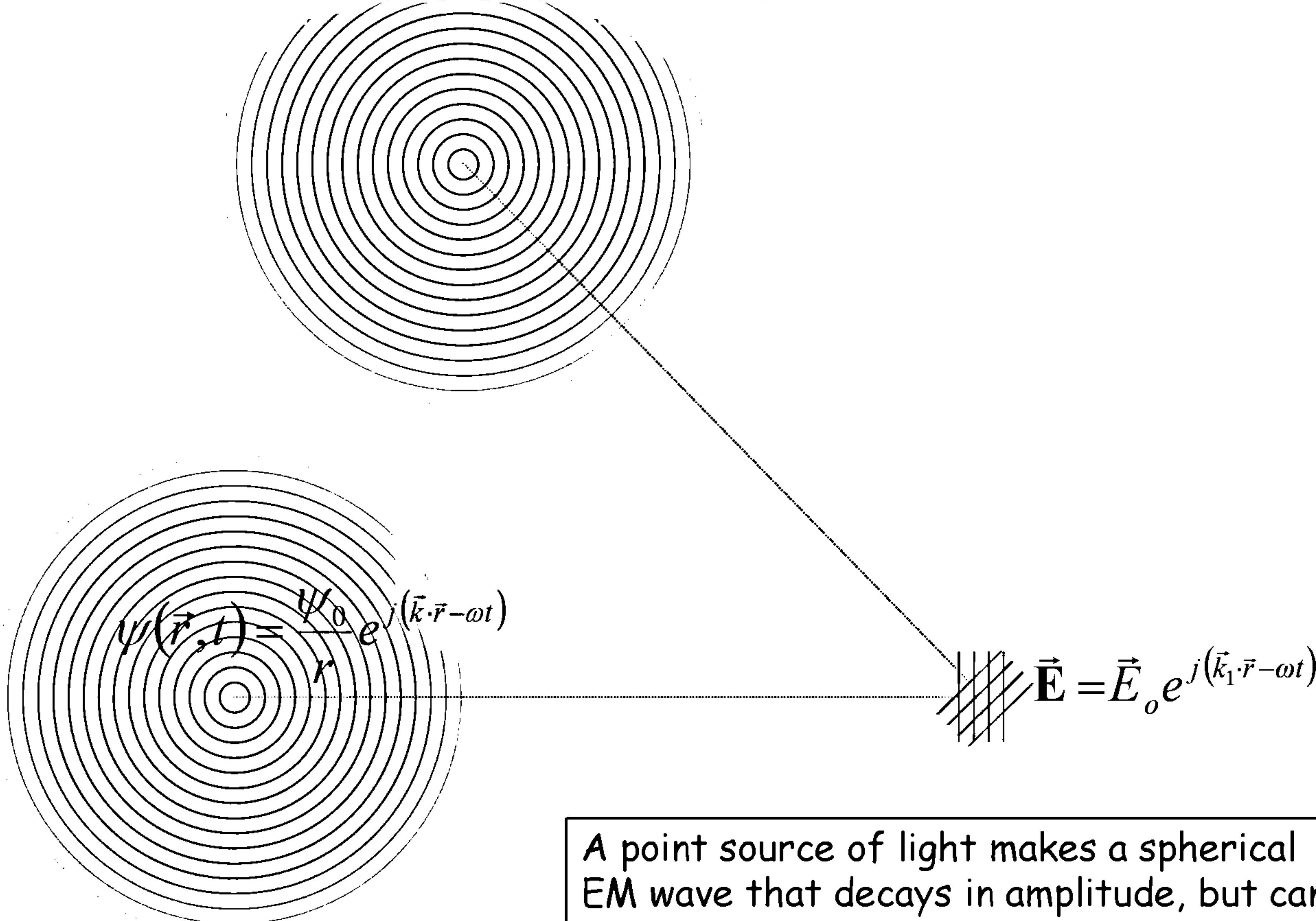
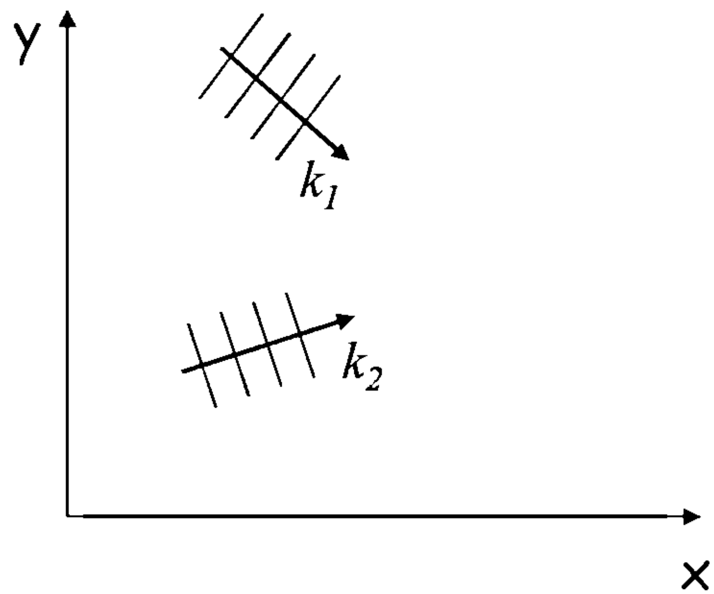


A point source of light makes a spherical EM wave that decays in amplitude, but can be approximated as a plane wave far from the source.

