed as follows:

$$\begin{aligned} R^h_{t+h} &= \frac{1}{h}\sum_{k=1}^{h} R_{t-h+k,t+k}\ , \ \text{ where} \\ R_{t-h+k,t+k} &= (1+R_{t-h+k,t-h+k+1}) \times \ldots \times (1+R_{t+k-1,t+k}) \end{aligned}$$

In the alternative without averaging I use last-to-last quarter consumption growth and returns as follows:

$$\begin{aligned} \frac{C^h_{t+h}}{C^h_t} &= \frac{c_{t+h}}{c_t} \\ R^h_{t+h} &= (1+R_{t,t+1}) \times \ldots \times (1+R_{t+h-1,t+h}) \end{aligned}$$

The excess returns vector (25×1) for a horizon h is given by $\mathbf{R}^{e,h}_{t+h} = \mathbf{R}^h_{t+h} - R^{f,h}_{t+h}\mathbf{1}_{25\times 1}$ where $\mathbf{R}^h_{t+h}$ and $R^{f,h}_{t+h}$ are respectively the returns of Fama-French portfolios and the t-bill rate computed by applying the above formulae. The moment function g is defined as follows:

$$\begin{aligned} g^h(b_0, b_1, \alpha, \phi) &= \begin{bmatrix} m^h_{t+h}(b_0, b_1, \phi)\mathbf{R}^{e,h}_{t+h} - \alpha \mathbf{1}_{25\times 1} \\ m^h_{t+h}(b_0, b_1, \phi) - 1 \end{bmatrix}, \ \text{ where} \\ m^h_{t+h} &= m^{0,h}_{t+h} \times Z_{t+h} = m^{0,h}_{t+h} \times Q'(P_{C^h_{t+h}/C^h_t}, \phi) \end{aligned}$$

[13]This is simply a matter of convention as long as returns and consumption growth are appropriately aligned. I can also use any other quarter, say, middle or first in each period. This does not affect the results in any way since all available observations with horizon h are used. Also I leave enough data at the end of the sample so that returns and consumption up to horizon h are available and the last time period is the same (2002Q4) across all samples with different h.

$$
\begin{aligned}
m^{0,h}_{t+h} &= b_0 + b_1 \log\left(\frac{C^h_{t+h}}{C^h_t}\right) \\
P_{C^h_{t+h}/C^h_t} &= \frac{1}{T}\sum_{k=1}^{T} I\left(\frac{C^h_{k+h}}{C^h_k} \leq \frac{C^h_{t+h}}{C^h_t}\right) \ , \quad Q(P) = P^{\phi},
\end{aligned}
$$

and where $I(\cdot)$ is the indicator function.[14] Note that the second equation in g normalizes the mean of m^h_{t+h} to 1 because the mean of the SDF is not identified when using excess returns (see Cochrane (2001)). It is preferable to normalize m rather than the coefficient b_0 because for various Z's the mean of the SDF may change making pricing errors incomparable across the models.[15] Note that g is linear in the parameters b_0, b_1 and α, and nonlinear in ϕ. The parameter α in the first equation allows for a non-zero intercept in the characteristic line. It evaluates how well the model fits the level of excess returns separately from the fit of the variation in risk premia. The moment condition used to identify the model is:

$$
Eg^h(b_0, b_1, \alpha, \phi) = 0
$$

To estimate the model I minimize the distance function which is the square root of a weighted average mean squared error:

$$
d^h(b_0, b_1, \alpha, \phi) = \sqrt{Eg^h(b_0, b_1, \alpha, \phi)'\mathbf{W}Eg^h(b_0, b_1, \alpha, \phi)},
$$

where $\mathbf{W}$ is 26×26 weighting matrix. In the first-stage GMM the weighting matrix is a unit matrix for the 25 return moments and a large number for the last (normalization) moment.[16] In the iterated (efficient) GMM the weighting matrix is equal to the inverse of the estimated variance-covariance matrix of the moment errors.

[14] This is the simplest way of estimating consumption growth cdf without imposing any distributional assumptions. A possible alternative would require an assumption about the distribution, e.g. log-normality, and estimation of the parameters of the distribution along with the asset pricing parameters of the model. While such a method would yield the parameters estimates for consumption growth distribution, it also is more complicated due to nonlinearity in the moment function. Because the model is already nonlinear in ϕ and requires search to estimate, I use a simpler method without distributional assumptions.

[15] In addition, during the estimation the values of $Q'(P)$ are computed to insure theoretical normalization $EQ' = EZ = 1$. For that I use numerical counterpart to the derivative of the decision weighting function $Q'(P_i) = (Q(P_i) - Q(P_{i-1})/(P_i - P_{i-1})$ where i and $i-1$ denote two adjacent ranked outcomes. Since $Q(0) = 0$ and $Q(1) = 1$ this insures that the multiplier function Z integrates to 1 in any sample and for any value of ϕ.

[16] The weight is large enough so that across different h the mean of SDF is 1 and the variation in the weight does not affect any of the results.

To find the optimal coefficients I confine ϕ in the interval between 0 and 1 and search over a fine grid. The search is necessary because there is no guarantee that distance function is globally convex in ϕ and, indeed, in some cases it was not. For every ϕ the system is linear in the remaining parameters and solution can be found analytically (see Cochrane (2001)). There are economic reasons to confine ϕ to be below 1.[17] The values above 1 may imply risk-seeking behavior so that the total SDF m may be increasing in consumption growth when there is not "enough" concavity in the utility of outcomes u (when the coefficient b_1 in the linearized version is not sufficiently negative). On the other hand, $\phi \leq 1$ is a sufficient condition for risk aversion (Quiggin 1993) provided u is non-convex. Since risky assets are typically observed to carry a risk premium, it is reasonable to assume that risk aversion is a representative behavior under "normal" circumstances. Despite this restriction the estimates of almost all specifications fall in the interval strictly between 0 and 1 with a few exceptions in the efficient GMM estimation when the constraint at 1 is binding.

