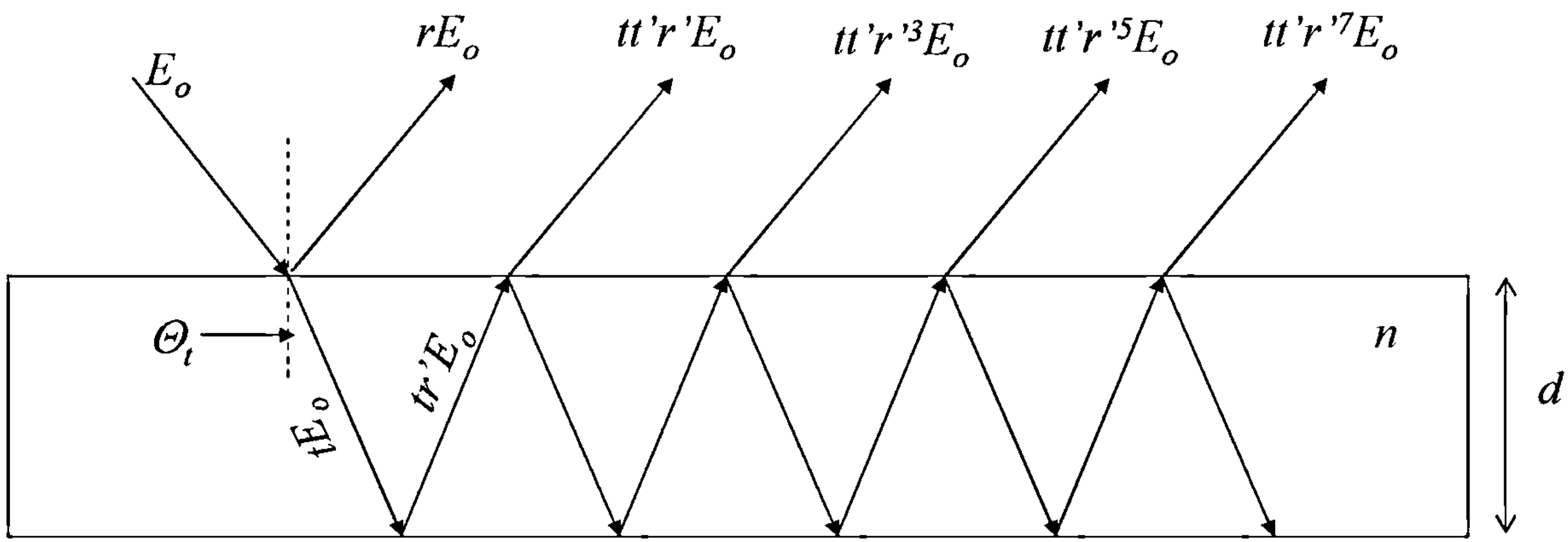
23. Multiple Beam Interference

Reflection coefficient: $r = \dfrac{E_r}{E_o}$ for internal: $r' = \dfrac{E_r}{E_o}$

Transmission coefficient: $t = \dfrac{E_t}{E_o}$ for internal: $t' = \dfrac{E_t}{E_o}$

Amplitude of each reflection:



Phase lag between reflections:

$$\delta = \frac{2\pi}{\lambda_o}\left(\Delta_p + \Delta_r\right) = \frac{2\pi}{\lambda_o}\left(2nd\cos(\theta_t) + 0\right)$$

$\downarrow$ $\downarrow$

(difference between first and rest will be covered by reflection amplitudes r and r' later)

Use complex notation: $E = E_o e^{i\omega t}$

Modify amplitude and phase for each reflected ray:

$$E_1 = rE_o e^{i\omega t}$$

$$E_2 = tt'r'E_o e^{i(\omega t - \delta)}$$

$$E_3 = tt'r'^3 E_o e^{i(\omega t - 2\delta)}$$

$$E_4 = tt'r'^5 E_o e^{i(\omega t - 3\delta)}$$

$$E_N = tt'r'^{(2N-3)} E_o e^{i(\omega t - (N-1)\delta)}$$

For $N >= 2$

Total reflected field:

$$E_R = rE_o e^{j\omega t} + \sum_{N=2}^{\infty} tt' r'^{(2N-3)} E_o e^{j(\omega t - (N-1)\delta)}$$

factor

$$E_R = E_o e^{j\omega t}\left[r + tt' r' e^{-j\delta} \sum_{N=2}^{\infty} r'^{(2N-4)} e^{-j(N-2)\delta} \right]$$

combine

$$E_R = E_o e^{j\omega t}\left[r + tt' r' e^{-j\delta} \sum_{N=2}^{\infty} \left(r'^2 e^{-j\delta} \right)^{(N-2)} \right]$$

This makes a geometric series:

$$\sum_{N=2}^{\infty} x^{(N-2)} = 1 + x + x^2 + \ldots$$

Which converges to: $= \dfrac{1}{1-x}$ for |x|<1

$$E_R = E_o e^{j\omega t}\left[r + tt'r'e^{-j\delta}\left(\frac{1}{1-r'^2 e^{-j\delta}}\right)\right]$$

