"Pricing Prices"*

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Abstract

Price quotes are a valuable commodity by themselves. This is a conundrum in the standard asset pricing framework. We study the value of access to accurate and timely prices in a market economy explicitly taking into account that in the U.S., exchanges have property rights in the price quotes they generate. The fact that typically large institutions and sophisticated individuals obtain real time price quotes motivates us to propose a simple model based on complementarity of private information on the fundamentals and information on price. We find that granting the public access to real time pricing data has benefits such as stimulating the role of stock market monitoring. Since the effect on liquidity can be negative, exchanges need to be able to charge a fee for this service. In other situations, the exchange can also benefit from free public disclosure of price quotes. We explicitly derive an equilibrium for differentially informed traders and a profit maximizing exchange. We confirm that, indeed, agents with the most precise private information will acquire real time price access. We outline several further empirical implications of our model.

Keywords: Real-time prices; Sale of Information; Property rights; Market Efficiency; Liquidity

JEL classification: G.14; G.20

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1 Introduction

The prices of financial assets are a valuable commodity in and by themselves. Sales of transaction data produced revenue of $167 million for the NYSE alone in 2004[1]. In the standard asset pricing theory (Huang-Litzenberger 1988, Cochrane 2005), this is a conundrum. On the other hand, Roll (1984) has shown that enough useful information is contained in market prices to predict the weather in Florida orange groves. Thus, access to accurate, transparent, and timely market prices has great value to both dealers (like orange juice futures traders) and producers (like orange growers who can undertake protective measures against a freeze). The value of the information contained in prices can best be measured by agent's willingness to pay for it (Grossman and Stiglitz 1980). We thus analyze the "price of prices" as an important characteristic of the financial market.

The fact that agents are willing to pay hundreds of thousands of dollars per year to obtain real time price access via Reuters or Bloomberg cannot easily be addressed within the standard asset pricing framework, where agents are typically modelled by preferences they have over lotteries, and financial markets are viewed as elaborate and complex, but nevertheless gambles. Nobody would pay for data on past or current outcomes of a (manipulation-free) gamble. We conclude that financial markets are more than a lottery, and that they produce high-quality market prices as a valuable output.

To study this issue, we explicitly take into account the U.S., financial exchanges have property rights in the price quotes they generate[2] (Mulherin et al., 1991). This was decided in a landmark 1905 Supreme Court verdict[3], in which "the court stated that it was unlikely the market for future sales was merely gambling since the prices are so important in business and farming."[4] We develop a model of strategic trading in the spirit of Kyle (1985). In our model, agents have access to information about the value of an asset. But since agents do not know the most recent price of the asset, they face a source of execution risk when trading in the markets. For instance, if an agent knows that the stock is worth $100, but does not know whether it is currently trading at $98 or $102, then the value of his information is

[1] According to the NYSE's financial statements. We expect similar magnitude of revenues for other exchanges such as the CBOT.

[2] This includes property rights in data derived from the exchange's price quotes. Certain limitations apply, as discussed below.

[3] Board of Trade of the City of Chicago v. Christie Grain and Stock Company, 198 U.S. 236 (1905)

[4] Quote taken from Mulherin et al. (1991)

greatly diminished. We solve for the unique linear equilibrium in an economy in which the exchange has property rights in market prices. Since prices are semi-strong efficient in the model, agents do not get an informational advantage over the market maker by purchasing real-time price access. The"price of prices" is positive only because real-time pricing data is a complement to the private information agents possess.

Our paper is closely related to the literature on the sale of information (Admati and Pfleiderer (1986, 1990)). In both cases, agents benefit from an improved forecast of asset mispricing $E(V_t - P_t)$. While the existing literature has focused on the acquisition of information on the fundamental value V_t, we find that agents can also improve their forecast of the anticipated price differential by observing P_t more precisely, which explains why they are willing to pay for a subscription to real-time price information. Note that the seller's incentives are different, since the exchange's primary business is facilitation of trade, not data sales. We explicitly consider this in our analysis. In the existing literature, indirect sale of information (e.g., via a mutual fund) is typically optimal (Admati and Pfleiderer (1990)). Instead, we find that exchanges sell real time price data directly. This is because *(i)* real time price data is only valuable in conjunction with private information, and thus indirect sale of information leads to zero revenue and *(ii)* truthful sale of pricing information by the exchange is incentive compatible, since the data is ex-post verifiable.

In the presence of a single informed trader, access to real time prices is valuable, but comes at the expense of lower market liquidity. This prevents the exchange from publicly disclosing its price quotes even though access to real time pricing information improves the effectiveness of financial markets as monitors (Holmstrom and Tirole, 1993). Indeed, we find that the exchange is indifferent between selling information or not. This is no longer the case when we introduce endogenous information acquisition. We find that sale of real time pricing information is beneficial because it eliminates the need to produce information that is already incorporated into prices. Furthermore, it increases the marginal utility of private information, which in turn increases the usefulness of market prices to monitor management.

When multiple informed agents compete in financial markets, they may find it in their interest to acquire real time pricing information even if, as a group, informed traders would benefit from abstaining to do so. Indeed, we show that the exchange maximizes its profits (and, simultaneously, market efficiency) by selling real time information to all informed agents[5]). Specifically, we show that the exchange always sells a signal on price information to all (or at least many) informed traders. Consistent with the availability of real time data, this signal is of high or infinite precision. Furthermore, we show conditions under which the exchange benefits from the public availability of a second signal of (considerably) lower precision. The exchange can benefit from the free disclosure of price information in two ways. Firstly, such a disclosure can level the playing field and thus improve market liquidity. Secondly, providing informed traders with more information can intensify the degree of competition among them, which in turn increases their willingness to pay for the high precision signal.

When informed agents differ along the precision of their respective private signals, we show that it is the agents with the most precise private information who have the highest marginal utility of accurate pricing information. At the same time, we prove that selling real time data to these agents has detrimental effects on market liquidity. The resulting tension leads to a challenging but interesting profit maximization problem for the exchange. The equilibrium "price of prices" can be characterized as a cost schedule when the cost increases in the precision of the current price signal. We show conditions under which an equilibrium exists and show that better informed agents indeed purchase more precise real time price data. This confirms the empirical fact that it is well informed, highly specialized traders who pay substantial amounts of money to obtain access to real time pricing data.

The "price of prices" that agents pay varies. Individual investors can receive real-time pricing data from a source such as Yahoo.com for a nominal fee (as of Dec. 2006, this fee is \$13.95 per month)[6], or they receive it for free from their broker (and indirectly pay by trading with the broker). For institutional investors who subscribe to professional data service through a company such as Reuters, they need to directly pay the exchange. In case of the NYSE, the fee is \$5,000 per month plus a \$60 fee for every display unit (as of Dec. 2006)). The latter fee is lowered by \$10 if the recipient can is willing to accept a five second delay.

We focus primarily on the timeliness of price quotes as one dimension of the economic value of prices. In addition, accurate pricing information is needed to implement advanced trading

[5]In contrast to Admati and Pfleiderer (1986, 1990), where sale to a subset of potentially informed agents is optimal. Our result is due to the exchange taking the impact of data sales on liquidity into account.

[6]As of January 2007, the NYSE is asking the SEC for permission to sell real time data to websites at \$100,000 a month.

strategies or detecting small arbitrage opportunities. Academic researchers are probably familiar with the "price of prices" from their own experience in acquiring costly data sets. Besides the dimensions previously mentioned, reliability and comprehensiveness determine the value of pricing data. Only large data sets without errors can lead to a maximum of information content being extracted from the data. It is thus not surprising that Wall Street firms invest in the maintenance and protection their own data-sets, as they are useful in their trading activities.

How to arrange the trading process in the most efficient way has been a central question in the microstructure literature. For this purpose, researchers have introduced a number of measures of market liquidity and informational efficiency. No single sufficient statistic for the quality of a market can be developed. We add the "price of prices" to the list of existing variables as a direct measure of the usefulness of information contained in market prices. The fact that the major exchanges receive a substantial portion of their total revenue from data sales indicates that, despite recent debates on market irrationality, the information content of stock prices is considerable.

In our paper, we establish the economic value of access to accurate and timely market prices. At the same time, it is striking that real time prices are so valuable, given that briefly delayed quotes are free. While we provide conditions under which the exchange benefits from the free disclosure of a noisy public signal, we view the question of an optimal delay time (typically 10 to 30 minutes, as seen in Table 1) as essentially an empirical one[7]. We believe that the value of pricing access for speculative trading is greatly diminished after a 15 minutes delay. Early studies such as such as Patell and Wolfson (1984), Jennings and Starks (1985), and Barclay and Litzenberger (1988) examine the speed of price response to various corporate announcements such as dividends, earnings, and equity offerings. They find that prices incorporate news within five to fifteen minutes, which perfectly corresponds to a high economic value of real-time prices, while a 15 minute delay will preclude significant trading profits. Kim et al. (1997) report a similar time frame for the reaction to analyst buy recommendations. Busse and Green (2002) provide some evidence that markets react to TV stock buy recommendations much quicker, often within seconds up to a minute. However,

[7]In what follows, we view the occurrence of a public news event as the origin of private information for informed agents, as in Kandel and Pearson (1995). We find it essential to explain the price of prices in a market that is efficient in the semi strong sense.

the response to sell recommendations is more gradual, "lasting fifteen minutes, perhaps due to the higher costs of short-selling." We conclude that a high price of real time quotes and a low price of quotes with a fifteen minute delay is consistent with the empirically observed speed of market reaction to news events.

This implies that private information on the fundamentals and pricing information are complementary. We propose a simple model based on information complementarity explaining the stylized facts described above. We explicitly derive an equilibrium for differentially informed traders. We show that in order to extract maximum profits, the exchange should sell price information according to a certain cost schedule with higher cost for higher precision. Our model has several further empirical implications.

This by no means implies that delayed price quotes are worthless. Indeed, they can be viewed as a public good, helping for instance in the valuation of large portfolio positions. A portfolio of liquidly traded securities in markets with good price discovery is much easier to value than, for instance, a portfolio of energy derivatives such as ENRON had[8]. Similarly, an agent engaging in optimal portfolio choice still values accurate price quotes with a short delay, as they will help him allocate funds more efficiently. In fact, the legal system has placed limitations unto the exchange's property rights in price quotes that align with this view. For instance, agents are allowed to use an exchange's settlement price as the basis for the creation of derivatives products[9].

This paper is structured as follows. We present our model in section 2. We solve for equilibrium and analyze its implications for the price of prices in section 3. We examine far more complex information allocations in section 4 and provide implications and conclusions of our analysis in section 5.

2 The Model

We consider a model of price formation in securities markets with strategic traders along the lines of Kyle (1985). However, we modify the model to explicitly incorporate the institutional fact that exchanges have the property rights to the prices they generate (Mulherin et al.

[8]The absence of transparent market prices have led people to doubt the valuations ENRON attributed to their portfolio.

[9]The New York Mercantile Exchange (NYMEX) filed and lost a lawsuit against the Intercontinental Exchange (ICE), trying to prohibit ICE from doing exactly that.

1991). Specifically, we assume that a single risky asset is traded at times $t = 1, \ldots, T$. In each time period t, the asset generates a random cash flow of δ_t, which we assume to be Normally distributed with mean zero and variance σ_δ^2. Similar to Admati and Pfleiderer (1988), the asset does not pay dividends, resulting in a "liquidation" value of the asset at the beginning of period t of

$$V_t = \sum_{s=1}^{t-1} \delta_s. \tag{1}$$

We assume that all δ_t's are jointly independent. The risk-free rate in this economy is normalized to zero.

We assume three types of risk-neutral agents in our model: Uninformed (liquidity) traders, informed traders, and competitive market makers. We assume that in each period t, a group of short-lived informed and liquidity traders are born just before markets open. They trade once when markets open at time t and realize their profits or losses. They then consume and leave the economy to make room for a new generation of traders.

The generation of noise traders collectively submit a demand of $u_t \sim N(0, \sigma_u^2)$ at time t. In addition, there is a generation of n_t agents that have access to information about the asset's cash flow δ_t prior to trading. Specifically, we assume that agent k receives a signal

$$S_{k,t} = V_{t+1} + \epsilon_{k,t} \tag{2}$$

about the value of the asset. The error terms $\epsilon_{k,t}$ are Normally distributed with mean zero and variance $\sigma_{\epsilon,k}^2$. We assume that the cross-sectional correlation of the error terms is zero. Based on their private information, agents may choose to submit buy or sell orders to an exchange, where the asset is traded.

Following Kyle (1985), we assume that prices are set by risk-neutral, competitive market makers working at the exchange, who observe the aggregate net order flow. As in and Admati and Pfleiderer (1988), the market makers observe the cash flow δ_t at the end of each trading round and set the market efficient price. We assume that the asset value at the end of period t, V_t, becomes publicly and costlessly available after m periods of trade. This assumption parallels the institutional feature that price quotes generally become public after a period of 15 or 20 minutes.

Thus, informed agents who want to trade on their private information face an execution risk. While they know the fundamental value of the company, they face uncertainty about the most recent market price. We assume that informed agent k who chooses not to acquire a costly price signal in period t, trades on a noisy lagged signal and therefore has a belief[10]

$$\hat{P}_{t,k} = V_t + \nu_{t,k}, \tag{3}$$

where the error terms $\nu_{t,k}$ are jointly independent and Normally distributed with mean zero and variance σ_ν^2.

Agents who wish to learn V_{t-1} after only $l < m$ lags of time need to pay an amount C_l to the exchange. Agents may thus update their beliefs to

$$\bar{P}_{t,k} = V_t + \eta_{t,k}. \tag{4}$$

By doing so, the agent's error variance is reduced to

$$\sigma_{\eta,k}^2 = \frac{l_k}{m}\sigma_\nu^2. \tag{5}$$

Thus, there is a very simple link between the timeliness and the accuracy of price quotes in our model, in the sense that more timely information improves its accuracy. For instance, an agent k who purchases real time access to price quotes ($l_k = 0$) observes V_{t-1} perfectly: $\sigma_{\eta,k}^2 = 0$.

Note that, we follow Admati and Pfleiderer (1988) by assuming that private information is short-lived and becomes known to market makers at the end of each period. We view our model as a "reduced form" of a more general setup of dynamic informed trades along the lines of Foster and Viswanathan (1996) and Back et al. (2000). In these models, agents learn each others' information by observing the current market price. To do so in reality, they need to obtain costly real time price access. In such a setting, the relationship between accuracy and timeliness of information will be more intricate. We choose not to follow this approach here for the risk of having technicalities distracting from the focus of the paper, namely the value of information contained in prices. Rather, the model we employ has a close resemblance to the literature on sale of information. We will provide a discussion on this issue below. Exploring the role of learning from prices in a model with more complex dynamic trading strategies is an exciting direction for future work.

[10] In other words, agents have noisy beliefs about V_t, since they only observe V_{t-m}.

3 Equilibrium and Analysis

We proceed to derive the equilibrium in our economy and analyze its general structure in this section. Since we simplified the dynamics of our analysis, equilibrium in any period $t > 1$ is independent of prior or future periods. We thus drop the time subscript (t) in our variables to reduce notational clutter.

For this purpose, we find it useful to introduce the following notation. Let

$$\Sigma = (\sigma_\delta, (\sigma_{\epsilon,i}, \sigma_{\eta,i})_{i=1,\dots,n}) \tag{6}$$

be the $(2n+1)$-dimensional vector that provides a sufficient statistic for the ex-ante information structure in period t of our economy.

We then define the functions

$$f(\Sigma) = \sum_{i=1}^{n_t} \frac{\sigma_\delta^2}{\sigma_\delta^2 + 2\sigma_{\epsilon,i}^2 + 2\sigma_{\eta,i}^2} \tag{7}$$

and

$$g(\Sigma) = \sum_{i=1}^{n} \left(\frac{\sigma_\delta^2}{\sigma_\delta^2 + 2\sigma_{\epsilon,i}^2 + 2\sigma_{\eta,i}^2} \right)^2 . \tag{8}$$

This allows us to state the equilibrium in our model as follows.

Proposition 1. *The unique linear equilibrium in our model is given by informed traders' demand of*

$$x_i = \beta_i (S_i - \bar{P}_i) \tag{9}$$

$$\beta_i = \frac{\sqrt{2}\sigma_u}{\sigma_\delta} \frac{1}{\sqrt{f(\Sigma) + g(\Sigma)}} \frac{\sigma_\delta^2}{\sigma_\delta^2 + 2\sigma_{\epsilon,i}^2 + 2\sigma_{\eta,i}^2} \tag{10}$$

and a market maker pricing rule

$$P = V + \lambda(\sum_{i=1}^{n} x_i + u) \tag{11}$$

$$\lambda = \frac{\sigma_\delta}{\sqrt{2}\sigma_u} \frac{\sqrt{f(\Sigma) + g(\Sigma)}}{1 + f(\Sigma)} \tag{12}$$

Note that the resulting equilibrium is similar to the one in Kyle (1985). The differences are due to the execution risk caused by the uncertainty of informed traders about the most recent market price, as captured by the parameter $\sigma_{\eta,i}^2$. If this is zero for all agents, our model collapses to a variant of Kyle (1985).

