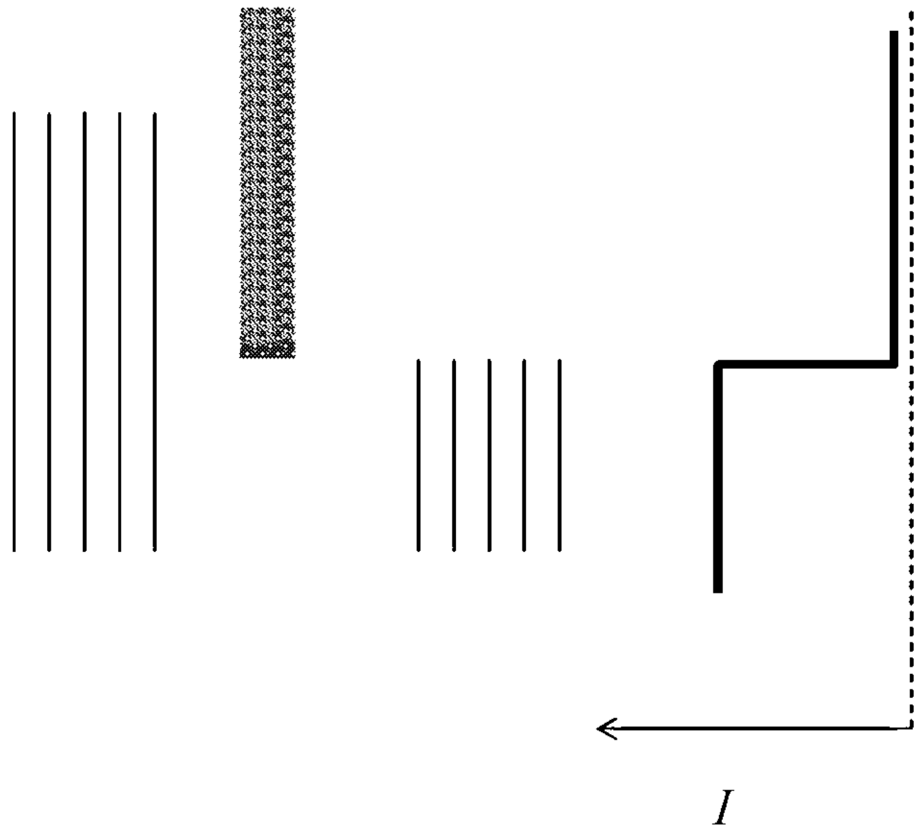
Fraunhofer Diffraction

Geometrical Optics:

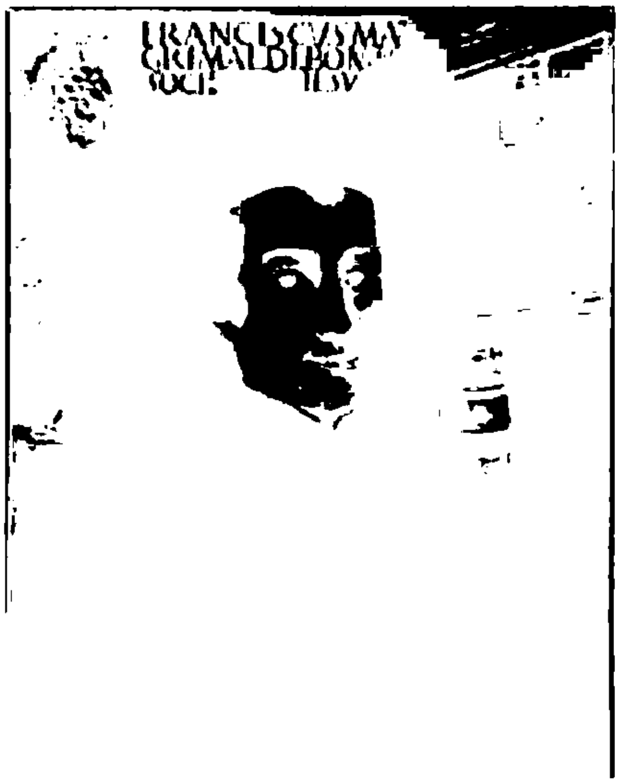
Bill Mageors



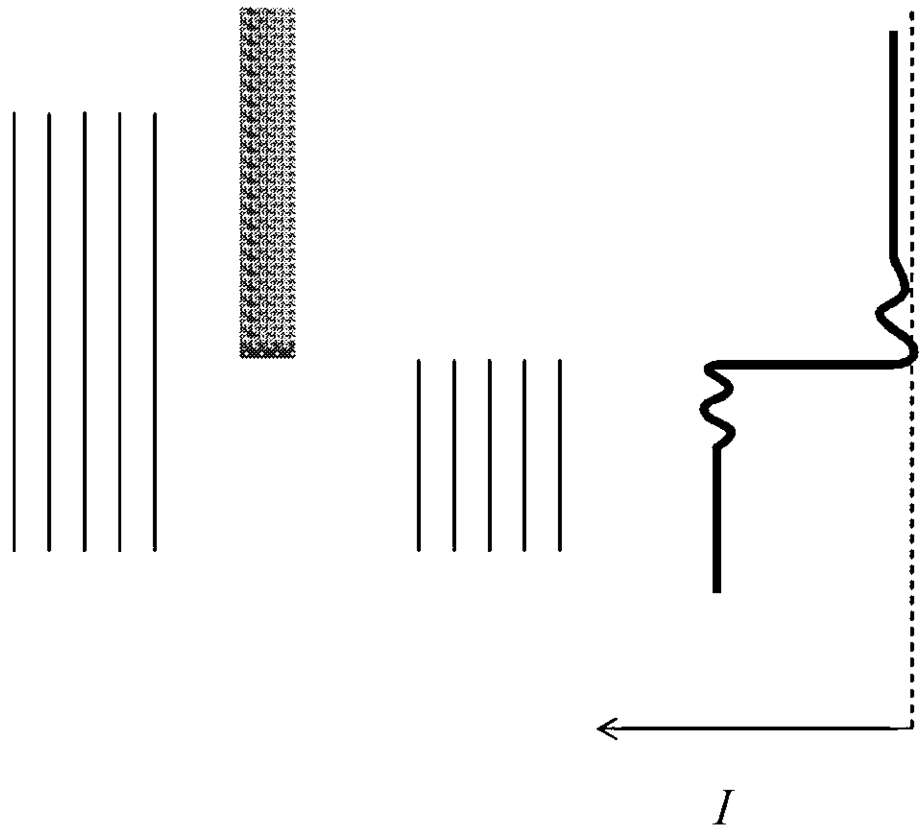
...light can't turn a corner.

Fraunhofer Diffraction

Physical Optics:



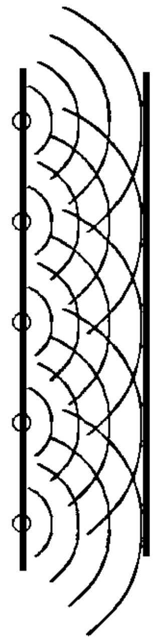
Francesco Maria Grimaldi



...actually, it can.

Huygens-Fresnel Principle

Every point on a wavefront may be regarded as a secondary source of spherical wavelets.



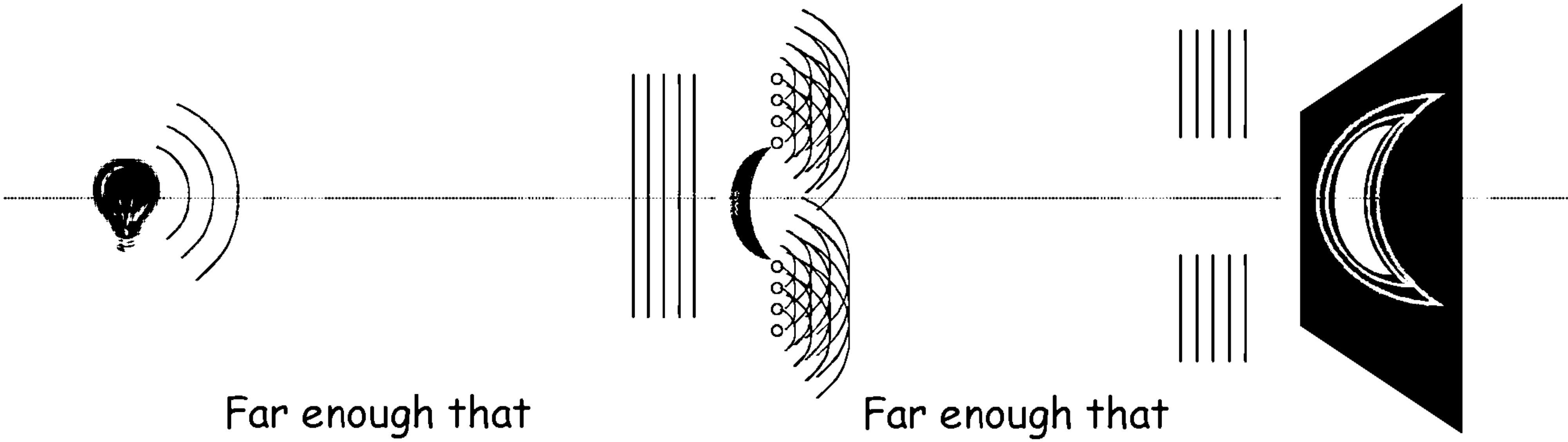
The propagated wave follows the periphery of the wavelets.

