35. Diffraction and Image Formation

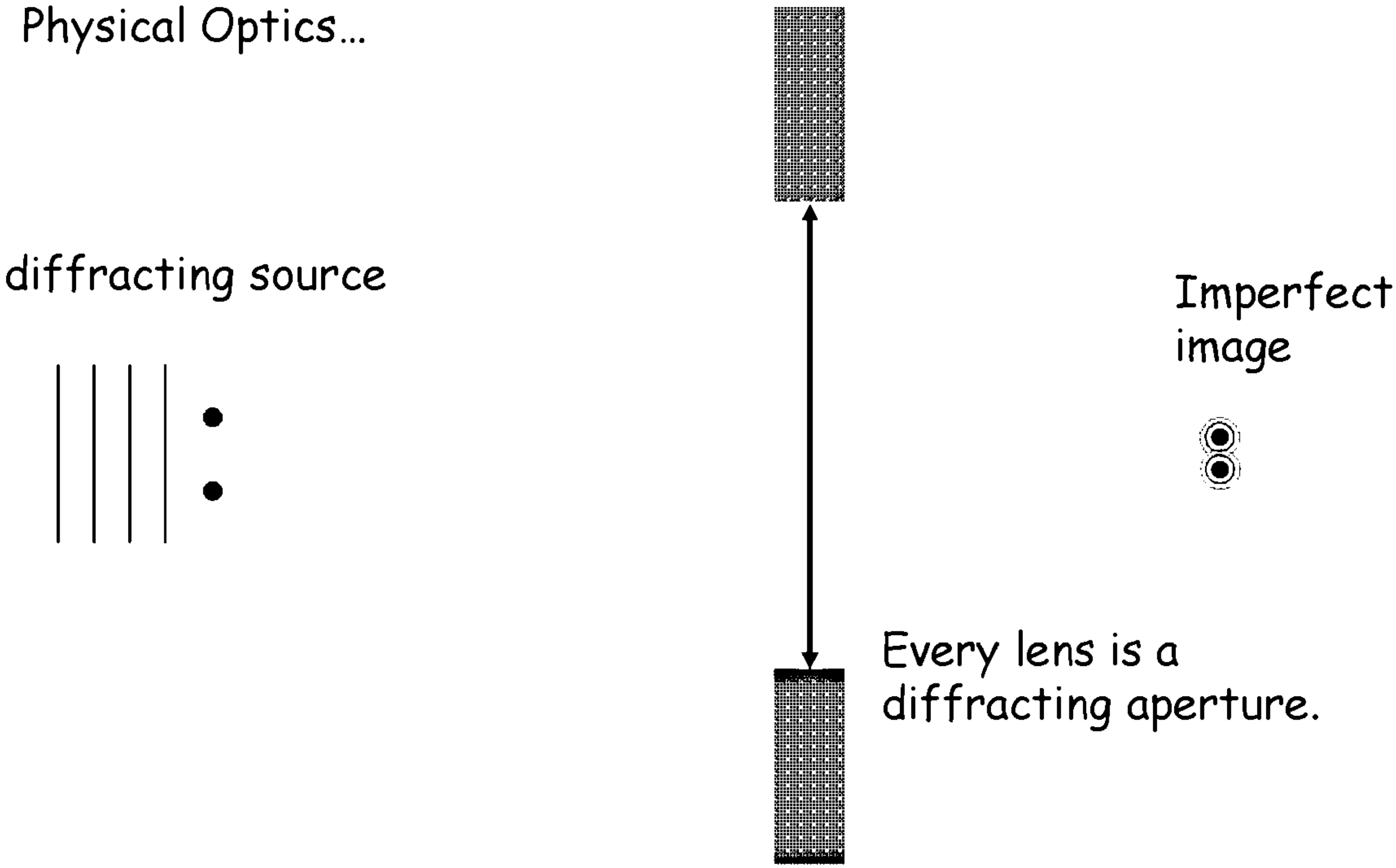
Where was modern optical imaging technology born?