The simple analytical structure of the model is a main reason for setting up the model the way we have done. Alternatively, one could assume that agents do not have a personalized signal about the last traded price, V_{t-1}, but rather observe the price history up until k lags before they are born. In this case, they may all use V_{t-k-1} as their best estimate of V_{t-1}. Such a model can still be solved, and leads to very similar economic behavior. However, we abandoned this line of modelling since the equilibrium price schedule necessarily implies a two-factor structure rather than the single-factor structure here. This added analytical complexity may obfuscate the economic intuition behind our paper.

We introduce two sets of new variables $\mu_{s,i}$ and $\mu_{p,i}$ that can be viewed as scaled precision of private and price information as

$$\mu_{s,i} = \frac{\sigma_\delta}{\sigma_{\epsilon,i}}, \qquad i = 1,..,n, \tag{13}$$

and

$$\mu_{p,i} = \frac{\sigma_\delta}{\sigma_{\eta,i}}, \qquad i = 1,..,n, \tag{14}$$

respectively. Specifically, $\mu_{s,i}$ measure the ratio of ex-ante uncertainty, σ_δ to agent i's error of private information, $\sigma_{\epsilon,i}$, while $\mu_{p,i}$ does the same with respect to agent i's error of price information, $\sigma_{\eta,i}$. In order to make the subsequent analysis more transparent, we also consider the following variables

$$\begin{aligned} x_i &= \frac{\sigma_\delta^2}{\sigma_\delta^2 + 2\left(\sigma_{\epsilon,i}^2 + \sigma_{\eta,i}^2\right)} \\ &= \left(1 + \frac{2}{\mu_{s,i}^2} + \frac{2}{\mu_{p,i}^2}\right)^{-1}, \qquad i = 1,..,n. \end{aligned} \tag{15}$$

Each of the variables x_i is analogous to a signal-to-noise ratio for the agent i. This variable lies between 0 (when agent i is uninformed) and 1 (when agent i is fully informed). Clearly, (15) is monotonically increasing and concave in both precision $\mu_{p,i}$ and $\mu_{s,i}$.

Expected profits for informed trader i are given by

$$\bar{\pi}_i = \overline{\pi}_K \; D(x)\left(x_i + x_i^2\right), \tag{16}$$

with the "structural factor"

$$D(x) = \frac{1}{1 + f(x)} \frac{1}{\sqrt{f(x) + g(x)}}, \tag{17}$$

where

$$f(x) = \sum_{j=1}^{n} x_j, \qquad g(x) = \sum_{j=1}^{n} x_j^2, \tag{18}$$

and are scaled by the expected insiders' payoff in Kyle (1985) model

$$\overline{\pi}_K = \frac{1}{2}\sigma_\delta \sigma_u. \tag{19}$$

Importantly, the structural factor $D(x)$ depends on the entire information distribution across the informed agents, while each of the the parameters $\{x_i, \quad i = 1, ..n\}$ characterizes the information set of a particular agent. The inverse market liquidity takes the form

$$\lambda(x) = \lambda_K \, \frac{\sqrt{2}\sqrt{f(x) + g(x)}}{1 + f(x)}, \tag{20}$$

with the standard inverse market depth parameter (Kyle 1985)

$$\lambda_K = \frac{\sigma_\delta}{2\sigma_u}. \tag{21}$$

There is a close resemblance to the well-established literature on the sale of information (see, among others, Admati and Pfleiderer 1986, 1990). This can best be seen when we examine the individual agent's trading strategy $x_i = \beta_i(S_i - \bar{P}_i)$. As in the literature on the sale of information, the trading strategy is linear in the anticipated price differential. From a modeling perspective, the difference is that the existing literature has focused on improving the agent's forecast of next period price (denoted S_i in our model). In this paper, however, we argue that the agent will not perfectly observe current price (denoted $\bar{P}_i$). Rather, the agent needs to purchase real time price data to improve his estimate about anticipated price appreciation of the asset. This similarity to the existing literature explains why some of the special cases we examine in section 3 bear a certain resemblance to existing results. At the same time, this enables us to adopt the intuition from existing models and apply it to our analysis.

Following this intuition, we can identify several dimensions along which we differ from the existing literature. Firstly, we consider the sale of data, not information. Since the data is ex-post verifiable, we can easily overcome concerns about the seller's incentives to sell information truthfully. "Lying" by the exchange is not a concern.

Indeed, the seller of information is an entirely different entity. In fact, it is not the exchange's primary business to sell data. Rather, the exchange needs to facilitate smooth trading to maximize listing fees and attract buyers and sellers. In what follows, we will specifically analyze how the sale of data impacts the behavior of the exchange and parameters of the trading system. It may even be the case that the exchange benefits from a free disclosure of real-time price data - something that could never happen in the existing literature on the sale of information.

Last but not least, the value of the information contained in real-time prices is probably highly limited for individual and passive investors. It is probably well-informed private individuals and institutions who benefit the most of real-time data access[11]. We capture this spirit in our model, where real-time price access is only valuable in conjunction with private information. This is in contrast to the literature on the sale of information, in which anyone can gain a valuable advantage from purchasing information.

Our paper differs from the existing literature on the sale of information in the following sense. In Admati and Pfleiderer (1986, 1990), it is often optimal to sell information indirectly (i.e., by setting up a mutual fund), as this (i) mitigates truth-telling constraints and (ii) often leads to higher revenue due to avoidance of competition among informed traders. In the case of exchanges selling real-time transaction data, this is typically done directly, since (i) the data is ex-post verifiable and (ii) individual agents need to combine real time pricing data with their own private information to reap trading profits. In the setting of our model, indirect sale of information would generate zero revenue, since the exchange does not have any private information it can combine with real time pricing data (by definition, the stock price already reflects all the exchange's information).

3.1 Analysis

We now proceed to analyze the equilibrium we derived above. Specifically, we investigate the exchange's decision to potentially sell pricing data, and the informed agents' incentives to acquire them. It is important to recognize that the sale of price and transactions data is not the prime business of an exchange. Certainly, the exchange cannot maximize revenue from data sales without taking the impact of such a sale on price discovery and market liquidity into account. Thus, as in Holmstrom and Tirole (1993), we assume that firms need

[11] Presumably, the information contained in real-time prices is also valuable to financial intermediaries, institutions who execute massive trades, and liquidity providers who may act as market makers. We abstract from these issues for the sake of simplicity.

to compensate all or at least some of uninformed traders for their expected losses to better informed market participants. Specifically, let $q \in [0,1]$ denote the fraction of uninformed trader's losses the exchange needs to reimburse (we can think of q as the exchange's shadow price of liquidity). We assume that the firm is willing to offer the exchange a (constant) listing fee Q, from which the firm subtracts the expected compensation of uninformed traders, which are given by $q\lambda\sigma_u^2$. In addition, the exchange can earn additional revenue by selling its pricing data to informed traders. Let $C(i) \geq 0$ denote the dollar amount that informed agent i is willing to spend on acquiring pricing data (given the exchanges pricing scheme $(c_1, \ldots, c_m)$). The exchange's problem is now to

$$\max_{c_1,\ldots,c_m} Q - q\lambda\sigma_u^2 + \sum_{i=1}^{n} C(i) \tag{22}$$

Alternatively, this maximization problem can be rationalized by the idea that exchanges compete for order flow, as in Huddart, Hughes, and Brunnermeier (1999). Thus, the effect of the exchange's sale of information on market liquidity directly affects the exchange's profit. The intuition is as follows. If sale of information reduces market depth, then the exchange has to compensate uninformed traders for higher trading costs to attract their orders (and vice versa). The parameter q will be dictated by the degree of competition, where $q = 1$ represents perfect competition among exchanges.

To guide our economic intuition, we conduct our analysis in three steps. First, we analyze the case of a monopolistic informed trader. Second, we allow any positive number of informed traders, n_t, with the same precision of their information. In the third and final step, we analyze the general case of multiple agents with different information quality. For the sake of economic intuition, we will start with the case of $m = 1$, i.e. the exchange either selling a signal of perfect precision, or no information[12]. We then proceed to investigate which agents purchase more informative signals if the exchange offers a menu of different signals at different prices. Furthermore, we focus on perfect competition among exchanges initially ($q = 1$), and revisit the more general case subsequently.

[12] We eliminate the first trading period, $t = 1$, from our consideration here, since there is no uncertainty about past transaction prices. Period 1 is identical with a standard Kyle model.

3.2 Monopolistic Informed Trader

We start with the case of a single informed trader ($n = 1$). This simplifies the analysis significantly. Indeed, proposition 1 implies that the individual agents expected profits with private information and price precisions μ_s and μ_p, respectively, are given by

$$\bar{\pi}_1 = \overline{\pi}_K \left(1 + \frac{1}{\mu_s^2} + \frac{1}{\mu_p^2}\right)^{-\frac{1}{2}}, \tag{23}$$

while the inverse of market liquidity, λ, is

$$\lambda = \lambda_K \left(1 + \frac{1}{\mu_s^2} + \frac{1}{\mu_p^2}\right)^{-\frac{1}{2}}. \tag{24}$$

It can be seen from these expressions that certainly the exchange cannot afford to disclose past price quotes freely, since an increase in the informed trader's price precision, μ_p, leads to an increase of λ and therefore to a decrease in market liquidity.

3.2.1 Optimal Sale of Pricing Data

Since we assume $m = 1$, a purchase of real-time pricing data would reduce the informed trader's uncertainty about V to zero, and thus increase his profits by

$$\bar{\delta} = \overline{\pi}_K \left(\left(1 + \frac{1}{\mu_s^2}\right)^{-\frac{1}{2}} - \left(1 + \frac{1}{\mu_s^2} + \frac{1}{\mu_p^2}\right)^{-\frac{1}{2}}\right). \tag{25}$$

Thus, the exchange either charges an amount of $c_1 = \bar{\delta}$, or does not sell information at all. However, the informed trader's willingness to pay for information is due to his increased trading profits, which in turn cause the market to become less liquid. Note that the extra revenue from the sale of information is exactly equal to the additional losses to liquidity traders. We have thus proved

Proposition 2. *In the case of a monopolistic insider and , perfect information about the most recent transaction price, V, is worth an amount of*

$$\bar{\delta} = \overline{\pi}_K \left(\left(1 + \frac{1}{\mu_s^2}\right)^{-\frac{1}{2}} - \left(1 + \frac{1}{\mu_s^2} + \frac{1}{\mu_p^2}\right)^{-\frac{1}{2}}\right). \tag{26}$$

Since this is exactly offset by a decrease in market liquidity, the exchange is indifferent between selling past pricing data and keeping the information to itself in a case of a full compensation for liquidity traders, ($q = 1$).

This proposition differs from results of Admati and Pfleiderer (1986), since the exchange needs to compensate the expected losses of liquidity traders. While the proposition establishes the value of pricing information, but questions remain. What are the welfare effects of the exchange selling pricing information? Are there situations in which the exchange can generate positive profits from the sale of data? We will proceed to answer these questions in turn.

First, we note that the informational efficiency of the stock market increases when the exchange supplies costly transaction data. The uncertainty about the asset's fundamental value that is not revealed in market prices, $Var(V+\delta|P)$, is found to decrease by an amount of

$$\frac{\sigma_\delta^2}{2}\left(\left(1+\frac{1}{\mu_s^2}\right)^{-1}-\left(1+\frac{1}{\mu_s^2}+\frac{1}{\mu_p^2}\right)^{-1}\right)\geq 0. \tag{27}$$

Therefore, while the exchange is indifferent between selling pricing data or not doing so in a case of a full compensation ($q=1$), society clearly prefers the additional welfare benefits of having more efficient markets. These benefits can originate from a mitigation of agency conflicts and improved executive compensation (Holmstrom and Tirole 1993) or from more efficient investment decisions (Fishman and Haggerty 1989). But the sale of pricing data has additional welfare implications that become clear once we allow for endogenous information acquisition.

3.2.2 Endogenous Information Production

We now investigate a situation in which the informed trader is not endowed ex-ante with information, but needs to engage in costly research to uncover it (Holmstrom and Tirole 1993, Verrecchia 1982). Specifically, we assume that, at a cost of $c_\epsilon(x)$, the agent can obtain a signal about the final payoff of the firm with precision x (the only change to the previous model is that the error variance $\sigma_{\epsilon,}^2=1/x$ is now endogenous). Similarly, let $c_\eta(y)$ denote the cost the agent needs to incur to obtain a signal about the most recent transaction price, V, with precision $y=1/\sigma_{\eta,1}^2$. We follow the literature in assuming that the cost functions c_ϵ and c_η are monotonically increasing, convex and differentiable.

In the absence of the exchange selling a signal to the informed, these assumptions assure that there exists a unique equilibrium at the information acquisition stage. In equilibrium, the informed investor acquires a positive signal precision about both the liquidation value of the asset and the most recent transaction price. Let $(\widehat{\sigma}^2_{\epsilon,1}, \widehat{\sigma}^2_{\eta,1})$ denote the agent's optimally chosen error variances.

We proceed to obtain some economic insights into the effects of allowing the (possibly costly) disclosure of transaction price data. Firstly, such a disclosure eliminates the need for the informed trader to engage in costly research about the current market price. This represents an immediate welfare gain of $c_\eta(1/\hat{\sigma}^2_{\eta,1})$, since spending resources to obtain information that society already possesses is redundant.

Furthermore, disclosing real time pricing data stimulates private information production, thus leading to improved market efficiency and stock market monitoring. Specifically, if they acquire real time pricing data, their error variance becomes $\tilde{\sigma}_{\eta,1} = 0 < \hat{\sigma}^2_{\eta,1}$. This in turn increases the marginal utility of private information, and thus induces the informed agent to acquire more precise signal about the asset's fundamental value. Let $1/\tilde{\sigma}^2_{\epsilon,1}$ denote the agent's optimally chosen precision of private information. It follows that $\tilde{\sigma}^2_{\epsilon,1} \leq \hat{\sigma}^2_{\epsilon,1}$. This will result in improved informational efficiency of the market.

We have thus established the welfare gains from disclosure of current transaction prices. However, since such a disclosure leads to more informed trade, market liquidity will deteriorate. Thus, the exchange cannot simply disclose this information freely, but needs to sell it. We summarize our results in the subsequent proposition.

Proposition 3. *A (potentially costly) disclosure of real-time pricing data increases the effectiveness of stock market monitoring by leading to more private information production. An additional welfare gain accrues since the informed agent does not need to produce information about current prices that is already known to the market.*

Again, since there is a monopolistic informed trader, the increase in market monitoring activity comes at the expense of larger losses to uninformed shareholders. The exchange thus needs to charge the informed trader for access to real time prices. Whether it will be profitable for the exchange to sell pricing data or not depends on the relative costs of producing private information about asset value versus information about current market price. The issue of a profitable sale of pricing information will become much more clear when we introduce competition among multiple informed traders below.

3.3 Multiple Informed Traders with Equal Precision

We proceed to analyze the case of multiple informed traders. When we introduce competition among informed traders, the economics of selling pricing data changes. This gives the exchange a new potential source of profits: Competition among informed agents can lead each one of them to acquire pricing information, even though informed traders as a whole may be worse off in the process.

For the sake of developing our intuition, we start with the case in which all informed traders have equal precision ex ante: $\mu_{p,2} = \sigma_\delta / \sigma_\nu$ and $\mu_s = \sigma_\delta / \sigma_\epsilon$ for all agents $i = 1 \dots n$. Assume that there are n_1 informed agents who have access to more precise costly time price quotes with $\mu_{p,1} \geq \mu_{p,2}$, and the remaining $n_2 = n - n_1$ agents do not. In what follows, we will refer to the n_1 agents who acquire the real time price information as "high types", and the n_2 who do not acquire that information as "low types". The signal-to-noise ratios for the high and low types of agents are

$$x_1 = \left(1 + \frac{2}{\mu_s^2} + \frac{2}{\mu_{p,1}^2}\right)^{-1}, \tag{28}$$

and

$$x_2 = \left(1 + \frac{2}{\mu_s^2} + \frac{2}{\mu_{p,2}^2}\right)^{-1}, \tag{29}$$

respectively. By construction, the signal-to-noise ratio of the high types is greater or equal than that of the low types

$$x_1 \geq x_2. \tag{30}$$

In what follows, we analyze how the exchange optimally chooses the precisions of both high and low price signals and the optimal cost of high signal.

