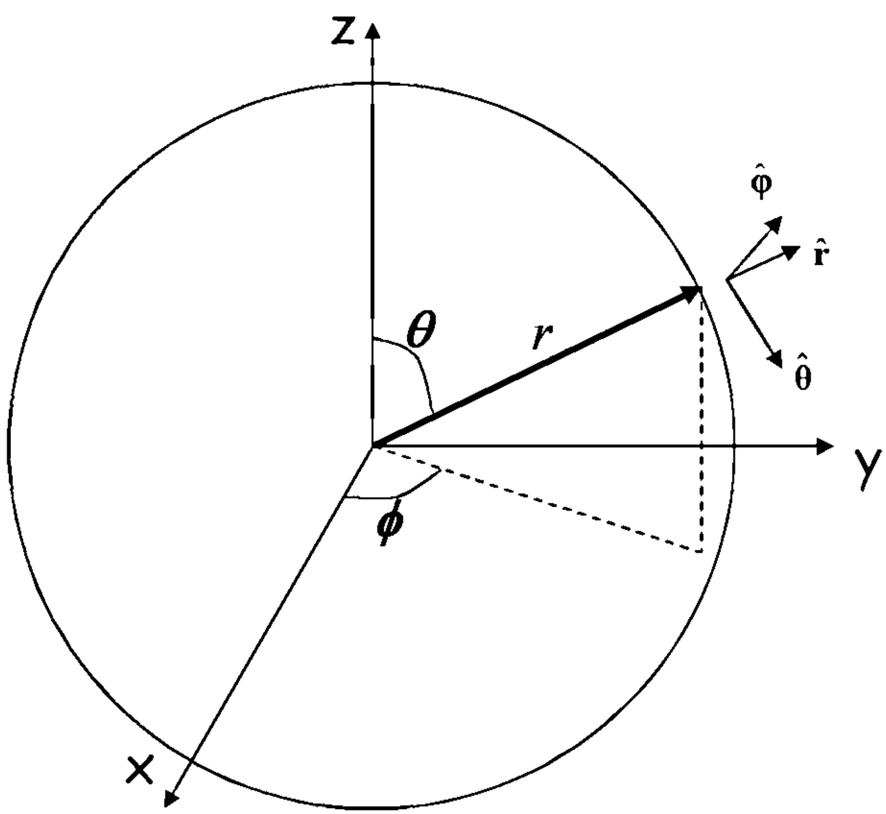
<u>EM spherical waves</u>

$$\nabla^2 \vec{\mathbf{E}} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

...in any coord. system

Differentials depend on coord system:



$$dl = dr\,\hat{r} + r d\theta\,\hat{\theta} + r\sin\theta\,d\varphi\,\hat{\varphi}$$

$$d\tau = r^2 \sin(\theta)\,dr\,d\theta\,d\varphi$$

$$\nabla s = \frac{\partial s}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial s}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial s}{\partial \varphi}\hat{\varphi}$$

1st derivatives | 2nd derivatives | "Laplacian"

Field:

$$\nabla s(x,y,z) \qquad \nabla\times\nabla s \rightarrow 0 \qquad \nabla\cdot\nabla s \rightarrow \nabla^2 s$$

$$\nabla\cdot\vec{\mathbf{V}}(x,y,z) \qquad \nabla\left(\nabla\cdot\vec{\mathbf{V}}\right)$$

$$\nabla\times\vec{\mathbf{V}}(x,y,z) \qquad \nabla\cdot\nabla\times\vec{\mathbf{V}} \rightarrow 0 \qquad \nabla\times\nabla\times\vec{\mathbf{V}}$$

$$\nabla\times\nabla\times\vec{\mathbf{V}} = \nabla\left(\nabla\cdot\vec{\mathbf{V}}\right) - \frac{\partial^2 V_x}{\partial x^2} - \frac{\partial^2 V_y}{\partial y^2} - \frac{\partial^2 V_z}{\partial z^2}$$

The vector spherical Laplacian:

(This is a good time for a scalar wave equation!)