Zeiss

Abbe

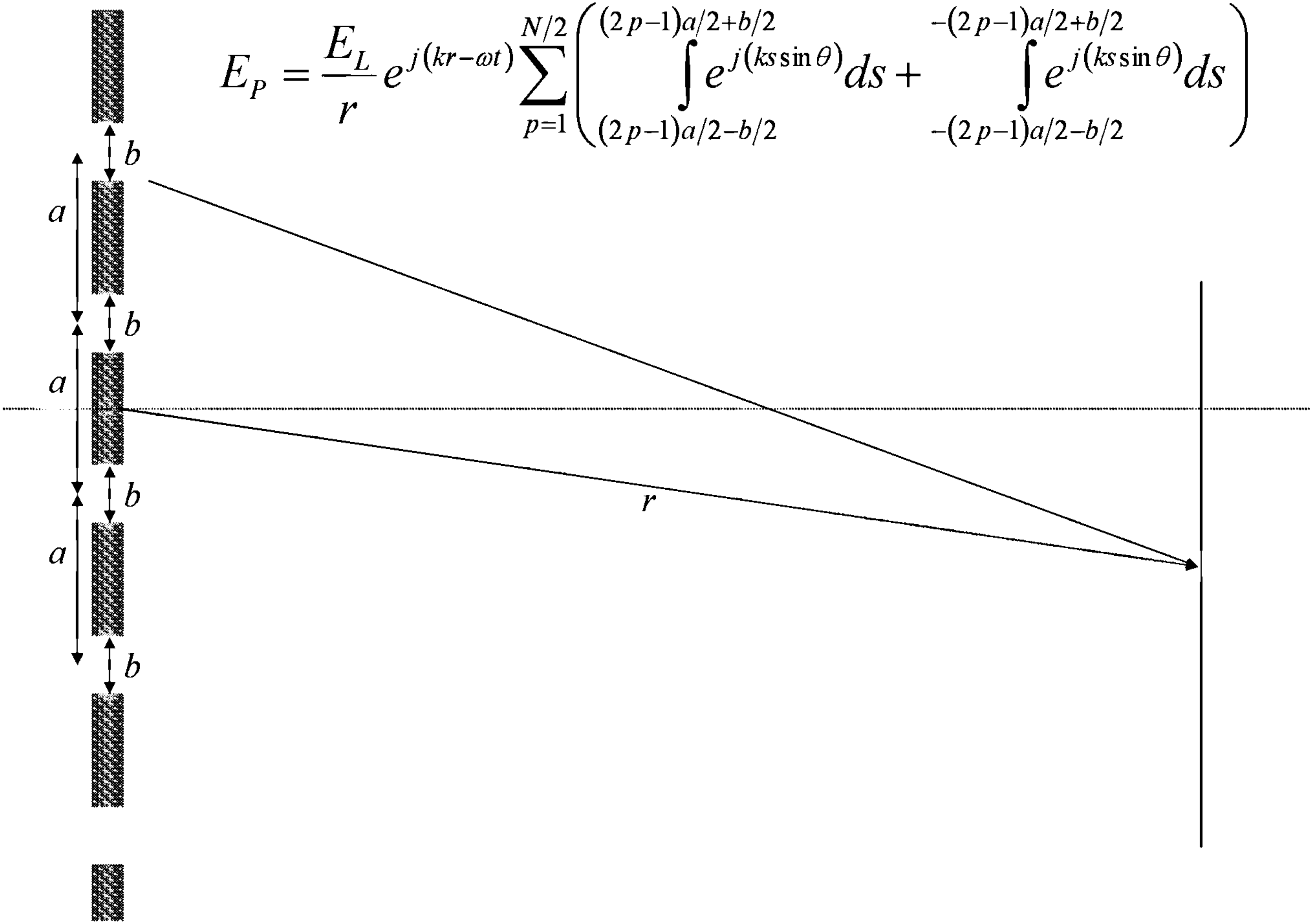
Schott

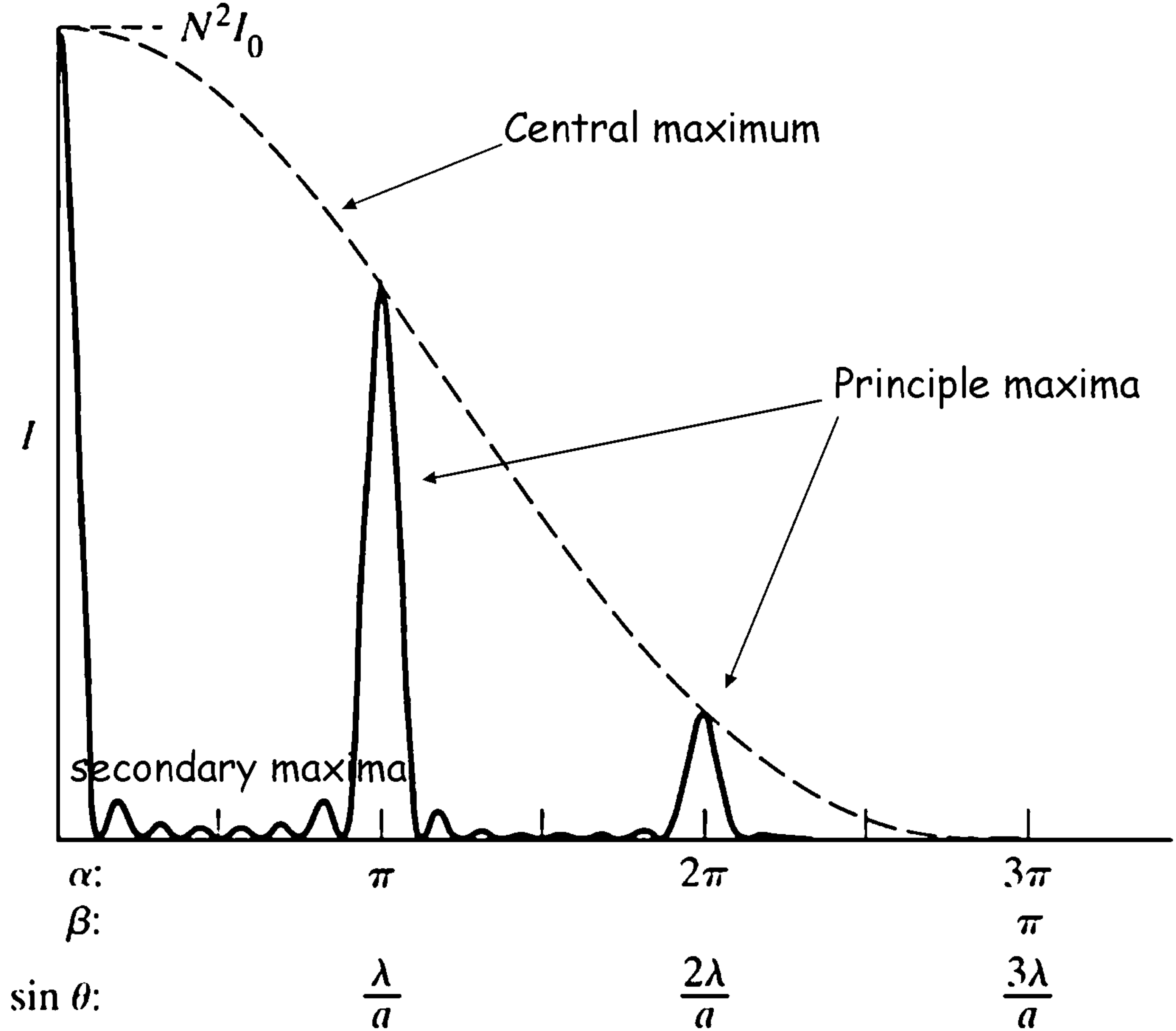
Physical Optics…



Multiple Slits

$$E_P = \frac{E_L}{r} e^{j(kr-\omega t)} \sum_{p=1}^{N/2} \left(\int_{(2p-1)a/2-b/2}^{(2p-1)a/2+b/2} e^{j(ks\sin\theta)} ds + \int_{-(2p-1)a/2-b/2}^{-(2p-1)a/2+b/2} e^{j(ks\sin\theta)} ds \right)$$





$N^2 I_0$
Central maximum
Principle maxima
I
secondary maxima
α:
π
2π
3π
β:
π
$\sin\theta$:
$\frac{\lambda}{a}$
$\frac{2\lambda}{a}$
$\frac{3\lambda}{a}$

Diffraction Grating

A special corner of multi-slit-space: $N \sim 10^4$, $a \sim \lambda$, $b \sim \lambda$

$b \sim \lambda$: central maximum is very large!