4 Data

The consumption is real quarterly series obtained from NIPA (Table 7.1) from the Bureau of Economic Analysis web site (www.bea.gov). I combine non-durables and services consumption in one series running from 1947Q1 to 2005Q4. I also compute a weighted average of the corresponding deflators from NIPA (Table 2.3.4) to be used for adjusting returns to real. Returns for 25 Fama-French portfolios and t-bill returns are obtained from Kenneth French data library on his web page at Dartmouth College.[18] The returns are for portfolios of stocks sorted into five quintiles by the value of market equity and by the value of book-to-market equity ratios. The returns are converted from monthly to quarterly to match consumption frequency.

Throughout the paper I use the numbering of portfolios using their quintiles on size and

[17] On the lower bound I use a small number since putting actual 0 forestalls the estimation because in this case Z is degenerate. In no case but one this constraint was binding (in the first stage estimation for $h = 2$ and the last-to-last quarter method).

[18] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

book-to-market ratio, for example S2B3 corresponds to second quintile on size and third quintile on book-to-market ratio. I also follow conventional terminology when referring to quintiles. Stocks in quintile B1 are *growth* stocks and those in quintile B5 are *value* stocks. Stocks in quintile S1 are *small* stocks and those in quintile S5 are *big* stocks. Returns are adjusted to real by using appropriate price deflators from NIPA. For estimation the samples of returns and consumption were cut-off at 2002Q4 so that the last observation is in the same quarter regardless of the horizon h.

5 Estimation results

One of the traditional ways to evaluate a linear asset pricing model is to use the R^2 in the cross-sectional regression of average realized returns vs. predicted expected returns based on covariances with factors. We can easily compute such a measure here from the model's moment conditions as follows (omitting subscripts for simplicity):

$$\begin{aligned} E m_0 Z R^e &= \alpha \\ E\{m_0 Z\} E R^e + E\{(m_0 Z - E\{m_0 Z\}) R^e\} &= \alpha \\ E R^e &= \alpha - E\{(m_0 Z - 1) R^e\} \end{aligned}$$

where the last equation obtains because of the normalization. We can use the estimated excess returns from the last equation for portfolios $i = 1, \ldots, 25$ and compute the R^2 for the cross-sectional regression as follows:

$$R_h^2 = 1 - \frac{\sigma^2(E\hat{R}_i^{e,h} - ER_i^{e,h})}{\sigma^2(ER_i^{e,h})}$$

where $E\hat{R}_i^{e,h}$ and $ER_i^{e,h}$ are, respectively, predicted and realized average excess returns on portfolio i. I report these R^2's along with the estimated coefficients of the model.

In this section I focus on the results using a fixed weighting matrix which is a unit matrix on the first 25 portfolio moments. There are two main reasons to focus on the estimation using fixed matrix rather than iterated GMM. First of all, the unit matrix evaluates the model on its ability to explain value and size premia which are economically interesting characteristics of the Fama-French portfolios. Alternative weighting matrices, while may be efficient from the statistical point of view, do not convey anything about the economic intuition underlying the results (Lettau and Ludvigson (2000)). Another reason is the econometric problem of small sample because the data in this study is quarterly and the number of moment conditions is relatively large compared to the time series dimension. However, for robustness check, all specifications of the model were estimated with efficient (iterated) GMM procedure until convergence. For brevity, these results are not reported in this section. The results for iterated procedure are reported for several specifications later in the paper in the robustness analysis.

Table 1 provides the first-stage GMM estimates of the model for horizons from 1 to 10 quarters for the "average" returns and consumption method. Table 2 reports the results when consumption and returns are computed from last-to-last quarter in the period h. Columns (1-6) provide results for the unrestricted model and columns (7-11) show the restricted case $\phi = 1$ (CCAPM). The last column (12) reports $\chi^2(1)$ statistic for a test that CCAPM is true, $\phi = 1$. Under the estimates, in parenthesis, the tables report standard errors and, in square brackets, the p-values of the distance tests $d^h = 0$ and $\chi^2(1)$ for the test $\phi = 1$. Standard errors are computed using Newey-West estimator for moment errors variance-covariance matrix with h+2 lags.[19] Distance test p-values for fixed weighting matrix are computed using the method in Jagannathan and Wang (1996).[20]

Examining the cross-sectional R^2's in table 2 we see that they initially increase with horizon h and then decline. This is consistent with the point emphasized by Jagannathan and Wang (2005) and Parker and Julliard (2005) that consumption-based model performs better at longer horizons, perhaps because consumption does not immediately adjust to returns. For all horizons above 1 quarter we see that the RDEU-based model performs significantly better than the CCAPM: it has higher R^2's and the distances (pricing errors) are lower. The R^2 peaks at 4 quarters at 69% for the model with averaging method and 62% for the end-to-end method.

[19]For lags from h to $2h$ the results are very similar to those reported.
[20]Parker and Julliard (2005) prove and adopt this method for non-linear models.

The estimates of ϕ are below 1 for all specifications with $h > 1$ and they are statistically different from 1 as indicated by the χ^2 tests in column 12. These estimates indicate that decision weighting function is concave and emphasizes outcomes in the left tail of the consumption growth distribution. Consumers therefore are averse to "bad" events, or recessions, and are paying more attention to these events. This feature of the utility corresponds to a form of risk aversion independent of the curvature in the utility. It also gives rise to an interesting time series behavior of the SDF which will be discussed later.

