

$r_{TE}$  being negative means a  $\pi$  phase shift.

In the previous calculations we could think of  $E_i$   $E_r$   $E_t$  as representing complex amplitude and phase:

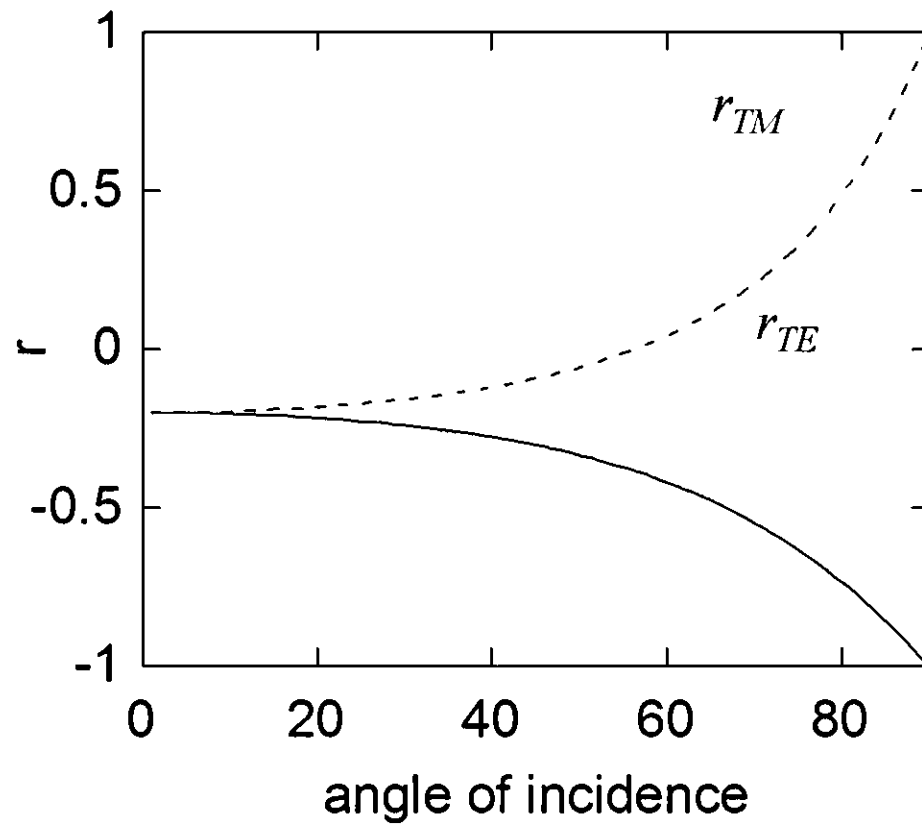
$$E_{oi} e^{j\varphi_i} \quad E_{or} e^{j\varphi_r} \quad E_{ot} e^{j\varphi_t}$$

$$r_{TE} = \frac{E_{or} e^{j\varphi_r}}{E_{oi} e^{j\varphi_i}} = \frac{E_{or}}{E_{oi}} e^{j(\varphi_r - \varphi_i)}$$

Since amplitudes are only positive, a negative value for  $r_{TE}$  is due to a phase difference. As long as  $r_{TE}$  is real, there will only be 0 and  $\pi$  phase shifts.

