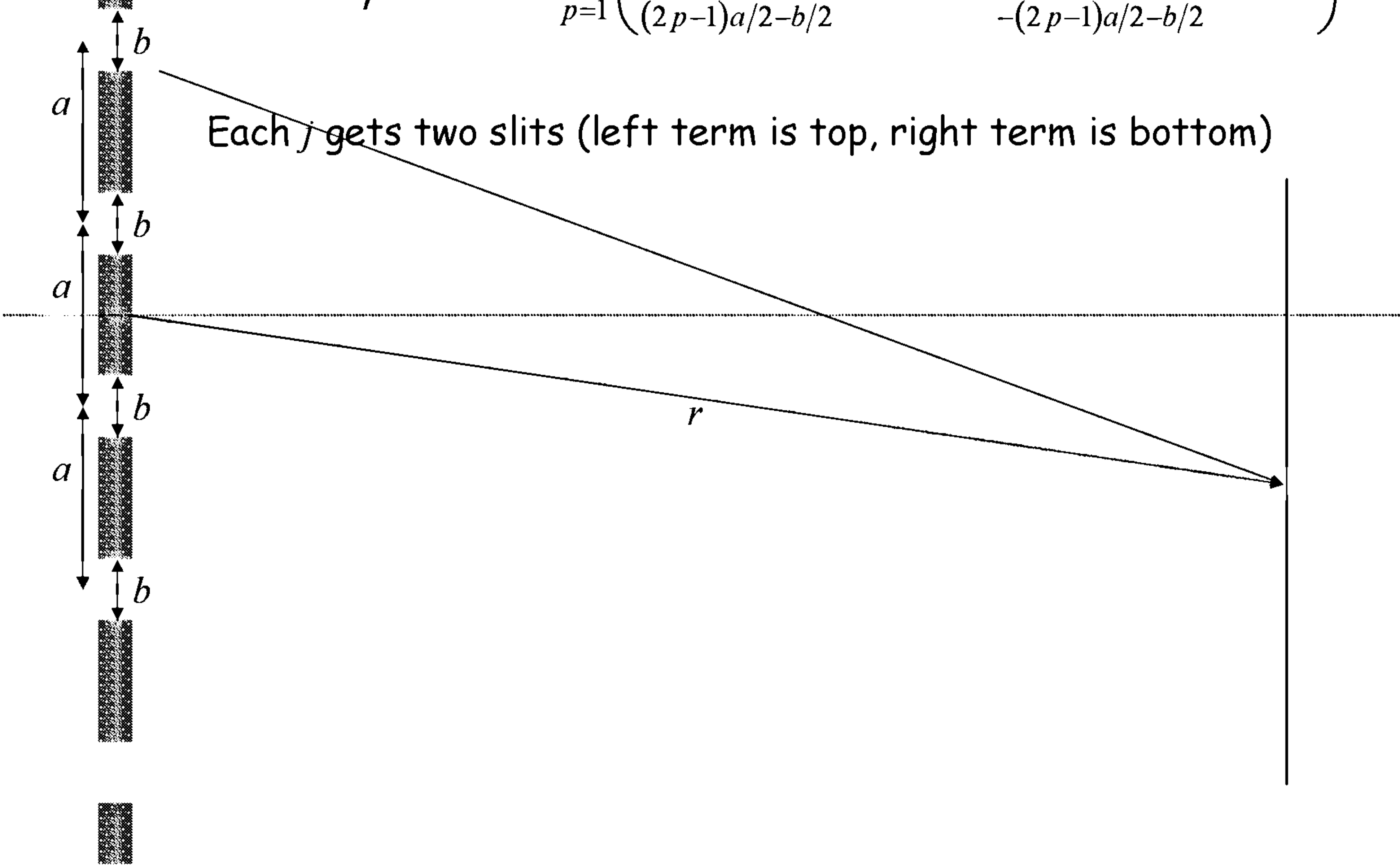
Multiple Slits

$$E_P = \frac{E_L}{r} e^{j(kr-\omega t)} \sum_{p=1}^{N/2} \left(\int_{(2p-1)a/2-b/2}^{(2p-1)a/2+b/2} e^{j(ks\sin\theta)} ds + \int_{-(2p-1)a/2-b/2}^{-(2p-1)a/2+b/2} e^{j(ks\sin\theta)} ds \right)$$