$a \sim \lambda$: principle maxima are highly separated!
(most don't exist)

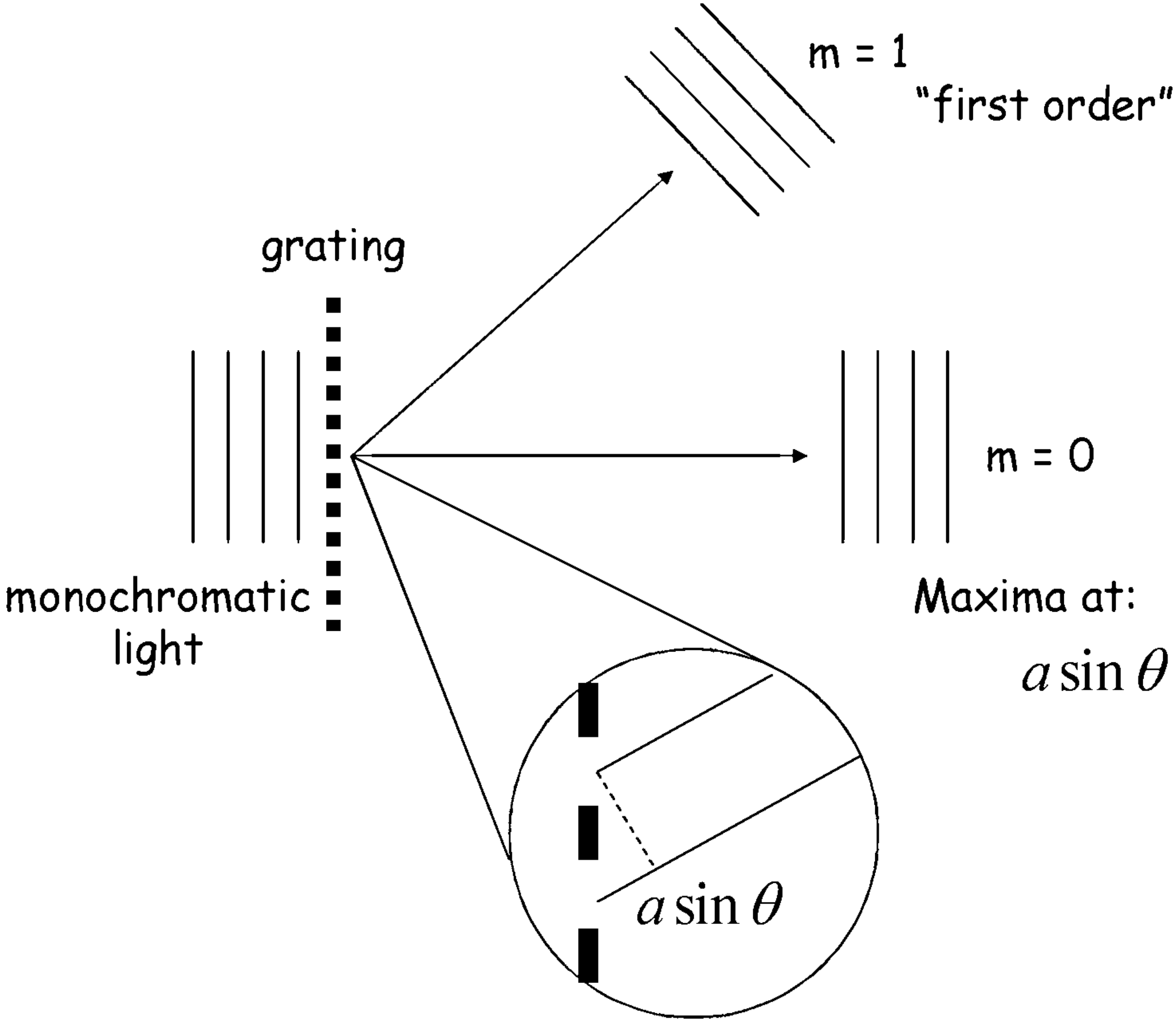
$N \sim 10^4$: Principle maxima are very narrow!
Secondary maxima are very low!

typical grating specs: 900 g/mm, 1 cm grating.