The expected profits for the high and low types are given by

$$\bar{\pi}^a (n_1) = \bar{\pi}_K \; D (n_1) \left(x_1 + x_1^2\right), \tag{31}$$

and

$$\bar{\pi}^{na} (n_1) = \bar{\pi}_K \; D (n_1) \left(x_2 + x_2^2\right), \tag{32}$$

respectively. The structural factor is

$$D (n_1) = \left(\left(1 + f (n_1)\right) \sqrt{f (n_1) + g (n_1)}\right)^{-1}, \tag{33}$$

where

$$
\begin{aligned}
f(n_1) &= nx_2 + n_1(x_1 - x_2), \\
g(n_1) &= nx_2^2 + n_1\left(x_1^2 - x_2^2\right).
\end{aligned} \tag{34}
$$

Since $x_1 > x_2$, the difference in expected profits, $\Delta = \bar{\pi}^a - \bar{\pi}^{na}$, is always positive. Consistent with the literature, we call Δ the informed agents' maximum willingness to pay for access to real time pricing information. The exchange now progresses choose the signal-to-noise ratios x_1 and x_2 and to set a profit-maximizing price of information. Since Q is constant, the exchange effectively maximizes the revenues net of a fraction q of the liquidity losses

$$
\begin{aligned}
&\max_c L_q(n_1, c), \\
L_q(n_1, c) &= n_1 c - q\lambda\sigma_u^2.
\end{aligned} \tag{35}
$$

The maximand $L(n_1, c)$ consists of two components. The first term, $n_1 c$, equals the revenue of the exchange from data sales, $R(n_1, c) = n_1 c$. The cost of information, c, is set to control how many agents find it optimal to acquire it, i.e. $n_1 \leq n$ is the maximum number of agents under which $\Delta \geq c$. The second component, $q\lambda\sigma_u^2$, is the compensation to uninformed traders. This reflects the fact that the exchange needs to maintain market liquidity, and can also be interpreted as the shadow cost of illiquidity. For the clarity of exposition, we first consider the case of full compensation ($q = 1$) and lift this restriction in the subsequent analysis. We define the exchange's profits of selling the price information as a marginal increase of the exchange's objective function due to the sell of the real-time price information. In our case, the exchange's profits are defined as

$$
\Delta L_q(n_1, c, x_1, x_2) = L_q(n_1, c, x_1, x_2) - L_q(0, c, x_1, x_2). \tag{36}
$$

The exchange optimizes its profits (36) with respect to the number of high types n_1 and signal-to-noise ratios x_1 and x_2, and this determines the optimal cost of a high signal c at equilibrium. To simplify the exposition, we first optimize (36) with respect to n_1 for arbitrary x_1 and x_2, and then consider the optimization with respect to the signal-to-noise ratios. We proceed with the following result.

Proposition 4. *In the case of full compensation ($q = 1$), the exchange maximizes profits by setting the price of information, c, such that all n informed agents in the economy purchase* *information. This occurs at a price of information of*

$$c = \overline{\pi}_K \, \frac{(x_1 - x_2)\,(1 + x_1 + x_2)}{(1 + nx_1)\,\sqrt{n\,(x_1 + x_1^2)}} \geq 0. \tag{37}$$

At this price, profits of the exchange are positive and amount to

$$\frac{\Delta L_1^*}{\overline{\pi}_K} = \sqrt{n}\,\left(x_2 + x_2^2\right) \left(\frac{1}{(1 + nx_2)\,\sqrt{x_2 + x_2^2}} - \frac{1}{(1 + nx_1)\,\sqrt{x_1 + x_1^2}} \right). \tag{38}$$

This objective-maximizing behavior by the exchange simultaneously maximizes the informational efficiency of the market.

From (38), it follows that the optimal profits of the exchange are monotonically increasing in the signal-to-noise ratio of the high signal x_1, implying that the optimal value is achieved for the maximal x_1. Note that equation (28) implies that the signal-to-noise x_1 is bounded

$$0 \leq x_1 \leq x_m = \left(1 + \frac{2}{\mu_s^2}\right)^{-1}, \tag{39}$$

and therefore the profits are maximized at $x_1^* = x_m$. Taking into account (28) and observing that x_1 monotonically increases in the precision $\mu_{p,1}$, we conclude that the exchange optimizes profits by selling a high signal of infinite precision ($\mu_{p,1} = \infty$), which can be interpreted as selling real time price information without any delay or noise added.

Importantly, the optimal expected profit of exchange (38) is non-monotonic in the signal-to-noise ratio of the low signal x_2. First of all, as stated in the above proposition, the profit (38) is non-negative for any $x_2 \in [0; x_1]$. Second, (38) takes on a value of zero for both boundary values of $x_2 = 0$ and $x_2 = x_1$. Indeed, the expression for the exchanges profits in (38) is the difference between the exchange's value function in case of everyone purchasing the high signal (which implies a signal to noise ratio of x_1) and everyone abstaining to purchase a signal, and thus having a signal to noise ratio of x_2. In case of $x_2 = 0$, traders do not have any information, and thus cannot benefit from purchasing pricing information. This explains why for $x_2 = 0$, the exchanges profits are zero. By contrast, if $x_2 = x_1$, the low signal equals the high one, and thus agents are not willing to purchase the high signal. Since the profit (38) is not identically zero and takes zero values on both sides of the segment $x_2 \in [0; x_1]$, the profit of exchange must achieve its maximal value inside of the interval $x_2 \in (0; x_1)$. This can be seen in Fig.1, which plots a typical profile of the profit function (38). In this case, the parameters are $n = 100$ and $x_m = 0.1$. Clearly, the function has a peak at $x_2 \simeq 0.01$. As we show in Appendix, for sufficiently large number of informed traders $n >> 1$, we have an approximate relation of

$$x_2^* \approx \frac{1 - \gamma(n, \mu_s)}{n - 2}, \tag{40}$$

with

$$\gamma(n, \mu_s) = \frac{8}{\sqrt{n-2}} \frac{n(n-1)^{5/2}}{(n-2)^{7/2}} \frac{1}{(1 + nx_m)\sqrt{x_m + x_m^2}}, \tag{41}$$

and $x_m(\mu_s) = \left(1 + \frac{2}{\mu_s^2}\right)^{-1}$. The approximation (40) with (41) works quite well for sufficiently large number of informed agents n. Combining (40) and (29), we finally obtain the following result for the optimal precision of the free price signal

$$\mu_{p,2}^* \approx \sqrt{\frac{2}{n-2}} \left(1 + \frac{1 + 2/\mu_s^2}{2(n-2)} - \frac{\gamma(n, \mu_s)}{2}\right). \tag{42}$$

One should note that (40) characterizes the optimal precision of the price information that is disseminated by the exchange for free, in terms of the two basic parameters characterizing the information structure of the market. Namely, the optimal precision of the free price signal depends only on the total number of informed agents n and on the amount of private information μ_s. In principle, this may allow one to make qualitative estimates regarding the amount of private information based on the well-known time delays of the free price information in various markets (see Table 1). In particular, it follows from (40) that the optimal precision of the low signal decreases in the total number of informed traders n and is non-monotonic in the amount of private information in the market captured by μ_s. In the limit, when n is large, the optimal price precision actually decreases in private information μ_s, implying that an exchange with higher amount of private information are expected to provide less precision and therefore longer time delays of free price information. We proceed to summarize our above findings.

Corollary 1. *In the case of full compensation ($q = 1$), the exchange maximizes profits by setting the price of information, c, such that all n informed agents in the economy purchase information. This occurs at a price of information of*

$$c = \overline{\pi}_K \frac{(x_m - x_2^*)(1 + x_m + x_2^*)}{(1 + nx_m)\sqrt{n(x_m + x_m^2)}} \geq 0. \tag{43}$$

The optimal high signal has infinite precision with the signal-to-noise ratio $x_1^* = x_m$*, whereas the optimal precision of the low signal is finite so that its optimal signal-to-noise* x_2^* *satisfies the condition*

$$0 \leq x_2^* \leq x_m. \tag{44}$$

The net profits of the exchange present a direct transfer of wealth from informed traders to the exchange. If exchanges compete, this wealth transfer can be passed on to customers, for instance, in the form of lower listing fees for companies. Note that we already assumed that exchanges compete for uninformed order flow, which necessitate them to compensate uninformed traders for their expected trading loss $\lambda\sigma_u^2$. So if the sale of pricing data leads to less liquid markets, then the proposition above implies that the revenue generated from data sales will be more than enough to offset the decrease in market liquidity. If, however, the sale of pricing data increases market liquidity, then it is immediately intuitive that the exchange will in fact generate positive profits. In both cases, we conclude that informed traders as a group would be better off without the sale of pricing data.

The above results indicate that (i) competition among these agents will lead to more informational efficiency and (ii) the exchange will make data accessible at prices affordable to all informed agents. To see if all informed agents will indeed purchase access to real time pricing data, or which ones will abstain from doing so, we proceed to study situations in which the quality of informed traders' signals varies.

We also point out the differences between our result and the literature on the sale of information. In that literature, the seller of information typically will only sell his signal to a fraction of the agents (Admati and Pfleiderer, 1986). In particular, the maximal profits are achieved by an indirect sell of private information, when there is effectively one representative informed agent (Admati and Pfleiderer, 1990). In our model, by contrast, the price information is typically sold to a finite number of agents ($n_1 > 1$). In particular, all potential customers obtain price information in the case of full compensation ($q = 1$). The difference is that in this case the exchange is not interested in squeezing profits from uninformed traders (whose losses it compensated). Instead, the exchange benefits from competition among informed traders. Their private information is a complement to the exchanges real time data.

Note that in general, the market illiquidity parameter λ displays non-trivial behavior in Kyle models of this type. With a single informed trader, more asymmetric information always leads to less liquidity. This is no longer the case when informed agents compete, since the increase in competition is a counter-acting force. Thus, market liquidity is typically maximal in market with either few or many informed agents. It is easy to see that an increase in the price of information can have an ambiguous effect on λ.[13]

The assumption of the full compensation for the liquidity traders $q = 1$ is not realistic. Indeed, the empirical evidence is that the liquidity traders on average loose money in the trading process. We now consider general values of $q \in [0; 1]$ for the rest of the paper. As in Admati and Pfleiderer (1986), the exchange might opt to sell only a noisy version of the most recent pricing data. We allow for this behavior and give conditions under which the exchange indeed wants to add noise to the data it sells.

Therefore, we assume that the high types may receive a noisy price signal with the precision $\mu_{p,1}$, while the low types receive a price signal with a lower precision $\mu_{p,2} < \mu_{p,1}$. To simplify the analysis, we first consider a limit when the precision of the delayed signal is zero, $\mu_{p,2} = 0$, and therefore the signal-to-noise ratio in the low state is zero, $x_2 = 0$. This is not very unrealistic since the speculative value of the "free" delayed signal is expected to be small. According to (16), the payoff of each high type agent is

$$\bar{\pi}_h = \frac{1}{\sqrt{n_1}} \frac{\sqrt{x_1}\sqrt{1+x_1}}{1+n_1x_1}, \tag{45}$$

with x_1 given by

$$x_1 = \left(1 + \frac{2}{\mu_s^2} + \frac{2}{\mu_{p,1}^2}\right)^{-1}, \tag{46}$$

and the low types make no profit, $\bar{\pi}_l = 0$. Therefore, the objective function of the exchange is given by

$$L_q = (1-q)\sqrt{n_1} \frac{\sqrt{x_1}\sqrt{1+x_1}}{1+n_1x_1}. \tag{47}$$

It is easy to see that the objective function (47) is non-monotonic in the effective "signal-to-noise ratio" parameter x_1. As we show in the Appendix, the function (47) achieves its maximum at

$$x_n^* = \frac{1}{n-2}. \tag{48}$$

Since the parameter x_1 given by (46) is limited by the precision of the private signal

$$x_1 \le x_c = \left(1 + \frac{2}{\mu_s^2}\right)^{-1}, \tag{49}$$

[13]I have moved the above paragraph here. It used to be before Theorem; did not seems to make sense.

the precision of the price signal that the exchange sells depends on the relation between (48) and (49). We obtain the following

Proposition 5. *When all traders have the same precision of private information, the exchange sells a single price signal with infinite precision (real time price data) to all traders if the number of traders is smaller than the critical number, $n \leq n_c$, and to a limited number of n_c traders if $n \geq n_c$, where the critical number is given by $n_c = 1 + \frac{2}{\mu_s^2}$.*

Note that the above result holds for any $q \in [0;1]$ provided that $x_2 = 0$. In the case of partial compensation $q \in (0;1)$, the exchange maximizes profits by setting the price of information, c, such that a finite number $n^* \leq n$ of informed agents in the economy purchase information. The optimal number n^* depends on the compensation ratio q and on signal to noise parameters x_1 and x_2. In particular, there exist the bounds $0 < q_c < 1$ and $q_c < q_m < 1$ such that $n^* = 1$ for $q \in (q_c; q_m)$, and $1 < n^* < n$ for $q \in (0; q_c)$.

The typical case is represented in Fig.2, where we present a two-dimensional contour plot of the exchange objective function $L_q(n_1)$ as a function of the compensation parameter $q \in [0;1]$ and the number of high type agents n_1. The total number of informed agents is $n = 1000$ and the high price signal has a precision $\mu_{p,1} = 10$. As can be seen in Fig.2, the critical value in this case is $q_c \simeq 0.6$.

4 Multiple Informed Traders with Heterogeneous Precision

We now analyze the general case in which agents have different precision of information, and the exchange can sell signals of different quality. The insider i's payoff is characterized by

$$\eta_i = \frac{\overline{\pi}_i^*(\mu)}{\overline{\pi}_K} = D(x)\left(x_i + x_i^2\right). \tag{50}$$

Using the scaled variables η_i, we obtain the following

Proposition 6. *Agents with more precise private information (smaller $\sigma_{\epsilon,i}$) value accurate price information more in the sense that they have a higher marginal utility of price information.*

The above proposition is consistent with the observation that the primary beneficiaries of real-time price information are well-informed traders and large sophisticated financial institutions. However, our result raises the following concern. If agents who already have superior private information benefit more from real time price information, then granting those agents access to price information might significantly increase the degree of information asymmetry in the market, and therefore deteriorate market liquidity. We analyze this issue below and confirm that it is indeed the case.

Suppose the exchange introduces a cost schedule for the information that depends on its precision. Let $C(\mu_{p,i})$ denote the cost that informed agent i needs to incur in order to acquire price information of precision $\mu_{p,i}$. We assume that the cost increases in the precision. The informed traders decide upon the optimal precision to acquire based on their marginal utility $p_\mu(\mu_{p,i}, \mu_{s,i}, x)$. Using (89), we obtain that the optimal amount of information acquired by trader i is defined by the following condition

$$\frac{2x_i^2}{\mu_{p,i}^4}(1+2x_i) D(x) = \frac{\partial}{\partial\left(\mu_{p,i}^2\right)} C(\mu_{P,i}). \tag{51}$$

From (51), it follows that agent i decides on the optimal amount of price information he acquires based on the cost schedule and the precision of his private signal. The optimal price precision $\mu_{p,i}^*$ is defined by the condition

$$\mu_{p,i}^* = \mu_{p,i}^*(\mu_{s,i}, x), \tag{52}$$

and depends on the precision of the private signal and the information distribution across all other agents. In principle, the condition (52) defines the amount of price information acquired by each agent in the economy.

Our goal is to analyze how access to costly pricing information affects aggregate market characteristics in the presence of a potentially large number of heterogeneously informed agents. For this purpose, we have to study how the distribution of informed agents is affected by the acquisition of costly price information. In other words, our analysis requires to derive the equilibrium distribution of informed agents with respect to precision of the price signals they acquire, given the precision of the private signals they already possess. First, we consider how the information acquisition affects market liquidity. The appendix derives the following:

Proposition 7. *When the informed agent i acquires additional price information, market liquidity increases or decreases depending on whether pricing information is acquired by the* *agents on the left or on the right side of the distribution. The acquisition of price information increases (decreases) market liquidity, if agent i's informativeness, as measured by x_i, is below (above) the critical value $x_e = \overline{x} + \frac{Var(x)}{\overline{x}} + \frac{1}{2}$.*

Here, $\bar{x} = E(x_i)$ denotes the sample mean of the signal to noise ratio x_i. The above result implies that the price information acquired by informed traders, affects market liquidity in an ambiguous way. For an individual informed agent acquiring the price information, the effect on market liquidity depends on the amount of information the agent already possesses represented by x_i. If the agent's ex-ante information is already above the critical value x_e, then the additional information on current price obtained by the agent increases the inverse market depth, λ, and thus decreases market liquidity. If the agent's signal to noise ratio is below x_e, the opposite occurs. Equation (94) shows that the critical value x_e depends on the sample average of all agents' signal-to-noise ratios plus a shift proportional to the sample variance. A simple intuition lies behind our finding. On the one hand, if the highly informed agents become even more informed, the information asymmetry in the market increases, leading to the reduction of the market liquidity. On the other hand, if the less informed agents become more informed, the information asymmetry decreases and therefore the market liquidity increases.