ψ : Our scalar wave which represents the field strength.

$$\nabla^2 \vec{\mathbf{E}} = \mu_o \varepsilon_o \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

$$\nabla^2 \psi = \mu_o \varepsilon_o \frac{\partial^2 \psi}{\partial t^2}$$

The spherical Laplacian:

$$\nabla^2 \psi = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 \psi}{\partial \varphi^2}$$

Unless you are a masochist ..... put the spherical source at the origin!!!!

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)$$

$$= \frac{1}{r^2}\left(2r\frac{\partial\psi}{\partial r} + r^2\frac{\partial^2\psi}{\partial r^2}\right)$$

$$= \frac{2}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial r^2}$$

$$= \frac{1}{r}\frac{\partial^2(r\psi)}{\partial r^2}$$

Put back into wave equation:

$$\frac{1}{r}\frac{\partial^2(r\psi)}{\partial r^2}=\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}$$

$$\frac{\partial^2(r\psi)}{\partial r^2}=\frac{1}{c^2}\frac{\partial^2(r\psi)}{\partial t^2}$$ Wave equation for $r\psi$

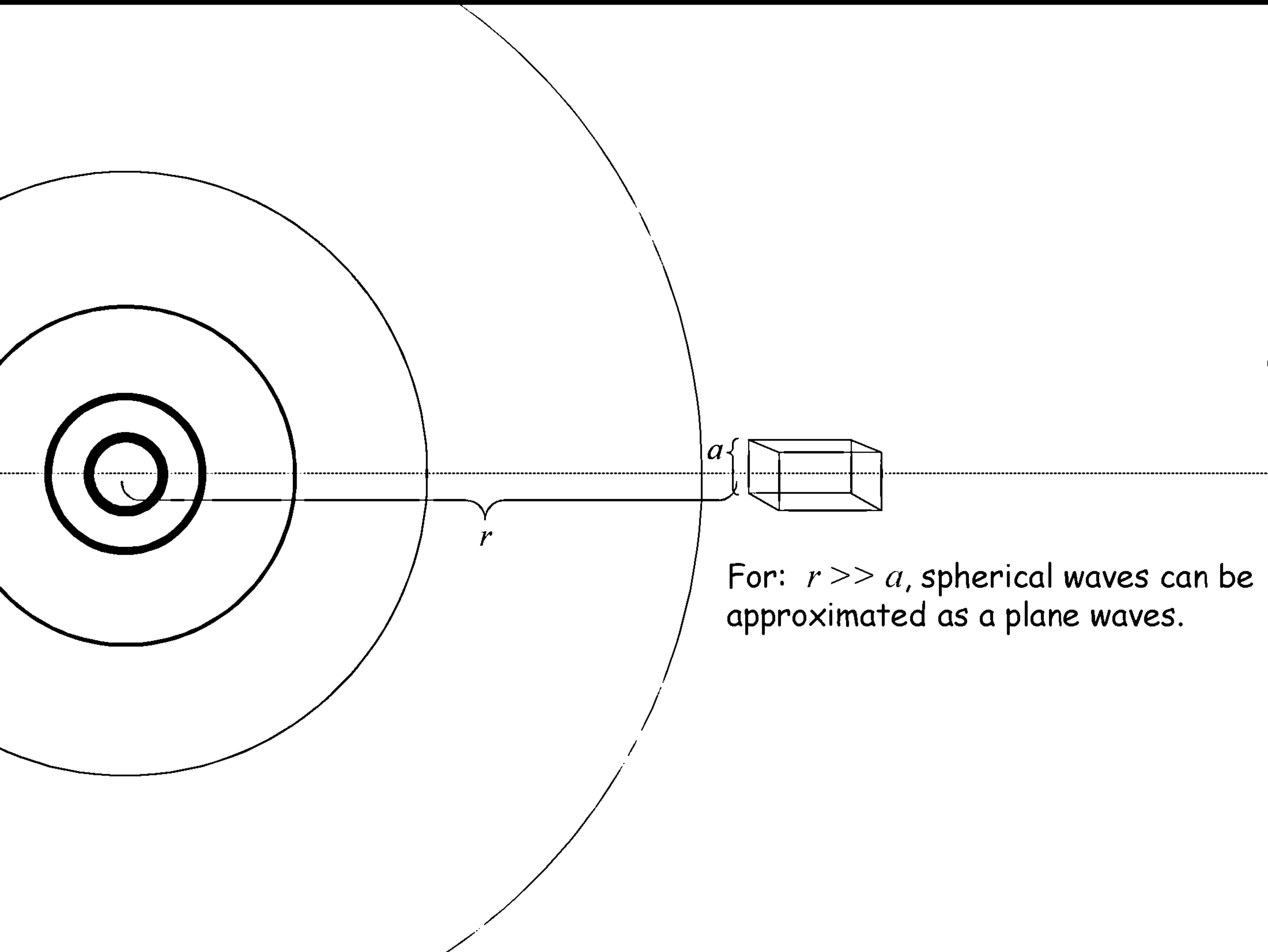
Guess a solution:

$$r\psi(r,t)=\psi_0 e^{j(k\cdot r-\omega t)}$$

Note: ψ (the field strength) and ψ_o (the field amplitude) don't have the same unit.

$$\psi(r,t)=\frac{\psi_0}{r}e^{j(k\cdot r-\omega t)}$$

As r increases, the field strength decreases



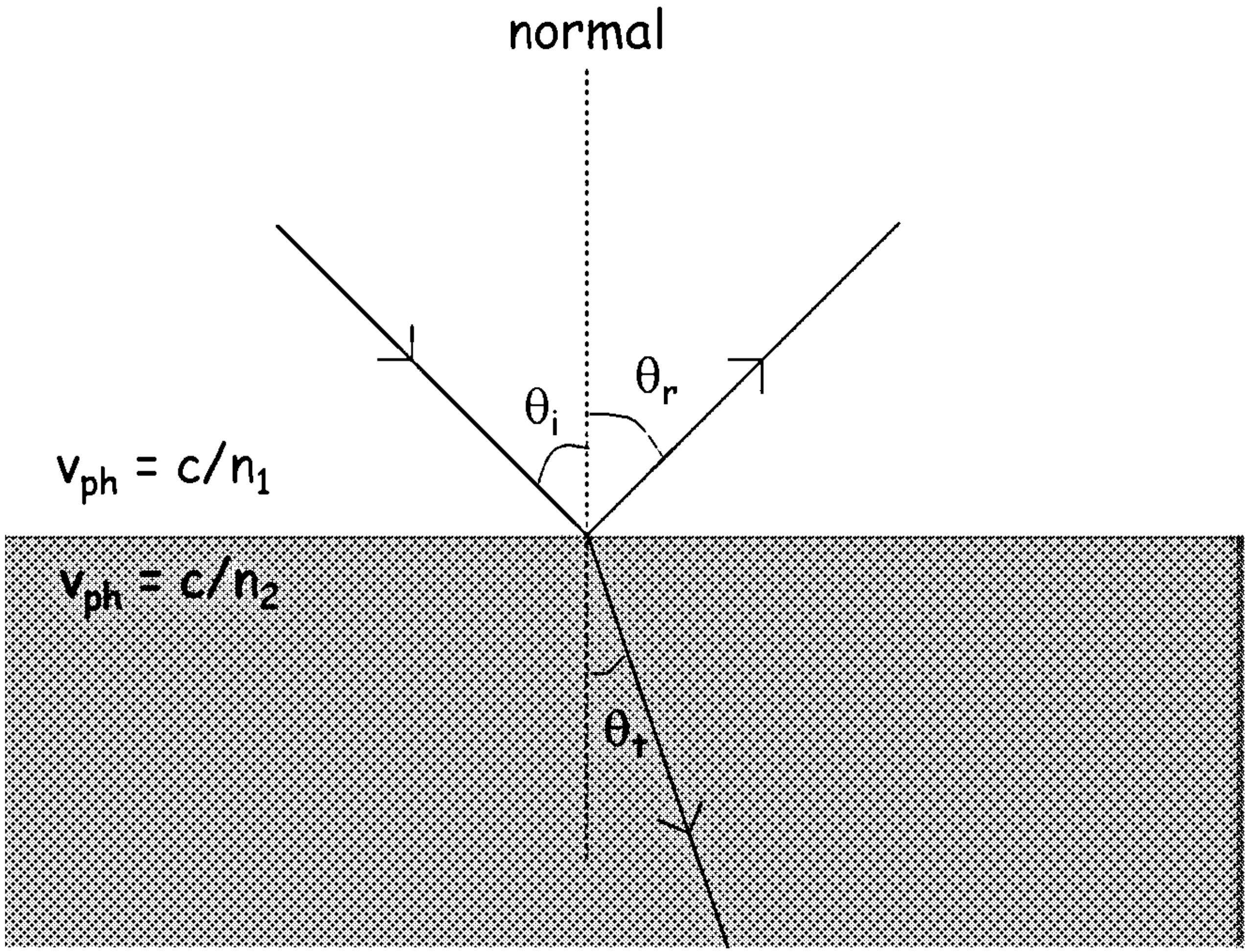
For: $r >> a$, spherical waves can be approximated as a plane waves.