By the way:

$$\left.\begin{aligned} tt' &= 1 - r^2 \\ r &= -r' \end{aligned}\right\}$$

Stoke's Relations

(I'll cover it later - I promise)

(You will wish I hadn't)

$$E_R = E_o e^{j\omega t}\left[\frac{r\left(1-e^{-j\delta}\right)}{1-r^2 e^{-j\delta}}\right]$$

$$I_R = \tfrac{1}{2}\varepsilon_o c E_R{}^2$$

...but when using complex notation for fields:

$I_R = \tfrac{1}{2}\varepsilon_o c\left|E_R\right|^2$ ...which means... $I_R = \tfrac{1}{2}\varepsilon_o c E_R E_R{}^*$

...where * means the complex conjugate (j -> $-j$)

$$I_R = \tfrac{1}{2}\varepsilon_o c E_o{}^2 e^{j\omega t}\left[\frac{r\left(1-e^{-j\delta}\right)}{1-r^2 e^{-j\delta}}\right] e^{-j\omega t}\left[\frac{r\left(1-e^{+j\delta}\right)}{1-r^2 e^{+j\delta}}\right]$$

$$I_R = \tfrac{1}{2}\varepsilon_o c E_o{}^2\left[\frac{r^2\left(1-e^{-j\delta}-e^{+j\delta}+1\right)}{1-r^2e^{-j\delta}-r^2e^{+j\delta}+r^4}\right]$$

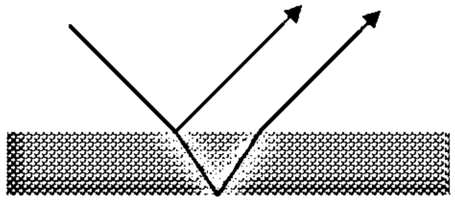
Use: $2\cos\delta = e^{i\delta}+e^{-i\delta}$

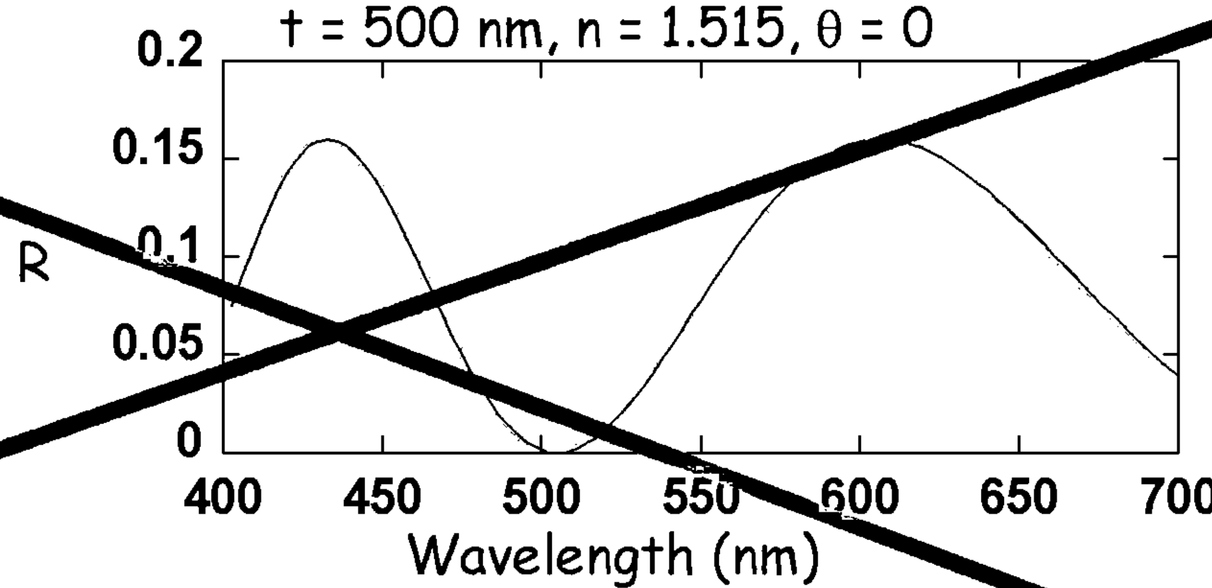
$$I_R = \tfrac{1}{2}\varepsilon_o c E_o{}^2\left[\frac{r^2(2-2\cos\delta)}{r^4-r^2(2\cos\delta)+1}\right]$$

$$I_o = \tfrac{1}{2}\varepsilon_o c E_o{}^2 \qquad\qquad R = I_R/I_o$$

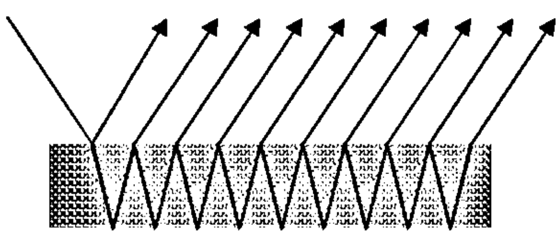
$$R = \frac{r^2(2-2\cos\delta)}{r^4-r^2(2\cos\delta)+1}$$

