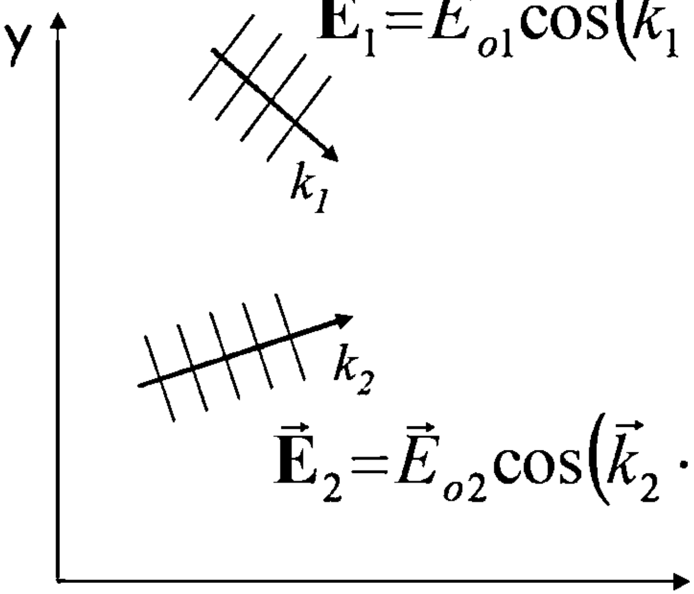


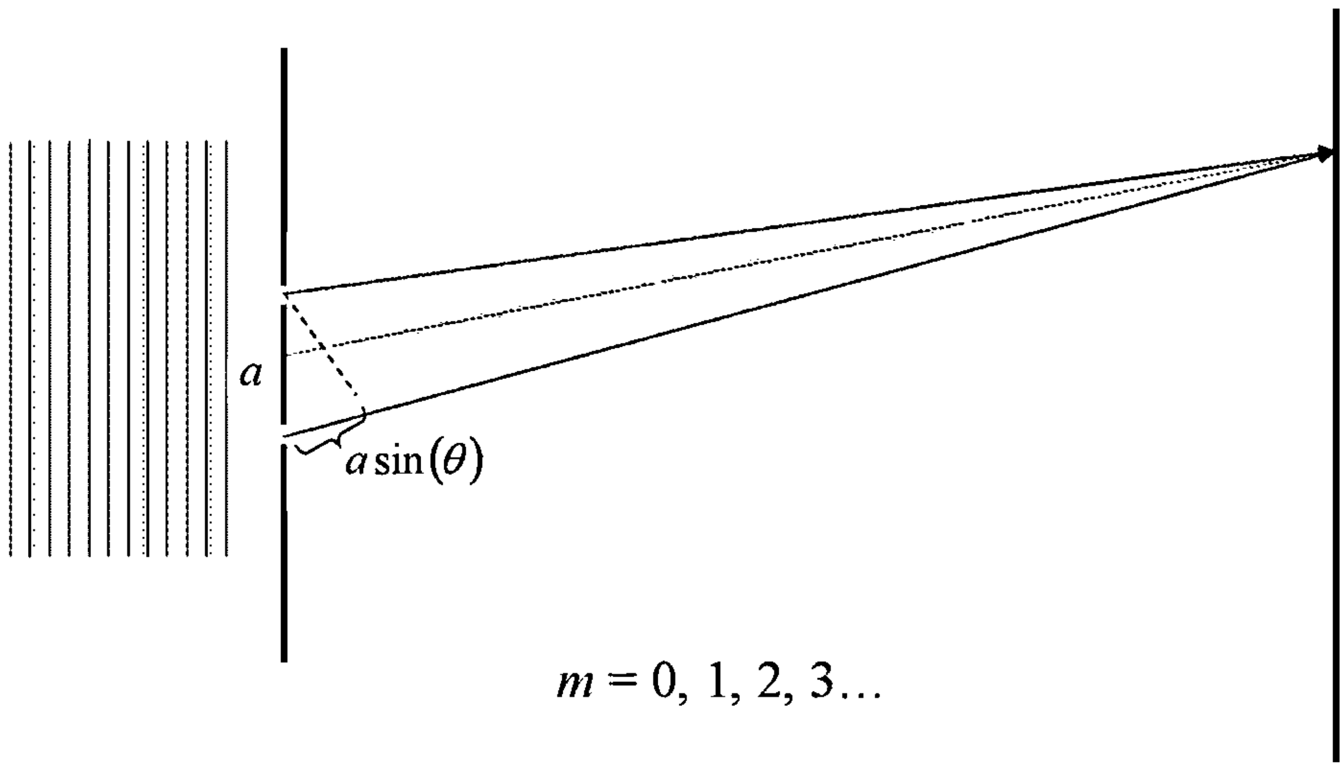
$$\vec{\mathbf{E}}_1 = \vec{E}_{o1}\cos\left(\vec{k}_1 \cdot \vec{r} - \omega t + \varepsilon_1\right)$$