Geometrical Optics: light is a "ray" that travels in straight lines

(the "ray" is essentially the wave vector of a plane wave)

Huygens' and Fermat's Principles

All of Geometrical optics boils down to…



Law of Reflection:

$$\theta_i = \theta_r$$

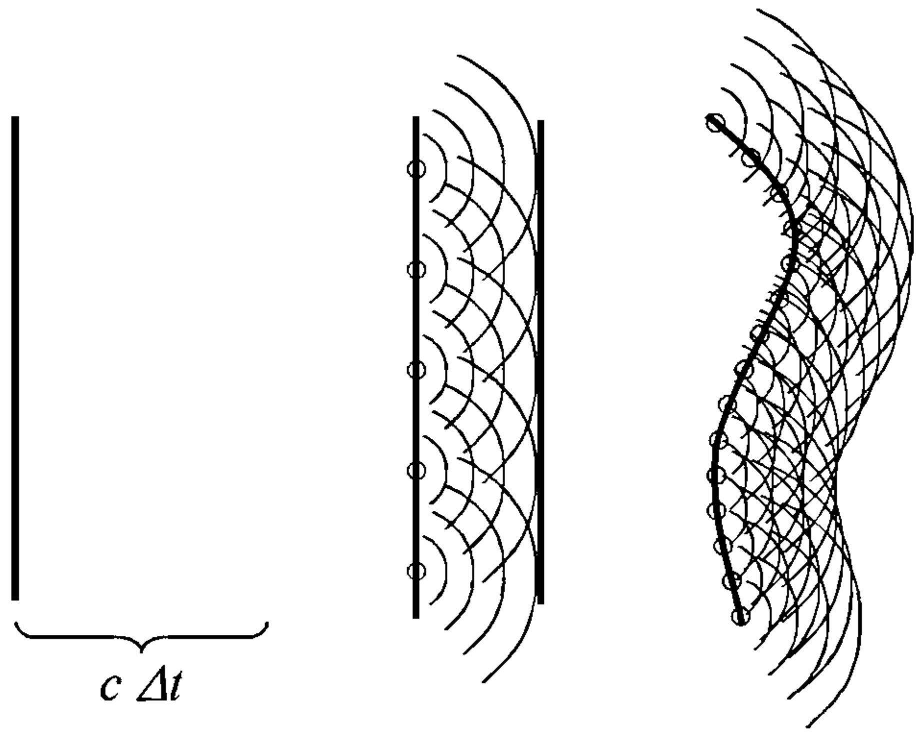
Snell's Law:

$$\frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{n_2}{n_1}$$

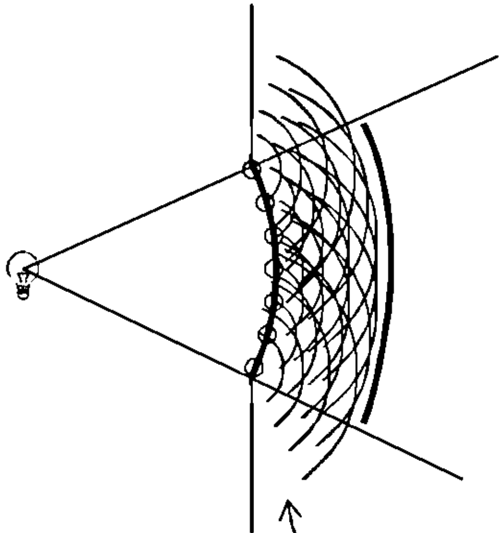
<u>Huygens' Principle</u>

Every point on a wavefront may be regarded as a secondary source of wavelets.

Planar wavefront:



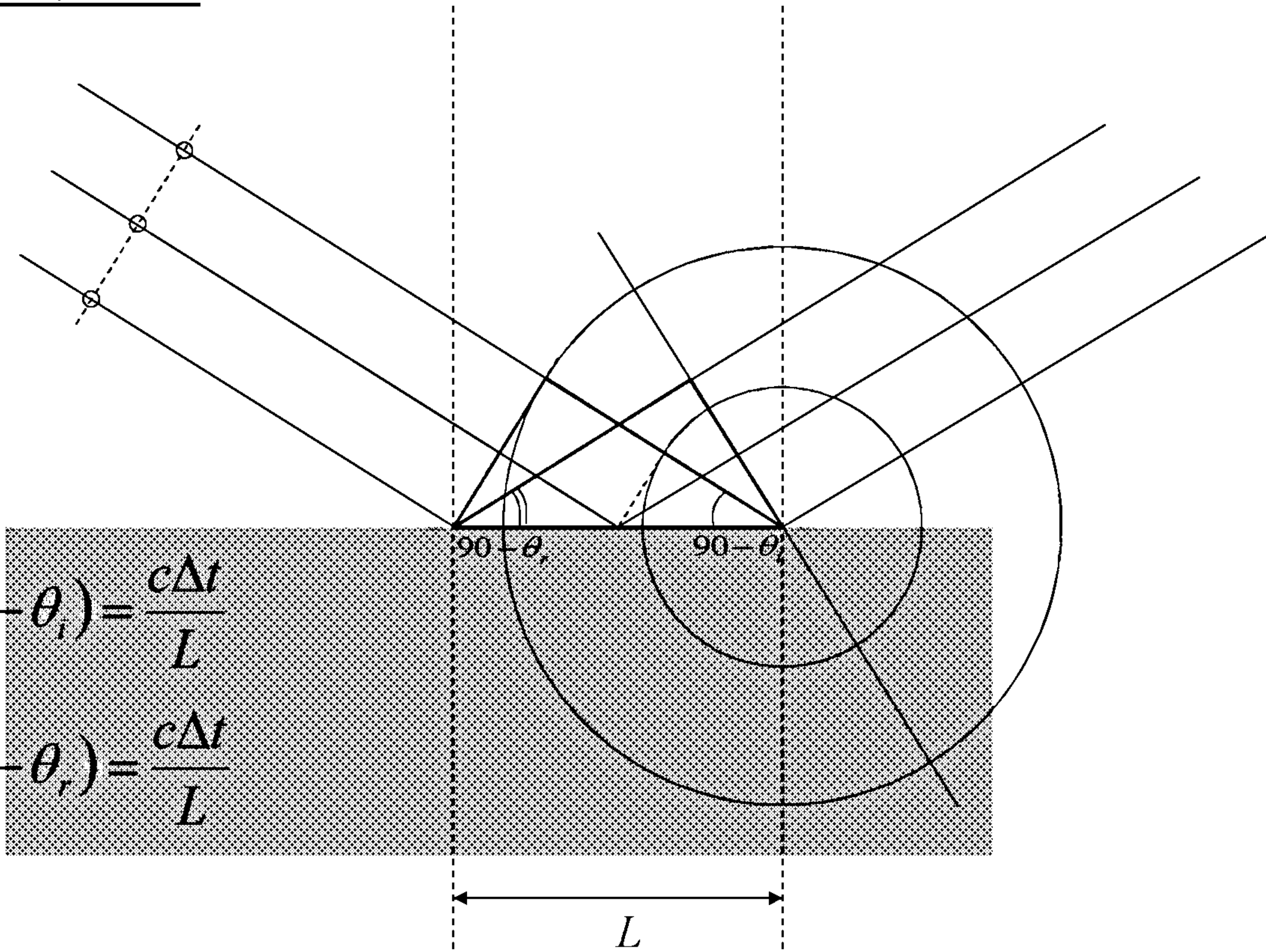
Huygens': Point source through an aperture:



In geometrical optics, this region should be dark.

Ignore the peripheral and back propagating parts!

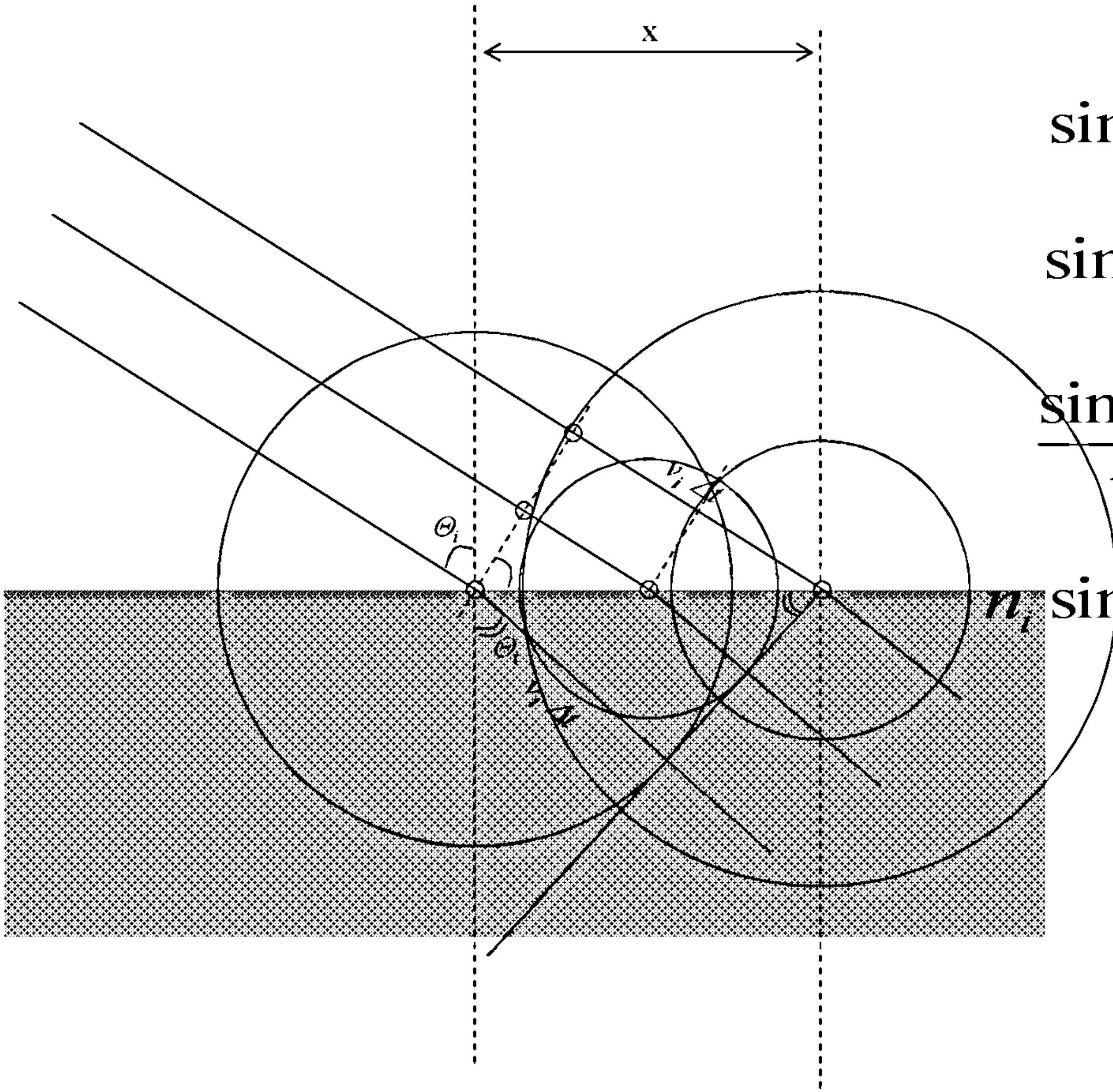
Huygens': Reflection



$$\cos(90-\theta_i)=\frac{c\Delta t}{L}$$

$$\cos(90-\theta_r)=\frac{c\Delta t}{L}$$