Plane Wave Interference

Q. What happens when two EM plane waves overlap?

A. Superposition!



Assumptions: E field only, 2 plane waves, same frequency, arbitrary directions.

$$\vec{\mathbf{E}}_1 = \vec{E}_{o1}\cos\left(\vec{k}_1 \cdot \vec{r} - \omega t + \varepsilon_1\right)$$

$$\vec{\mathbf{E}}_2 = \vec{E}_{o2}\cos\left(\vec{k}_2 \cdot \vec{r} - \omega t + \varepsilon_2\right)$$

(same wave*number*, but different direction and different phase)

$$\vec{\mathbf{E}}_{sum} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$$

We observe the irradiance:

$$E_e = I_{sum} = \left\langle \left| \vec{\mathbf{S}} \right| \right\rangle = \varepsilon_o c \left\langle \vec{\mathbf{E}}_{sum} \cdot \vec{\mathbf{E}}_{sum} \right\rangle$$

$$= \varepsilon_o c \left\langle \left(\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 \right) \cdot \left(\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 \right) \right\rangle$$

$$= \varepsilon_o c \left\langle \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_2 + 2\vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 \right\rangle$$

$$= \varepsilon_o c \left\langle \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_1 \right\rangle + \varepsilon_o c \left\langle \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_2 \right\rangle + 2\varepsilon_o c \left\langle \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 \right\rangle$$

Interference term

$$= \underbrace{I_1 + I_2}_{} + I_{12}$$

Original beam intensities

$$I_{12} = 2\varepsilon_o c \left\langle \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 \right\rangle$$

Interference term:

$$I_{12} = 2\varepsilon_o c \left\langle \vec{E}_{o1} \cos\left(\vec{k}_1 \cdot \vec{r} + \varepsilon_1 - \omega t\right) \cdot \vec{E}_{o2} \cos\left(\vec{k}_2 \cdot \vec{r} + \varepsilon_2 - \omega t\right) \right\rangle$$

$$I_{12} = 2\varepsilon_o c \vec{E}_{o1} \cdot \vec{E}_{o2} \left\langle \cos\left(\underbrace{\vec{k}_1 \cdot \vec{r} + \varepsilon_1}_{\alpha} - \omega t\right) \cos\left(\underbrace{\vec{k}_2 \cdot \vec{r} + \varepsilon_2}_{\beta} - \omega t\right) \right\rangle$$

Isolate $cos(\omega t)$ by using: $cos(x-y) = cos(x)\, cos(y) + sin(x)\, sin(y)$

$$I_{12} = 2\varepsilon_o c \vec{E}_{o1} \cdot \vec{E}_{o2} \langle (\cos(\alpha)\cos(\omega t) + \sin(\alpha)\sin(\omega t))$$

$$(\cos(\beta)\cos(\omega t) + \sin(\beta)\sin(\omega t)) \rangle$$

$$I_{12} = 2\varepsilon_o c\vec{E}_{o1}\cdot\vec{E}_{o2}\Big[\cos(\alpha)\cos(\beta)\left\langle\cos^2(\omega t)\right\rangle +$$

$$+\sin(\alpha)\sin(\beta)\left\langle\sin^2(\omega t)\right\rangle +$$

$$+\left(\cos(\alpha)\sin(\beta)+\sin(\alpha)\cos(\beta)\right)\left\langle\sin(\omega t)\cos(\omega t)\right\rangle\Big]$$

Over many cycles:

$$\left\langle\sin^2(\omega t)\right\rangle = \tfrac{1}{2} \qquad \left\langle\cos^2(\omega t)\right\rangle = \tfrac{1}{2} \qquad \left\langle\sin(\omega t)\cos(\omega t)\right\rangle = 0$$

$$I_{12} = \varepsilon_o c\vec{E}_{o1}\cdot\vec{E}_{o2}\left[\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)\right]$$

$$I_{12} = \varepsilon_o c \vec{E}_{o1} \cdot \vec{E}_{o2} \cos(\alpha - \beta)$$

$$I_{12} = \varepsilon_o c \vec{E}_{o1} \cdot \vec{E}_{o2} \cos\left(\vec{k}_1 \cdot \vec{r} + \varepsilon_1 - \vec{k}_2 \cdot \vec{r} - \varepsilon_2\right)$$

$$I_{12} = \varepsilon_o c \vec{E}_{o1} \cdot \vec{E}_{o2} \cos\Big(\underbrace{\left(\vec{k}_1 - \vec{k}_2\right) \cdot \vec{r} + (\varepsilon_1 - \varepsilon_2)}\Big)$$

phase difference: δ

$$I_{sum} = I_1 + I_2 + \varepsilon_o c \vec{E}_{o1} \cdot \vec{E}_{o2} \cos(\delta)$$

$$\delta = \left(\vec{k}_1 - \vec{k}_2\right) \cdot \vec{r} + (\varepsilon_1 - \varepsilon_2)$$

Result: A constant irradiance (the sum) modified by a sinusoidal term that depends on their relative phase

ε defines the origin

Interference problems usually consider two paths to a point P.

$\boldsymbol{k} \cdot \boldsymbol{r}$ counts waves in space.

Total phase difference:

$$\delta = \frac{\Delta_{path} + \Delta_{phase}}{\lambda_o} 2\pi$$

$\Delta_{path} =$ Optical path difference

$\Delta_{phase} =$ Phase difference (in meters)

Constructive	Destructive
$\delta = 2m\pi$	$\delta = (2m+1)\pi$
$\Delta_p + \Delta_r = m\lambda_o$	$\Delta_p + \Delta_r = (m + \frac{1}{2})\lambda_o$

All $m = 0, 1, 2, 3, \ldots$