$$\vec{\mathbf{E}}_2 = \vec{E}_{o2}\cos\left(\vec{k}_2 \cdot \vec{r} - \omega t + \varepsilon_2\right)$$

$$I_{sum} = I_1 + I_2 + \varepsilon_o c \vec{\mathbf{E}}_{o1} \cdot \vec{\mathbf{E}}_{o2} \cos(\delta)$$

$$I_{sum} = 4 I_0 \cos^2\left(\frac{\delta}{2}\right)$$

$$\delta = \left(\vec{\mathbf{k}}_1 - \vec{\mathbf{k}}_2\right) \cdot \vec{\mathbf{r}} + \left(\varepsilon_1 - \varepsilon_2\right)$$



$m = 0, 1, 2, 3\ldots$

Maxima at:

$$m\lambda = a\sin(\theta_{max})$$

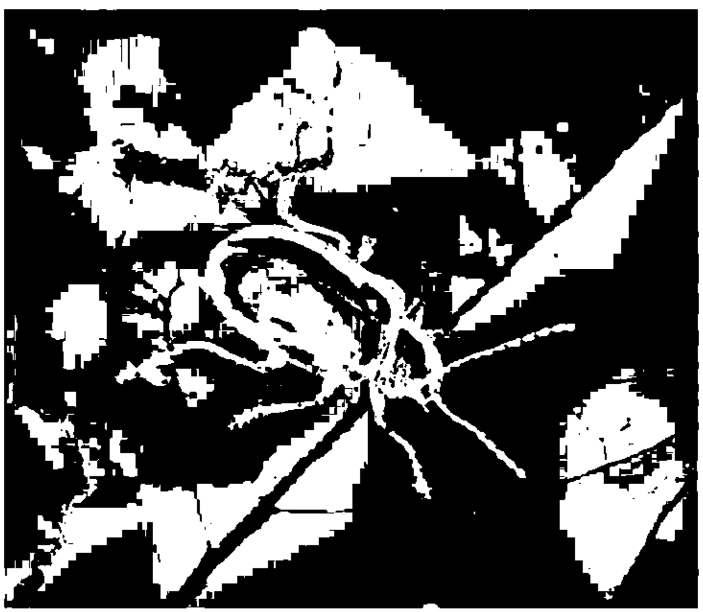