Together, propositions 6 and 7 define an interesting economic tension for the exchange as seller of pricing data. While the sale of real time data to privately well-informed agents is particularly profitable, the liquidity implications of doing so are problematic. Thus, optimal sale of data in the presence of heterogeneously informed traders becomes a far more complex problem to study. We proceed with the case of a binary distribution in agents' precision of private information before we analyze the general case.

4.1 Binary distribution of private precision

In order to analyze the equilibrium acquisition of price information, we first examine the case of a binary distribution of the precision of agents' private information. Agents either have a high precision of $\mu_{s,1}$ or a lower precision of $\mu_{s,2} < \mu_{s,1}$. We assume that there are n_1 agents of type one and n_2 agents of type two [14]. Suppose that the exchange can sell two

[14] Note that the signals obtained by diferent agents are uncorrelated.

types of price signals with precision $\mu_{p,1}$ and $\mu_{p,2}$, such that $\mu_{p,1} > \mu_{p,2}$. Analogous to the private signals, we will refer to the price signals as "high" and "low" precision.

What we have in mind is a small distortion of our previous case of homogeneously informed agents. Assume that instead of having n agents of homogeneous precision μ_s as in the previous section, we have n_1 agents of precision $\mu_{s,1} = \mu_s + \xi$ and n_2 agents of precision $\mu_{s,1} = \mu_s - \xi$. In this case, the exchange may no longer want to sell signals of infinite precision, but instead offer two precision, $\mu_{p,1}$ and $\mu_{p,2}$.

Suppose the informed traders can acquire the high or low signal on price at a cost C_1 and $C_2 < C_1$, respectively. In what follows, we derive the relation between $\{C_k,\ k = 1, 2\}$ and $\{\mu_{p,k},\ k = 1, 2\}$ that maximizes the revenues of the exchange, and can therefore be viewed as an optimal cost schedule. Based on the signal complementarity results discussed above, we analyze a separating equilibrium when the high and low types of informed traders buy price signals with high and low precision, respectively. This is possible because the agents of high type have higher precision of their private signals and due to the signal complementarity are willing to pay more for the additional price signals. At equilibrium, the exchange sets the costs $\{C_k,\ k = 1, 2\}$ so that the informed agents of corresponding type are indifferent between acquiring a costly precise price signal or getting a free noisy signal. In what follows, we assume that these free noisy signals yield the residual effective payoffs $\pi_{r,1}$ and $\pi_{r,2} < \pi_{r,1}$ for the high and low types, respectively.

Taking into account (50), we obtain the following revenue maximizing conditions for the two types of informed traders[15]

$$\begin{aligned} x_{1,1} + x_{1,1}^2 &= \frac{C_1}{D(x)} + e_1, \\ x_{2,2} + x_{2,2}^2 &= \frac{C_2}{D(x)} + e_2, \end{aligned} \tag{53}$$

[15] Note that we have that the structural factor $D(x)$ is also affected by the agents' strategies with respect to acquiring the price information. However, these distortions are small for the sufficiently large number of informed agents and scale as $1/n$. Since we are primarily interested in the case of large number of agents, we neglect these effects.

with $e_k = \pi_{r,1}/D(x)$, $k = 1, 2$, and

$$x_{1,1} = \left(1 + \frac{2}{\mu_{s,1}^2} + \frac{2}{\mu_{p,1}^2}\right)^{-1}, \qquad (54)$$
$$x_{2,2} = \left(1 + \frac{2}{\mu_{s,2}^2} + \frac{2}{\mu_{p,2}^2}\right)^{-1}.$$

The incentive compatibility conditions take the form

$$x_{1,2} + x_{1,2}^2 \leq \frac{C_2}{D(x)} + e_1, \qquad (55)$$
$$x_{2,1} + x_{2,1}^2 \leq \frac{C_1}{D(x)} + e_2,$$

where

$$x_{1,2} = \left(1 + \frac{2}{\mu_{s,1}^2} + \frac{2}{\mu_{p,2}^2}\right)^{-1}, \qquad (56)$$
$$x_{2,1} = \left(1 + \frac{2}{\mu_{s,2}^2} + \frac{2}{\mu_{p,1}^2}\right)^{-1}.$$

Intuitively, the conditions (55) ensure that it is indeed optimal for the agent with high precision of private information to acquire the high precision signal on price and vice versa, thus ensuring a separating equilibrium. Solving (53), we obtain the following relation

$$\frac{\mu_{p,1}^2}{\mu_{p,2}^2} = \frac{\left(\sqrt{\frac{1}{4} + \frac{C_2}{D(x)} + e_2} - \frac{1}{2}\right)^{-1} - 1 - \frac{2}{\mu_{s,2}^2}}{\left(\sqrt{\frac{1}{4} + \frac{C_1}{D(x)} + e_1} - \frac{1}{2}\right)^{-1} - 1 - \frac{2}{\mu_{s,1}^2}}. \qquad (57)$$

With the notations $\Delta C_k = \frac{C_k}{D(x)} - e_k$, $k = 1, 2$, and $y = \Delta C_1 / \Delta C_2$, (57) finally yields

$$\frac{\mu_{p,1}^2}{\mu_{p,2}^2} = \Theta(y), \qquad (58)$$
$$\Theta(y) = y \frac{\sqrt{\frac{1}{4} + \frac{C_2}{D(x)} + e_2} + \frac{1}{2} - \left(1 + \frac{2}{\mu_{s,2}^2}\right)\Delta C_2}{\sqrt{\frac{1}{4} + y\Delta C_2 + 2e_1} + \frac{1}{2} - y\left(1 + \frac{2}{\mu_{s,1}^2}\right)\Delta C_2}.$$

Analogously, solving (55), we obtain the conditions

$$\Omega\left(\frac{C_1}{D}\right) \leq 0, \qquad (59)$$
$$\Omega\left(\frac{C_2}{D}\right) \geq 0,$$

with

$$\Omega(z) = \phi(z+e_2) - \phi(z+e_1) - 2\left(\frac{1}{\mu_{s,2}^2} - \frac{1}{\mu_{s,1}^2}\right), \tag{60}$$

$$\phi(x) = \left(\sqrt{\frac{1}{4}+x} - \frac{1}{2}\right)^{-1}.$$

In order to analyze the resulting equilibrium, we choose the base case parameters given in the following table.

$\mu_{s,1}$	$\mu_{s,2}$	e_1	e_2	ΔC_2
5	3	2	1	0.2

Table of Parameter Values

In Figure 3, we present a plot of the incentive compatibility condition $\Omega(z)$. As follows form Figure 3, the base case parameters are consistent with the incentive compatibility constraint. In Figure 4, we plot the optimal schedule defined by (58) as a function of the scaling parameter $y = \Delta C_1/\Delta C_2$. As follows from Figure 4, the optimal schedule can be approximated by a segment of the straight line in the region of parameters under consideration. This implies that the optimal schedule for the bimodal distribution is approximately quadratic. Making use of this result, we will assume that the optimal cost schedule is quadratic also in the case of multiple types of informed traders with differential private information. We analyze this case in the following subsection.

4.2 Optimal Acquisition of Price Information

We now consider a continuous distribution of the private precision and a continuous set of the precision of the acquired price signals. According to the previous analysis, we restrict ourselves to the cost schedules quadratic in the price precision

$$C(\mu_p) = B\mu_p^2, \tag{61}$$

where B is a positive real constant. The motivation for the quadratic cost schedule is based on the analysis done in the previous subsection. Another reason for choosing (61) is that the linear cost schedule may lead to the situations where there is no equilibrium. This has a simple intuition, since the economics of a problem is defined in terms of the marginal cost being a derivative of (61). In case of the linear cost schedule, the marginal cost is a constant, which may lead to corner solutions of the optimization problem for the objective function of the exchange.

Substituting the schedule (61) into (51), we obtain[16]

$$\frac{x_i^2}{\mu_{p,i}^4}\left(1+2x_i\right)D\left(x\right)=\frac{B}{2}. \tag{62}$$

Combining (51) and (15), we obtain the following equilibrium condition

$$\Phi\left(\mu_{p,i}^2,\mu_{s,i}^2\right)=\varepsilon, \tag{63}$$

with effective marginal benefit

$$\Phi\left(\xi,\eta\right)=\eta^2\frac{\left(\xi+\eta+\frac{3}{2}\xi\eta\right)}{\left(\xi+\eta+\frac{1}{2}\xi\eta\right)^3}. \tag{64}$$

and cost schedule parameter

$$\varepsilon=\frac{2B}{D\left(x\right)}. \tag{65}$$

The relation (63) defines the precision of price signal $\mu_{p,i}$ acquired by the informed agent with a precision of private signal $\mu_{s,i}$.

Note that since the arguments of (64) represent precision, we are only interested in solutions of (63) consisting of positive values of $\mu_{S,i}$ and $\mu_{p,i}$. The pair of equations (63) and (64) has positive solutions only if $\varepsilon\in[0,1]$. Economically, this means that the equilibrium cost schedule parameter has to be sufficiently small. Otherwise, the marginal cost and benefits of acquiring the information are not be balanced, and there is no equilibrium. This is illustrated in Figure 5, where the function $\Phi\left(\mu_p^2,\mu_s^2\right)$ is plotted against its arguments. Clearly, $\Phi\left(\mu_p^2,\mu_s^2\right)\leq 1$ for positive values of μ_p and μ_s, and therefore (63) only has solutions if $\varepsilon\in[0,1]$. Figure 5 also illustrates an important signal complementarity property of our model. Namely, the marginal benefit of information acquisition for agent i, $\Phi\left(\mu_{p,i}^2,\mu_{s,i}^2\right)$, increases in the precision of his private signal $\mu_{s,i}$. In other words, more informed agents have higher marginal benefits for the additional price signal.

In Figure 6, we present solutions $\mu_{p,i}\left(\mu_{s,i},\varepsilon\right)$ of (63) for three values of the cost schedule parameter $\varepsilon=\frac{2B}{D(x)}=0.3,0.5,0.8$. We observe that the traders with more private information

[16]See the previous footnote.

also buy a price signal with higher precision. This is consistent with Figure 5 and the results of proposition 6. As the agents' private precision goes to infinity, the precision of the acquired information on price remains bounded, $\mu_{p,i} < \mu_{p,\max}(\varepsilon) < \infty$. As it follows from Figure 6, the upper bound $\mu_{p,\max}(\varepsilon)$ decreases in the cost schedule parameter ε. This has a simple intuition, since when the information on price becomes more costly, the agents are less willing to purchase it. These results are summarized in

Proposition 8. *When the exchange sells pricing information according to a quadratic cost schedule, then an equilibrium exists if the cost schedule parameter ε is sufficiently low. The equilibrium precision of the acquired signals decreases in the cost schedule parameter ε.*

4.3 Equilibrium distribution of price precision

We study the equilibrium distribution of price precision acquired by informed traders. In what follows, we show that the distribution is characterized by a continuous probability density function (p.d.f.)[17].

Since the function (64) can not be inverted analytically the closed form fully analytic solutions are not available, and part of this work relies on numerical simulations.

We assume that the precision of private signals is uniformly distributed across the informed agents with the finite support $\mu_s \in [0, \mu_{s\max}]$. For the base case simulations, we adopt $\mu_{s\max} = 10$. First we derive the p.d.f. $\rho(\mu_p)$ for the equilibrium distribution of the acquired price signal μ_p. This p.d.f. is uniquely defined provided that there is a relation $\mu_s = \Gamma(\mu_p)$, and the function $\Gamma(\cdot)$ is monotonic. As is illustrated by Figure 6, this is indeed the case. The details of the derivation are given in the appendix.

The p.d.f. is presented in Figure 7. The cost schedule parameter $\varepsilon = \frac{2B}{D(x)}$ is taken

[17] This does not contradict the fact that the total number N of agents in our model economy is finite (possibly large), and therefore their payoffs are non-zero. The reason is that according to (52), the amount of price information acquired by each informed agent is completely defined by the precision of his private signal and the cost schedule parameter. In other words, (52) can be viewed as a mapping of private information distribution onto the resulting distribution of price information. More specifically, this mapping is presented in Figure 4. Note that for each realization of private information distribution, we can construct the corresponding distribution of price information using the mapping (52). For finite number of agents, each realization of the distribution of private information is represented by a discrete distribution of the precisions. However, the resulting distribution is continuous if we average over the possible realizations of initial discrete distribution. This also reflects the ergodicity property of our model.

$\varepsilon = 0.5$. Dashed line illustrates the p.d.f. of the corresponding uniform distribution. As one can see, the resulting p.d.f. $\rho(\mu_p)$ of the distribution of price precision has a finite support $\mu_p \in [0, \mu_{p\max}]$ with $\mu_{p\max} = 1.38$. Also, the p.d.f. $\rho(\mu_p)$ is right-skewed in comparison to the uniform one. The reason is the complementarity between the private and price signals in our model, meaning that the agents with higher precision of their private signals also tend to acquire more precise information on price. Therefore, the p.d.f. increases for the large price precision. To summarize, we make the following observation. When the exchange sells pricing information according to a quadratic cost schedule, then the equilibrium distribution of the precision of the acquired signals is right-skewed. This is due to the complementarity between the private signals and the signals on price.

From analysis of Figure 6 it also follows that the cost schedule parameter $\varepsilon = \frac{2B}{D(x)}$ is uniquely defined given the distribution of the precision of private signals $\rho(\mu_p)$. Indeed, as it follows from Figure 6, for each $\varepsilon \in [0, 1]$ we can uniquely define the equilibrium distribution of the precision of acquired price signals. From this distribution and making use of the definition (50), we can derive the structural factor $D(x)$, and therefore define the slope of cost schedule B as

$$B(\varepsilon) = \frac{\varepsilon D(x(\varepsilon))}{2}. \tag{66}$$

As a result, we obtain a unique value of the slope $B(\varepsilon)$ for each cost schedule parameter ε. The results are presented in Figure 8. Importantly, the function $B(\varepsilon)$ is monotonically increasing and goes to infinity as $\varepsilon \to 1$. This illustrates that the equilibrium exists for any positive slope of the cost schedule B. Since the function $B(\varepsilon)$ is monotonically increasing, the inverse function $\varepsilon = \varepsilon(B)$ is well defined. Therefore, the slope B uniquely defines the equilibrium scaling parameter ε. Since the scaling parameter ε completely defines the equilibrium distribution, it follows that the slope B also uniquely defines the equilibrium distribution of the price precision.

In Figure 10, we present the inverse market depth parameter $\lambda(\varepsilon)$ as a function of the effective slope ε. Clearly, the inverse market depth monotonically decreases in ε and therefore also monotonically decreases in the cost slope B. This has a simple intuition. As the price information becomes more costly, less agents acquire the information and therefore the information asymmetry decreases. In the limit when B goes to infinity, the agents do not acquire any price information, their profits go to zero and therefore the liquidity losses go to zero. In other words, in the limit when $B \to \infty$ we have $\lambda(\varepsilon) \to 0$, and market becomes infinitely deep.

One should note that when the effective slope increases, the support of the p.d.f. of price precision distribution decreases and the "bump" on the right tale of the distribution becomes more narrow. This means that as the cost slope B increases, the average agent acquires less price information. At the same time, when the slope is high, there is a small fraction of highly informed agents who are still willing to acquire the information with a very high precision. These agents contribute to the narrow and high peak on the right tale of Figure 7. However, the fraction of such agents is small and the information asymmetry decreases as the cost slope increases.

The above results are summarized as follows. When the exchange sells pricing information according to a quadratic cost schedule, the equilibrium exists for arbitrary non-zero slope B of the cost schedule. As the slope B increases, the agents acquire less information on price, their profits decrease, and the market liquidity increases.