Huygens': Refraction



$$\sin(\theta_i) = \frac{v_i \Delta t}{x}$$

$$\sin(\theta_t) = \frac{v_t \Delta t}{x}$$

$$\frac{\sin(\theta_i)}{v_i} = \frac{\sin(\theta_t)}{v_t}$$

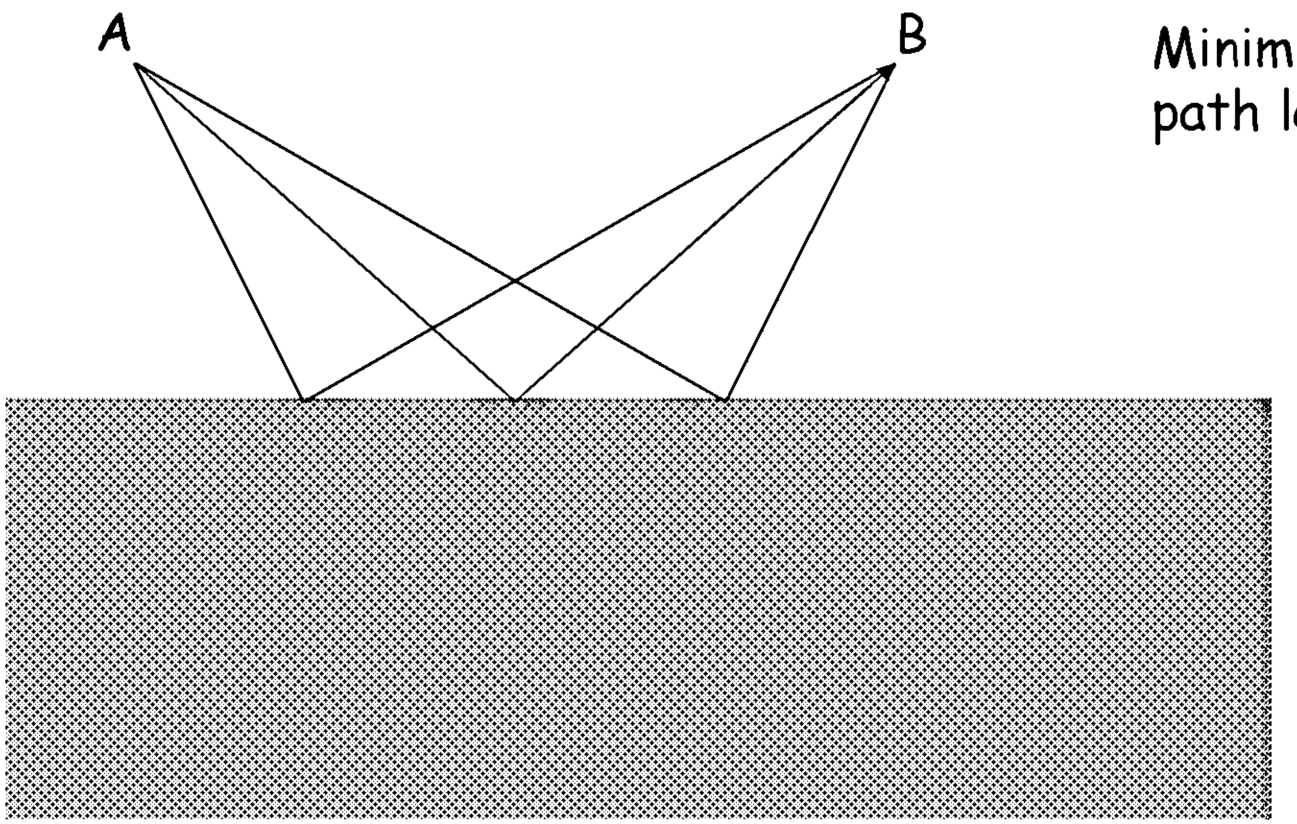
$$n_i \sin(\theta_i) = n_t \sin(\theta_t)$$