Each j gets two slits (left term is top, right term is bottom)



$$E_P = \frac{E_L}{r} e^{j(kr-\omega t)} \sum_{p=1}^{N/2} \left(\int_{(2p-1)a/2-b/2}^{(2p-1)a/2+b/2} e^{j(ks\sin\theta)} ds + \int_{-(2p-1)a/2-b/2}^{-(2p-1)a/2+b/2} e^{j(ks\sin\theta)} ds \right)$$

1. integrate and apply limits

2. let: $\beta = \frac{1}{2} kb \sin\theta \quad \alpha = \frac{1}{2} ka \sin\theta$

3. factor α's from β's

4. apply Euler's formula

5. $(\ \ ) = \frac{2j\sin(\beta)}{jk\sin\theta} \sum_{p=1}^{N/2} 2\cos((2p-1)\alpha)$

$$(\quad) = \frac{4\sin(\beta)}{k\sin\theta}\,\mathrm{Re}\left[\sum_{p=1}^{N/2} e^{j(2p-1)\alpha}\right]$$

Dude! That's a geometric series:

$$\sum_{p=1}^{N/2} e^{j(2p-1)\alpha} = e^{j\alpha} + e^{3j\alpha} + e^{5j\alpha} + \ldots + e^{(N-1)j\alpha}$$

But a proper geometric series looks like this:

$$\sum_{p=0}^{n} ar^{p} = \frac{a\left(r^{n+1}-1\right)}{r-1}$$

$$[\;]=\sum_{p=1}^{N/2}\left(e^{j\alpha}\right)^{2p-1}$$

$$[\;]=\sum_{p=1}^{N/2}e^{j\alpha}\left(e^{j\alpha}\right)^{2p-2}$$

$$[\;]=\sum_{p=1}^{N/2}e^{j\alpha}\left(e^{2j\alpha}\right)^{p-1}$$

$$[\;]=\sum_{p=0}^{N/2-1}e^{j\alpha}\left(e^{2j\alpha}\right)^{p}$$

$$[\;]=\frac{e^{j\alpha}\left(e^{2j\alpha(N/2)}-1\right)}{e^{2j\alpha}-1}$$

$$[\ ]=\frac{e^{jN\alpha}-1}{e^{j\alpha}-e^{-j\alpha}}$$

$$[\ ]=\frac{\cos(N\alpha)+j\sin(N\alpha)-1}{\cos(\alpha)+j\sin(\alpha)-\cos(-\alpha)-j\sin(-\alpha)}$$

$$[\ ]=\frac{\cos(N\alpha)+j\sin(N\alpha)-1}{2j\sin(\alpha)}$$

$$[\ ]=\frac{j\cos(N\alpha)-\sin(N\alpha)-j}{-2\sin(\alpha)}$$

What were we calculating again?

$$(\ \ ) = \frac{4\sin(\beta)}{k\sin\theta}\mathrm{Re}\left[\sum_{p=1}^{N/2} e^{j(2p-1)\alpha}\right]$$