The coefficients b_1 are almost always negative and this sign is consistent with declining marginal utility of u.[21] Interestingly, the absolute value of these coefficients, which are linked to the curvature (second derivative) of u, is lower in the RDEU-based model than in the CCAPM. This is because the decision weights function Q provides additional channel for risk aversion and the curvature in the outcome utility does not have to be as strong. Despite relatively high R^2's the models have statistically significant pricing errors and the p-values of the distance tests are small in almost all specifications. The intercepts α are sizable and this suggests that the risk premium puzzle persists: neither RDEU-based model nor CCAPM can fully explain the levels of risk premia on stocks relative to T-bills. For RDEU this is consistent with the conclusion in Epstein and Zin (1991) that the first order risk aversion in the RDEU can only partially resolve the risk premium puzzle.

Table 3 shows pricing errors for 25 Fama-French portfolios for RDEU-based and standard CCAPM for the horizon of 4 quarters using estimates from Table 1 (for average returns and consumption method). The table also reports the averages of absolute pricing errors within quintiles. The errors are generally lower for the RDEU model than for CCAPM. The improvement is especially considerable for small growth portfolios B1S1, B1S2 and B1S3. Pricing value stocks in quintile 5 is also better under RDEU except for large value stocks in B5S5. Figure 1 plots the realized average returns against predicted returns for the two models. As evident from the figure and indicated by the R^2, the RDEU-based model makes more consistent predictions about excess returns than CCAPM. The figure also shows

[21] The high standard errors indicate that value function is relatively flat around the estimates. Formal tests (not reported) reject $b_1 = 0$ in almost all specifications.

considerable improvement in pricing smaller growth stocks (the two portfolios with the lowest average realized returns).

The results presented so far indicate that RDEU-based CCAPM performs considerably better than the standard CCAPM. The pricing errors, either measured by the cross-sectional R^2 or by the value of Hansen-Jagannathan distance are lower for the RDEU model. The estimates indicate that it is important to allow for the risk aversion to downside risk which goes beyond the standard mechanism of declining marginal utility in the expected utility. In the rest of the paper I investigate the economic intuition behind the success of the RDEU-based model and also demonstrate the robustness of main findings.

6 Exploring the results: Downside risk

In this section I explore the economic intuition behind the results for the RDEU-based model and consider time variation of the associated SDF and its implications for cross-sectional risk premia. Consider the stochastic discount factor in the RDEU-based model:

$$m = \frac{u'(c_{t+1})}{u'(c_t)} Z\left(P\left(\frac{c_{t+1}}{c_t}\right)\right) = m_0 Z$$

This SDF is a product of the standard consumption-based SDF m_0 from the CCAPM and a multiplier Z which depends on the cdf of consumption growth. A successful candidate SDF must be sufficiently volatile, be sufficiently negatively correlated with stock returns and must exhibit time-varying risk aversion provided that we accept that consumption risk does not exhibit substantial predictable variation over time. The model of Campbell and Cochrane (1999) is perhaps the most parsimonious theory that satisfies these requirements via the mechanism of external habit. It is useful to draw parallels to their model.

In Campbell and Cochrane (1999) the SDF for the power utility function is determined as a function of consumption and external habit process x_t:

$$m = \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} \left(\frac{s_{t+1}}{s_t}\right)^{-\gamma} = m_0 \times \left(\frac{s_{t+1}}{s_t}\right)^{-\gamma}, \quad \text{where} \quad s_t = \frac{c_t - x_t}{c_t}$$

Note that similar to the RDEU case, the habit SDF is also a product of the standard consumption-based SDF m_0 and another function. All the action in the external habit model comes from the variation in the consumption surplus ratio s_t. First of all, the multiplier function increases the volatility of the SDF. More importantly however, it helps to generate the right time series properties for the SDF. In particular, the surplus ratio falls during recessions because habit stock responds with lag to consumption changes. As a result, the SDF increases in recessions well beyond the increase implied in the marginal utility m_0. This provides the mechanism for cyclical variation in risk premia. The intuition provided by the habit model is elegant and consistent with the behavior of asset prices, but it is not testable directly because the habit process x_t is unobservable.

Consider now what happens in the RDEU when consumption falls in recessions. The cumulative probability of low consumption realizations is small, i.e. when ranked relative to other possible outcomes these are "bad" events. Rank-dependency in the utility function allows for such events to be "singled out" by the consumers through the weighting transformation. The function Z acts as a multiplier on the probabilities of the events: it is high where consumers pay more attention and low where they pay less attention to their utility. The estimates of the model imply that the transformation is such that "bad" events in the left tail receive more weight at the expense of the "good" events in the right tail. Therefore, the estimates imply that the multiplier Z in the SDF must rise during recessions and decline during "normal" or "good" times. This dynamics is akin to the habit model just reviewed but with an advantage that all components of the model depend on observable consumption.

Figure 2 shows the time series behavior of the SDF for the RDEU (solid line) and CCAPM (dashed line) using estimates of both models for 4 quarters horizon from table 1.[22] Note that both the SDF and the multiplier function Z on the lower panel have a pronounced business-cycle pattern. They increase considerably during recessions and then decline and remain low during the periods of expansions with average or high consumption growth. The comparison with the corresponding SDF from CCAPM (dashed line) indicates that the main differences occur during recessions. Therefore the key to better pricing exhibited by the RDEU-based

[22]The graphs are similar across various specifications of the model estimated in tables 1 and 2. The SDFs for the models using last-to-last quarter method are somewhat more volatile but exhibit the same cyclical patterns.

model is that it can distinguish bad times in a more refined way than does the standard CCAPM. The downside consumption risk is important to consumers and it is reflected in asset returns.