Figure 9 illustrates the dependence of the exchange revenue from selling the price information $R(\varepsilon)$ on the effective slope parameter ε. Importantly, the dependence is non-monotonic in the slope parameter. Namely, $R(\varepsilon)$ has a maximum of $R_m \approx 0.54$ at $\varepsilon_m \approx 0.67$. This implies that the slope of equilibrium cost schedule that maximizes the revenues of the exchange B_m is given by condition $B_m = B(0.67)$. Comparing with Figure 6, we obtain $B_m \approx 0.59$. The non-monotonic dependence of the revenues on the slope has a simple intuition. Namely, the increasing of the slope has an ambiguous effect on the revenues. On the one hand, the revenues increase when the slope increases, since the informed traders who acquire the price information are paying more. On the other hand, when the slope is too high, there are less agents who acquire the price information, causing the revenues to decrease. Therefore, we obtain the following result. When the exchange sells pricing information according to a quadratic cost schedule, the revenues depend on the slope B in a non-monotonic way. In particular, there exists a finite value B_m that maximizes the revenues.

According to the discussion of section 3, the objective of the exchange is not only to increase the revenues, but also to provide a sufficient level of liquidity. As a limiting case, we may assume that the exchange completely compensates the liquidity traders for their losses. This leads to the maximization problem given by (35). One can show that (35) is always non-positive and is monotonically increasing in the slope of the cost schedule B. In the limit $B \to \infty$, no information is being acquired, all profits go to zero and $L \to 0$. The dependence $L_q(\varepsilon)$ for the base case parameters is represented in Figure 11. As we have discussed above, we assume that the liquidity traders get compensated for some fraction q of their losses by the exchange, and the objective of exchange takes the form

$$\begin{aligned} & \max_B L_q(\varepsilon(B)), \qquad (67) \\ L_q(\varepsilon(B)) &= R(\varepsilon(B)) - q\lambda(\varepsilon(B))\,\sigma_u^2, \end{aligned}$$

where we took into account that the function $B(\varepsilon)$ is invertible. In Figure 11, we present the dependence of the objective $L_q(\varepsilon)$ defined by (67) for $q = 0.2$. As follows from Figure 11, $L_q(\varepsilon)$ has an internal maximum for $\varepsilon_L \approx 0.75 < 1$. Comparing with Figure 8, we conclude that the corresponding cost slope is $B_L = B(0.75) \approx 0.77$.

In Figure 12, we present the results indicating that the optimal slope defined above decreases when the amount of private information represented by a "size" of distribution of the precision of private signals $\mu_{s,m}$ in the market increases. This has a simple intuition. When the number of informed traders with high precision of private information increases, there are two market forces that affect the equilibrium cost. On the one hand, with the number of highly informed traders increasing, there are more agents who value the price information higher and therefore are willing to pay more potentially increasing the equilibrium cost. We will refer to this a "direct" effect of increasing the amount of private information in the market. On the other hand, when the number of informed agents increases, each agent has lower value for the additional information, since he expects to have less informational advantage over the others and each agent profits from the informational asymmetry between him and the rest of the market. This will effectively reduce the equilibrium cost schedule other things equal. We will refer to this as an "indirect" effect. Technically, the indirect effect is related to the dependence of the structural factor $D(x)$ on the entire information structure of the market, while the direct effect has to do with the profit dependence on the information set of each agent. According to Figure 12, the indirect effect is more important in the range of parameters under consideration.

Note that in fact the solution process described above leads to a fixed-point type solution for both the equilibrium distribution with respect to the acquired precision, and the optimal cost schedule set by the exchange. Indeed, we first define the functional form of the distribution of the precision, which depends on both the slope of the cost schedule B and the higher moments of the distribution itself contained in the structural factor $D(x)$. Then we choose the parameter of the distribution in order to match the structural factor $D(x)$ and the slope B. When this is done, we effectively "close the loop" and make the solution self-consistent, i.e. the true equilibrium. We also define both equilibrium structural factor $D(x)$ and the slope of the cost schedule B_{eq} optimal for the exchange.

Technically, the solution of such two-dimensional fixed point problem could be a difficult task, especially since one of the "variables" is actually a function $D(x)$. The reason why this is feasible in our case is that the solution actually depends only on a combination $\varepsilon = \frac{2B}{D(x)}$, which means that we have a special "scaling property" in this model. For this reason, the fixed-point problem becomes essentially one-dimensional and therefore more tractable. However, it still requires extensive numerical simulations.

5 Conclusion and Extensions

Agents who want access to real time price quotes pay considerable amounts for this service. This fee can range from a nominal amount for private accounts up to hundreds of thousands of dollars annually for the most reliable and comprehensive service from Reuters or Bloomberg. We derive the "price of prices" in a Kyle-type setting where we explicitly account for the fact that in the U.S., exchanges have property rights in the price quotes they generate. In the process, we document the beneficial welfare effects of granting individuals access to real-time pricing data (such as an increase in price informativeness). Since doing so may impact market liquidity in a detrimental way, the exchange needs to charge for this service.

When informed agents have equal precision of private information, and the exchange takes the liquidity impact of data sales on uninformed traders fully into account, we show the following. The exchange finds it optimal to sell a high precision signal (often of infinite precision) to informed traders. Due to the impact of data sales on liquidity, the exchange typically needs to charge for this service, and informed traders are willing to acquire it. We also provide conditions under which the exchange benefits when a second, lower precision signal is made publicly available. We interpret the first behavior as the sale of real time pricing data, while the second action corresponds to the free dissemination of a noisy signal of current price as in the publication of delayed price quotes. Empirical evidence shows that the speculative value of public information is greatly diminished after 5 to 15 minutes. This might possibly explain why U.S. exchanges delay free quotes by 10 to 30 minutes.

Roll (1984) has shown that prices contain substantial amounts of useful information. We follow Grossman and Stiglitz (1980) in postulating that the value of this information to traders is best measured by agents' willingness to pay for it. We observe that the major exchanges receive substantial amounts of their revenue from data sales. This allows us to conclude that the information content of market prices is of an economically significant magnitude. Our aim is to provide a simple model that explains the price of prices and to understand the economics of the sale of real time price data by exchanges such as the NYSE. Our model has several empirical implications.

Our analysis suggests that the price of prices is intimately related to the amount and extent of informed trading in a market. We predict that the price of prices is higher in markets in which more private information is present, as real time price information acts as a complement to private information. A measure such as the probability of informed trade, PIN, endeavors to capture what we are after. Note, however, that PIN is a relative measure of the percentage of informed trades, while we consider the absolute amount, so we would need to consider the multiple of PIN by trading volume. This reasoning implies that a market with substantial informed trading will have a significant price of prices. This might explain both the high price of real time data access and the significant amount of revenue produced by a major exchange such as the NYSE. Conversely, exchanges with less informed trade have a lower price of prices. For instance, one might compare markets for different securities. We conjecture that privately informed trading is less pronounced in bond markets (due to the bonds' concave pay-offs and lower market liquidity), our theory predicts that the price of prices is lower in such markets.

One of the most interesting aspects of the delay times reported in table 1 is the variation within Chinese markets. The mainland Chinese exchanges in Shanghai and Shenzhen provide price quotes for free. Hong Kong, on the other hand, has the longest delay in the sample, 60 minutes. We conjecture that a substantial amount of price discovery and informed trade takes place at Hong Kong, where market liquidity is greater. Thus, the price of prices may be more significant here. And while we do not claim that we can explain the differences completely, our results do indicate that even free disclosure of real time price information is in the interest of an exchange. If the expected revenue from selling real time data is low, then the exchange may abstain from doing so.

Our analysis in section 4 makes the interaction between the amount of private information and the price of prices more precise. Specifically, we show that the slope of the optimal cost schedule is indeed decreasing in the amount of private information in the market. Given the right data, this could lead to a more specific empirical test of our model. We show that the exchange typically finds it optimal to sell pricing information to a significant number of informed traders. In particular, real time price data is sold to all informed agents when the precision of their information is equal and uninformed traders are sufficiently compensated for their losses. In general, we find that it is the better informed agents who acquire more precise pricing information. This seems to coincide with our beliefs of which agents do purchase real time pricing data through services such as Bloomberg or Reuters. But it could certainly be tested given micro-level data.

Another interesting dimension of the price of prices is variations within a single exchange. As aforementioned, the NYSE offers a substantial discount to customers who are willing to accept a 5 second delay in prices. This can be explained by our model. As shown in section 4, when information precision differs across agents, the privately better informed ones purchase considerably more pricing information even in the presence of a super-linear price schedule such as a quadratic. Since the difference in precision of price information between a zero and a five second delay is very significant, we can rationalize a substantial price difference. Our result critically depends on competition among informed traders, which puts the purchaser of even marginally delayed information at a significant disadvantage. Modelling the specific dynamics of informed trade that enable us to say more about different periods of delay remains an interesting opportunity for further work.

Several interesting aspects remain to be explored in future research. On the one hand, one could analyze the role of liquidity providers and market making more explicitly. Certainly, institutions which attempt to create or unwind significant asset positions will benefit from observing current market demand by subscribing to real time prices. Alternatively, in a model of imperfect competition among market makers, each market maker may obtain information about order flow directed to other market makers through real time price access. This will be an interesting direction for future research. On the other hand, one could envision a model with richer, more complex dynamic trading strategies. For instance, the model of Back, Cao and Willard (2000) could be extended to allow for different classes of agents: some receive real time price access and some do not. Doing so will yield interesting insights into the price of prices, but will probably come at a hefty analytical expense.

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Table 1: **Descriptive Statistics: Delay Times of Major International Exchanges**

This table documents the delay times after which price quotes are freely available. Note that all the major exchanges make price quotes publicly available after a delay time ranging from 10 to 60 minutes, A notable exception are the exchanges in the emerging economies of China (Shanghai and Shenzhen Stock Exchanges) and India (Real-time data is available for the National Stock Exchange of India for users who sign in with Yahoo.com
Of particular interest is the cross-sectional variation within the major U.S. exchanges. Delay times vary from 10 minutes at the CBoT (who was first awarded property rights in price quotes in 1905) and CME, 15 minutes at NASDAQ, 20 minutes at the NYSE up to 30 minutes at NYMEX.
Data Source: Yahoo.com

Country	Exchange	Delay Time
USA	American Stock Exchange	20 minutes
USA	Chicago Board of Trade	10 minutes
USA	Chicago Mercantile Exchange	10 minutes
USA	NASDAQ	15 minutes
USA	New York Mercantile Exchange	30 minutes
USA	NYSE	20 mintes
USA	OTC Bulletin Board	20 minutes
USA	Pink Sheets	15 minutes
Canada	Toronto Stock Exchange	15 minutes
China	Shanghai & Shenzhen Exchanges	real-time (0 minutes)
France	Paris Stock Exchange	15 minutes
Germany	Frankfurt Stock Exchange & XETRA	15 minutes
Hong Kong	Hong Kong Stock Exchange	60 minutes
India	National Stock Exchange	15 minutes (real-time for users who sign in)
Italy	Milan Stock Exchange	20 minutes
Korea (South)	Korea Stock Exchange	20 minutes
Netherlands	Amsterdam Stock Exchange	15 minutes
Spain	Madrid Stock Exchange & Fixed Income Market	15 minutes
Switzerland	Swiss Exchange	30 minutes
Taiwan	Taiwan Stock Exchange	20 minutes
UK	London Stock Exchange	20 minutes

6 Appendix: Proofs

Proof of Proposition 1

The expected payoff of agent i is given by

$$\overline{\pi}_i = E_t\left[\beta_i\left(S_i - \widetilde{P}_i\right)\left(V - P - \lambda\beta_i\left(S_i - \widetilde{P}_i\right)\right)\right], \tag{68}$$

with

$$\begin{aligned} P &= V + \sigma_\delta \widetilde{\varepsilon}_\delta, \\ S_i &= V + \sigma_{s,i}\widetilde{\varepsilon}_{s,i}, \\ \widetilde{P}_i &= P + \sigma_{p,i}\widetilde{\varepsilon}_{p,i}. \end{aligned} \tag{69}$$

Here, $\widetilde{\varepsilon}_\delta$, $\widetilde{\varepsilon}_{s,i}$ and $\widetilde{\varepsilon}_{p,i}$ are standard Normal random variables. Assuming that all signals are uncorrelated, we obtain

$$\overline{\pi}_i = \beta_i\sigma_\delta^2\left(1 - \lambda\beta_i\frac{\sigma_\delta^2 + \sigma_{\eta,i}^2 + \sigma_{\varepsilon,i}^2}{\sigma_\delta^2} - \lambda\sum_{j\neq i}\beta_j\right). \tag{70}$$

The FOC applied to (70) yields

$$\beta_i = \frac{1}{2\lambda}\frac{\sigma_\delta^2}{\sigma_\delta^2 + \sigma_{\eta,i}^2 + \sigma_{\varepsilon,i}^2}\left(1 - \lambda\sum_{j\neq i}\beta_j\right). \tag{71}$$

Solving (71), we obtain

$$\beta_i = \frac{1-\lambda q}{\lambda}\frac{\sigma_\delta^2}{\sigma_\delta^2 + \sigma_{\eta,i}^2 + \sigma_{\varepsilon,i}^2}, \tag{72}$$

with

$$\begin{aligned} q &= \frac{1}{\lambda}\frac{1}{1+f}, \\ f &= \sum_{j=1}^{n}\frac{\sigma_\delta^2}{\sigma_\delta^2 + 2\sigma_{\eta,i}^2 + 2\sigma_{\varepsilon,i}^2}. \end{aligned} \tag{73}$$

Since the losses of liquidity traders equal the aggregate payoffs of the informed agents, the he liquidity parameter λ satisfies

$$\begin{aligned} \lambda\sigma_u^2 &= \frac{\sigma_\delta^2}{\lambda}\frac{g}{(1+f)^2}, \\ g &= \sum_{j=1}^{n}\left(\frac{\sigma_\delta^2}{\sigma_\delta^2 + 2\sigma_{\eta,i}^2 + 2\sigma_{\varepsilon,i}^2}\right)^2. \end{aligned} \tag{74}$$

Combining (72), (73) and (74), we obtain, the results of the proposition.

Q.E.D.

Proof of Proposition 2

Proposition 2 simply follows from proposition 1 by setting the number of informed traders in period t, n_t, equal to one. Then we compare the single informed traders expected payoff $\pi_{1,t}$ depending on whether he acquires pricing information or not. The difference is found to be always positive and is denoted as δ_t in proposition 2. With a single informed trader, any additional profits he makes need to come at the expense of the uninformed traders, which implies that the exchange, which needs to compensate the uninformed traders expected losses, is exactly indifferent between selling real-time pricing information and not selling it.

Q.E.D.

Proof of Proposition 3

Let $h_{\epsilon,1} = 1/\sigma^2_{\epsilon,1}$ denote the agent's precision of private information and, similarly, let $h_{\nu,1} = 1/\sigma^2_{\nu,1}$ denote the agent's precision of pricing information. The agent's marginal utility of private information is given as

$$\frac{\partial \bar{\pi}_1}{\partial h_{\epsilon,1}} = \frac{\sigma_\delta^2}{4h_{\epsilon,1}\sigma_{u,1}\left(1/h_{\epsilon,1} + 1/h_{\nu,1} + \sigma_\delta^2\right)^{3/2}}. \tag{75}$$

We first show that the marginal utility is increasing in the agent's precision of pricing information, $h_{\nu,1}$. Indeed, the second-order cross derivative is given

$$\frac{\partial^2 \bar{\pi}_1}{\partial h_{\nu,1}\partial h_{\epsilon,1}} = \frac{3\sigma^2_{\delta,}}{8h_{\epsilon,1}h_{\nu,1}\sigma_u\left(1/h_{\epsilon,1} + 1/h_{\nu,1} + \sigma_\delta^2\right)^{5/2}}, \tag{76}$$

which is positive. Next, we show that the agent's two-dimensional information acquisition problem is convex, which concludes our proof. We first obtain

$$\frac{\partial^2 \bar{\pi}_1}{\partial h^2_{\epsilon,1}} = \frac{h_{\nu,1}\sigma_\delta^2\left(h_{\nu,1} + 4h_{\epsilon,1} + 4h_{\epsilon,1}h_{\nu,1}\sigma_\delta^2\right)}{8h^2_{\epsilon,1}\sigma_u\left(1/h_{\epsilon,1} + 1/h_{\nu,1} + \sigma_\delta^2\right)^{1/2}\left(h_{\epsilon,1} + h_{\nu,1} + h_{\epsilon,1}h_{\nu,1}\sigma_\delta^2\right)}, \tag{77}$$

as well as

$$\frac{\partial^2 \bar{\pi}_1}{\partial h^2_{\nu,1}} = \frac{h_{\epsilon,1}\sigma_\delta^2\left(h_{\epsilon,1} + 4h_{\nu,1} + 4h_{\epsilon,1}h_{\nu,1}\sigma_\delta^2\right)}{8h^2_{\nu,1}\sigma_u\left(1/h_{\epsilon,1} + 1/h_{\nu,1} + \sigma_\delta^2\right)^{1/2}\left(h_{\epsilon,1} + h_{\nu,1} + h_{\epsilon,1}h_{\nu,1}\sigma_\delta^2\right)}, \tag{78}$$

We now compute the determinant of the Hessian matrix $D^2\bar{\pi}_1$, which is found to be

$$\frac{\sigma_\delta^2\left(h_{\epsilon,1} + h_{\nu,1} + 4h_{\epsilon,1}h_{\nu,1}\sigma_\delta^2\right)}{16\sigma_u^2\left(h_{\epsilon,1} + 4h_{\nu,1} + h_{\epsilon,1}h_{\nu,1}\sigma_\delta^2\right)^4} > 0 \tag{79}$$

Thus, the agent's maximization problem is concave, and the solution thus is unique.

