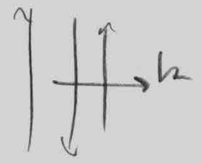
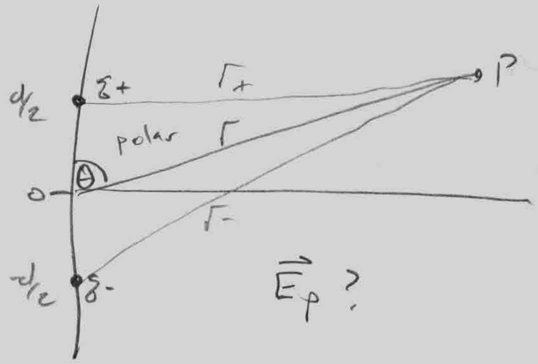
Rayleigh Scattering



$q_+ = q\cos(\omega t)$

$q_- = -q\cos(\omega t)$



plane wave + dielectric sphere = oscillating dipole moment

Use potentials : $\vec{E} = \nabla V$

$$V_P = \frac{1}{4\pi\epsilon_0}\left[\frac{q\cos(\omega(t - r_+/c))}{r_+} + \frac{q\cos(\omega(t - \frac{r_-}{c}))}{r_-}\right]$$

Simplify the 4 parts:

$$r_+ = \sqrt{\left(\frac{d}{2}\right)^2 + r^2 - 2\frac{d}{2}r\cos\theta}$$

$$\frac{1}{r_+} = \left(r^2 + \left(\frac{d}{2}\right)^2 - rd\cos\theta\right)^{-1/2}$$

expand for small d (around $d=0$)

$$= (r^2)^{-1/2}(d-0)^0 + -\frac{1}{2}(r^2 + 0 - 0)^{-3/2}(\varnothing + \varnothing - r\cos\theta)(d-0)^1$$

$$+ \ldots$$

$$\frac{1}{r_+} \simeq \frac{1}{r} + \frac{1}{2}r^{-3}r\cos\theta\, d = \frac{1}{r} + \frac{1}{2}r^{-2}d\cos\theta$$

$$\boxed{\frac{1}{r_+} \simeq \frac{1}{r}\left(\overset{a\ +\ b}{1 + \frac{d\cos\theta}{2r}}\right)} \quad \mathrm{I}$$

similar $$\boxed{\frac{1}{r_-} = \frac{1}{r}\left(\overset{a\ -\ b}{1 - \frac{d\cos\theta}{2r}}\right)} \quad \mathrm{II}$$

$$\cos\left(\omega\left(t - \frac{r_+}{c}\right)\right) \approx \cos\left(\omega\left(t - \frac{1}{c}(r)\left(1 + \frac{d\cos\theta}{2r}\right)^{-1}\right)\right)$$

$\frac{1}{1+x}$ expand around $x=0 \rightarrow 1-x$

$\frac{1}{1-x}$ " " " $\rightarrow 1+x$

$$\approx \cos\left(\omega\left(t - \frac{r}{c}\left(1 - \frac{d\cos\theta}{2r}\right)\right)\right)$$

$\cos(a+b)$
$= \cos(a)\cos(b) - \sin(a)\sin(b)$

$$\approx \cos\left(\omega t - \frac{\omega r}{c} + \frac{\omega d\cos\theta}{2c}\right)$$

$$\approx \cos\left(\omega t - kr + \frac{kd}{2}\cos\theta\right)$$

$d \ll \lambda$, so kd is small. expand around $kd = \emptyset$

$$\approx \cos(\omega t - kr) - \tfrac{1}{2}\sin(\omega t - kr)\cos\theta(kd)$$

$$\cos\left(\omega\left(t - \frac{r_+}{c}\right)\right) \approx \overset{c}{\cos(\omega t - kr)} \overset{-}{-} \overset{d}{\frac{kd}{2}\sin(\omega t - kr)\cos\theta} \quad \text{III}$$

$$\cos\left(\omega\left(t - \frac{r_-}{c}\right)\right) \approx \overset{c}{\cos(\omega t - kr)} + \overset{d}{\frac{kd}{2}\sin(\omega t - kr)\cos\theta} \quad \text{IV}$$

$$V_p = \frac{1}{4\pi\epsilon_0}\frac{q}{r}\left[(a+b)(c-d) - (c+d)(a-b)\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r}\left[\cancel{ac} - ad + bc - \cancel{bd} - \cancel{ac} + cb - da + \cancel{db}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r}\left\{2bc - 2ad\right\}$$

$$= \frac{2}{4\pi\epsilon_0}\frac{q}{r}\left[\frac{d\cos\theta}{2r}\cos(\omega t - kr) - \frac{kd}{2}\cos\theta\sin(\omega t - kr)\right]$$

$$= \frac{2qd\cos\theta}{4\pi\epsilon_0 r \cancel{2}}\left[\cancel{\frac{1}{r}\cos(\omega t - kr)} - k\sin(\omega t - kr)\right]$$

$\emptyset$

at large r

dipole moment

$$V_p = -\frac{qd\,k\cos\theta}{4\pi\epsilon_0 r}\sin(\omega t - kr)$$

$$\nabla V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta}$$

$$= -\frac{qdk\cos\theta}{4\pi\epsilon_0}\left[\cancel{-r^{-2}\sin(\omega t - kr)} - \frac{k}{r}\cos(\omega t - kr)\right]\hat{r}$$

$\emptyset$

$$\nabla V = \frac{qdk^2\cos\theta}{4\pi\epsilon_0 r}\cos(\omega t - kr)\hat{r} + \cancel{\frac{qdk\sin\theta}{4\pi\epsilon_0 r^2}\sin(\omega t - kr)}$$

$\emptyset$

Done? No!

Electrostatics: $\vec{E} = -\nabla V$

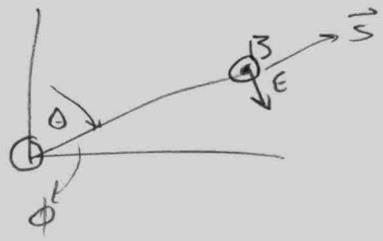
Electrodynamics: $\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$ vector potential

$$\frac{\partial \vec{A}}{\partial t} = -\frac{\mu_0 q d c^2 k^2}{4\pi r}\cos(\omega t - kr)(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})$$

$$\frac{qdk^2}{4\pi\epsilon_0}$$

$$\vec{E}_p = -\frac{qdk^2}{4\pi\epsilon_0 r}\sin\theta\cos(\omega t - kr)\,\hat{\theta}$$

$$\vec{B}_p = \nabla\times\vec{A} = -\frac{\mu_0 q d c k^2}{4\pi r}\sin\theta\cos(\omega t - kr)\,\hat{\phi}$$



$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

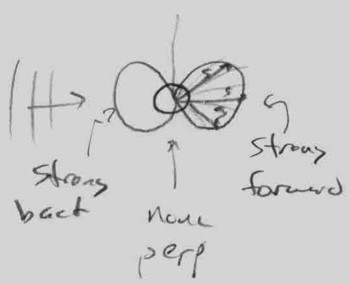
$$\cancel{\langle|\vec{S}|\rangle = \left(\frac{(qd)^2\mu_0 c^3 k^4}{32\pi^2}\right)\frac{\sin^2\theta}{r^2}\,\hat{r}}$$

$$\langle\vec{S}\rangle = \frac{(qd)^2 c k^4}{32\epsilon_0\pi^2}\,\frac{\sin^2\theta}{r^2}\,\hat{r}$$

r-dependence

Ec down w/ r^2 as w/ spherical wave

θ dependence



Φ dependence

none