We can further explore the economic intuition of the downside risk now relating it to the cross-section of returns. Consider again the pricing equation implied by the RDEU model:

$$
\begin{aligned}
E\left\{\frac{u'(c_{t+1})}{u'(c_t)} Z\left(P\left(\frac{c_{t+1}}{c_t}\right)\right) R^e_{t+1}\right\} &= 0 \quad \text{, or supressing time subscripts:} \\
E\left\{m_0 Z R^e\right\} &= 0
\end{aligned}
$$

Denote $\mu_0 = E m_0$ and recall that $EZ \equiv 1$. We can write a decomposition of excess returns starting from the above pricing equation as follows:

$$
\begin{aligned}
0 &= E\left\{m_0 Z R^e\right\} \\
0 &= E\left\{((m_0 - \mu_0) + \mu_0)((Z-1)+1)R^e\right\} \\
0 &= \mu_0 E R^e + E\left\{(m_0 - \mu_0)R^e\right\} + \mu_0 E\left\{(Z-1)R^e\right\} + E\left\{(m_0 - \mu_0)(Z-1)R^e\right\}
\end{aligned}
$$

The last equation implies the following decomposition of excess return:

$$
ER^e = \underbrace{-(1/\mu_0)E\left\{(m_0 - \mu_0)R^e\right\}}_{\text{Consumption risk}} \; \underbrace{-E\left\{(Z-1)R^e\right\}}_{\text{Downside risk}} \; -(1/\mu_0)E\left\{(m_0 - \mu_0)(Z-1)R^e\right\}
$$

The first term is the familiar covariance of excess return with consumption-based SDF m_0. In the case of CCAPM this is the only term that determines expected returns. For CCAPM with downside risk two more terms appear. The first one is a covariance of the return with Z. Recall that Z is always positive and is higher during recessions in the estimated model. Therefore stocks that tend to have low returns in recessions (negative excess returns) will have a negative covariance with Z and a positive downside risk premium. The third term is a partially centered third moment which does not appear to have immediate economic interpretation. It captures the comovements of excess return with the product of the demeaned SDF components.

Using the estimates of the model we can compute the implied consumption risk premium and downside risk premium for 25 test portfolios. Table 4 shows the components of risk premium for the model estimated over 4 quarters horizon. Note that growth stocks typically have slightly negative downside risk component (except big companies in portfolio B1S5). On the other hand, value stocks have a more negative covariance with Z and command a higher downside risk premium. The change in average risk premium going from growth quintile (1) to value quintile (5) is roughly 4.8%. Consumption risk premium captures only a small fraction of the variation in risk across value and growth portfolios, the same change in the average consumption risk premium is only 0.4%. Consumption risk exhibits slightly more variation than downside risk within some quintiles when it comes to explaining the size premium.

The RDEU-based model emphasizes the distinction between value and growth stocks in their correlation with downside risk. Value stocks do relatively poorly in recessions and must have a higher risk premium. This intuition is consistent with previous empirical research on conditional CCAPM by Lettau and Ludvigson (2000). Unlike the conditional model however, the model with downside risk aversion explicitly links risk premia on assets with consumer aversion towards recession risk.

In summary, the successful performance of the CCAPM with aversion to downside risk has intuitive economic interpretation. The time series of the SDF of this model show strong counter-cyclical patterns similar to the external habit formation model but relying only on observable components. The risk premia on test portfolios are explicitly linked to the consumer risk attitude and the differences in returns are justified in part by their different exposure to the downside consumption risk.

7 Some robustness checks

In this section I verify that the main results are robust to the choice of portfolios and sample period. I first estimate the model on a subset of 9 portfolios from the original 25. I select 9 portfolios that represent value and size premium: B1S1, B1S3, B1S5, B3S1, B3S3, B3S5, B5S1, B5S3, B5S5. Table 5 reports the results for these portfolios using the average of the returns and consumption over the planning period.[23] Panel A shows first stage estimation with unit matrix and panel B shows the efficient estimates. From Panel A we can see that with fewer portfolios the cross-sectional R^2's became even slightly better and the distances are lower which indicates somewhat lower pricing errors for this subset of portfolios. The coefficient estimates are consistent with those in Table 1. The iterated GMM estimates are quite stable across various horizons but their R^2 are lower. This is not surprising because the weighting matrices in the iterated GMM do not necessarily emphasize the risk premium across growth and value stocks. Another issue with iterated GMM is a small sample problem of estimating variance-covariance of the moment errors when the number of moments is relatively large compared to the time series dimension (see Altonji and Segal (1996) and discussion in Lettau and Ludvigson (2000) and references therein). For this reason, the estimation reported in previous section with the unit matrix is preferred to the estimates with an estimated weighting matrix.

Another robustness exercise is to consider an alternative subsample. Several earlier studies of CAPM and conditional CCAPM have considered a shorter time period from 1963Q3 to 1999Q4 (Fama and French (1993), Lettau and Ludvigson (2000) and Parker Julliard (2005)). This is a much shorter sample than is used in the present paper and it does not include the returns from the bust of the "dot com" bubble. Table 6 shows the results of this estimation. Due to shorter sample the R^2 and distances deteriorate somewhat. This is particularly apparent in the iterated GMM where the weighting matrix has to be estimated and there are only 147 observations available for 25 moment conditions. The results from the iterated GMM estimation are therefore the least reliable here. The first stage estimates appear to be consistent with previous results.

Several other robustness checks were attempted and results were consistent with the original specification. I considered alternative Fama-French portfolios with 3 groups on book-to-market and 2 groups on size, the above two robustness checks were also performed

[23]The results for last-to-last quarter returns and consumption are similar (not reported) and available upon request.

using last-to-last quarter computation of returns and consumption growth. I conclude that overall the aversion to downside risk appears to be a very robust result found across different specifications and samples.

8 Conclusions

Aversion to bad events is a natural response of human choice in risky situations. The standard expected utility model does not separate risk aversion from satiation in the absence of risk because concavity of the utility function is the only feature which implies risk aversion. The RDEU model disentangles these distinct aspects of behavior and it turns out to be important in modeling risk premia of assets. The consumption asset pricing model based on the RDEU performs considerably better than the standard CCAPM. The model implies that aversion to consumption downturns, modeled separately from the declining marginal utility, represents an empirically relevant feature.

The success of the model is important for two reasons. First the model provides a directly observable and intuitive economic link between the consumer risk attitude and the variation in risk premia over time and in the cross-section. Second, the results demonstrate that a non-expected utility model like RDEU may be successfully applied in an empirical study in finance outside of experimental laboratory settings. This will hopefully encourage more work on such applications.