Augustin-Jean Fresnel

Huygens, just add the wavelets considering interference!

(That's a sweet ascot by the way.)

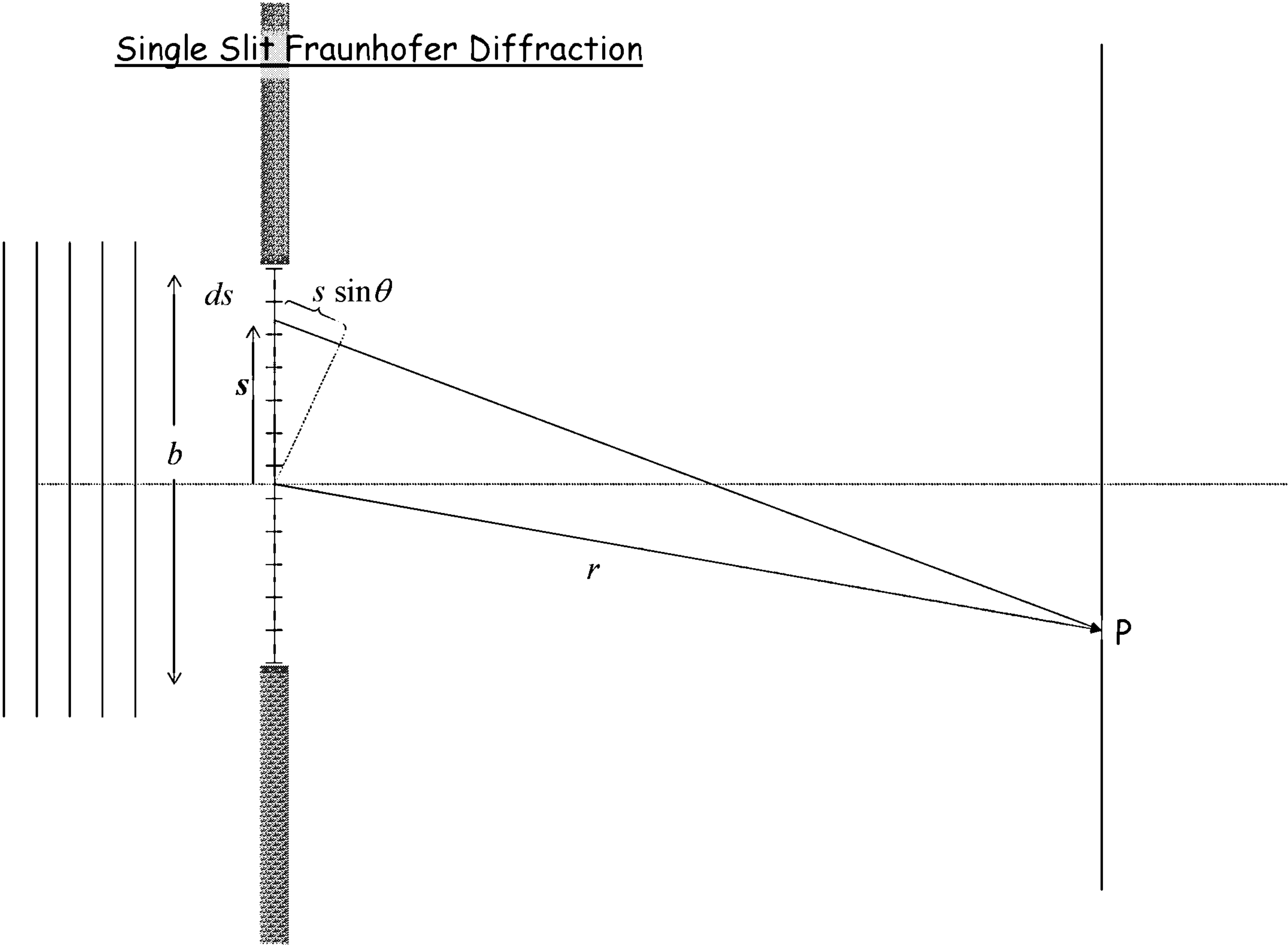
Far-Field Diffraction
(a.k.a. Fraunhofer Diffraction)