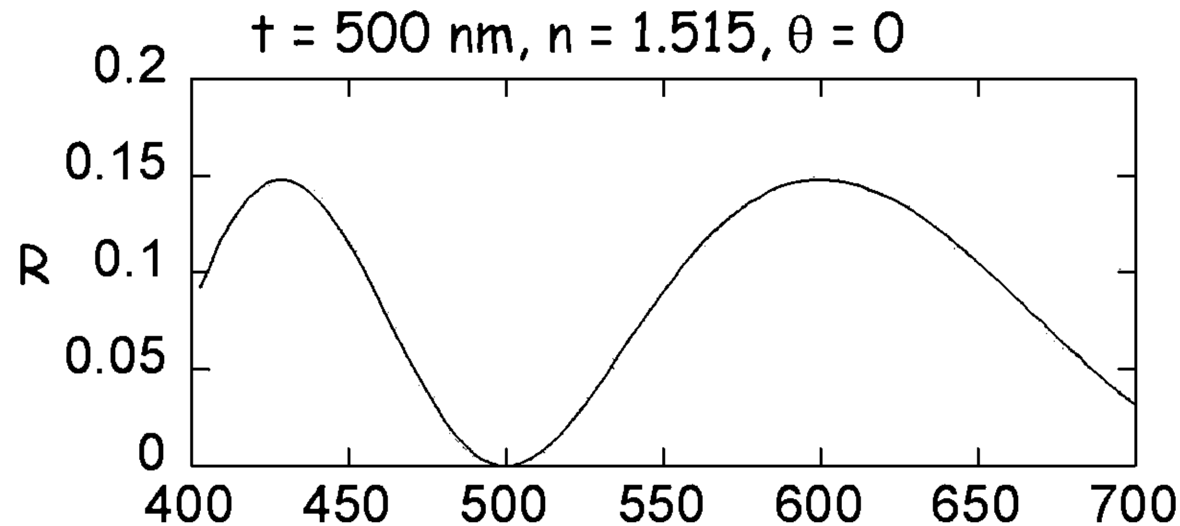
Double Beam





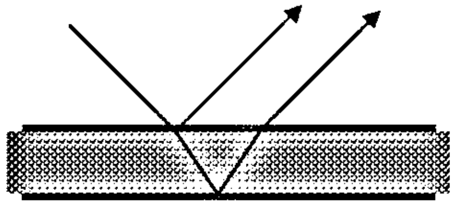
Multiple Beam

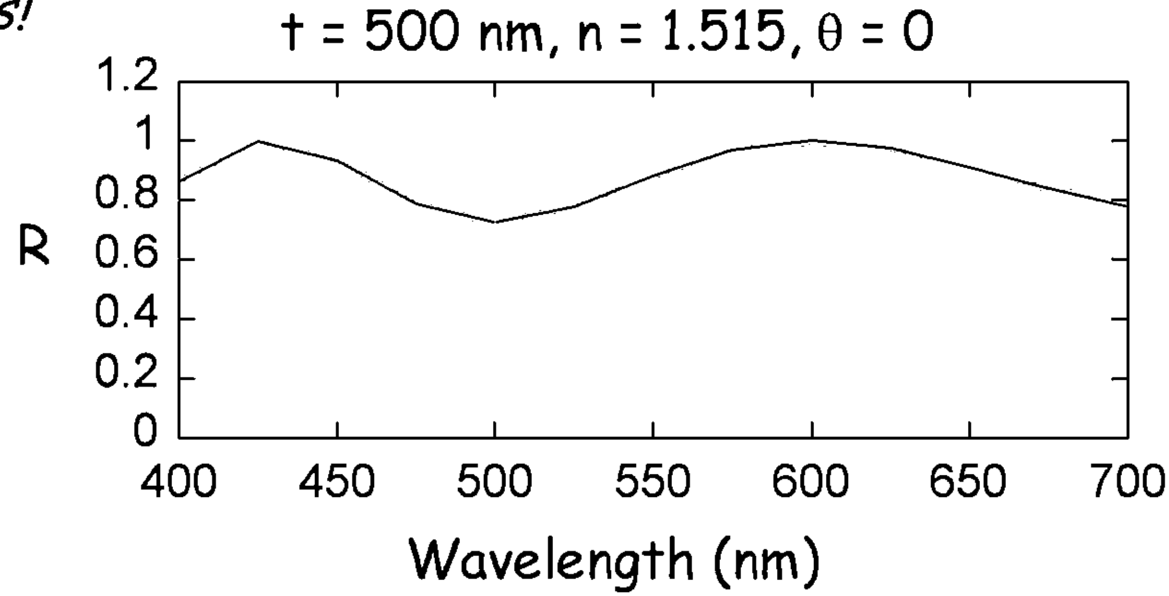




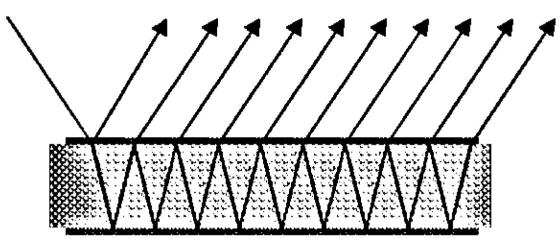
Add 50% reflective surfaces!