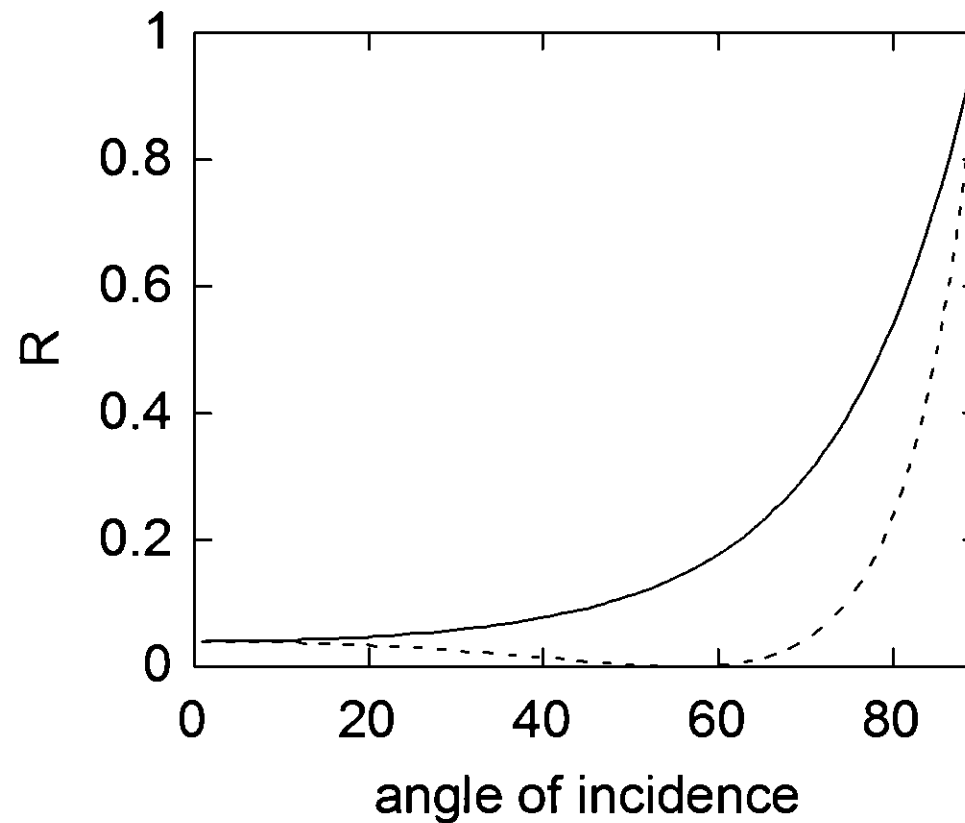
SIMILAR DERIVATION FOR  $r_{TM}$

$$r_{TM} = \frac{-n^2 \cos \theta_i + \sqrt{n^2 - \sin^2 \theta_i}}{n^2 \cos \theta_i + \sqrt{n^2 - \sin^2 \theta_i}}$$



What is observed? Irradiance ( $\text{W}/\text{m}^2$ ) not the E field, so define:

$$\text{Reflectance: } R = \frac{E_{e-ref}}{E_{e-inc}} = \left( \frac{E_r}{E_i} \right)^2 = r^2$$



Transmission? We could do all that again, or:

$$t_{TE} = \frac{E_t}{E_i}$$

$$E_i + E_r = E_t$$

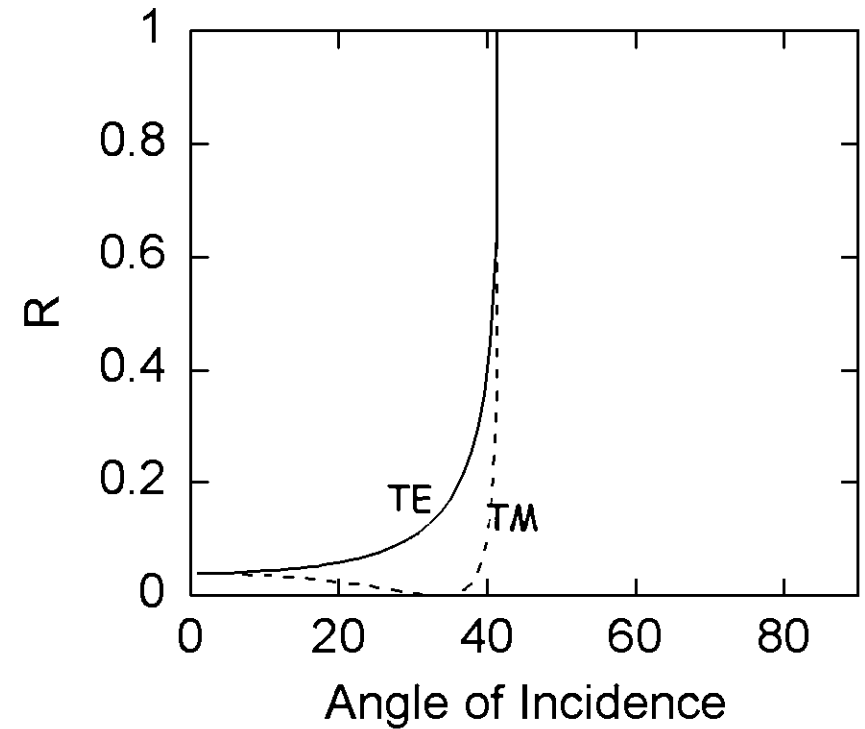
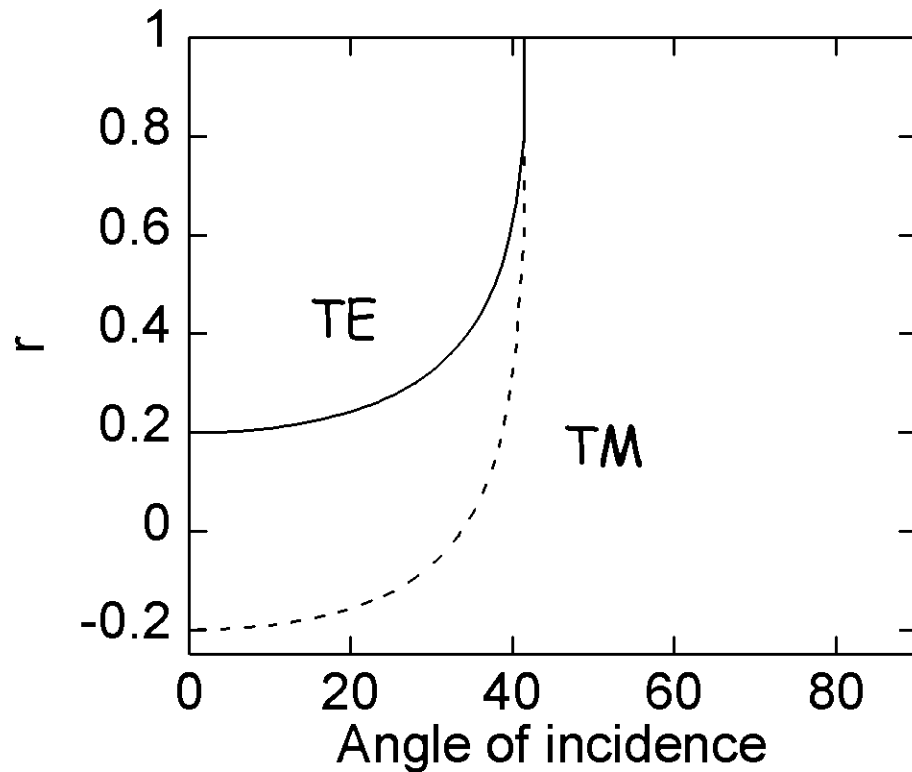
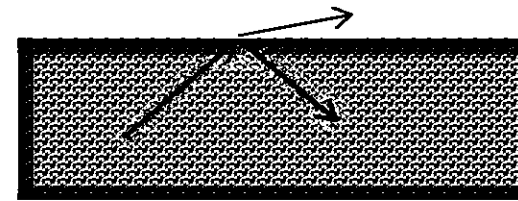
$$1 + \frac{E_r}{E_i} = \frac{E_t}{E_i}$$