$$m\lambda \approx a\frac{y_{max}}{s}$$

$$I_p = 4 I_0 \cos^2\left(\frac{\pi a}{s\lambda} y\right)$$

Thin Film Interference

$$I_{12}$$

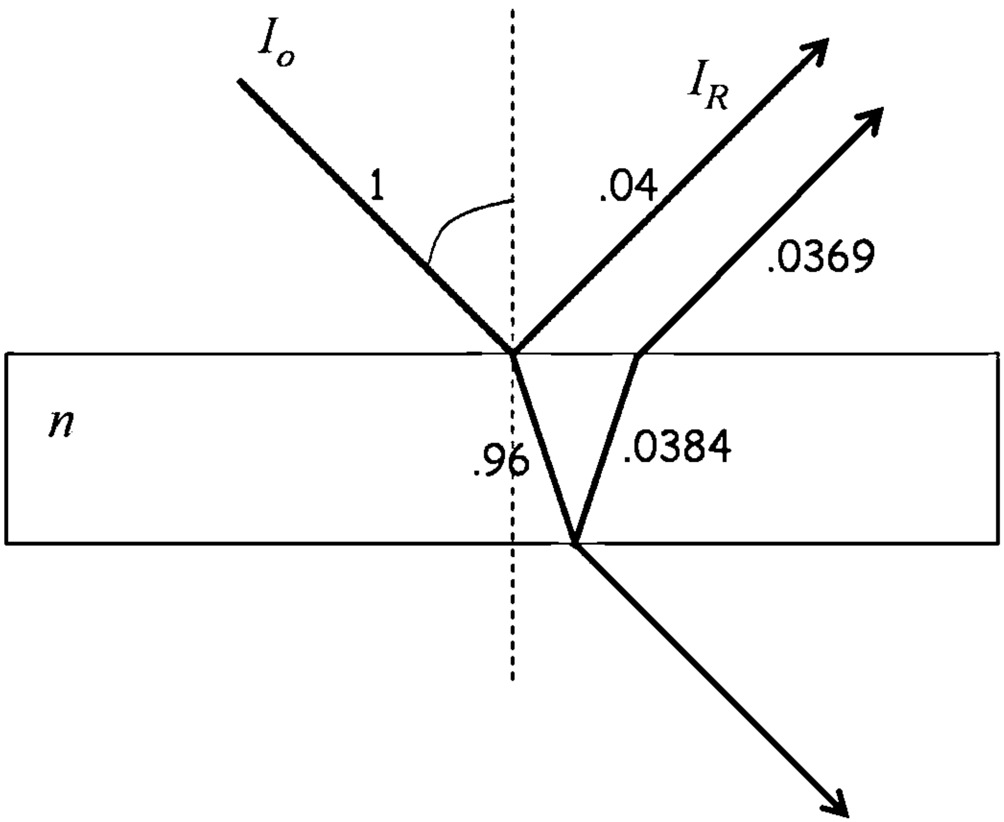
opal



dogbane beetle

peacock

Reflection from a dielectric slab:



Reflectance:

$$R = \frac{I_R}{I_o} = \left(\frac{n-1}{n+1}\right)^2$$

(near normal incidence)

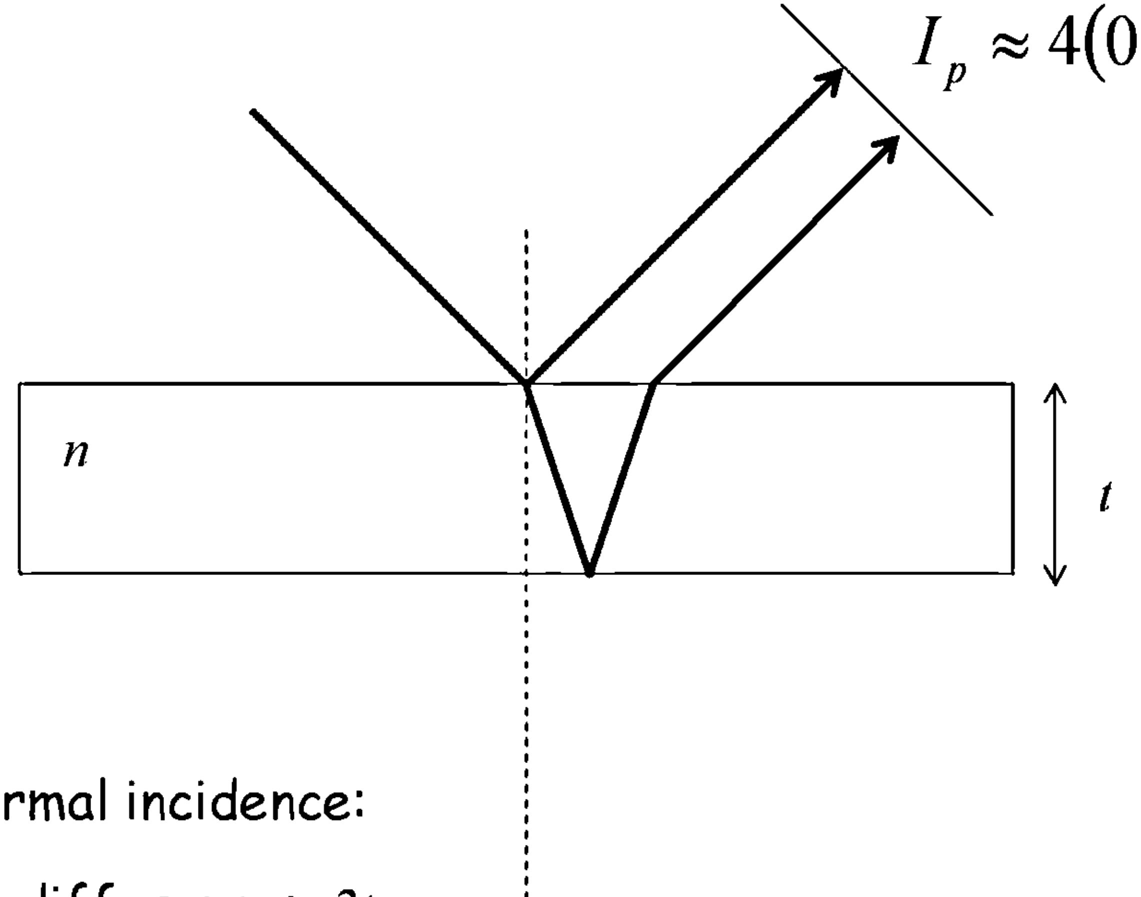
(n = relative index)

R = 0.04 for glass

"amplitude division"

Dielectric Films

$$I_p \approx 4(0.04 I_0)\cos^2\left(\frac{\delta}{2}\right)$$



What is δ?