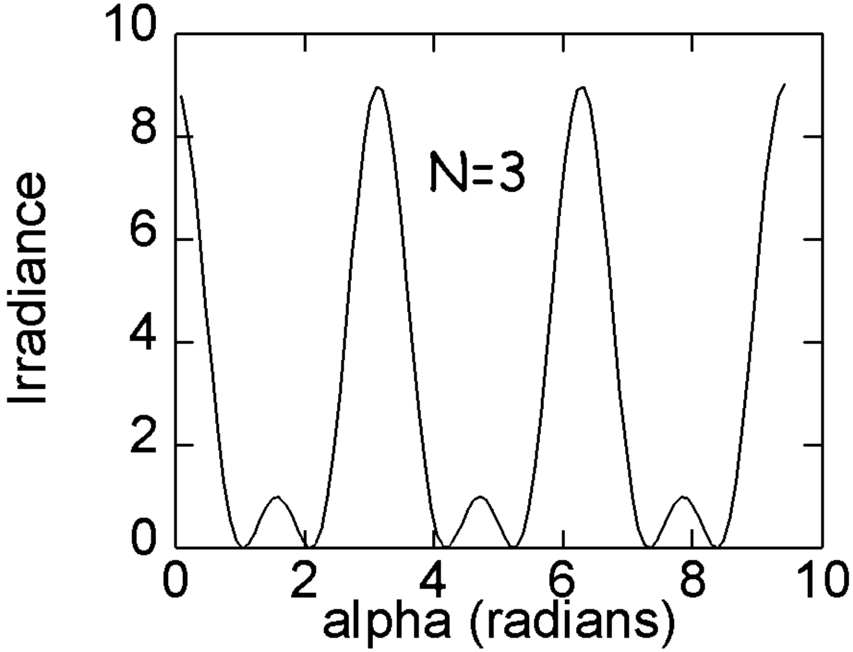
$$(\ \ ) = 2b\frac{\sin(\beta)}{\beta}\frac{\sin(N\alpha)}{2\sin(\alpha)}$$

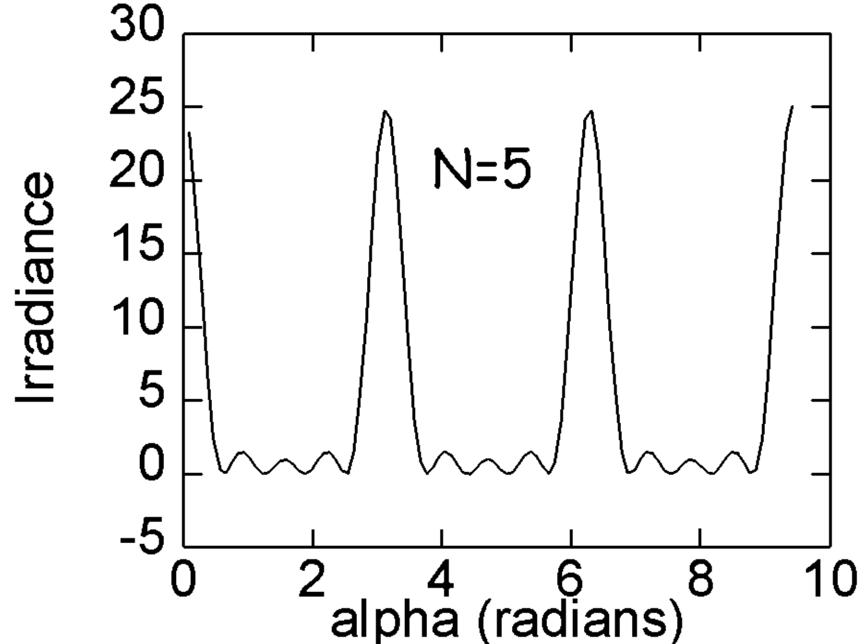
$$E_P = \frac{E_L b}{r}e^{j(kr-\omega t)}\frac{\sin(\beta)}{\beta}\frac{\sin(N\alpha)}{\sin(\alpha)}$$

$$I_P = I_0\left(\frac{\sin(\beta)}{\beta}\right)^2\left(\frac{\sin(N\alpha)}{\sin(\alpha)}\right)^2$$