$$1 + r_{TE} = t_{TE}$$

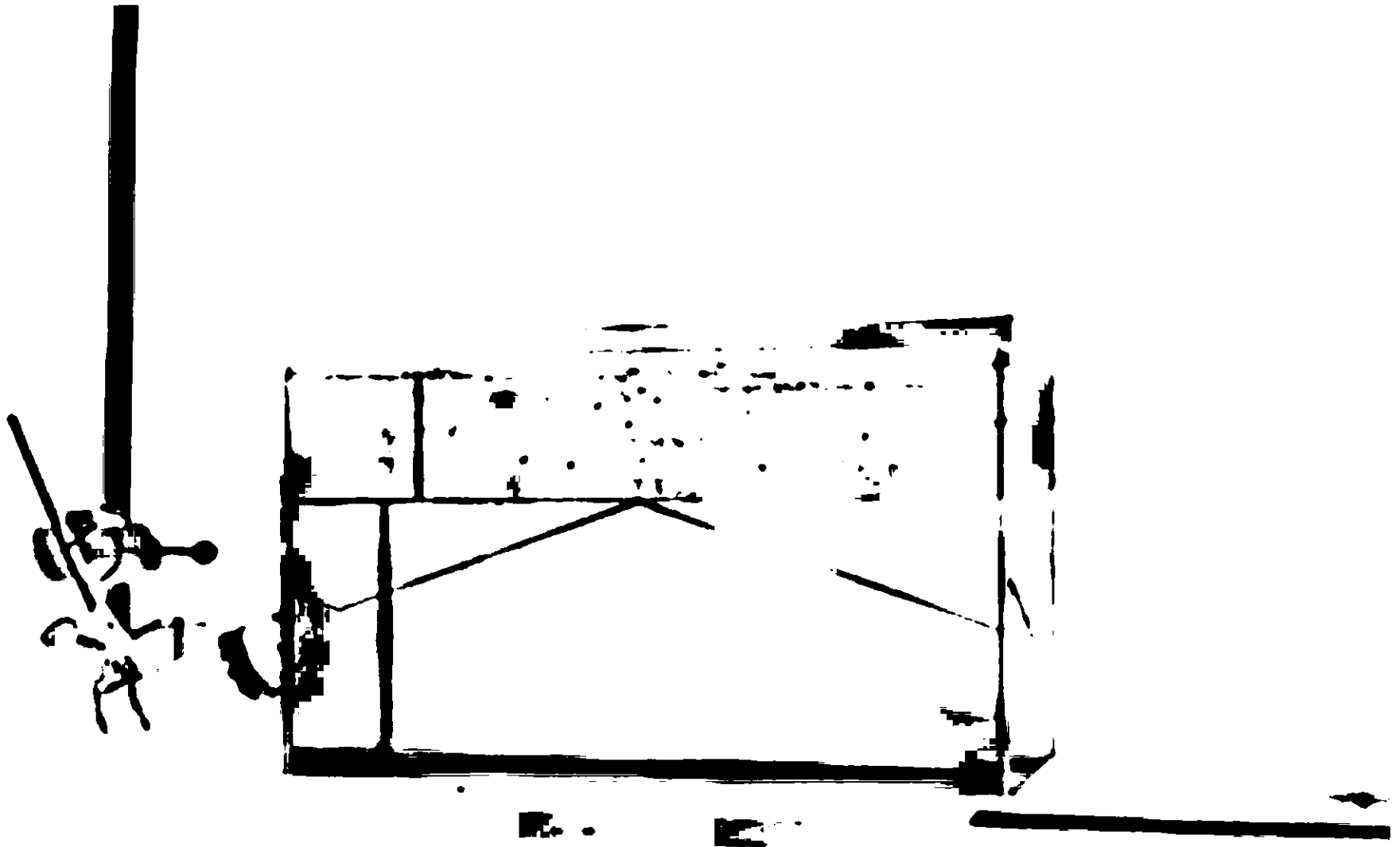
...get transmission coefficients from the reflection coefficients already derived.

This is NOT a conservation law! There is no law of conservation of the E field!