Far enough that source illumination is a plane wave.

Far enough that diffracted wavelets are plane wave.

Single Slit Fraunhofer Diffraction



Get the amplitudes right!

field amplitude

spherical wavelets:

$$dE_P = \left(\frac{dE_o}{r}\right)e^{j(kr-\omega t)}$$

field

As ds goes to zero, dE_o must go to zero:

$$dE_o = E_L ds$$

field amplitude per unit width

$$dE_P = \left(\frac{E_L ds}{r}\right)e^{j(kr-\omega t)}$$

Get the phases right! (just like Young's Double Slit)

$$dE_P = \left(\frac{E_L ds}{r}\right) e^{j(kr - \omega t + ks\sin\theta)}$$

Integrate over the slit:

$$\int_{-b/2}^{+b/2} dE_P = \frac{E_L}{r} e^{j(kr-\omega t)} \int_{-b/2}^{+b/2} e^{j(ks\sin\theta)} ds$$

$$E_P = \frac{E_L}{r} e^{j(kr-\omega t)} \left. \frac{e^{j(ks\sin\theta)}}{jk\sin\theta} \right|_{-b/2}^{+b/2}$$

$$E_P = \frac{E_L}{r} e^{j(kr-\omega t)} \frac{1}{jk\sin\theta} \left(e^{j(kb\sin\theta/2)} - e^{-j(kb\sin\theta/2)} \right)$$

new variable: $\beta = \tfrac{1}{2} kb \sin\theta$

$$E_P = \frac{E_L}{r} e^{j(kr-\omega t)} \frac{b}{2j\beta}\left(e^{j\beta} - e^{-j\beta}\right)$$

$$E_P = \frac{E_L b}{r} \frac{\sin\beta}{\beta} e^{j(kr-\omega t)}$$

Calculate irradiance:

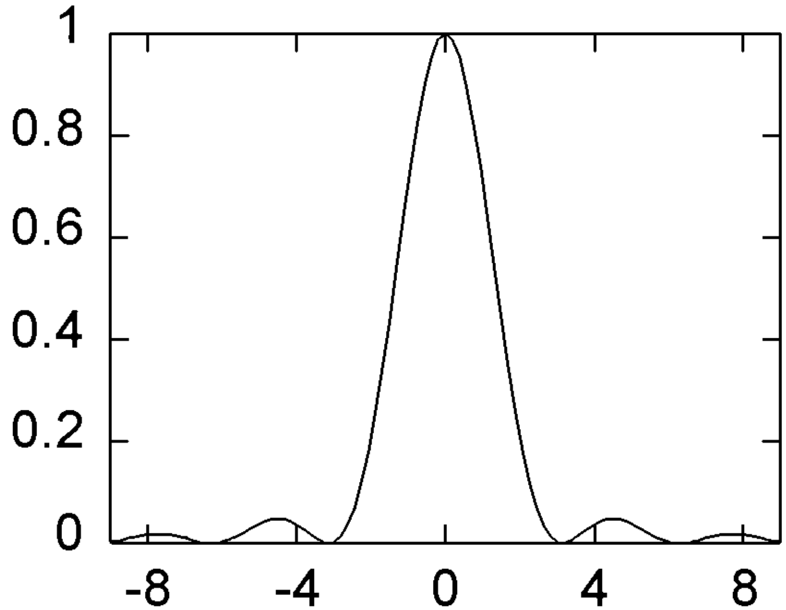
$$I = \frac{\varepsilon_o c}{2} {E_P}^2$$

field at r

$$I = \underbrace{\frac{\varepsilon_o c}{2}\left(\frac{E_L b}{r}\right)^2}_{\text{an irradiance}} \frac{\sin^2\beta}{\beta^2} \longrightarrow \operatorname{sinc} x = \frac{\sin x}{x}$$