Assume near normal incidence:

Physical path difference: $2t$

Optical path difference: $\Delta_p = 2tn$

phase difference: $\delta = \frac{\Delta_p}{\lambda_o} 2\pi$

Wait!! Reflections can cause phase shifts!

First reflection: π phase shift

Second reflection: no phase shift

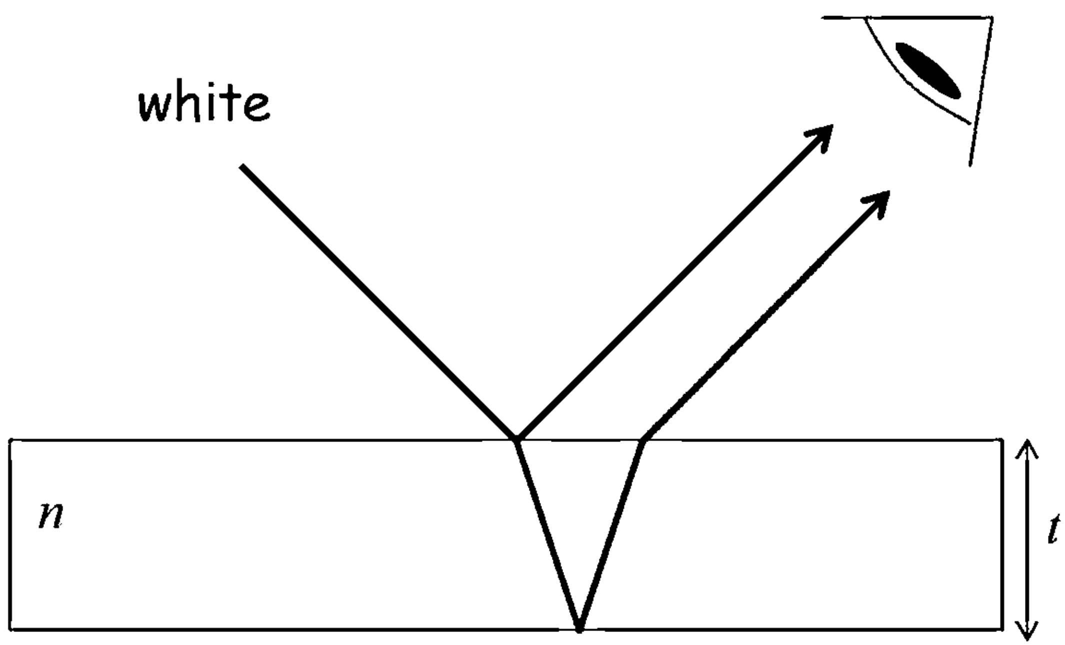
How does this lead to color?

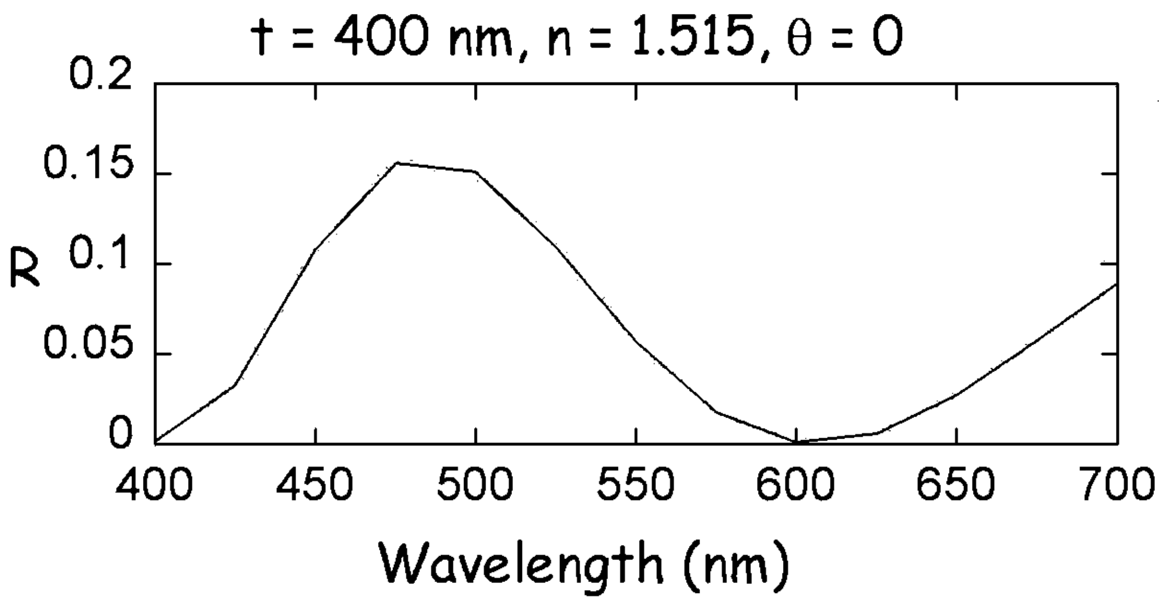
"Reflection" path difference: $\Delta_r = \frac{1}{2}\lambda_o$