Fermat's Principle

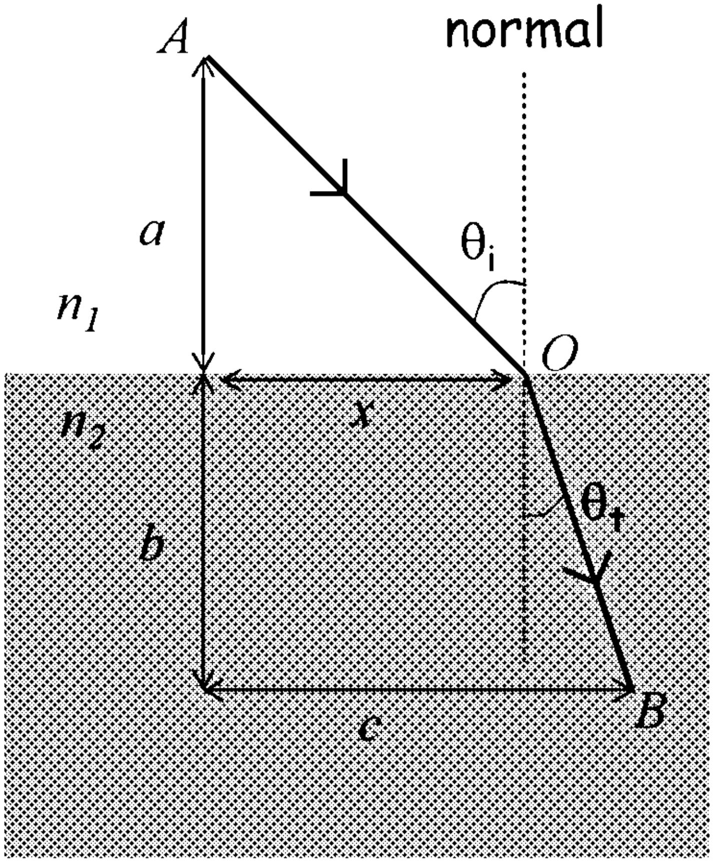
The path a beam of light takes between two points is the one which is traversed in the least time.

Isotropic medium: constant velocity.

Minimum time = minimum path length.



Fermat's: Refraction



$$t = \frac{AO}{v_i} + \frac{OB}{v_t}$$

$$t = \frac{\sqrt{a^2 + x^2}}{v_i} + \frac{\sqrt{b^2 + (c-x)^2}}{v_t}$$

$$\frac{dt}{dx} = \frac{x}{v_i \sqrt{a^2 + x^2}} - \frac{c-x}{v_t \sqrt{b^2 + (c-x)^2}}$$

$$\frac{dt}{dx} = \frac{\sin(\theta_i)}{v_i} - \frac{\sin(\theta_t)}{v_t} = 0$$

$$n_i \sin(\theta_i) = n_t \sin(\theta_t)$$

Huygens' Principle and Fermat's principle provide a qualitative (and quantitative) proof of the law of reflection and refraction within the limit of geometrical optics.