The second part of the proposition is obvious. The sale of information that is already in possession of the exchange eliminates the costly expenditure of resources by the agent to uncover it again.

Q.E.D.

Proof of Proposition 4

The exchange maximizes its revenue net of liquidity costs, $L_1(n_1, c) = n_1 c - \lambda\sigma_u^2$, by setting $c = \bar{\pi}^a - \bar{\pi}^{na}$ as a function of the number of informed agents who choose to acquire real-time pricing data. Thus, implicitly, the exchange can control how many agents will do so. We can show that the objective function of he exchange $L(n_1)$ is negative and is monotonically increasing in n_1. The objective function of the exchange is given by

$$L_1(n_1) = n_1\left(\bar{\pi}^a(n_1) - \bar{\pi}^{na}(n_1)\right) - \lambda\sigma_u^2. \tag{80}$$

Since the total amount of profits of the informed agents equals the total losses of liquidity traders, we have

$$\lambda\sigma_u^2 = n_1\bar{\pi}^a(n_1) + (n - n_1)\,\bar{\pi}^{na}(n_1), \tag{81}$$

which after substitution into the exchange objective function yields

$$L_1(n_1) = -n\bar{\pi}^{na}(n_1). \tag{82}$$

From (32) with (33), it follows that $\bar{\pi}^{na}(n_1)$ monotonically decreases in n_1. Therefore, the objective function (82) is maximized by the maximal value of $n_1 = n$. The optimal value of (82) is therefore

$$L_1^* = L_1(n) = -n\bar{\pi}^{na}(n). \tag{83}$$

Combining (83) with (33) and substituting into (36), we obtain the second result of the proposition. The optimal profits are positive since $x_1 \geq x_2$.

It is straightforward to show that price informativeness in the market is higher with all agents acquiring price information (the equilibrium strategy for the exchange) as opposed to no agent acquiring price information.

Q.E.D.

Proof of Corollary 1

Follows from the text.

Q.E.D.

Proof of Proposition 5

Differentiating (47) with respect to x_1, we obtain the FOC

$$\left(x_1 + \frac{1}{2}\right)(1 + n_1 x_1) - n_1 x_1 (1 + x_1) = 0, \tag{84}$$

which has a unique solution

$$x_1^* = \frac{1}{n_1 - 2}. \tag{85}$$

Taking into account (49), we conclude that (85) holds if the number of high type agents is sufficiently large

$$n_1 \geq n_c + 2 = 3 + \frac{2}{\mu_s^2}. \tag{86}$$

If (86) does not hold, the optimal precision of the price signal is infinite. Summarizing the above, we have

$$\begin{aligned} x_1^* &= \frac{1}{n_1 - 2}, \qquad n_1 \geq n_c + 2, \\ x_1^* &= \frac{1}{n_c}, \qquad n_1 \leq n_c + 2. \end{aligned} \tag{87}$$

Note that the difference between our result and the corresponding one parallel to (Admati and Pfleiderer, 1990) is that our critical number contains an extra term proportional to the inverse relative private precision $\frac{2}{\mu_s^2}$, which vanishes in the limit of infinite precision of private information, but may be quite large for sufficiently low private precision. Taking (85) and (86) into account and combining with (47), we obtain the objective of the exchange optimized with respect to the precision of the price signal, in the form

Q.E.D.

Proof of Proposition 6

The marginal value of the information for the i-th informed trader is given by

$$\begin{aligned} \frac{\partial \eta_i}{\partial s_i} &= \frac{\partial \eta_i}{\partial x_i}\frac{\partial x_i}{\partial s_i}, \\ \frac{\partial x_i}{\partial s_i} &= -2x_i^2 \leq 0. \end{aligned}$$

Following the setting of the previous subsection, the marginal value of the price information for the k-th informed trader is given by

$$p\left(s_i, z_i, x\right) = \frac{\partial \eta_i}{\partial s_i} = -2x_i^2\left(1+2x_i\right) D\left(x\right). \tag{88}$$

Since $\mu_{p,i} = \frac{1}{s_i}$ and $\mu_{s,i} = \frac{1}{z_i}$, the marginal value is also defined in terms of signal precision, in the form

$$\begin{aligned} p_\mu\left(\mu_{p,i}, \mu_{s,i}, x\right) &= \frac{\partial \eta_i}{\partial \mu_{p,i}} = -\left(\frac{1}{\mu_{p,i}}\right)^2 \frac{\partial \eta_i}{\partial s_i} \\ &= \frac{2x_i^2}{\mu_{p,i}^2}\left(1+2x_i\right) D\left(x\right). \end{aligned} \tag{89}$$

Note that (88) is obtained by the partial differentiation of (50) and taking into account that the "structural factor" $D\left(x\right)$ only depends on the information distribution across the agents, and not on the actions of a particular agent. This is consistent with the optimization scheme of the previous subsection, where the agents' willingness to pay was defined by comparing their expected payoffs in the states with the high and low information quality assuming that the distribution of the information (characterized in that case only by a number of the agents in a high information state n_1) is not affected by the actions of the informed agent. We also have

$$\begin{aligned} \frac{\partial \eta_i}{\partial \mu_{p,i}} &= \frac{\partial \eta_i}{\partial x_i}\frac{\partial x_i}{\partial \mu_{p,i}}, \\ \frac{\partial x_i}{\partial \mu_{p,i}} &= 2\mu_{p,i}^2 x_i^2 \geq 0, \\ \frac{\partial x_i}{\partial \mu_{s,i}} &= 2\mu_{s,i}^2 x_i^2 \geq 0. \end{aligned}$$

Differentiating the marginal profits with respect to the precision of the private signal $\mu_{s,i}$ and taking into account that the structural factor $D\left(x\right)$ does not change, we obtain

$$\begin{aligned} J_i &= \frac{\partial^2 \eta_i}{\partial \mu_{s,i}\partial \mu_{p,i}} = 4\mu_{p,i}^2\mu_{s,i}^2 D\left(x\right)\ x_i^2 \frac{\partial}{\partial x_i}\left(x_i^2\left(1+2x_i\right)\right) \\ &= 8\mu_{p,i}^2\mu_{s,i}^2 F\left(x\right)\ x_i^3\left(1+3x_i\right) \geq 0, \end{aligned} \tag{90}$$

implying that the marginal profits of the informed traders with respect to the price precision increase in the precision of their private signals.

Q.E.D.

Proof of Proposition 7

Making use of (12), we obtain

$$\begin{aligned} \frac{\partial \lambda}{\partial \mu_i} &= 2x_i^2 s_i^2 \frac{\partial \lambda}{\partial x_i}, \\ \frac{\partial \lambda}{\partial x_i} &= \frac{1}{\sqrt{2}\beta_0}\left(\frac{1}{2}\frac{1+2x_i}{\left(1+f\right)\sqrt{f+g}} - \frac{\sqrt{f+g}}{\left(1+f\right)^2}\right). \end{aligned} \tag{91}$$

From (91), it follows that

$$\begin{aligned} \frac{\partial \lambda}{\partial \mu_i} &\geq 0, \qquad x_i \geq x_e, \\ \frac{\partial \lambda}{\partial \mu_i} &\leq 0, \qquad x_i \leq x_e, \end{aligned} \tag{92}$$

where

$$x_e = \frac{\frac{1}{2} f(x) + g(x) - \frac{1}{2}}{1 + f(x)}. \tag{93}$$

Recalling the definition of $f(x)$ and $g(x)$ from (50), we obtain from (93) in the large-N limit

$$x_e = \frac{1}{2} + \frac{\overline{x^2}}{\overline{x}} = \overline{x} + \frac{Var(x)}{\overline{x}} + \frac{1}{2}, \tag{94}$$

with

$$\begin{aligned} \overline{x} &= \frac{f(x)}{n} = \frac{1}{n} \sum_{j=1}^{n} x_j, \\ \overline{x^2} &= \frac{g(x)}{n} = \frac{1}{n} \sum_{j=1}^{n} x_j^2. \end{aligned} \tag{95}$$

Combining (92) and (94), we finally arrive at

$$\begin{aligned} \frac{\partial \lambda}{\partial \mu_i} &\geq 0, \qquad x_i \geq \overline{x} + \frac{Var(x)}{\overline{x}} + \frac{1}{2}, \\ \frac{\partial \lambda}{\partial \mu_i} &\leq 0, \qquad x_i \leq \overline{x} + \frac{Var(x)}{\overline{x}} + \frac{1}{2}. \end{aligned} \tag{96}$$

Q.E.D.

Proof of Proposition 8
Follows immediately from the text.
Q.E.D.

Proof of Proposition 9
Since the p.d.f. $\rho_s(\mu_s)$ of the precision of private signals across the agents is uniform, we have for the p.d.f. of the precision of price signals across the agents $\rho_p(\mu_p)$

$$\begin{aligned} \rho_p(\mu_p) &= \rho_s(\mu_s(\mu_p)) \frac{d\mu_s(\mu_p)}{d\mu_p} \\ &= \rho_s(\Gamma(\mu_p)) \frac{d\Gamma(\mu_p)}{d\mu_p}, \end{aligned} \tag{97}$$

with the function $\Gamma(\mu_p)$ defined from the equilibrium condition (63), as an isolevel curve of the function (64). Differentiating (64), we obtain

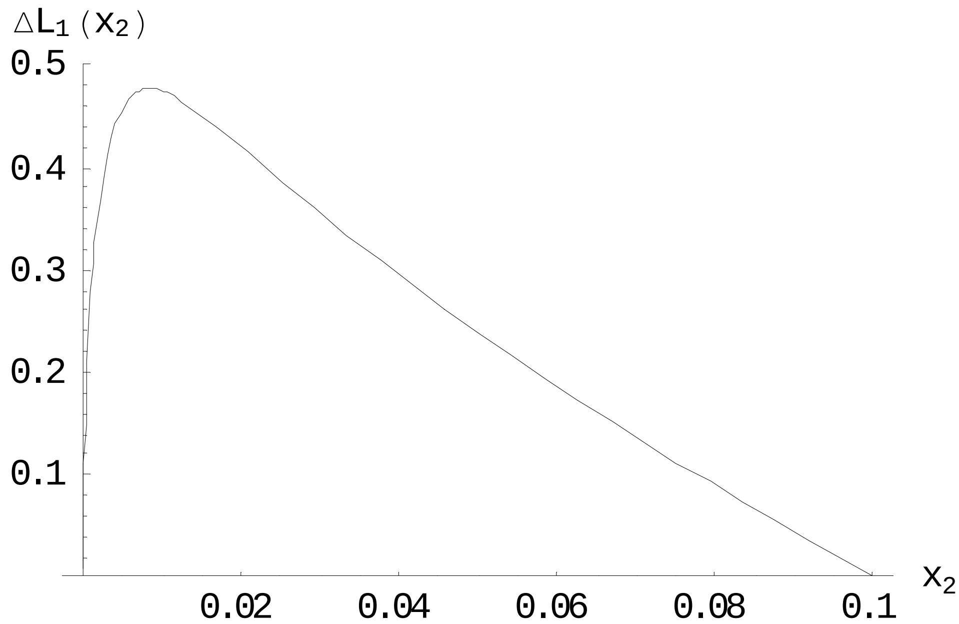
$$\frac{d\Gamma(\mu)}{d\mu} = \frac{\Gamma^3(\mu)}{\mu^2} \frac{m_1}{m_2 m_3 - m_4}, \tag{98}$$

where

$$\begin{aligned} m_1 &= \mu + \Gamma(\mu) + 2\mu\Gamma(\mu), \\ m_2 &= \mu + \Gamma(\mu) + \frac{3}{2}\mu\Gamma(\mu), \\ m_3 &= \mu + \Gamma(\mu) + \frac{1}{2}\mu\Gamma(\mu), \\ m_4 &= \frac{1}{2}\mu\Gamma^2(\mu), \end{aligned} \tag{99}$$

and the above conditions are satisfied simultaneously with (64). This is done using numerical simulations.
Q.E.D.

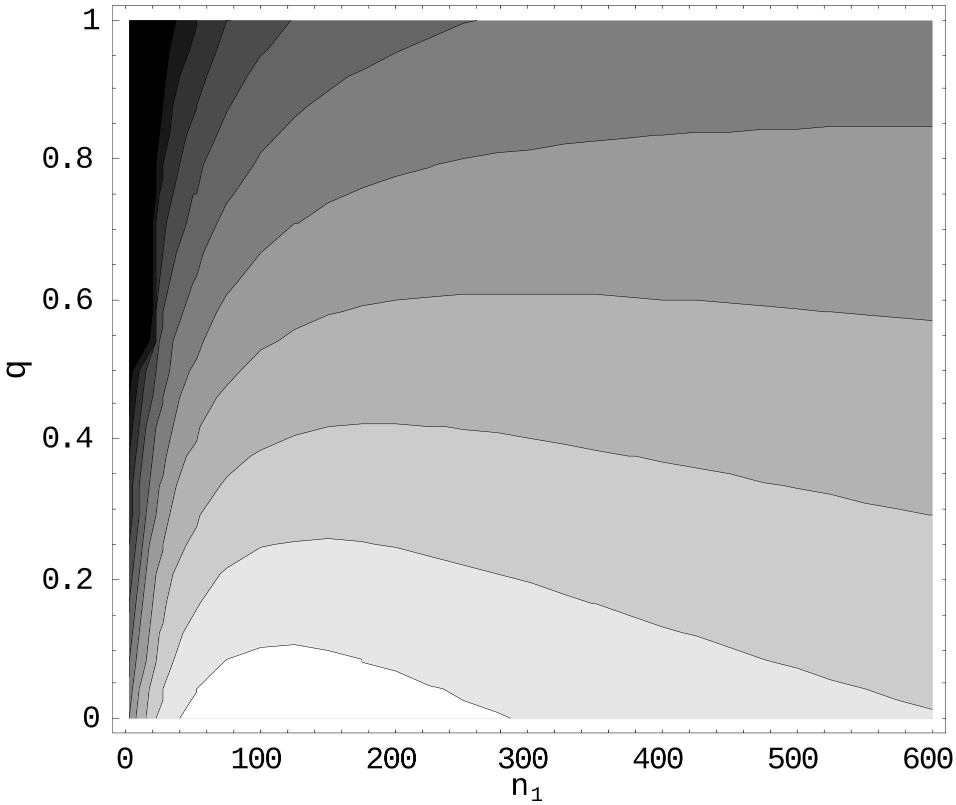
Figure 1: Profit of exchange as a function of the signal-to-noise of low signal.



This figure shows a typical profile of the profit function of the exchange as a function of a signal-to-noise ratio of the low signal, x_2. In this case, the parameters are $n = 100$ and $x_m = 0.1$. Clearly, the profit function has a peak at $x_2^* \cong 0.01$. This is consistent with the estimate obtained in the text.

Importantly, the above figure illustrates that the profit of exchange is maximal at some finite positive value of the signal-to-noise ratio of the low signal. This means that the optimal precision of the low signal being disseminated by the exchange for free, is also greater than zero, consistent with the stylized facts on the price information dissemination policies discussed in the text.