Total phase difference: $$\delta = \frac{\Delta_p + \Delta_r}{\lambda_o} 2\pi$$

Constructive	Destructive
$\delta = 2m\pi$	$\delta = (2m+1)\pi$
$\Delta_p + \Delta_r = m\lambda_o$	$\Delta_p + \Delta_r = (m + \frac{1}{2})\lambda_o$

All $m = 0, 1, 2, 3, \ldots$





$$I_p = 0.16 I_0 \cos^2\left(\frac{\delta}{2}\right)$$

$$I_p = 0.16 I_0 \cos^2\left(\pi \frac{\Delta_p + \Delta_r}{\lambda_o}\right)$$

$$I_p = 0.16 I_0 \cos^2\left(\pi \frac{2tn + \frac{\lambda_o}{2}}{\lambda_o}\right)$$

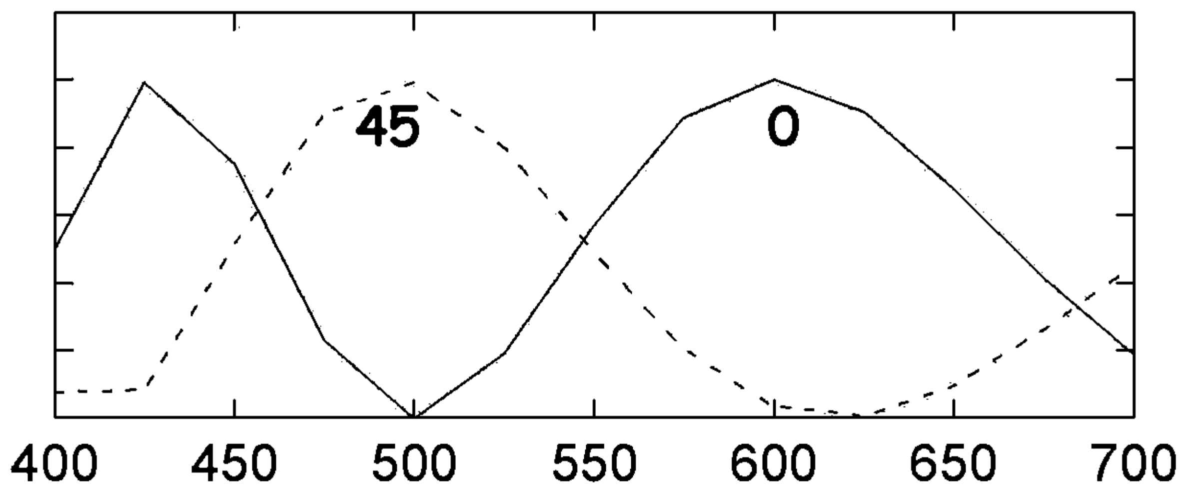
$$I_p = 0.16 I_0 \cos^2\left(\frac{2tn\pi}{\lambda_o} + \frac{\pi}{2}\right)$$

For varying angle of incidence:

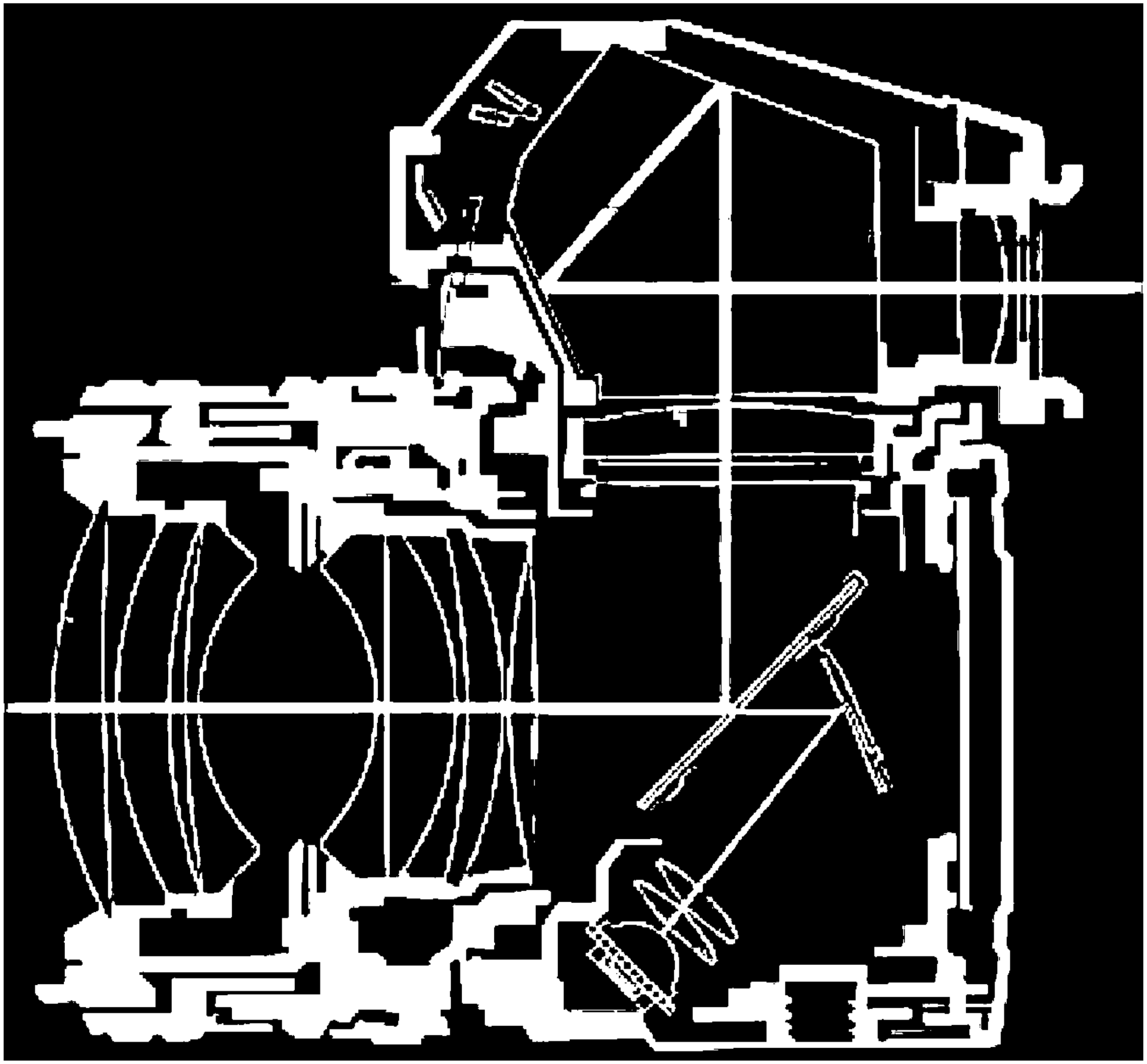
$$\Delta_p = 2tn\cos\left(\sin^{-1}\left(\sin(\theta_i)/n\right)\right)$$

$$I_p = 0.16 I_0 \cos^2\left(\frac{2tn\pi\cos\left(\sin^{-1}\left(\sin(\theta_i)/n\right)\right)}{\lambda_o} + \frac{\pi}{2}\right)$$

normal and 45 degrees



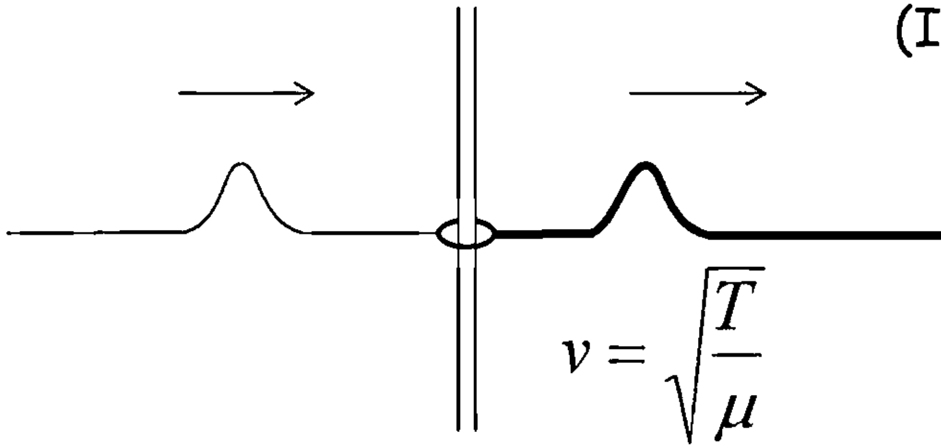
4% per interface adds up!