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Table 1: GMM estimates of the model for various planning horizons. Consumption and returns are averaged over the planning period. The weighting matrix is a unit matrix for the first 25 moments and a large number for the moment normalizing the mean of the SDF to 1.

h	*CCAPM with downside risk* $(0 < \phi \leq 1)$						*CCAPM* $(\phi = 1)$					$\chi^2(1)$ test
	R^2	b_0	b_1	α	ϕ	Dist.	R^2	b_0	b_1	α	Dist.	$\phi = 1$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	0.01	1.14	-27.3	0.021	1.00	0.031	0.01	1.14	-27.3	0.021	0.031	0.00
		(1.07)	(182.6)	(0.006)	(0.96)	[0.000]		(0.37)	(71.0)	(0.006)	[0.000]	[1.00]
2	0.14	1.18	-28.8	0.030	0.59	0.060	0.10	1.71	-68.2	0.033	0.062	6.44
		(0.63)	(79.9)	(0.015)	(0.37)	[0.006]		(0.48)	(44.5)	(0.012)	[0.002]	[0.01]
3	0.67	0.96	5.7	0.059	0.35	0.058	0.17	2.24	-79.0	0.045	0.091	2.56
		(0.45)	(80.1)	(0.021)	(0.35)	[0.002]		(0.64)	(38.9)	(0.023)	[0.012]	[0.11]
4	0.69	1.19	-16.3	0.069	0.41	0.076	0.23	2.76	-83.7	0.059	0.120	28.18
		(0.82)	(62.7)	(0.038)	(0.35)	[0.025]		(0.80)	(35.7)	(0.033)	[0.027]	[0.00]
5	0.62	1.61	-33.4	0.088	0.50	0.109	0.26	3.25	-85.1	0.079	0.153	6.40
		(1.03)	(46.8)	(0.039)	(0.33)	[0.022]		(0.97)	(34.6)	(0.040)	[0.026]	[0.01]
6	0.54	2.20	-47.6	0.115	0.60	0.150	0.26	3.65	-83.2	0.107	0.191	24.02
		(1.11)	(36.4)	(0.039)	(0.33)	[0.019]		(1.02)	(30.2)	(0.046)	[0.050]	[0.00]
7	0.48	2.64	-52.7	0.144	0.64	0.193	0.26	4.05	-81.9	0.140	0.231	102.17
		(1.22)	(32.7)	(0.041)	(0.32)	[0.009]		(1.17)	(29.8)	(0.048)	[0.046]	[0.00]
8	0.42	3.01	-55.0	0.178	0.66	0.242	0.24	4.32	-77.8	0.177	0.276	117.84
		(1.32)	(30.2)	(0.041)	(0.32)	[0.006]		(1.21)	(27.4)	(0.049)	[0.042]	[0.00]
9	0.33	3.49	-58.4	0.217	0.70	0.303	0.22	4.50	-72.5	0.218	0.328	446.41
		(1.47)	(28.6)	(0.043)	(0.34)	[0.004]		(1.25)	(25.3)	(0.052)	[0.046]	[0.00]
10	0.28	3.77	-57.0	0.263	0.73	0.362	0.20	4.58	-66.7	0.260	0.381	52.63
		(1.52)	(26.2)	(0.043)	(0.34)	[0.002]		(1.28)	(23.5)	(0.053)	[0.037]	[0.00]

Table 2: GMM estimates of the model for various planning horizons. Consumption growth and returns are measured from end to end of the planning period. The weighting matrix is a unit matrix for the first 25 moments and a large number for the moment normalizing the mean of the SDF to 1.

h	*CCAPM with downside risk* $(0 < \phi \leq 1)$						*CCAPM* $(\phi = 1)$					$\chi^2(1)$ test
	R^2	b_0	b_1	α	ϕ	Dist.	R^2	b_0	b_1	α	Dist.	$\phi = 1$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	0.04	1.32 (0.87)	-62.1 (138.4)	0.019 (0.008)	1.00 (0.59)	0.030 [0.001]	0.04	1.32 (0.41)	-62.1 (78.0)	0.019 (0.006)	0.030 [0.001]	0.00 [1.00]
2	0.21	3.23 (6.87)	209.3 (426.8)	0.032 (0.017)	0.08 (0.17)	0.056 [0.008]	0.14	1.76 (0.47)	-73.7 (43.2)	0.032 (0.014)	0.059 [0.004]	0.20 [0.65]
3	0.30	1.53 (0.71)	-48.9 (50.8)	0.038 (0.029)	0.63 (0.27)	0.080 [0.037]	0.22	2.18 (0.60)	-75.6 (36.7)	0.042 (0.024)	0.085 [0.011]	11.65 [0.00]
4	0.62	0.97 (0.40)	3.6 (54.7)	0.073 (0.033)	0.32 (0.31)	0.080 [0.009]	0.29	2.53 (0.71)	-73.2 (32.4)	0.055 (0.034)	0.110 [0.020]	29.50 [0.00]
5	0.55	1.68 (0.63)	-38.8 (28.1)	0.080 (0.045)	0.54 (0.24)	0.114 [0.036]	0.31	2.90 (0.86)	-72.9 (31.1)	0.070 (0.044)	0.141 [0.023]	13.62 [0.00]
6	0.57	1.58 (0.78)	-30.5 (31.5)	0.065 (0.071)	0.44 (0.26)	0.138 [0.087]	0.28	3.13 (0.95)	-68.0 (28.4)	0.090 (0.049)	0.180 [0.016]	95.43 [0.00]
7	0.44	2.32 (1.10)	-49.4 (29.8)	0.065 (0.085)	0.54 (0.39)	0.188 [0.013]	0.24	3.25 (0.96)	-61.4 (24.8)	0.114 (0.052)	0.220 [0.011]	53.84 [0.00]
8	0.51	1.85 (1.21)	-32.5 (30.5)	0.090 (0.056)	0.42 (0.32)	0.208 [0.048]	0.24	3.48 (1.02)	-58.9 (23.2)	0.136 (0.056)	0.259 [0.008]	14.45 [0.00]
9	0.49	2.31 (1.04)	-40.1 (21.7)	0.115 (0.055)	0.48 (0.26)	0.246 [0.088]	0.24	3.81 (1.10)	-59.3 (22.4)	0.165 (0.061)	0.300 [0.016]	119.03 [0.00]
10	0.42	2.78 (0.96)	-44.9 (19.3)	0.156 (0.060)	0.54 (0.22)	0.300 [0.079]	0.23	4.01 (1.16)	-57.0 (21.5)	0.203 (0.062)	0.345 [0.014]	34.04 [0.00]