N = 9,000

a = 1.11 microns $\qquad$ λ = 0.633 microns!

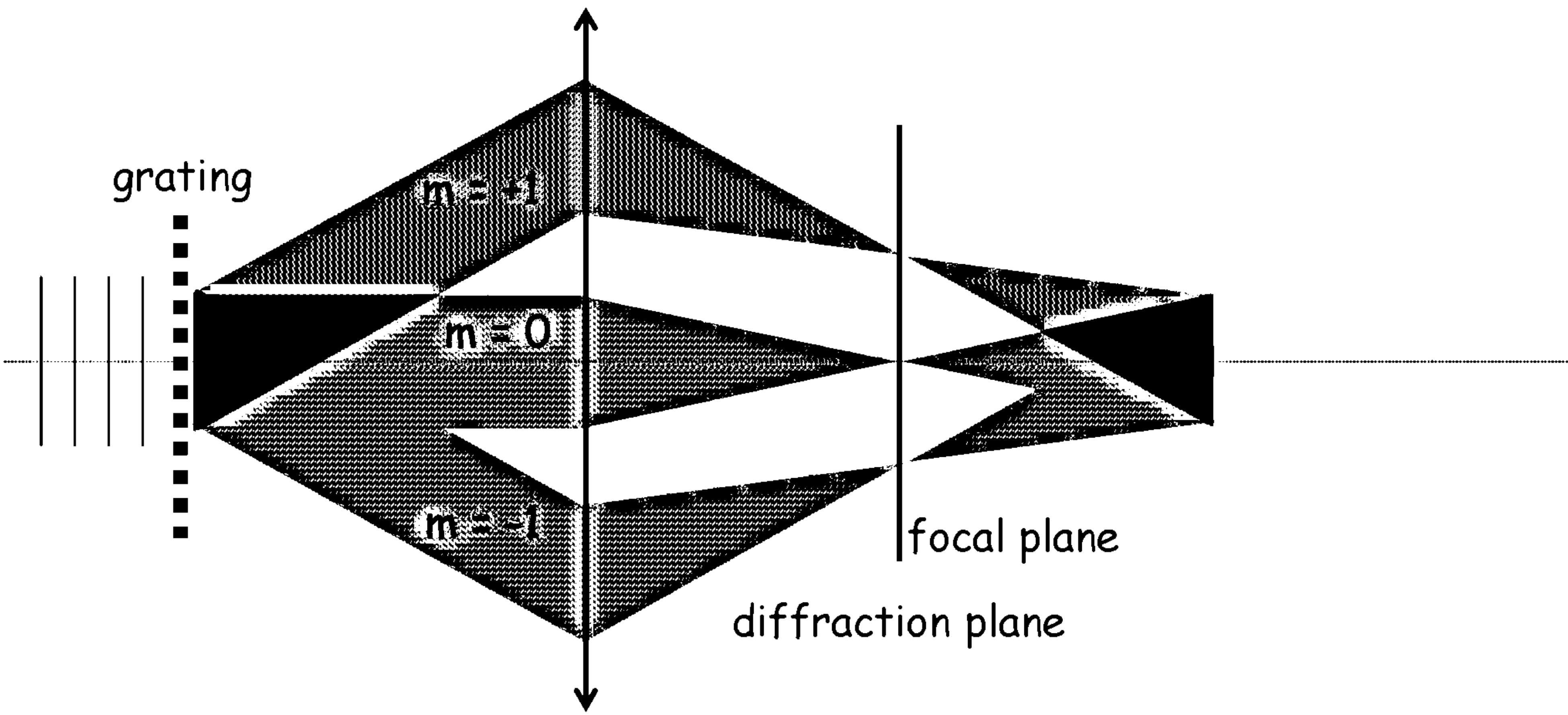
b = 1.11 microns



Maxima at:

$$a \sin\theta = m\lambda \quad m = 0, 1, 2, 3,$$

Abbe Theory of Image Formation



Abbe Theory of Image Formation

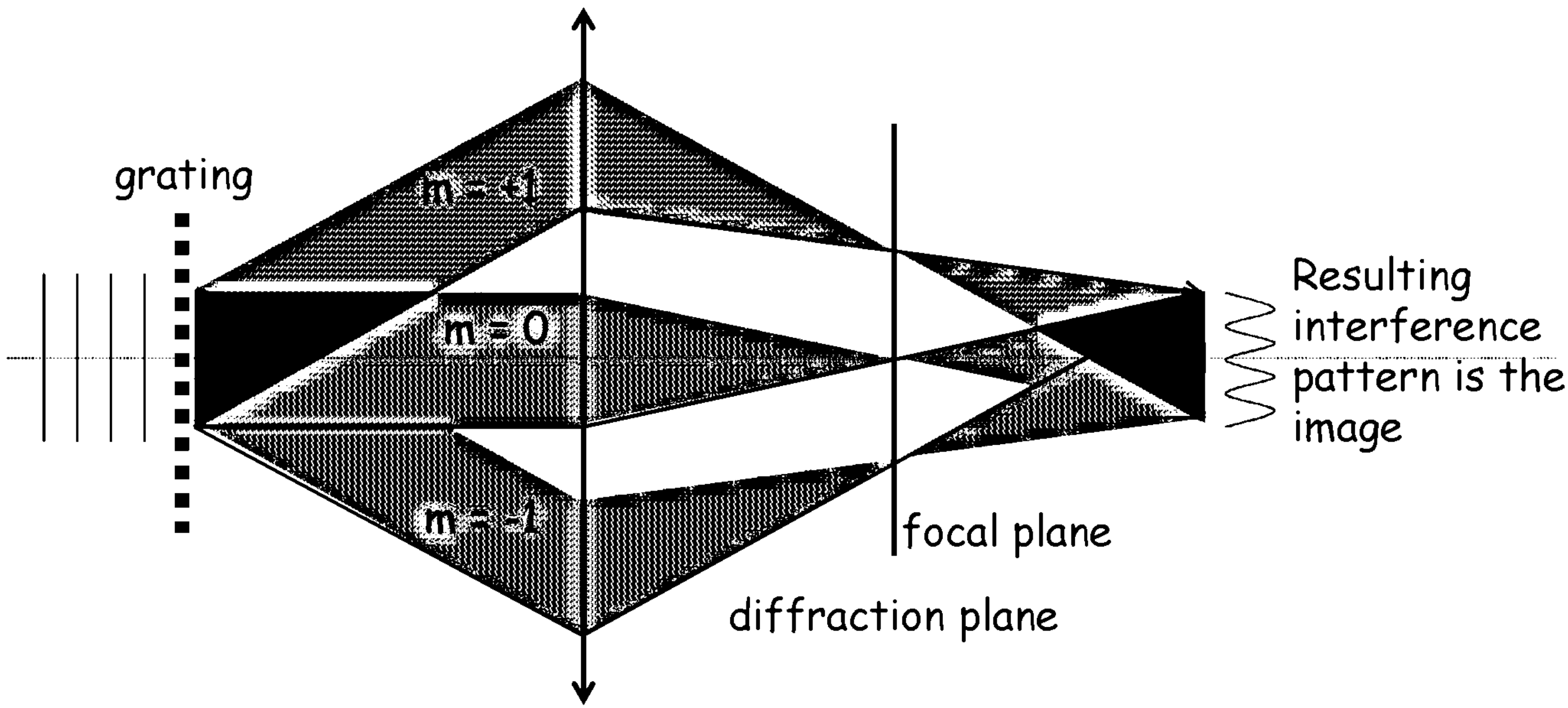
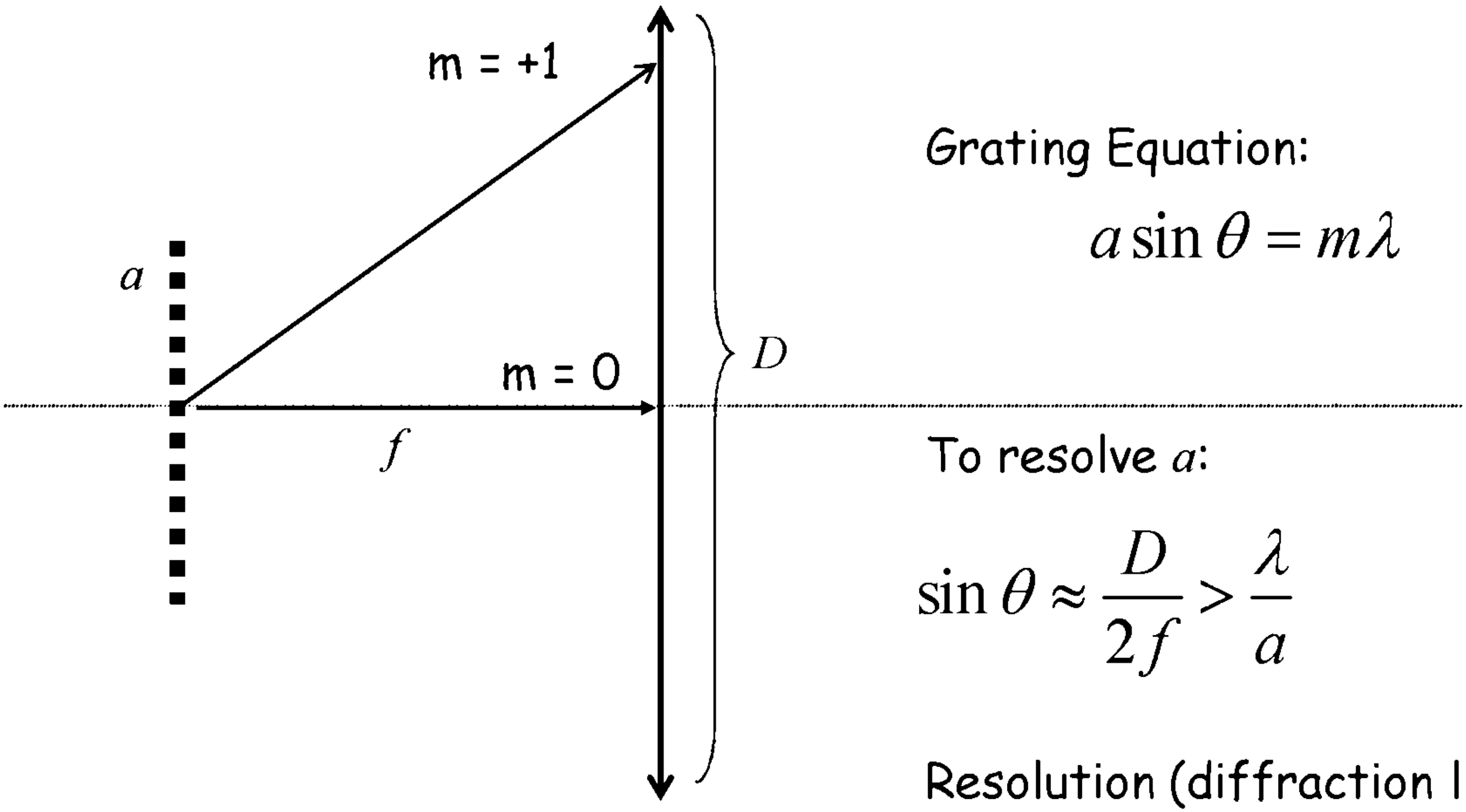