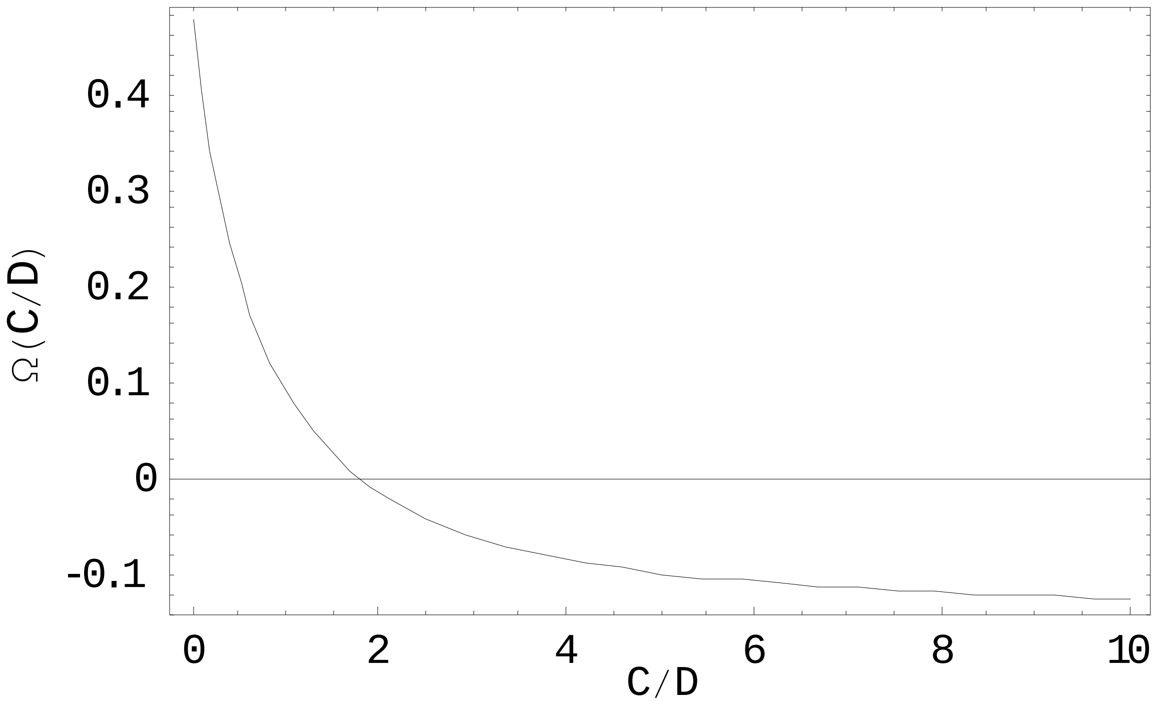
Figure 2: Exchange objective as a function of the compensation parameter and the number of high type agents



Two-dimensional contour plot of the exchange objective function as a function of the compensation parameter q ∈ [0, 1] and the number of high type agents n_1.

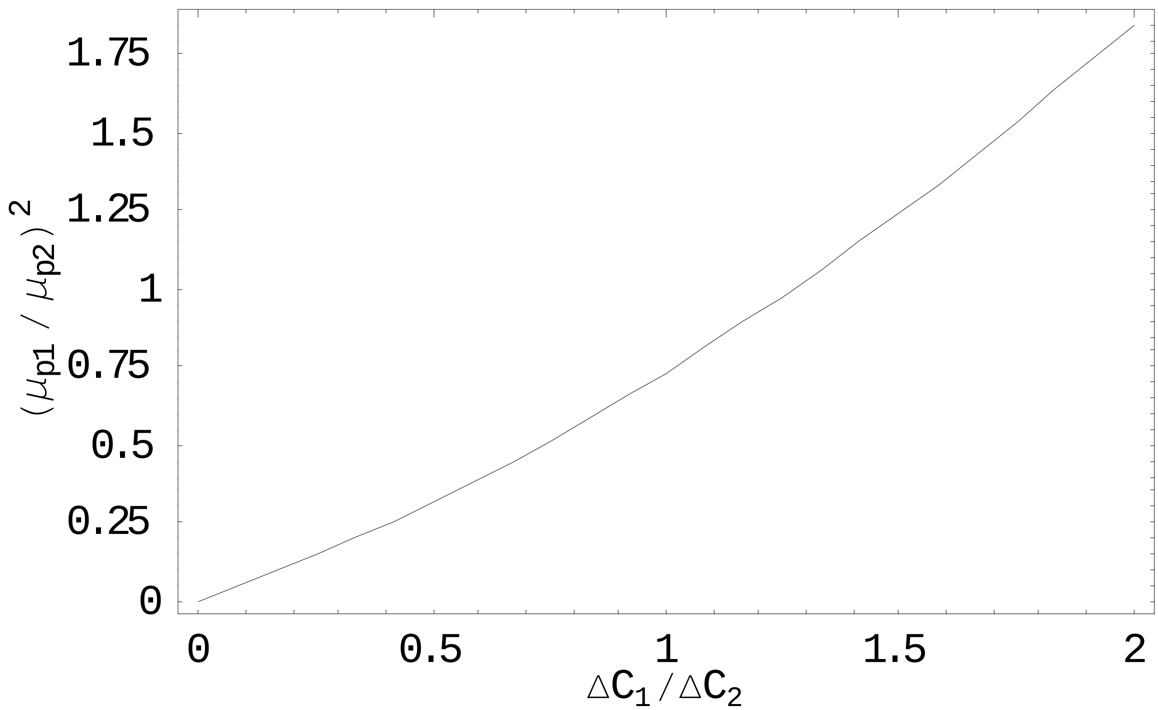
The total number of informed agents is n=1000 and the high price signal has a precision of $\mu_{p,1} = 10$. As is clear from Fig.2, the critical value in this case is $q_c \cong 0.6$.

Figure 3: Incentive Compatibility Conditions with Binary Distribution of Precision



This figure depicts the incentive compatibility conditions for the revenue maximizing cost schedule $C\left(\mu_{s,i}\right)$. These conditions ensure a separating equilibrium in the case of a binary distribution of the precision of private information, μ_s. On the horizontal axis, we plot the revenue maximizing cost schedule C scaled by the structural equilibrium parameter D. On the vertical axis, we plot the corresponding incentive compatibility constraint $\Omega(C/D)$. Specifically, the incentive compatibility constraint requires that $\Omega(C_1/D) \leq 0$ and $\Omega(C_2/D) \geq 0$. According to Fig.1, these conditions are satisfied for $C_1/D \geq 2$ and $C_2/D \leq 2$, and are therefore satisfied for our base case parameters $\mu_{S,1} = 5$, $\mu_{S,2} = 3$, $e_1 = 2$, $e_2 = 1$, and $\Delta C_2 = 1/5$.

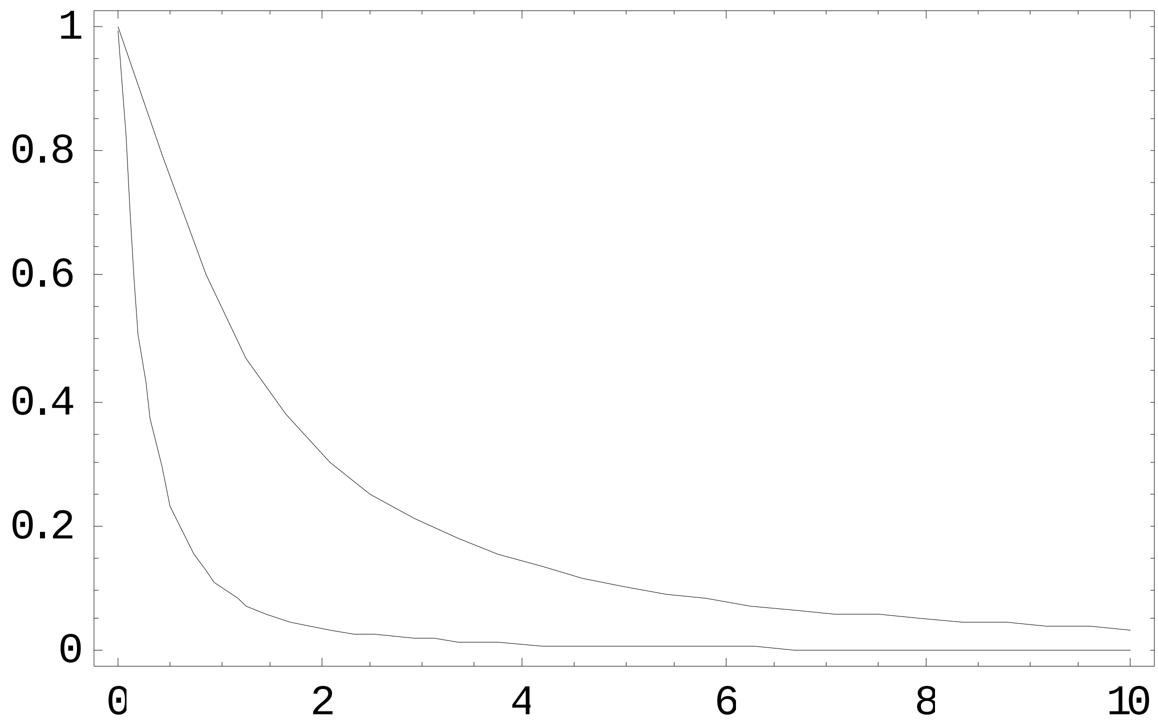
Figure 4: Revenue Maximizing Cost Schedule with Binary Distribution of Precision



We characterize the revenue maximizing cost schedule $\Delta C_i\left(\mu_{p,i}\right)$ - shown on the horizontal axis as a fraction – as a function on the precision of private information, μ_p. This plot is properly scaled to suit the binary distribution of agents' precision of private information. In other words, for a given ratio $\Delta C_1 / \Delta C_2$ between cost of obtaining more precise price information as opposed to less precise, there is a corresponding ratio of precisions of private information $\mu_{P,1} / \mu_{P,2}$.

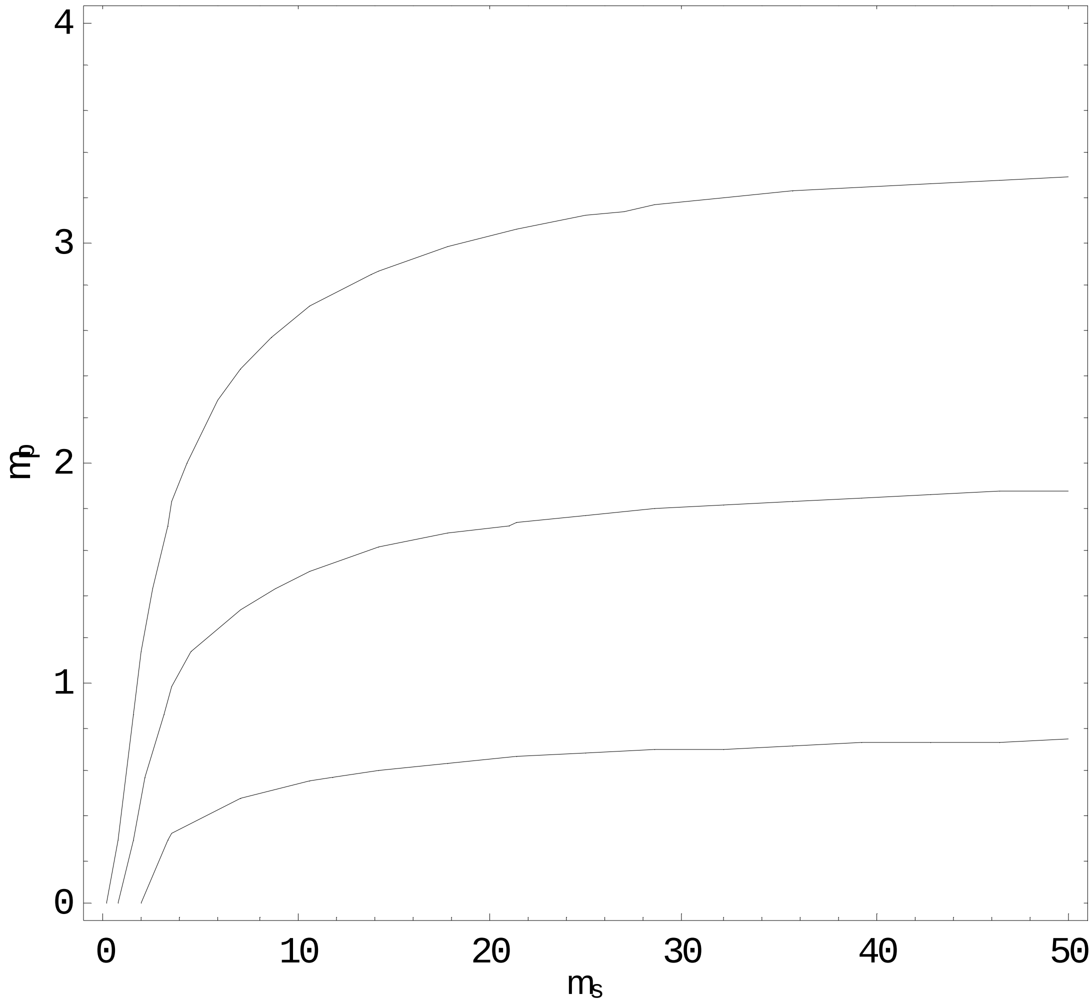
Note that $\Delta C_i\left(\mu_{p,i}\right)$ is monotonically increasing in $\mu_{p,i}$, and can be approximated by a linear function in the range of $\mu_{p,i}$ that belongs to our base case set of parameters and satisfies the incentive compatibility condition. Thus our choice of plotting $(\mu_{P,1} / \mu_{P,2})^2$. As before, our parameters are $\mu_{S,1} = 5$, $\mu_{S,2} = 3$, $e_1 = 2$, $e_2 = 1$, and $\Delta C_2 = 1/5$.

Figure 5: Marginal Value of Price Information as a Function of Private Information



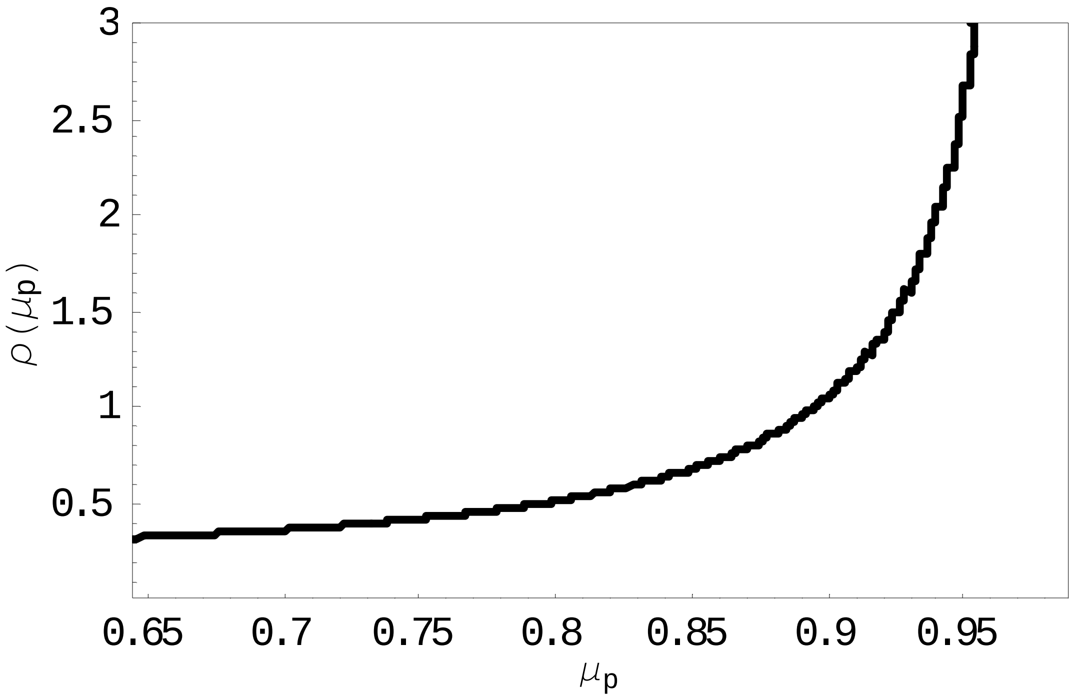
This figure shows the marginal value of information, $\Phi(\mu_p, \mu_s)$, as a function of the price precision μ_p, for two precisions of private signal μ_s. The lower and upper curves correspond to parameter values of $\mu_s = 0.5$ and $\mu_s = 5$, respectively. Clearly, the marginal value $\Phi(\mu_p, \mu_s)$ increases in the precision of the private signal, reflecting the fact that the two kinds of signals are complementary in our model. Furthermore, note that $0 \le \Phi(\mu_p, \mu_s) \le 1$ for any positive precisions, and therefore the conditions for equilibrium derived in the text can only be satisfied within the parameter range $\varepsilon \in [0,1]$.