"sinc"
"sampling function"
"sine cardinal"

$$I = I_o \operatorname{sinc}^2 \beta$$



Minima at:

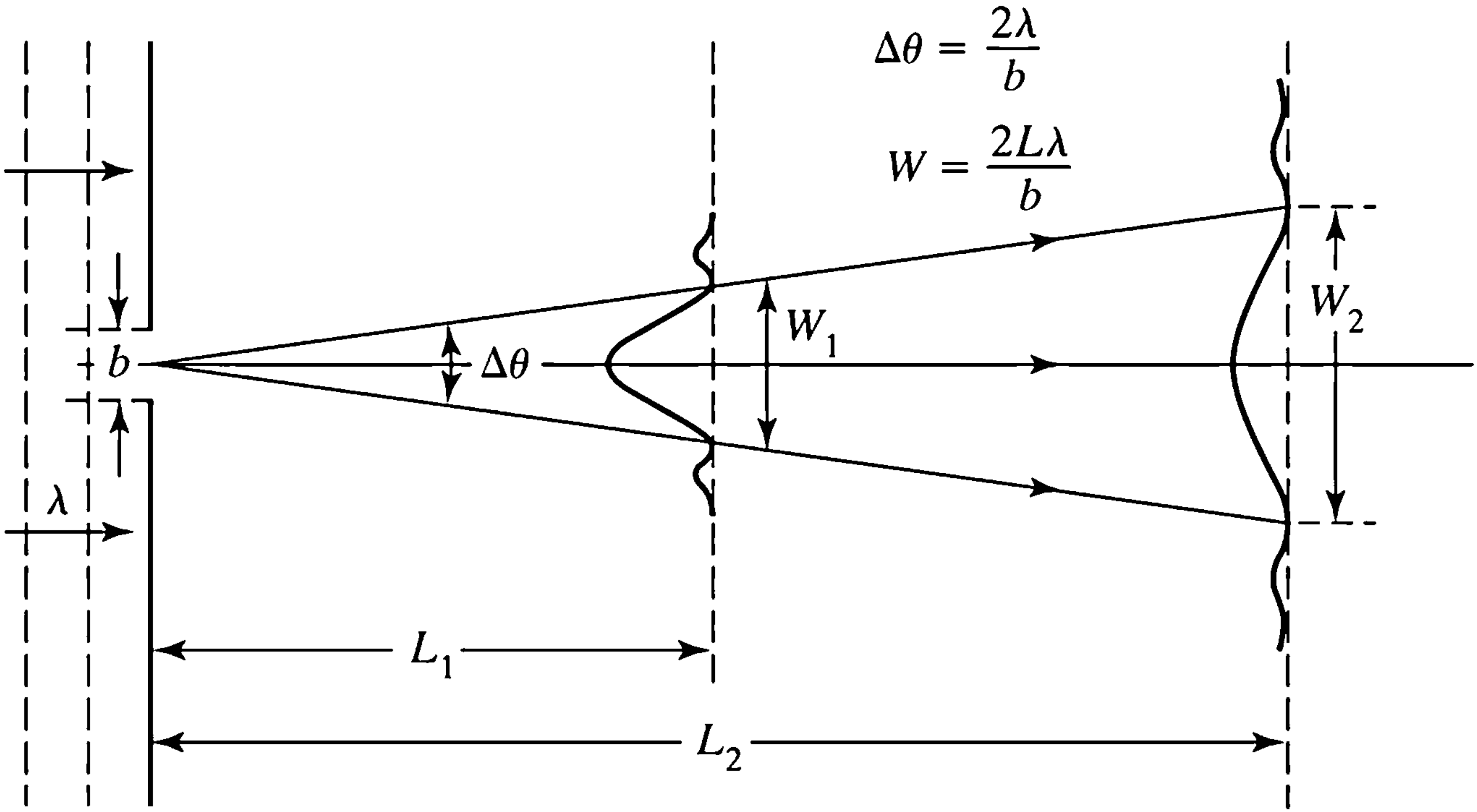
$$\beta = m\pi \qquad \mathrm{m} = \pm 1, \pm 2, \ldots$$

$$m\lambda = b\sin\theta$$

Maxima at:

$$\beta = \tan\beta$$ (graph it!)

Beam spreading



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