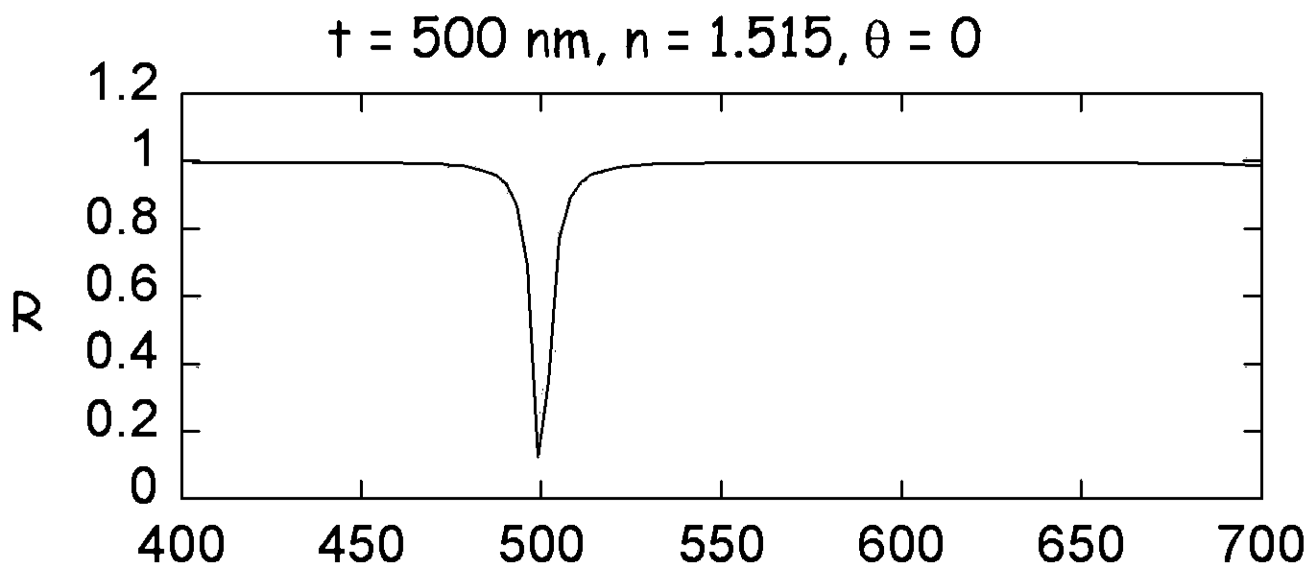
Double Beam



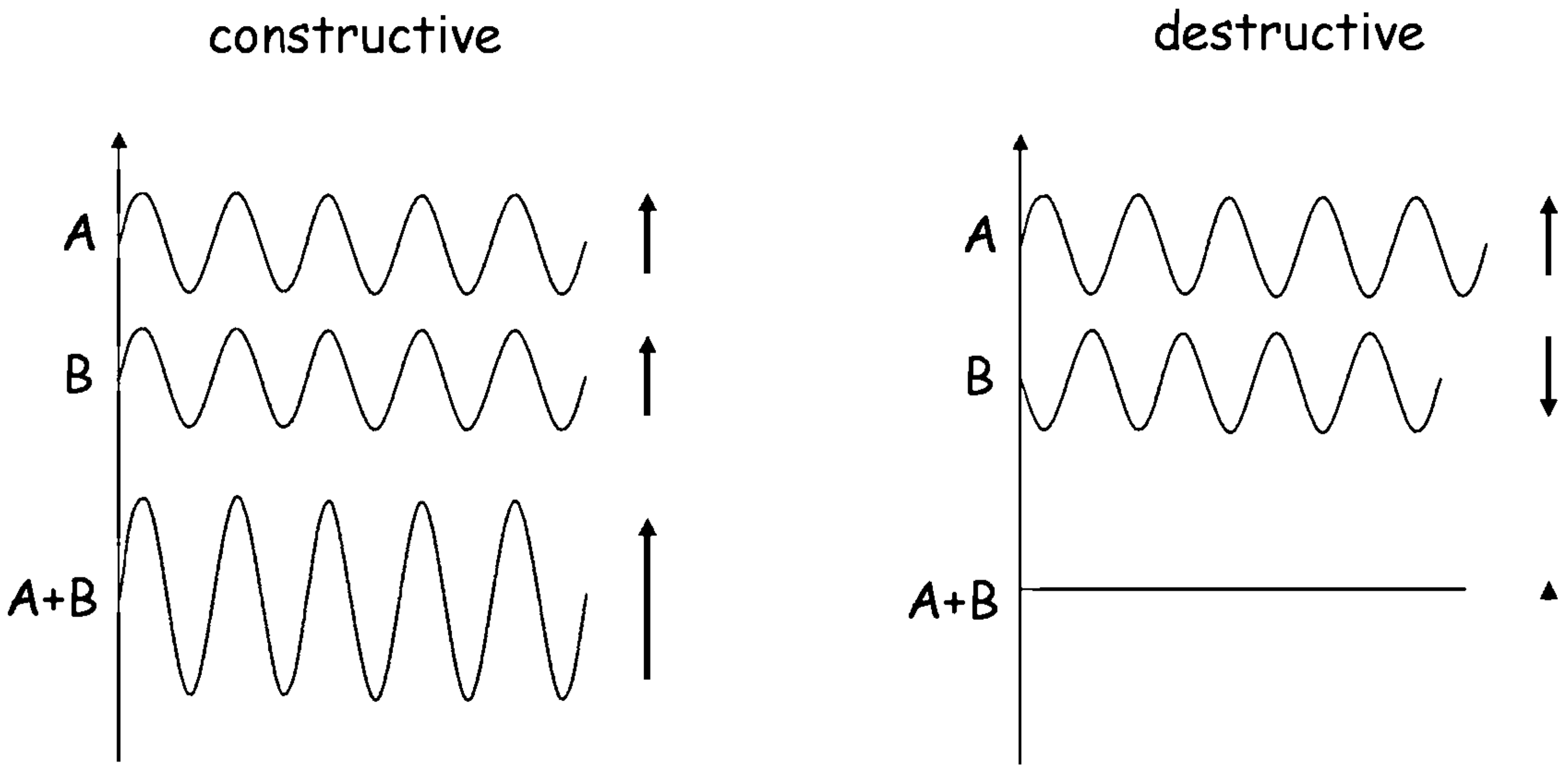


Multiple Beam



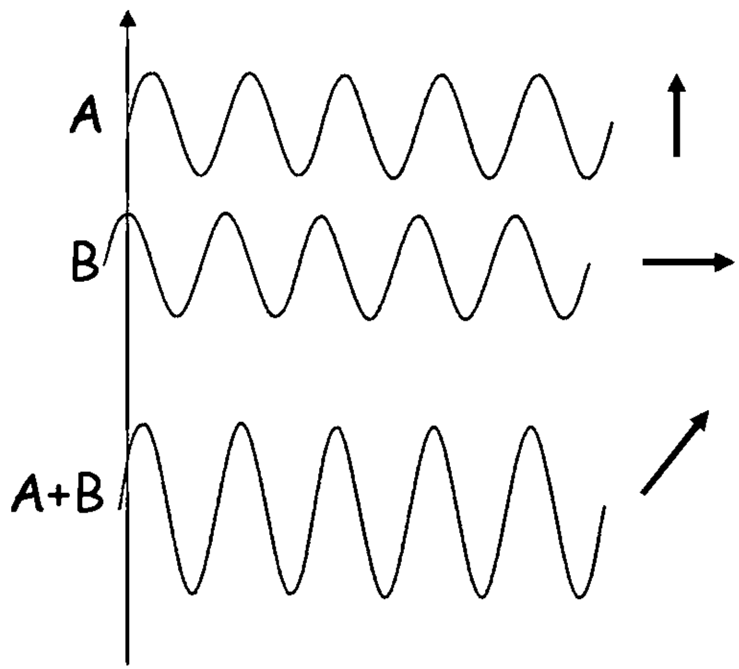


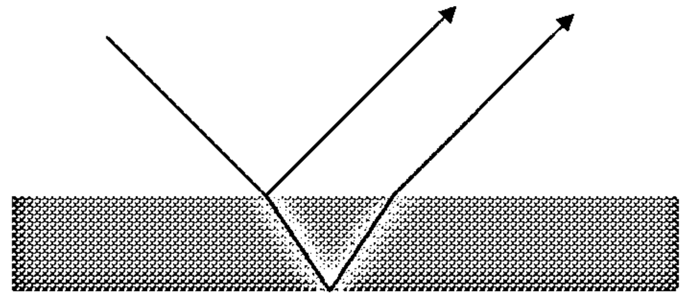
Phasors: Describe amplitude and phase with vector length and direction.



Phasors: Describe amplitude and phase with vector length and direction.