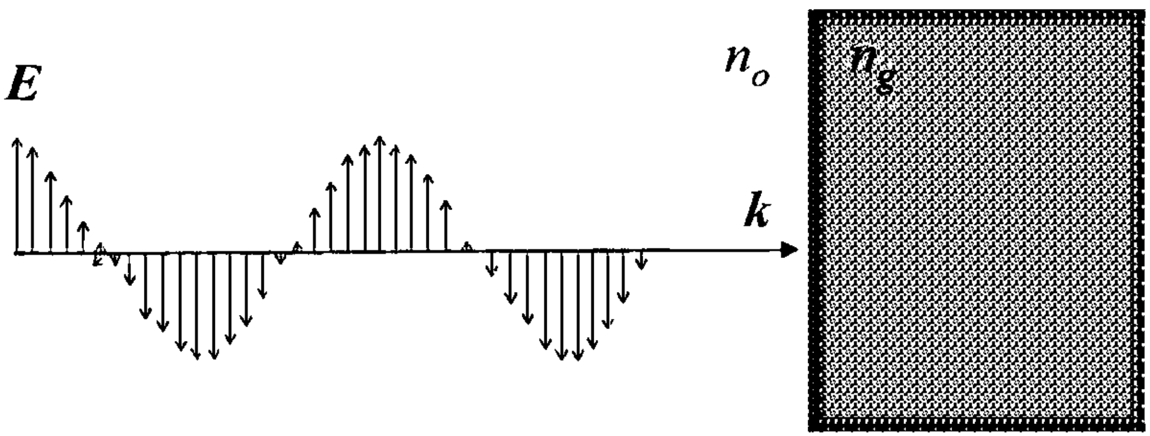
Goal: R = 0, T = 1

$$\sqrt{T_1\mu_1} = \sqrt{T_2\mu_2}$$

(Impedance Matching)