Figure 6: Equilibrium price precision as a function of private precision



This figure depicts the equilibrium precision of acquired pricing information, $\mu_p(\mu_s)$, as a function of the precision of private information μ_s. This is done for several values of the cost schedule parameter $\varepsilon = \frac{2B}{D}$. The upper curve corresponds to $\varepsilon = 0.8$, the middle curve to $\varepsilon = 0.5$, and the lower curve to $\varepsilon = 0.3$. Note that as the cost parameter increases, the maximal acquired price precision goes down.

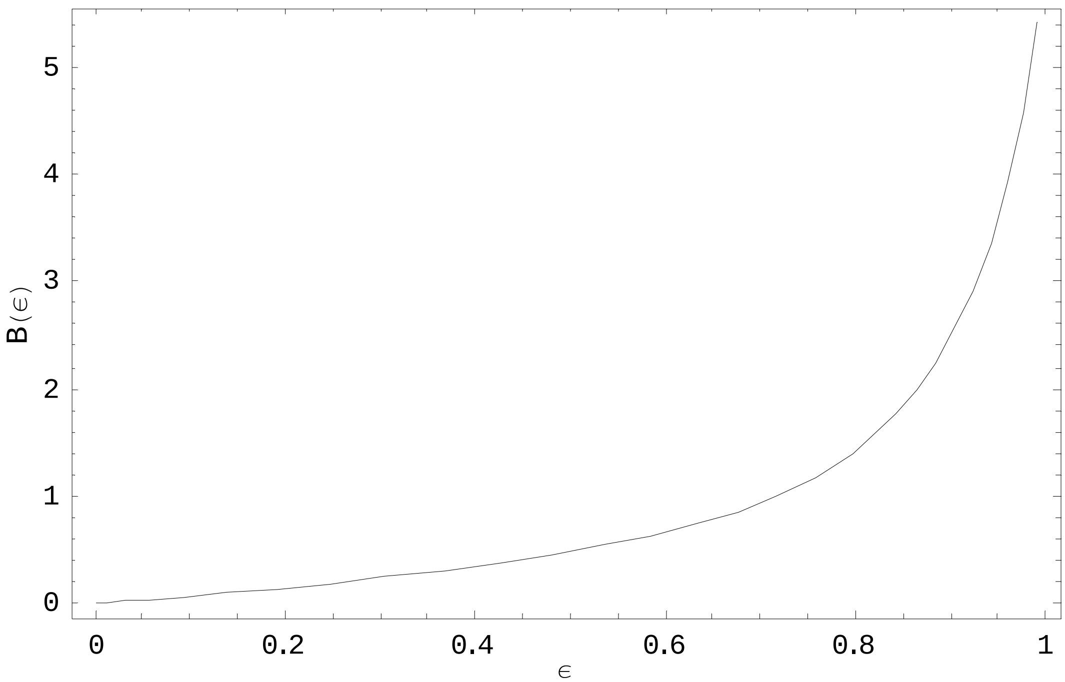
Figure 7: Equilibrium probability density of price precision



This figure plots the equilibrium probability density of the distribution of price precision in the case when the precision of private signals is uniformly distributed across the informed agents. Here, the cost schedule parameter is given by $\varepsilon = \frac{2B}{D} = 0.5$ and the distribution of the precisions of the private signals is uniform with $\mu_{\max} = 10$.

As one can see, the distribution is right-skewed in comparison to the uniform distribution of private information. The reason is that the agents with higher precisions of their private signals also tend to acquire more precise information on price. For this reason, the p.d.f. increases for the large price precisions.

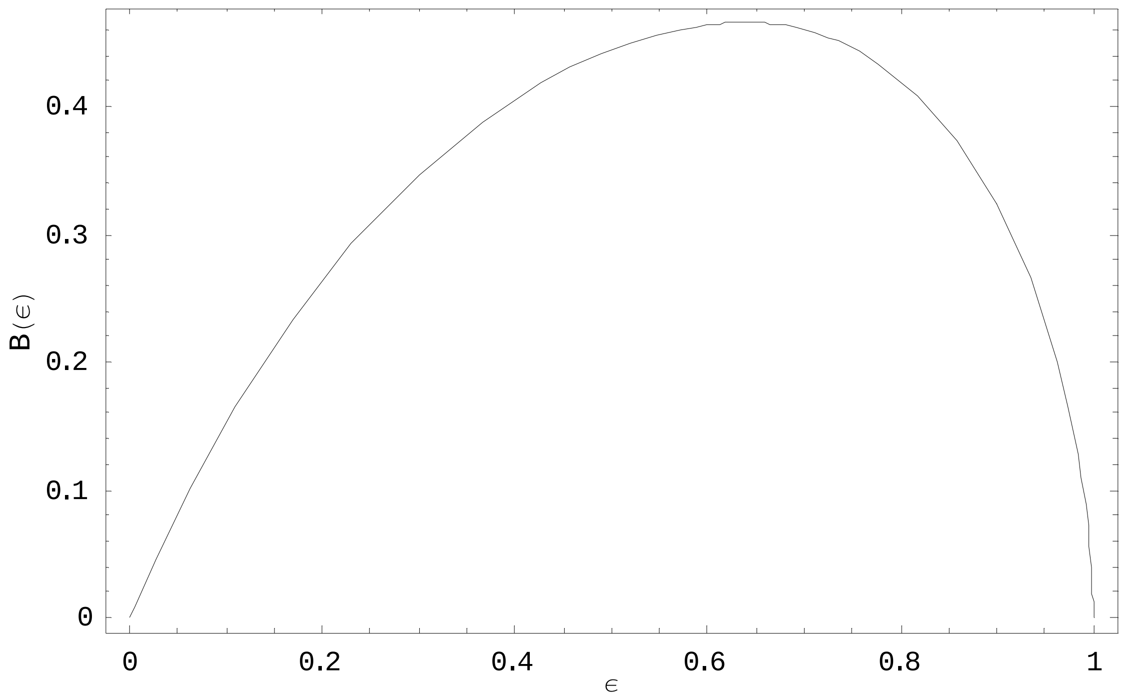
Figure 8: Equilibrium cost slope as a function of the cost schedule parameter



This plot illustrates the dependency of the slope of the cost function, B, as a function of the cost schedule parameter ε. An equilibrium only exists if $\varepsilon \in [0;1]$. As before, the distribution of the precisions of the private signals is uniform with $\mu_{s,\max} = 10$.

Note that $B(\varepsilon)$ is monotonically increasing and, therefore, the inverse function is uniquely defined. Since the effective slope parameter completely defines the equilibrium, this implies that the equilibrium is uniquely defined by a cost slope B.

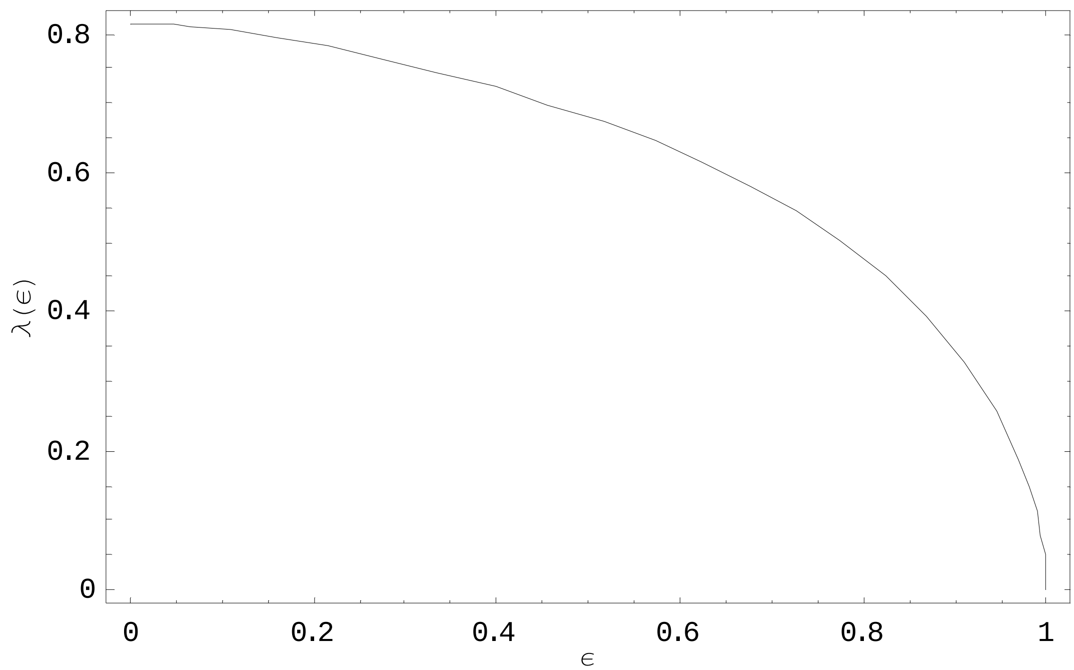
Figure 9: Exchange revenue as a function of the effective slope parameter



Here, we demonstrate how the exchange's revenues, $R(\varepsilon)$, depend on the cost schedule parameter ε . An equilibrium only exists if $\varepsilon \in [0;1]$, thus our choice of a parameter range. The distribution of the precisions of the private signals is uniform with $\mu_{s,\max} = 10$.

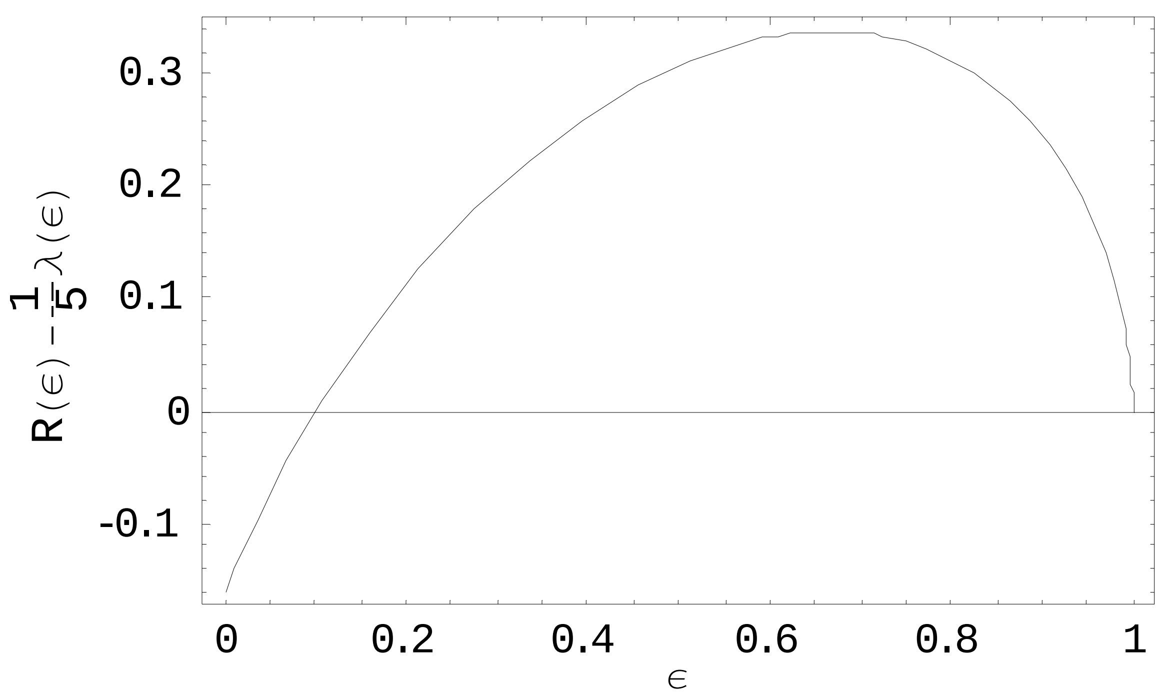
Note that $R(\varepsilon)$ has a maximum of $R_{\max} \approx 0.54$ at $\varepsilon_m \approx 0.67$. This implies that the slope of equilibrium cost schedule B that maximizes the revenues of the exchange is given by condition $B_{eq} = B(\varepsilon_m) \approx B(0.67)$. As can be seen in figure 4, we obtain $B_{eq} \approx 0.5951$.

Figure 10: Inverse market depth as a function of the effective slope parameter



This figure graphs the inverse of market depth, $\lambda(\varepsilon)$, against the effective slope parameter ε. The equilibrium only exists if $\varepsilon \in [0;1]$. The distribution of the precisions of the private signals is uniform with $\mu_{s,\max} = 10$. It can be seen that $\lambda(\varepsilon)$ is monotonically decreasing in the entire range $\varepsilon \in [0;1]$.

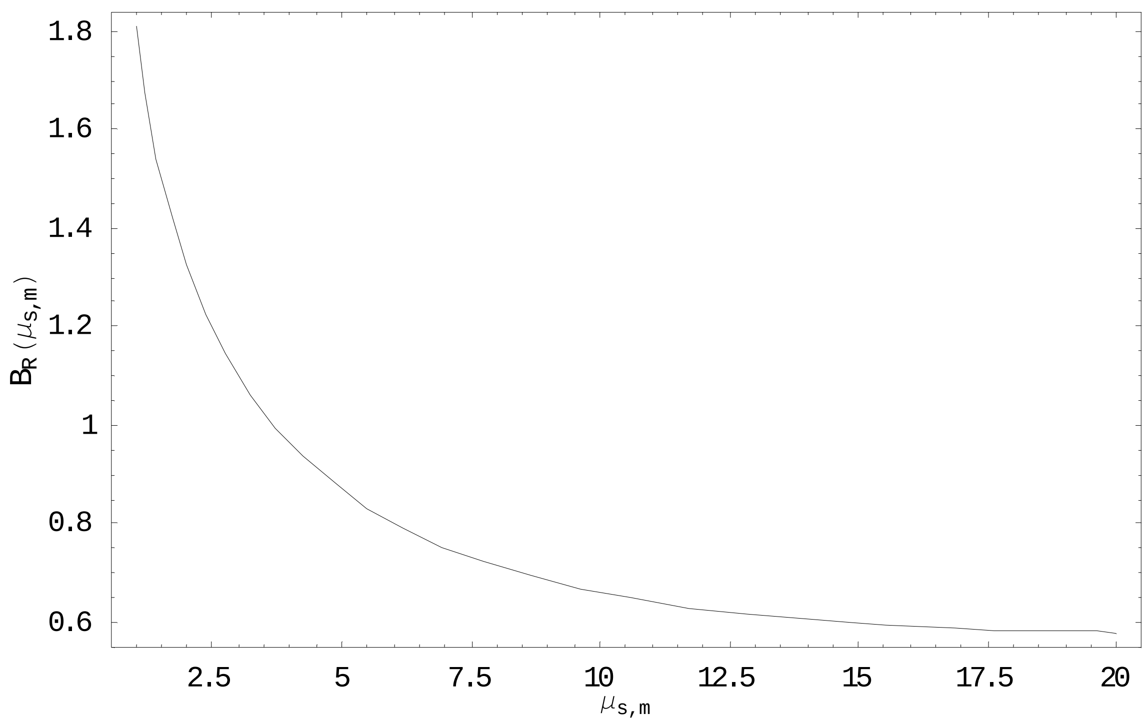
Figure 11: Exchange objective as a function of the effective slope parameter



The previous plot, figure 8, showed exchange revenue ignoring the impact that sale of pricing data has on market liquidity. By contrast, here we illustrate how the exchanges true objective function, $L(\varepsilon) = R(\varepsilon) - q\lambda(\varepsilon)$, varies with the cost schedule parameter ε. The term $q\lambda(\varepsilon)$ is the expected compensation the exchange needs to provide to uninformed traders – we can refer to it as the exchanges shadow cost of illiquidity.
In this case, the weight is q = 0.2. The equilibrium only exists if $\varepsilon \in [0;1]$. The distribution of the precisions of the private signals is uniform with $\mu_{s,\max} = 10$.

Note that $L(\varepsilon)$ has a maximum of $L_{\max} \approx 0.41$ at $\varepsilon_L \approx 0.75$. This implies that the slope of equilibrium cost schedule B that maximizes $L(\varepsilon)$ is given by condition $B_L = B(\varepsilon_L) \approx B(0.75)$. As can be seen in figure 4, this corresponds to a value of $B_L \approx 0.77$.

Figure 12: Revenue maximizing slope of the cost schedule as a function of the maximum precision of the private information



Here, we show how the exchange's revenue maximizing slope of the cost schedule, $B_R\left(\mu_{s,m}\right)$, depends on the maximum precision of the private information $\mu_{s,m}$.

Note that $B_R\left(\mu_{s,m}\right)$ monotonically decreases in $\mu_{s,m}$, implying that the indirect effect on the agents' strategies due to the collective contribution of all other agents, prevails in the range of parameters under consideration.