Table 3: Pricing errors for 25 Fama-French portfolios. Pricing errors are in percentage points computed as average realized return minus the return predicted by the model using estimates from Table 1 for the horizon of 4 quarters. Averages are computed using absolute value of pricing errors in a given quintile.

	A: RDEU-based CCAPM					
	Small	2	3	4	Big	Abs. Average
Growth	-2.54	-1.02	1.08	1.63	-1.90	1.64
2	0.24	1.22	0.36	-0.65	-0.96	0.69
3	0.58	0.27	-0.52	1.68	-0.57	0.72
4	1.86	1.78	0.76	-2.43	-1.48	1.66
Value	2.39	1.63	0.04	0.09	-3.54	1.54
Abs. Average	1.52	1.18	0.55	1.29	1.69	

	B: CCAPM					
	Small	2	3	4	Big	Abs. Average
Growth	-8.08	-4.32	-1.92	-0.51	-1.83	3.33
2	-1.40	-0.02	0.22	-0.23	-0.38	0.45
3	0.21	1.04	0.41	2.39	0.61	0.93
4	2.75	2.69	1.79	-0.22	-0.55	1.60
Value	2.89	2.69	2.21	1.08	-1.54	2.08
Abs. Average	3.06	2.15	1.31	0.89	0.98	

Table 4: Components of the risk premium. This table shows the components of excess return (in percentage points) due to exposure to downside risk and consumption risk for 25 Fama-French portfolios for the horizon of 4 quarters using estimates from Table 1.

	A: Downside Risk Premium					
	Small	2	3	4	Big	Average
Growth	-0.06	-0.36	-0.92	-0.44	2.06	0.06
2	2.60	1.06	2.20	1.37	1.23	1.69
3	2.93	3.79	3.20	2.08	2.16	2.83
4	4.12	3.31	3.57	5.30	2.99	3.86
Value	4.80	4.49	5.15	4.26	4.97	4.73
Average	2.88	2.46	2.64	2.51	2.68	

	B: Consumption Risk Premium					
	Small	2	3	4	Big	Average
Growth	1.51	0.90	0.67	0.60	0.82	0.90
2	1.39	0.84	0.92	0.54	0.46	0.83
3	1.18	1.18	0.96	0.68	0.60	0.92
4	1.24	1.00	1.05	1.28	0.90	1.09
Value	1.54	1.31	1.25	1.26	1.23	1.32
Average	1.37	1.05	0.97	0.87	0.80	

Table 5: Robustness check with a subset of portfolios. The models are estimated on 9 Fama-French portfolios (B1S1, B1S3, B1S5, B3S1, B3S3, B3S5, B5S1, B5S3, B5S5). Consumption and returns are averaged over the planning period. The weighting matrix is a unit matrix for the first 25 moments and a large number for the moment normalizing the mean of the SDF to 1.

h	CCAPM with downside risk ($0 < \phi \leq 1$)						CCAPM ($\phi = 1$)					$\chi^2(1)$ test
	R^2	b_0	b_1	α	ϕ	Dist.	R^2	b_0	b_1	α	Dist.	$\phi = 1$
A: Fixed Weights (Unit Matrix) GMM												
3	0.68	1.08	-10.8	0.051	0.38	0.040	0.08	1.91	-57.7	0.048	0.067	1.15
		(0.58)	(75.9)	(0.026)	(0.32)	[0.000]		(0.76)	(46.9)	(0.023)	[0.003]	[0.28]
4	0.70	1.19	-16.2	0.063	0.40	0.053	0.11	2.31	-62.2	0.063	0.090	12.64
		(0.82)	(63.6)	(0.040)	(0.31)	[0.002]		(0.83)	(38.3)	(0.029)	[0.002]	[0.00]
5	0.63	1.60	-33.4	0.080	0.48	0.076	0.11	2.58	-59.9	0.086	0.118	13.31
		(1.36)	(65.3)	(0.050)	(0.35)	[0.008]		(1.06)	(38.9)	(0.038)	[0.003]	[0.00]
6	0.55	2.08	-45.0	0.107	0.55	0.105	0.09	2.70	-53.5	0.117	0.149	4.21
		(1.48)	(52.3)	(0.050)	(0.33)	[0.010]		(1.22)	(37.4)	(0.046)	[0.003]	[0.04]
7	0.47	2.39	-47.5	0.134	0.57	0.137	0.07	2.78	-47.6	0.151	0.181	6.24
		(1.56)	(45.1)	(0.047)	(0.32)	[0.005]		(1.28)	(33.5)	(0.045)	[0.001]	[0.01]
B: Iterated GMM												
3	0.66	1.28	-36.7	0.017	0.41	0.533	0.07	2.12	-71.4	0.060	0.437	1.04
		(0.37)	(31.0)	(0.017)	(0.227)	[0.000]		(0.53)	(31.1)	(0.018)	[0.000]	[0.31]
4	-2.64	1.05	-14.9	-0.033	0.13	0.752	0.10	2.04	-49.2	0.083	0.462	250.53
		(0.53)	(22.0)	(0.010)	(0.112)	[0.000]		(0.51)	(22.6)	(0.020)	[0.000]	[0.00]
5	0.58	1.12	-8.5	0.038	0.35	0.547	0.04	1.82	-30.2	0.096	0.577	33.49
		(0.51)	(30.8)	(0.025)	(0.244)	[0.000]		(0.61)	(22.0)	(0.025)	[0.000]	[0.00]
6	0.51	1.31	-13.7	0.030	0.50	0.707	-0.08	2.00	-28.9	0.080	0.613	10.47
		(0.66)	(25.5)	(0.026)	(0.274)	[0.000]		(0.72)	(21.3)	(0.031)	[0.000]	[0.00]
7	0.42	0.79	9.5	0.075	0.34	0.786	-0.01	1.32	-8.2	0.115	0.717	17.83
		(0.94)	(47.3)	(0.026)	(0.361)	[0.000]		(0.74)	(19.4)	(0.032)	[0.000]	[0.00]