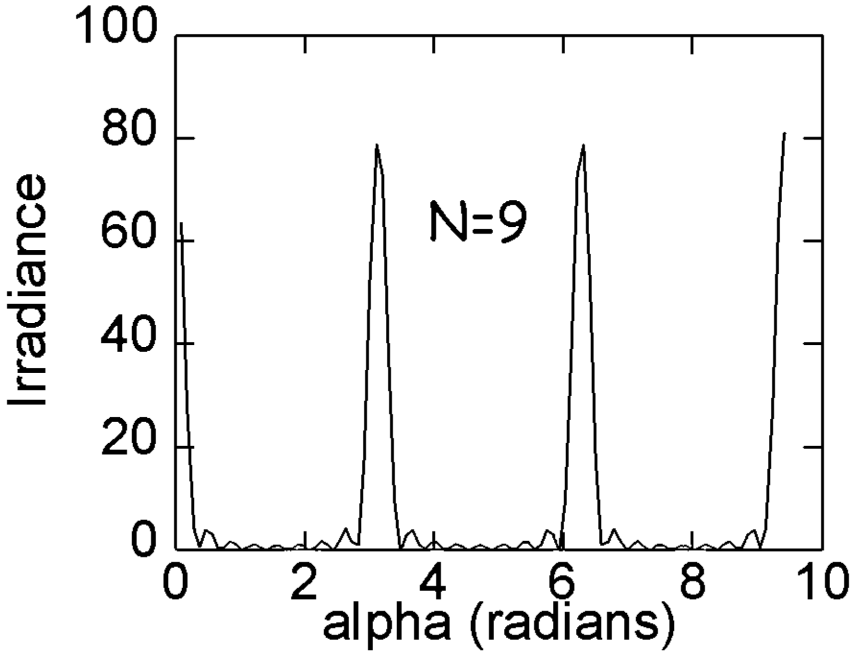
Single slit diffraction

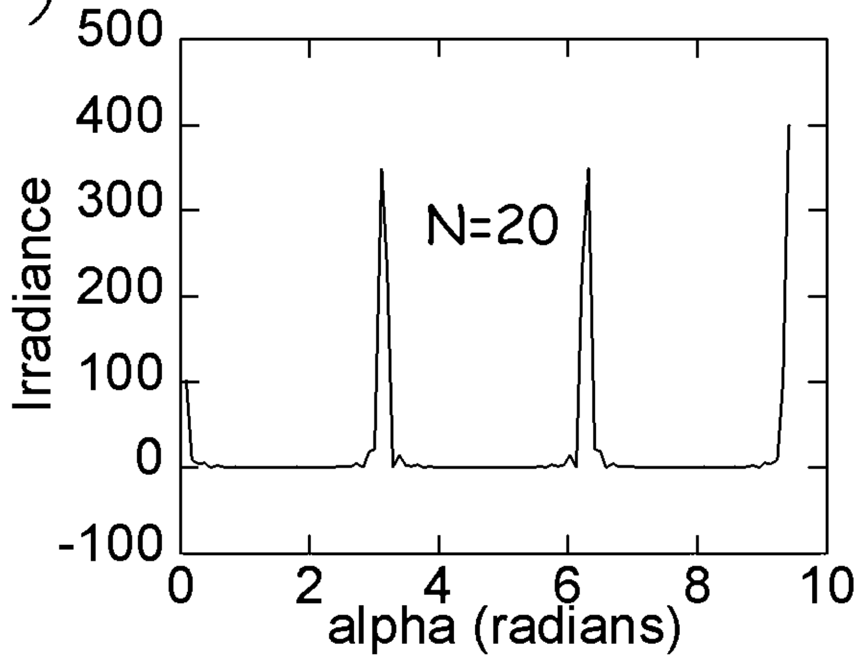
Multi slit interference



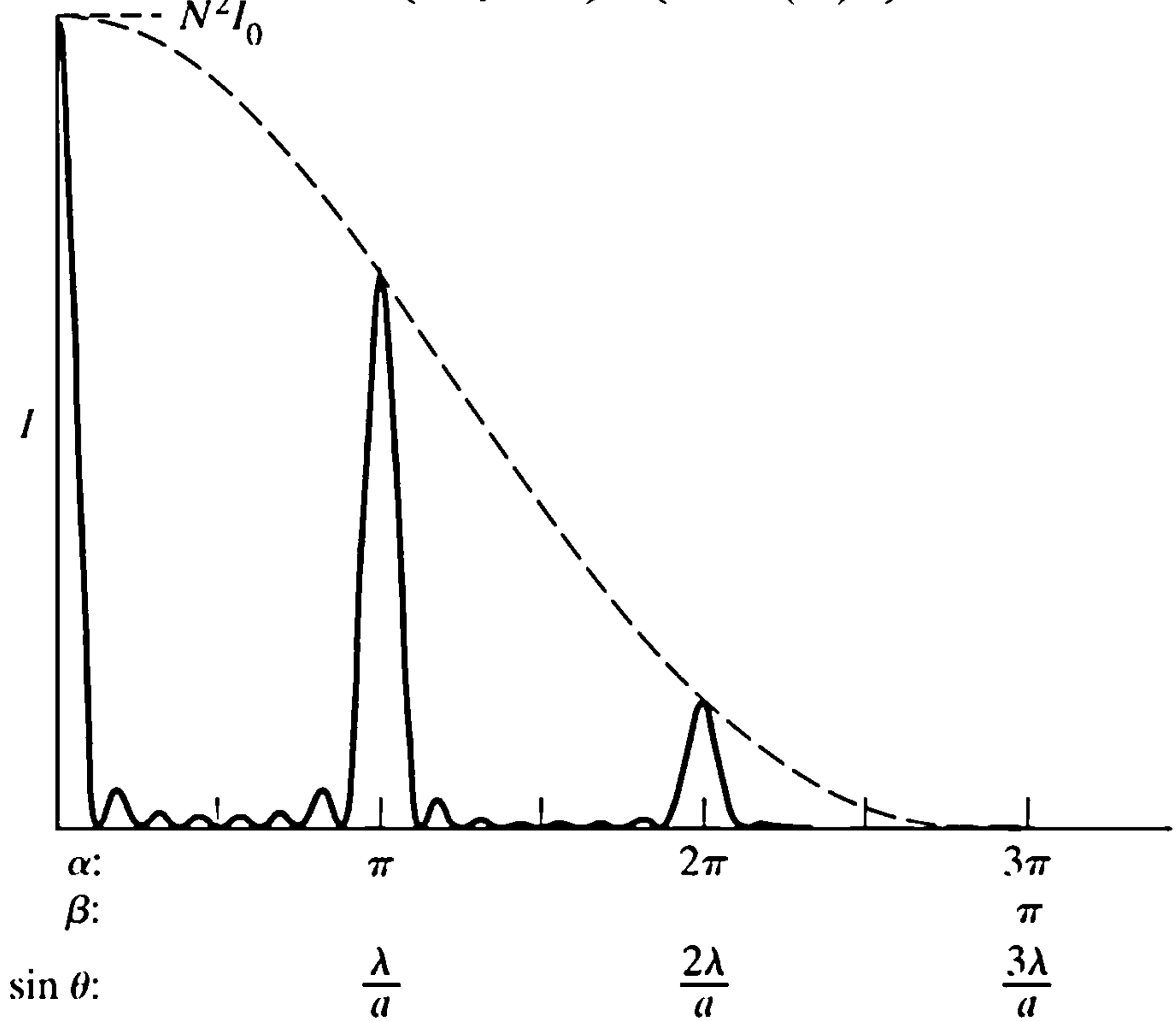


$$\left(\frac{\sin(N\alpha)}{\sin(\alpha)}\right)^2$$





$$I_P = I_0\left(\frac{\sin(\beta)}{\beta}\right)^2\left(\frac{\sin(N\alpha)}{\sin(\alpha)}\right)^2$$



Diffraction Grating

A special corner of multi-slit-space: $N \sim 10^4$, $a \sim \lambda$, $b \sim \lambda$

$b \sim \lambda$: central maximum is very large!