Image formation requires a lens large enough to capture the first order diffraction.



Grating Equation:

$$a\sin\theta = m\lambda$$

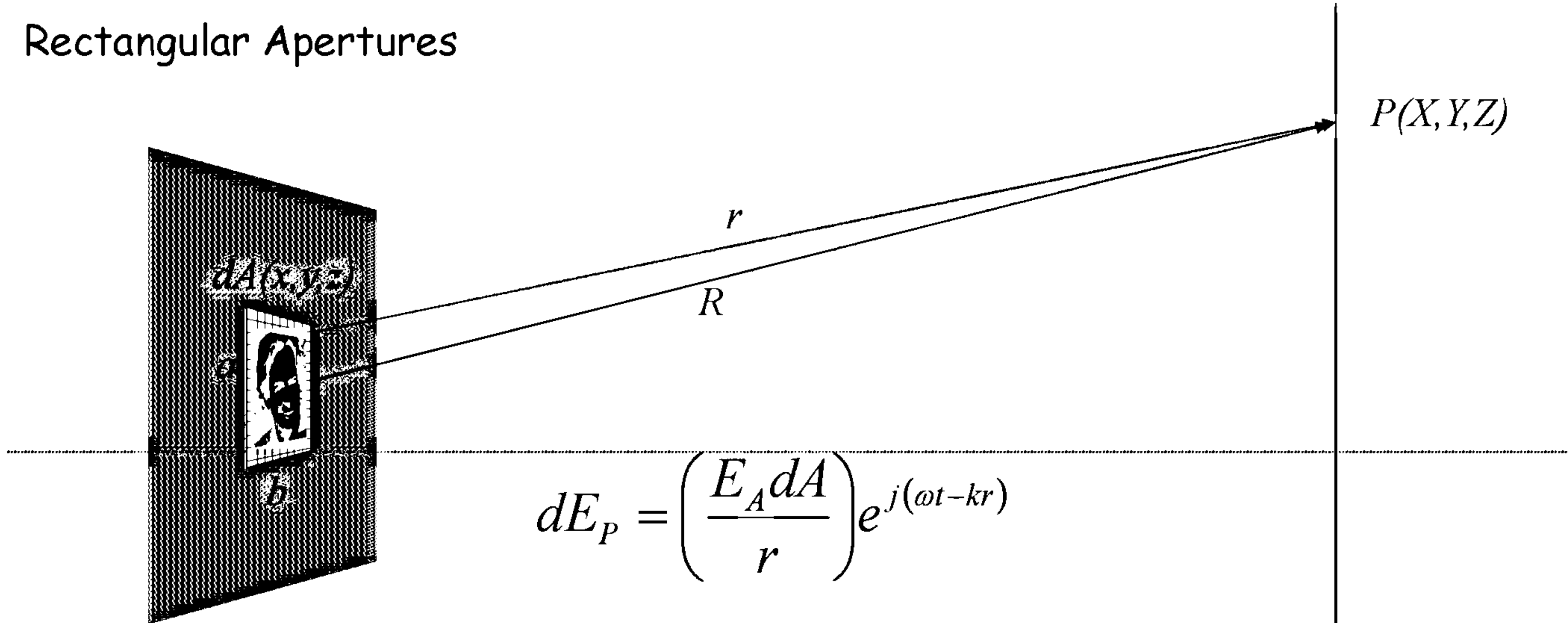
To resolve a:

$$\sin\theta \approx \frac{D}{2f} > \frac{\lambda}{a}$$

Resolution (diffraction limited):

$$a \approx \frac{2\lambda f}{D}$$

Rectangular Apertures



$$dE_P = \left(\frac{E_A dA}{r}\right) e^{j(\omega t - kr)}$$

$$E_P = \left(\frac{E_A}{R}\right) e^{j(\omega t - kR)} \iint_{aperture} e^{jk(Xx+Yy)/R} dA$$

Rather than an aperture, consider an object:

$$E_P = \frac{1}{R} e^{j(\omega t - kR)} \iint_{aperture} E_{Feynman} e^{jk(Xx+Yy)/R} dA$$

Remember, the integral is over the aperture area:

$$E_P = \frac{1}{R} e^{j(\omega t - kR)} \iint_{aperture} E_{Feynman} e^{jk(Xx+Yy)/R} dxdy$$

Let's rearrange that a little it (this is where the magic happens):

$$E_P = \frac{1}{R} e^{j(\omega t - kR)} \iint_{aperture} E_{Feynman} e^{j\left[\left(\frac{kX}{R}\right)x + \left(\frac{kY}{R}\right)y\right]} dxdy$$

THAT'S A FOURIER TRANSFORM!!

$$E_P(X,Y,Z) = \mathcal{F}\{E_{Feynman}\}$$

Where does diffraction put the spatial frequencies in $E_{Feynman}$?

$$k_x \equiv \frac{kX}{R} \qquad k_y \equiv \frac{kY}{R}$$