Table 6: Robustness check for a subsample. Data is from 1963Q3 to 1999Q4. Consumption and returns are averaged over the planning period. The weighting matrix is a unit matrix for the first 25 moments and a large number for the moment normalizing the mean of the SDF to 1.

	CCAPM with downside risk ($0 < \phi \leq 1$)						CCAPM ($\phi = 1$)					$\chi^2(1)$ test
h	R^2	b_0	b_1	α	ϕ	Dist.	R^2	b_0	b_1	α	Dist.	$\phi = 1$
A: Fixed Weights (Unit Matrix) GMM												
3	0.54	3.46	-156.7	0.045	0.86	0.066	0.50	4.09	-181.7	0.044	0.069	1.46
		(1.53)	(82.1)	(0.046)	(0.38)	[0.089]		(1.71)	(98.6)	(0.045)	[0.274]	[0.23]
4	0.54	3.99	-143.0	0.060	0.86	0.091	0.51	4.62	-160.6	0.063	0.095	1.32
		(1.75)	(74.9)	(0.067)	(0.39)	[0.143]		(2.03)	(87.6)	(0.056)	[0.287]	[0.25]
5	0.52	4.15	-122.8	0.082	0.83	0.123	0.46	4.87	-138.0	0.085	0.131	43.88
		(1.82)	(63.2)	(0.064)	(0.36)	[0.105]		(2.00)	(68.9)	(0.057)	[0.238]	[0.00]
6	0.48	4.13	-103.5	0.109	0.80	0.161	0.38	4.72	-111.4	0.112	0.176	98.48
		(2.02)	(55.2)	(0.054)	(0.38)	[0.071]		(1.79)	(51.8)	(0.050)	[0.190]	[0.00]
7	0.43	4.23	-91.9	0.138	0.79	0.205	0.33	4.67	-94.7	0.141	0.224	171.52
		(2.09)	(50.3)	(0.051)	(0.37)	[0.041]		(1.85)	(46.4)	(0.050)	[0.138]	[0.00]
B: Iterated GMM												
3	0.22	1.75	-44.2	0.015	1.00	1.700	0.21	1.76	-44.4	0.015	1.694	0.00
		(0.53)	(26.1)	(0.010)	(0.262)	[0.000]		(0.35)	(18.5)	(0.010)	[0.000]	[1.00]
4	0.34	1.08	-5.4	-0.033	0.50	2.490	0.20	1.81	-36.0	-0.030	2.051	9.81
		(0.45)	(27.2)	(0.009)	(0.255)	[0.000]		(0.43)	(16.7)	(0.011)	[0.000]	[0.00]
5	0.09	1.26	-9.9	-0.027	1.00	2.432	0.09	1.26	-9.8	-0.026	2.439	0.00
		(0.94)	(31.6)	(0.010)	(0.426)	[0.000]		(0.42)	(14.3)	(0.010)	[0.000]	[1.00]
6	-1.26	-2.24	115.1	0.081	0.69	4.073	-0.83	-11.47	303.7	-0.077	5.220	18.42
		(0.37)	(11.5)	(0.006)	(0.209)	[0.000]		(2.03)	(40.3)	(0.032)	[0.000]	[0.00]
7	-3.15	-17.46	410.7	0.057	1.00	2.562	-0.07	0.67	8.7	-0.106	10.717	0.00
		(1.61)	(27.5)	(0.021)	(0.238)	[0.000]		(0.47)	(12.9)	(0.003)	[0.000]	[1.00]