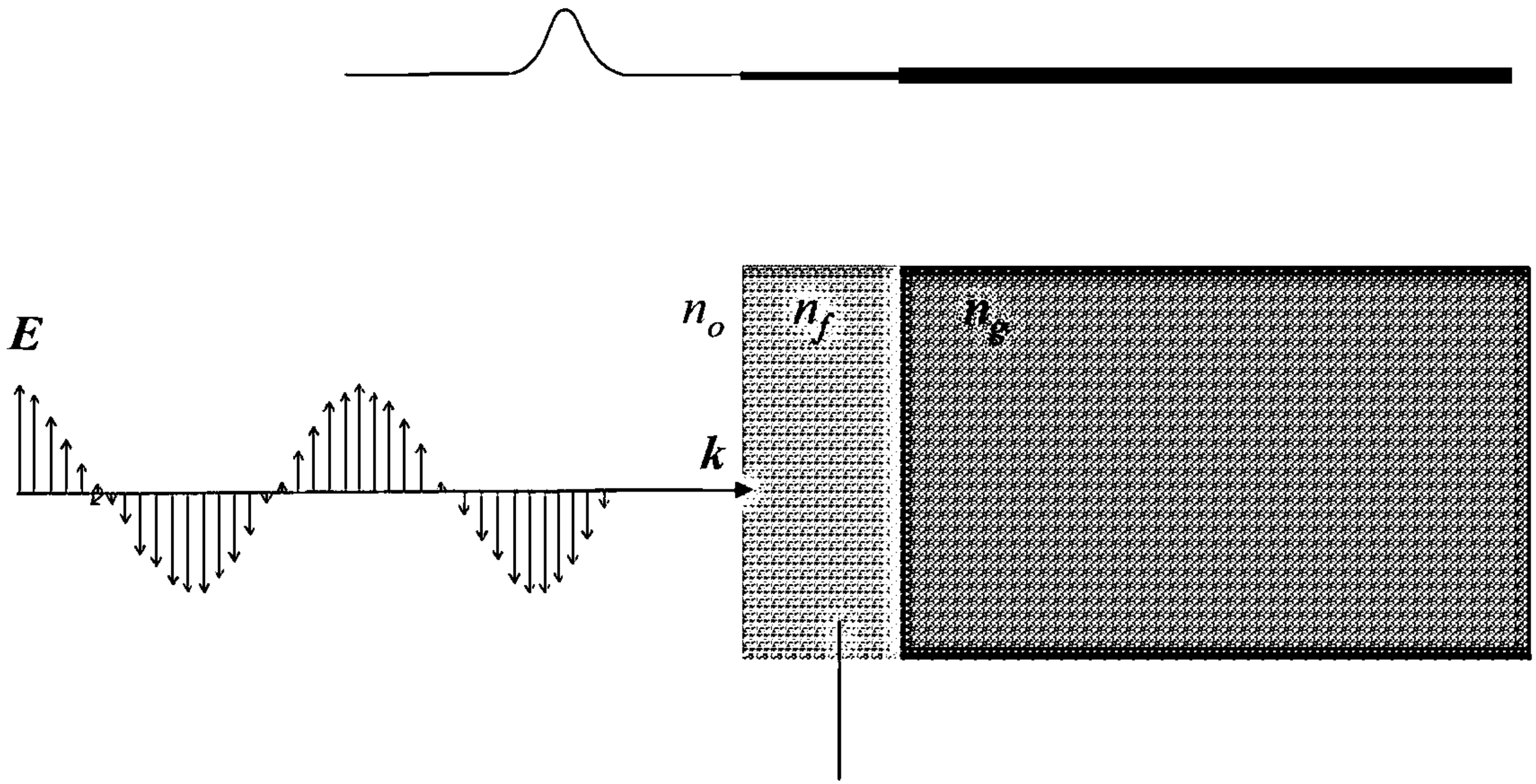


$$v = \sqrt{\frac{T}{\mu}}$$



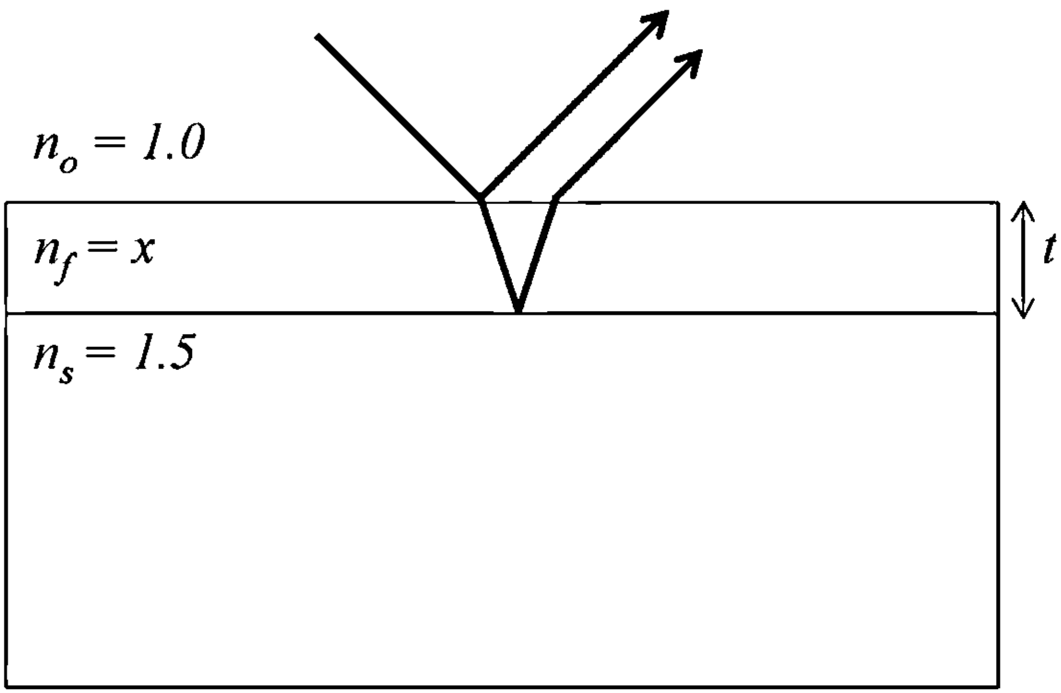
$$v = \frac{c}{n} = \frac{\left(\varepsilon_o\mu_o\right)^{-\frac{1}{2}}}{n}$$

Can only adjust n



Antireflection coating

Antireflection coating: two requirements



1. Destructive interference:

$$\Delta_p + \Delta_r = \left(m + \tfrac{1}{2}\right)\lambda_o$$

$$2tn_f + 0 = \left(0 + \tfrac{1}{2}\right)\lambda_o$$

$$t = \frac{\lambda_o}{4n_f}$$

1. Equal amplitudes:

$$\frac{\frac{n_f}{n_o}-1}{\frac{n_f}{n_o}+1}=\frac{\frac{n_s}{n_f}-1}{\frac{n_s}{n_f}+1}$$

$$\left(\frac{n_f}{n_o}-1\right)\left(\frac{n_s}{n_f}+1\right)=\left(\frac{n_f}{n_o}+1\right)\left(\frac{n_s}{n_f}-1\right)$$

$$\frac{n_s}{n_o}-\frac{n_s}{n_f}+\frac{n_f}{n_o}-1=\frac{n_s}{n_o}+\frac{n_s}{n_f}-\frac{n_f}{n_o}-1$$

$$\frac{n_s}{n_f}=\frac{n_f}{n_o}$$

$$n_f=\sqrt{n_s n_o}$$

Reflections from the front and back surfaces of a dielectric slab result in two plane waves with an optical path difference and therefore exhibit interference. The interference effects can be engineered to minimize reflectance or produce pretty colors.