$a \sim \lambda$: principle maxima are highly separated!
(most don't exist)

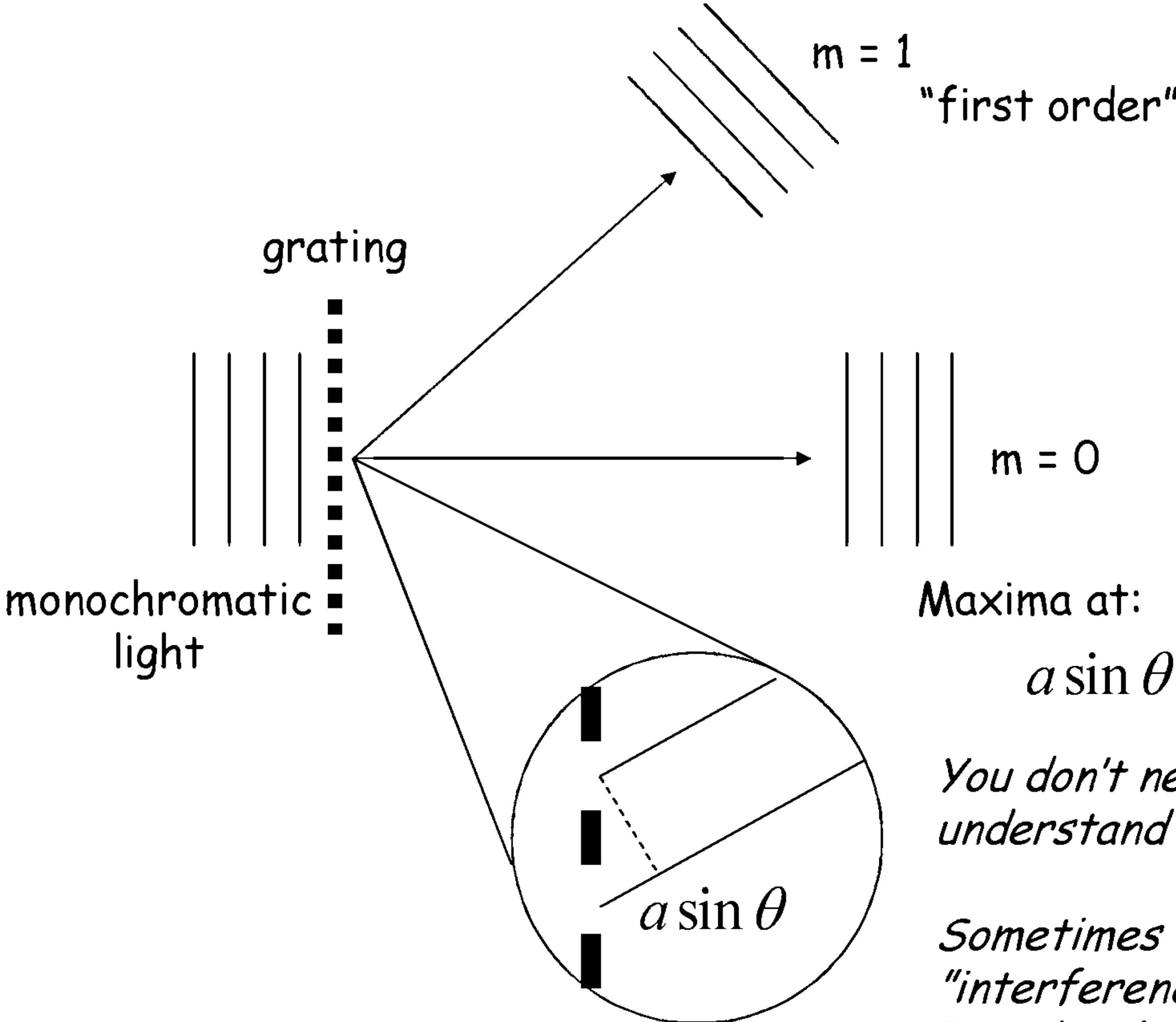
$N \sim 10^4$: Principle maxima are very narrow!
Secondary maxima are very low!

typical grating specs: 900 g/mm, 1 cm grating.

N = 9,000

a = 1.11 microns $\qquad$ λ = 0.633 microns!

b = 1.11 microns



Maxima at:

$$a \sin\theta = m\lambda \quad m = 0, 1, 2, 3,$$

You don't need diffraction to understand a diffraction grating!!!!

Sometimes they are called "interference gratings", but that's based on how they are made, not how they work.