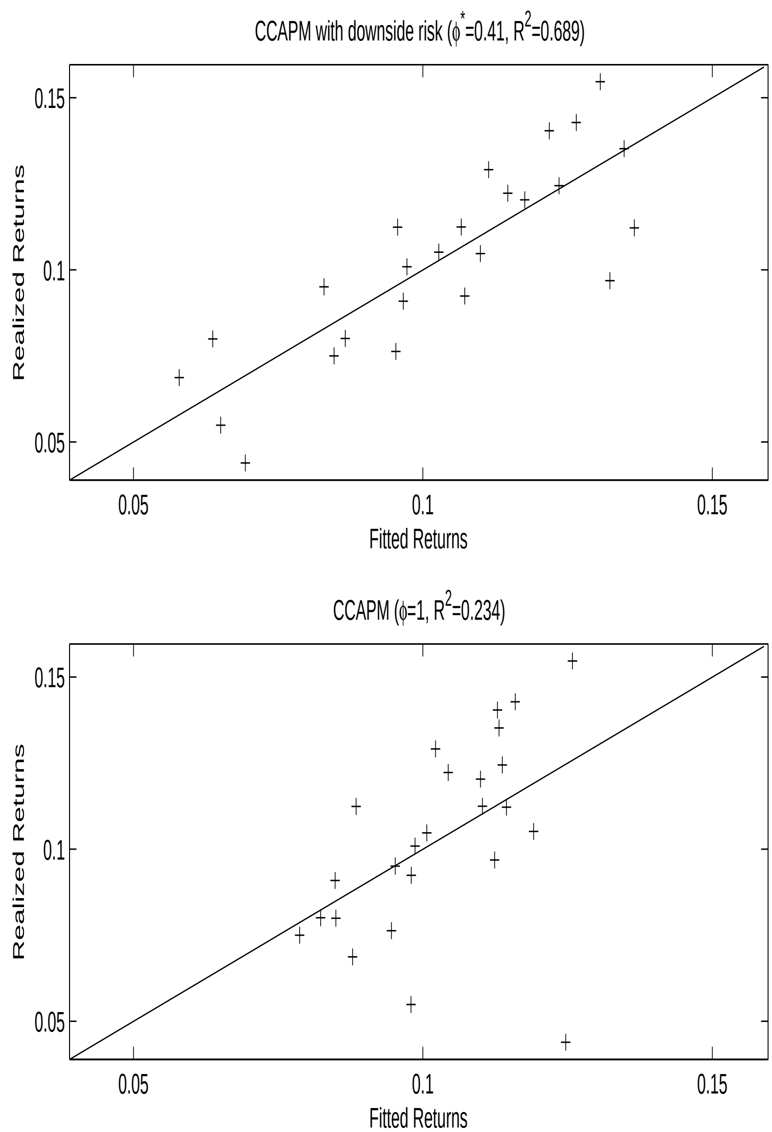

Figure 1: Fitted vs average realized returns. The predicted returns are computed using the estimates from table 1 for the horizon of 4 quarters.

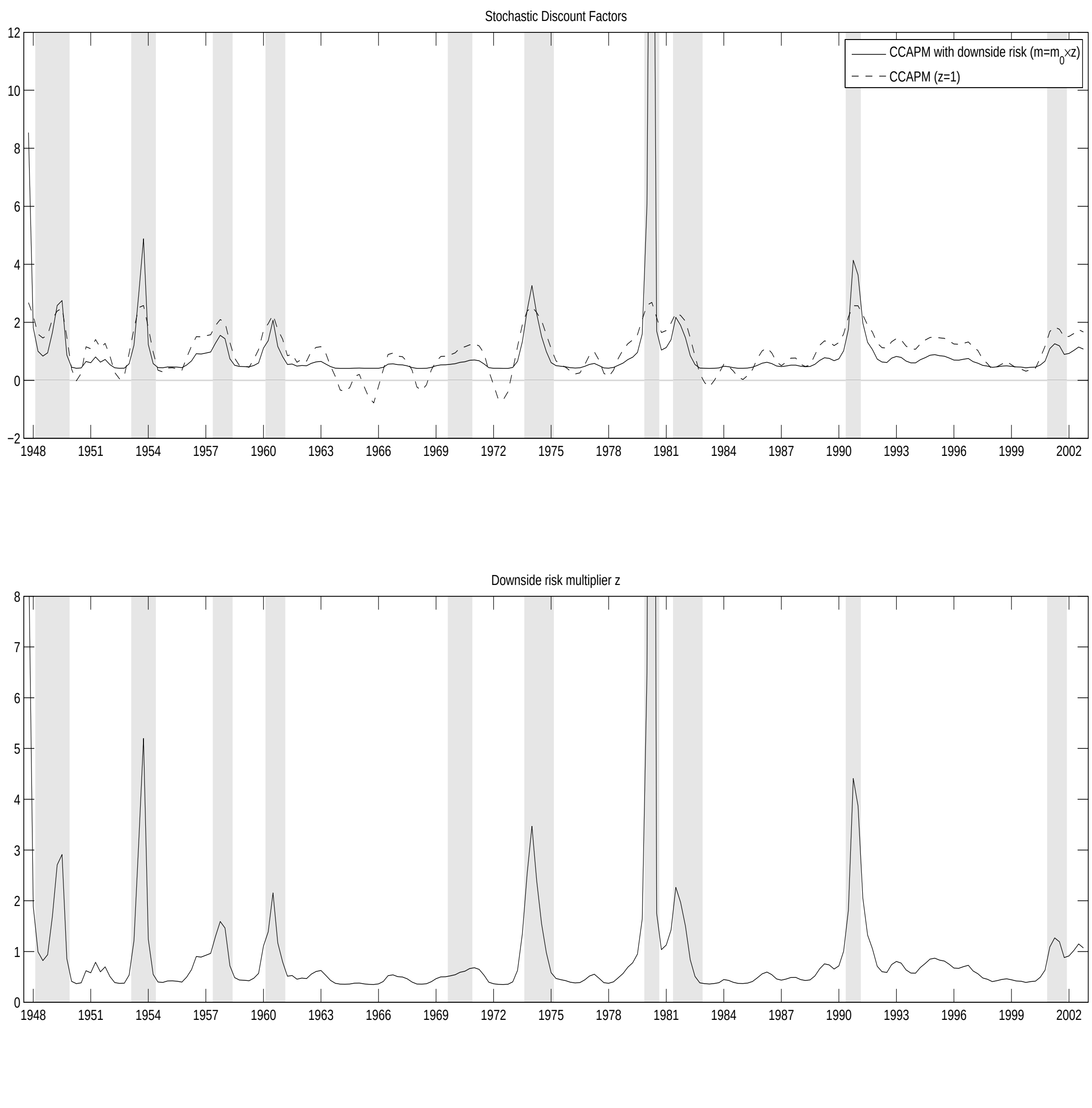

Figure 2: Stochastic discount factors. The figure shows the time series of stochastic discount factors (upper panel) and downside risk multiplier funciton Z (lower panel). Solid line on upper panel indicates SDF of the unrestricted model with downside risk and dashed line is the standard CCAPM. The SDFs are constructed using corresponding estimates from table 1 for the horizon of 4 quarters. Shaded areas indicate NBER recession quarters.