in between

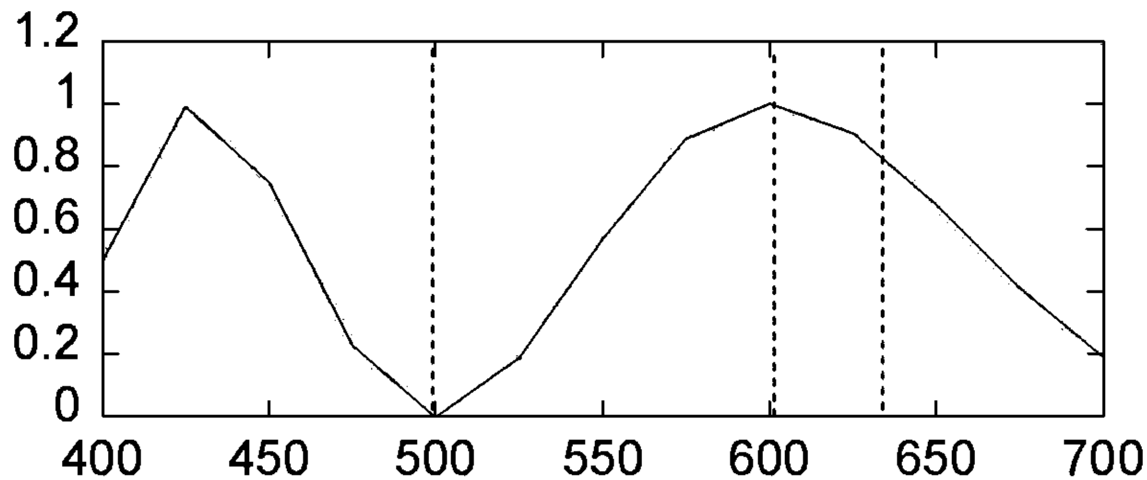


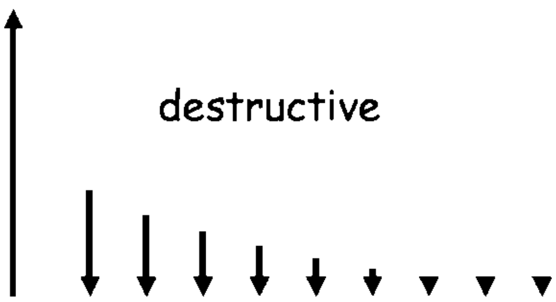


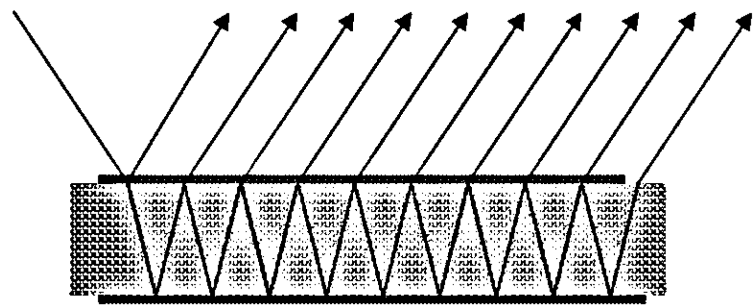
constructive:

destructive:

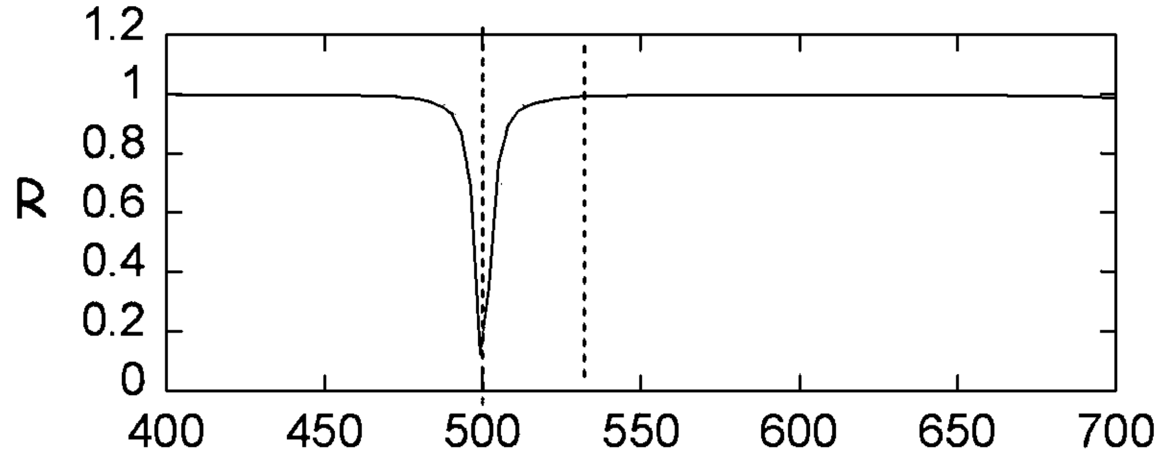
$\lambda/20$ off constructive:



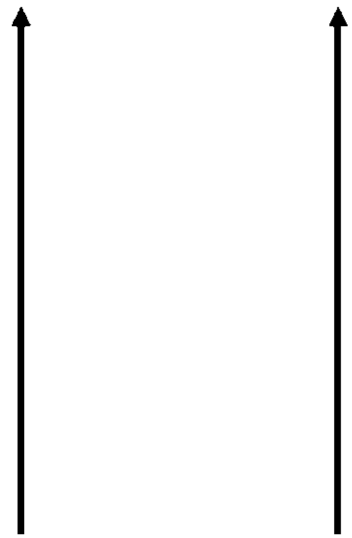

destructive



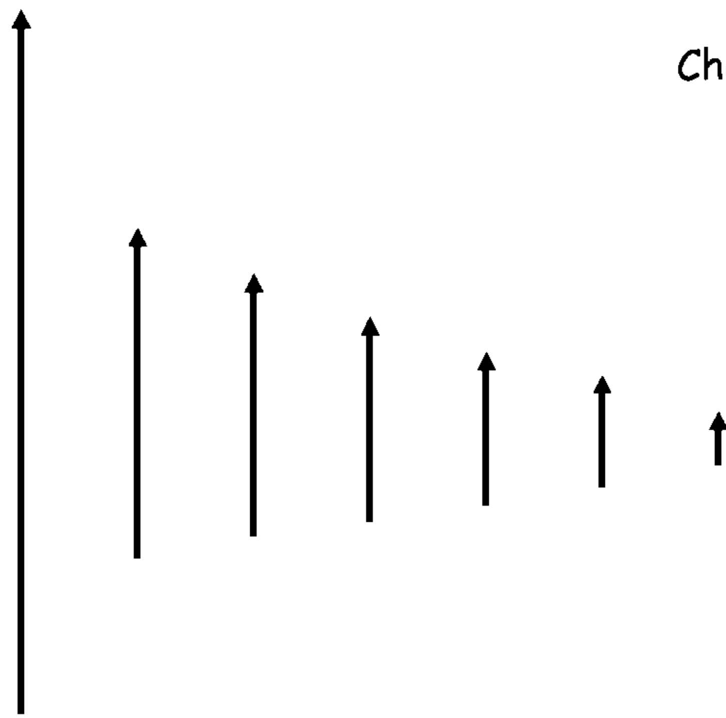

λ/20 off destructive


1.2
1
0.8
0.6
0.4
0.2
0
R
400
450
500
550
600
650
700

Double Beam

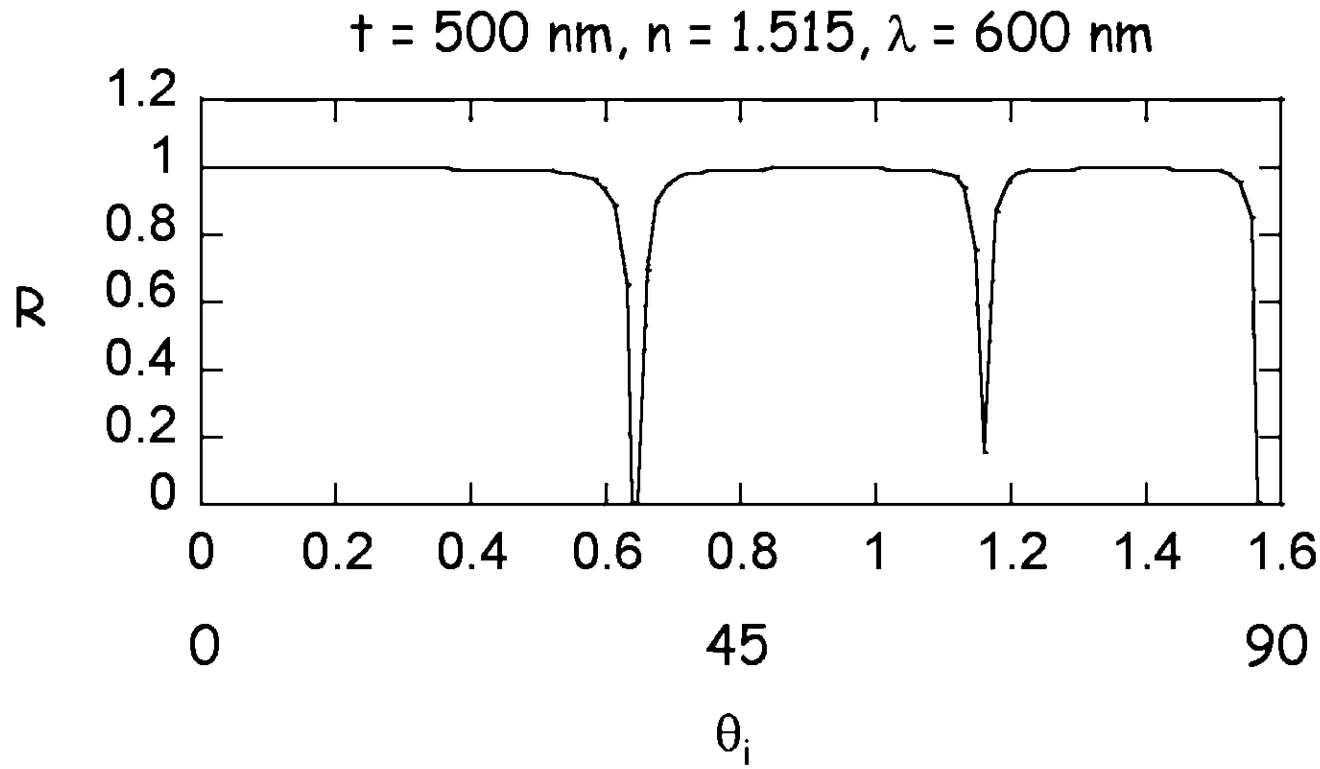


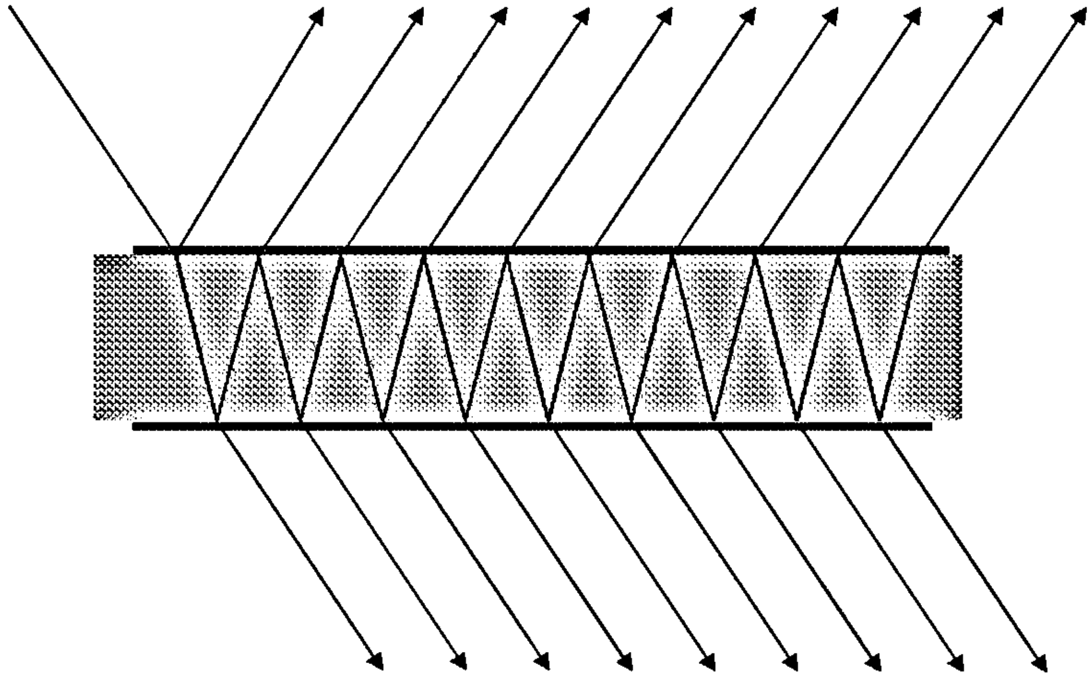
Multiple Beam



Changing ***phase***, not time

Multiple beam interference also causes sharp peaks with angle of incidence:





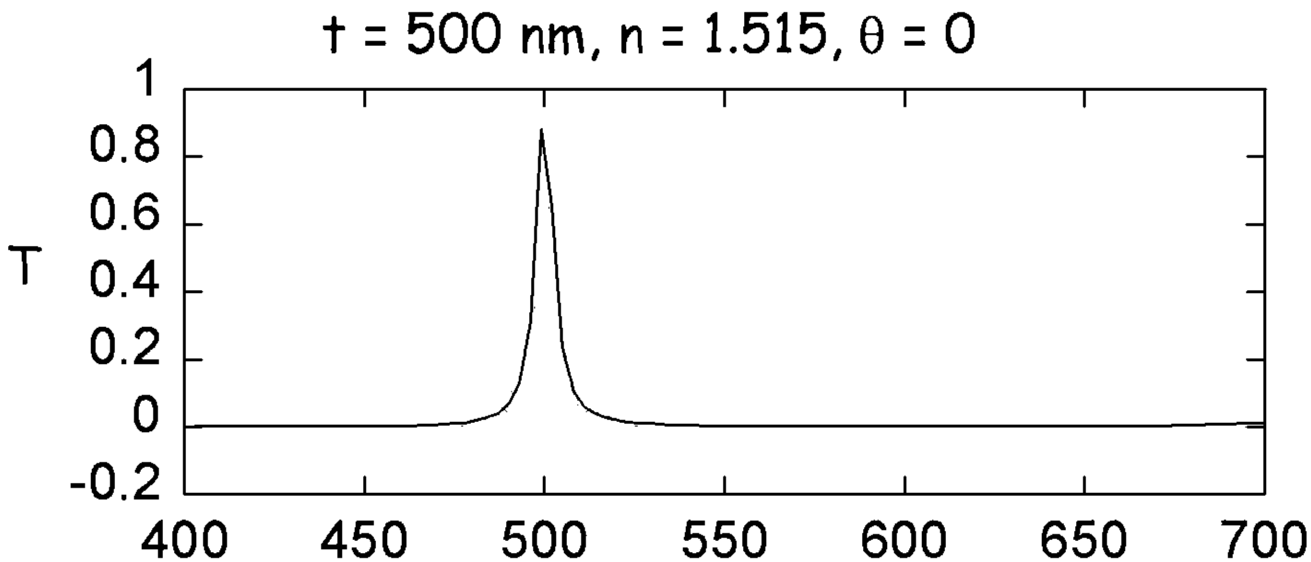
$$R + T = 1$$

$$\frac{r^2\left(2 - 2\cos\delta\right)}{r^4 - r^2\left(2\cos\delta\right) + 1} + T = 1$$

$$T = \frac{r^4 - r^2\left(2\cos\delta\right) + 1}{r^4 - r^2\left(2\cos\delta\right) + 1} - \frac{r^2\left(2 - 2\cos\delta\right)}{r^4 - r^2\left(2\cos\delta\right) + 1}$$

$$T = \frac{r^4 - 2r^2 + 1}{r^4 - r^2(2\cos\delta) + 1}$$

$$T = \frac{\left(1 - r^2\right)^2}{r^4 - r^2(2\cos\delta) + 1}$$



Multiple beams interfere constructively and destructively with a much sharper phase dependence than double beam interference since the multiple interfering components dephase at different rates.