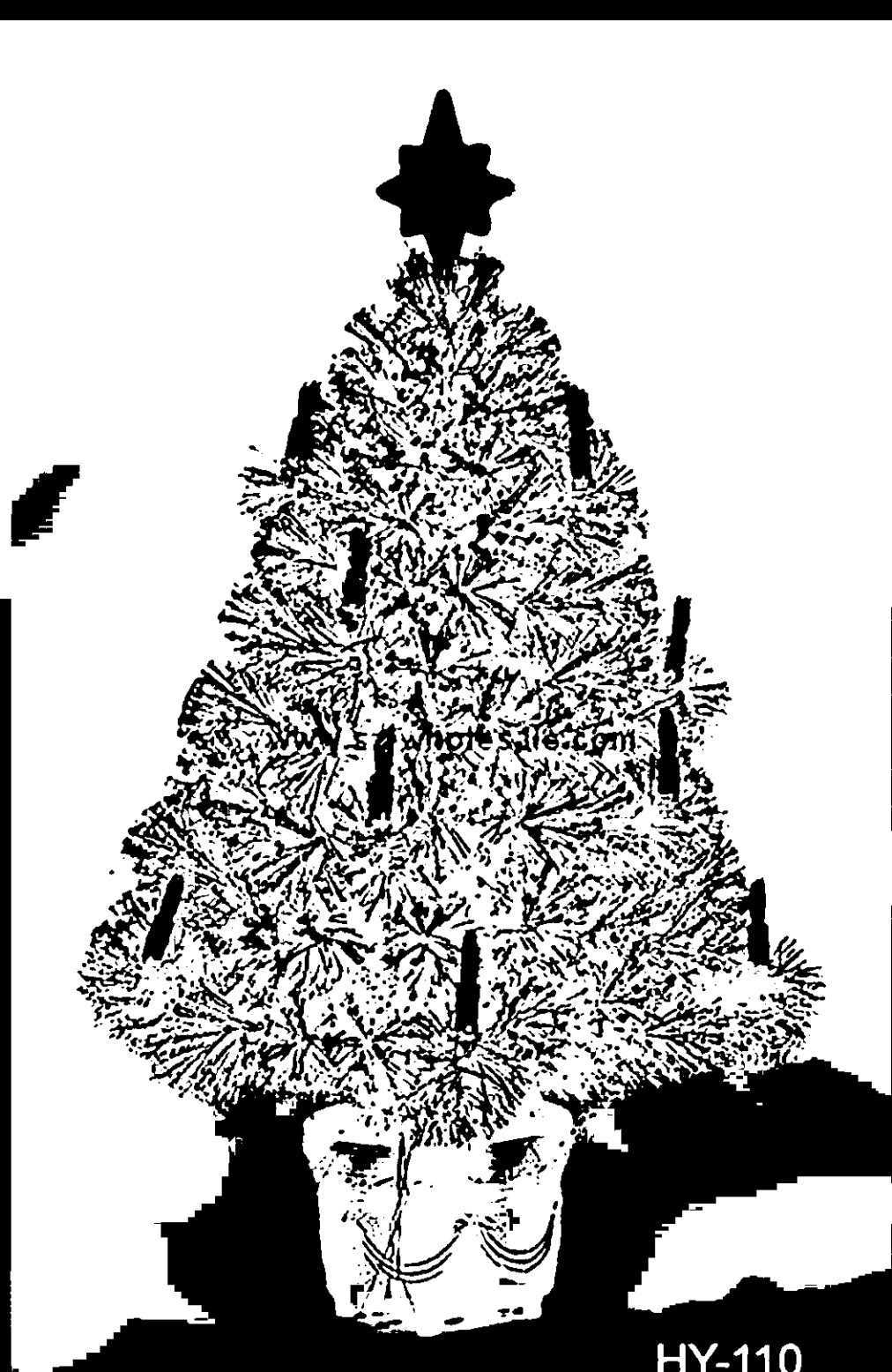
internal reflection: light incident in a high index travelling into a low index,  
 $n = n_2/n_1 = 0.667$



# The Milky Fishtank!

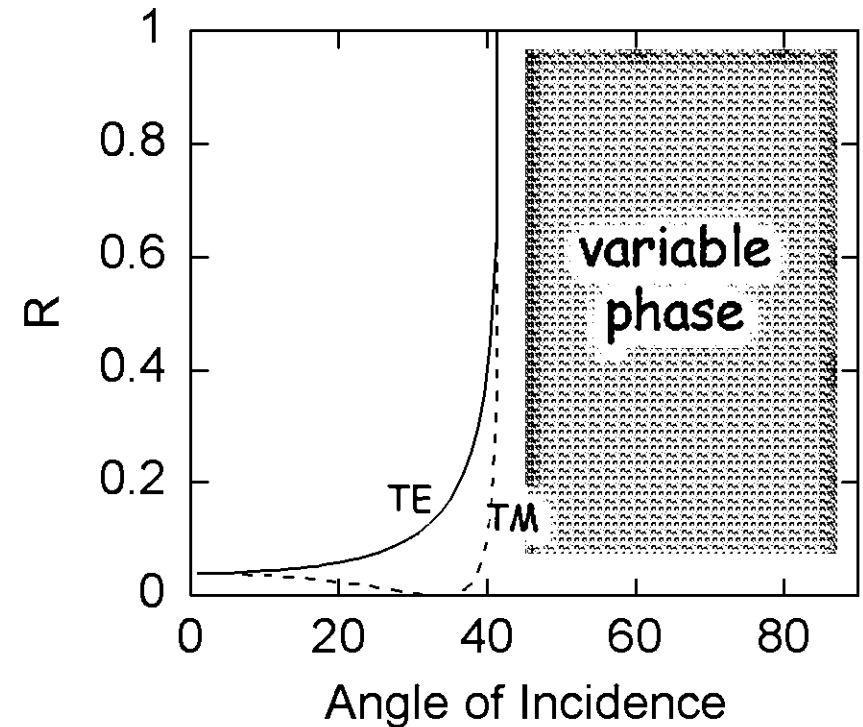
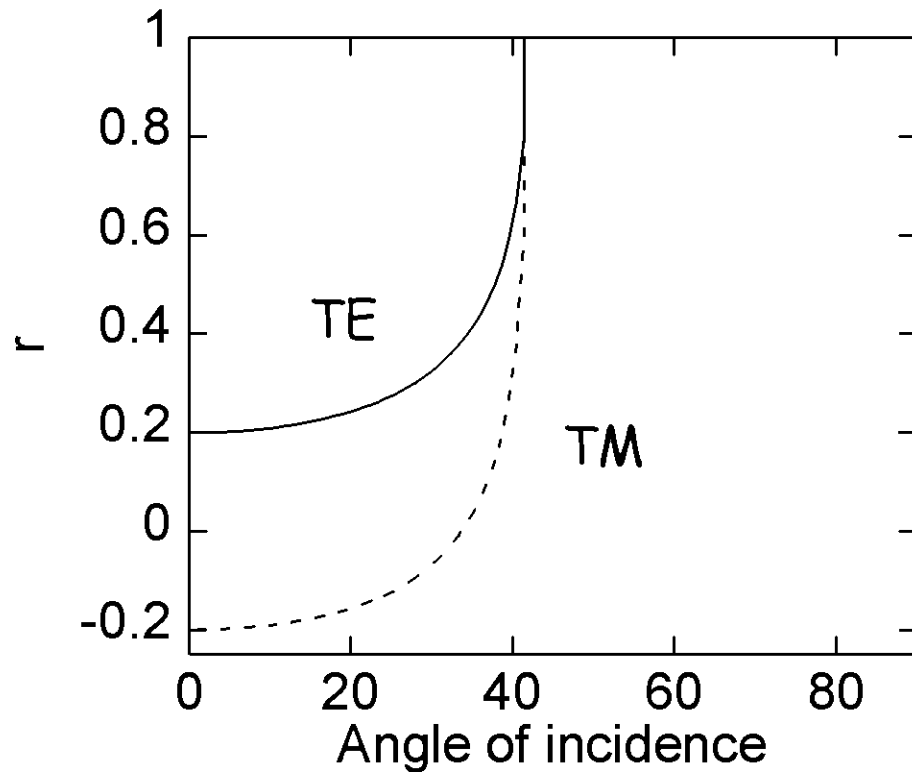
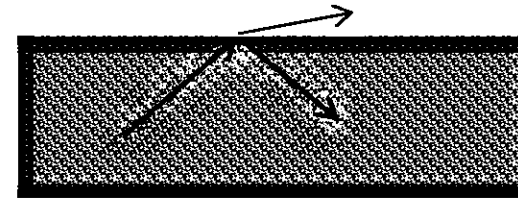






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internal reflection: light incident in a high index travelling into a low index,  
 $n = n_2/n_1 = 0.667$



write  $r$  in this form:

$$r = \frac{e^{-j\alpha}}{e^{j\alpha}}$$

Why? Because:

$$E_r = rE_i e^{j(kz - \omega t)} \hat{x}$$

$$E_r = \frac{e^{-j\alpha}}{e^{j\alpha}} E_i e^{j(kz - \omega t)} \hat{x}$$

$$E_r = e^{-2j\alpha} E_i e^{j(kz - \omega t)} \hat{x}$$

$$E_r = E_i e^{j(kz - \omega t - 2\alpha)} \hat{x}$$

phase shift =  $-2\alpha$

Great, but what is  $\alpha$ ?

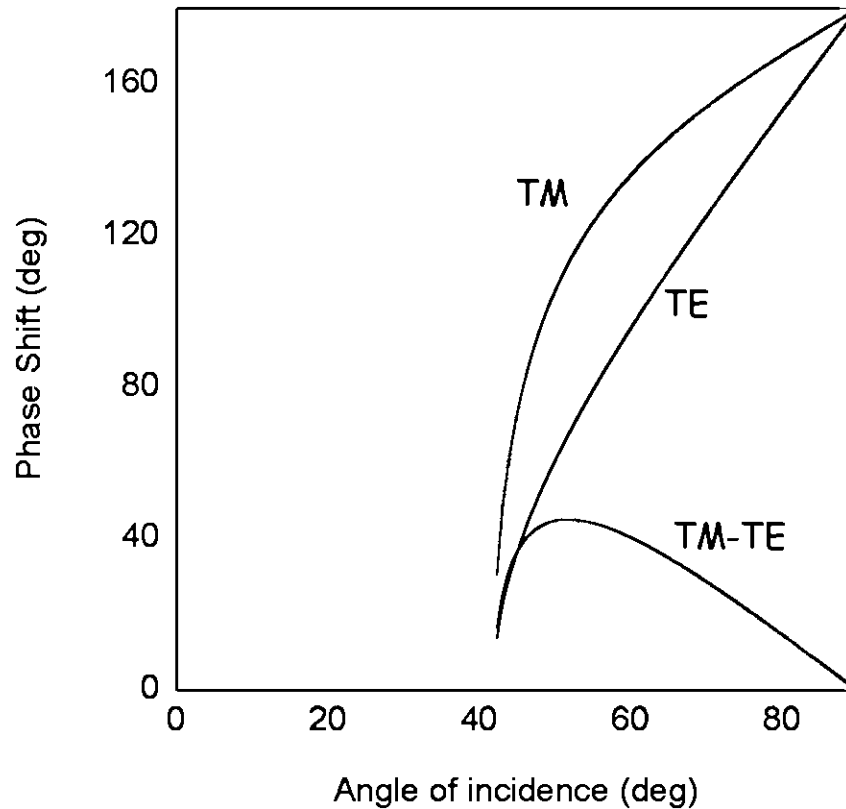
$$\begin{aligned} r &= \frac{e^{-j\alpha}}{e^{j\alpha}} \\ &= \frac{\cos(-\alpha) + j \sin(-\alpha)}{\cos(\alpha) + j \sin(\alpha)} \\ &= \frac{\cos(\alpha) - j \sin(\alpha)}{\cos(\alpha) + j \sin(\alpha)} \end{aligned}$$

TE:       $\cos(\alpha) = \cos \theta_i$                        $\sin(\alpha) = \sqrt{\sin^2 \theta_i - n^2}$

$$\tan(\alpha) = \frac{\sqrt{\sin^2 \theta_i - n^2}}{n^2 \cos \theta_i}$$

Phase shift upon total internal reflection ( $\phi$ ):

$$\tan\left(\frac{\phi}{2}\right) = \frac{\sqrt{\sin^2 \theta_i - n^2}}{\cos \theta_i}$$



Tunable phase lag based only on dielectric reflections!