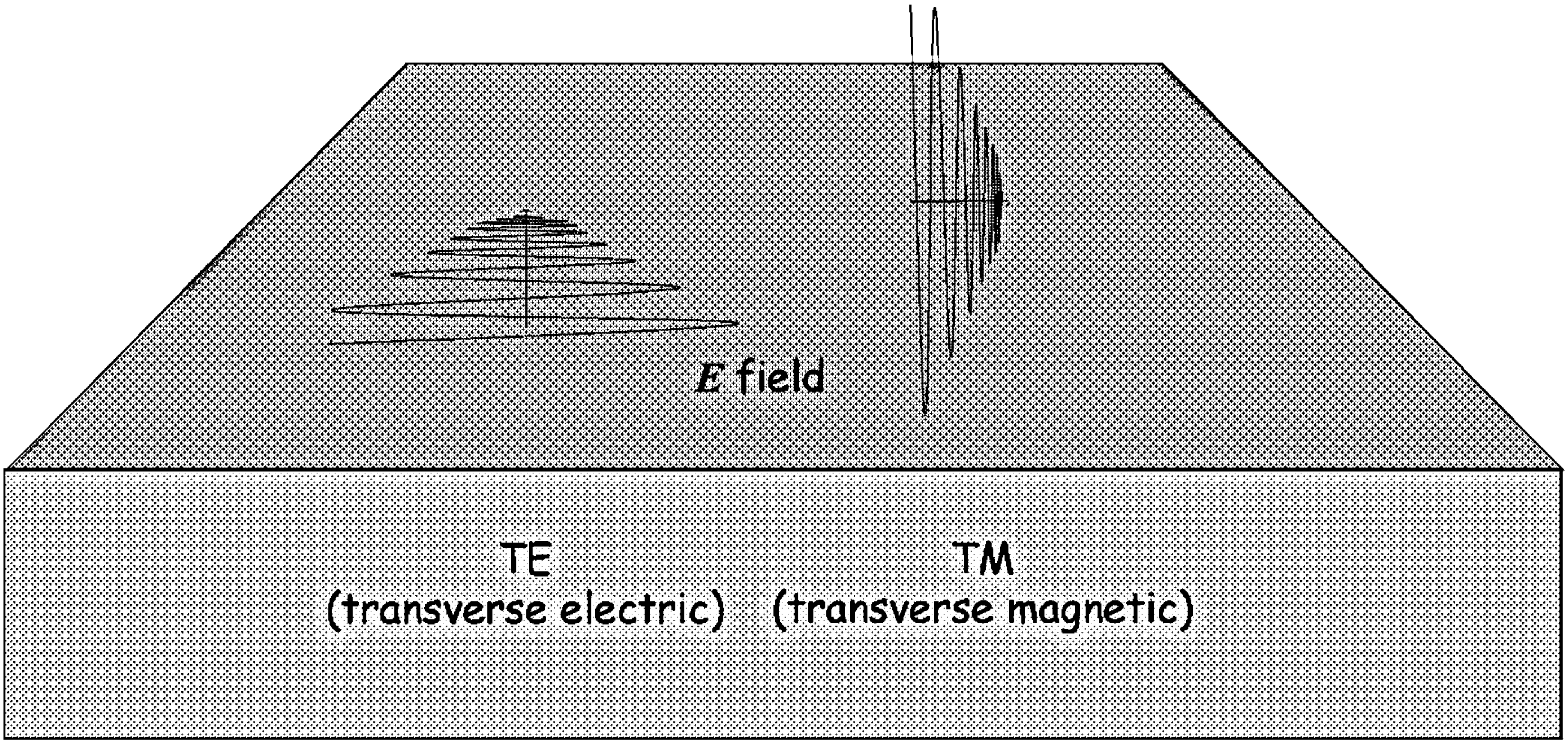
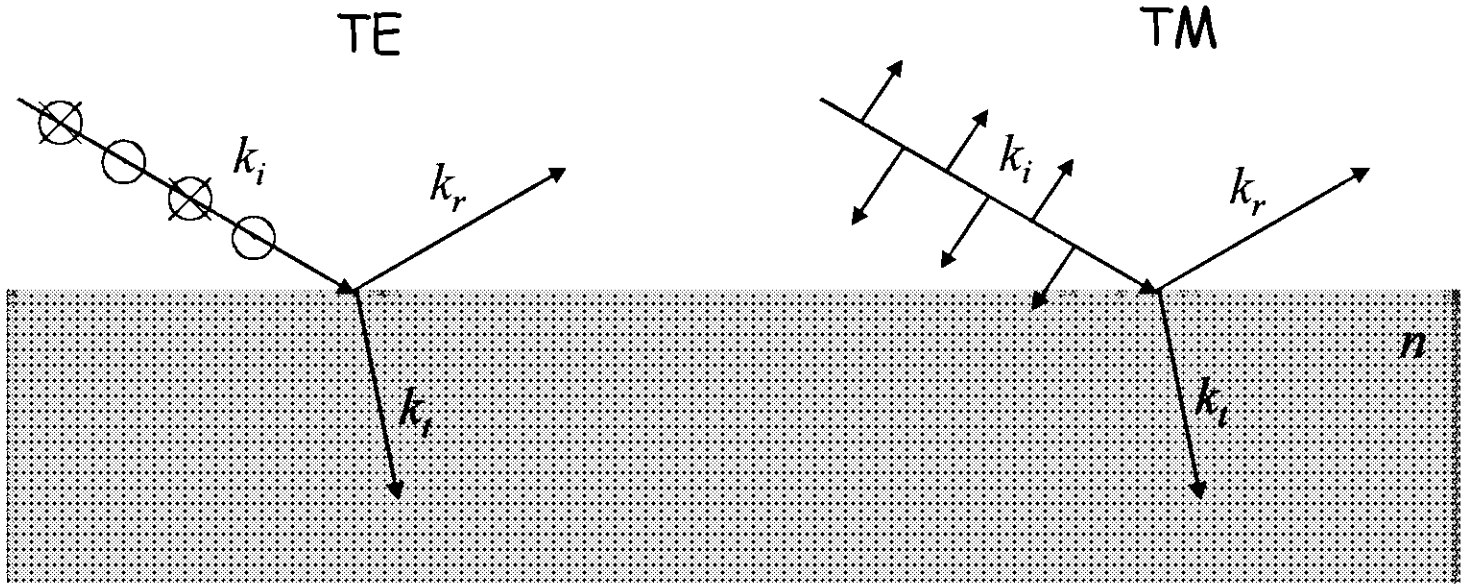
Light at a Plane Dielectric Interface





Assume:

A plane wave is incident: $\vec{\mathbf{E}}_{inc} = \vec{E}_{oi} e^{j\phi_i} e^{j(\vec{k}_i \cdot \vec{r} - \omega t)}$

A plane wave is reflected: $\vec{\mathbf{E}}_{ref} = \vec{E}_{or} e^{j\varphi_r} e^{j(\vec{k}_r \cdot \vec{r} - \omega_r t)}$

A plane wave is transmitted: $\vec{\mathbf{E}}_{trans} = \vec{E}_{ot} e^{j\varphi_t} e^{j(\vec{k}_t \cdot \vec{r} - \omega_t t)}$

What are the relative amplitudes, wave vectors, frequencies, and phases?

Relationship between fields at the interface should not depend on position or time:

$$\left(\vec{k}_i \cdot \vec{r} - \omega_i t\right) = \left(\vec{k}_r \cdot \vec{r} - \omega_r t\right) = \left(\vec{k}_t \cdot \vec{r} - \omega_t t\right)$$

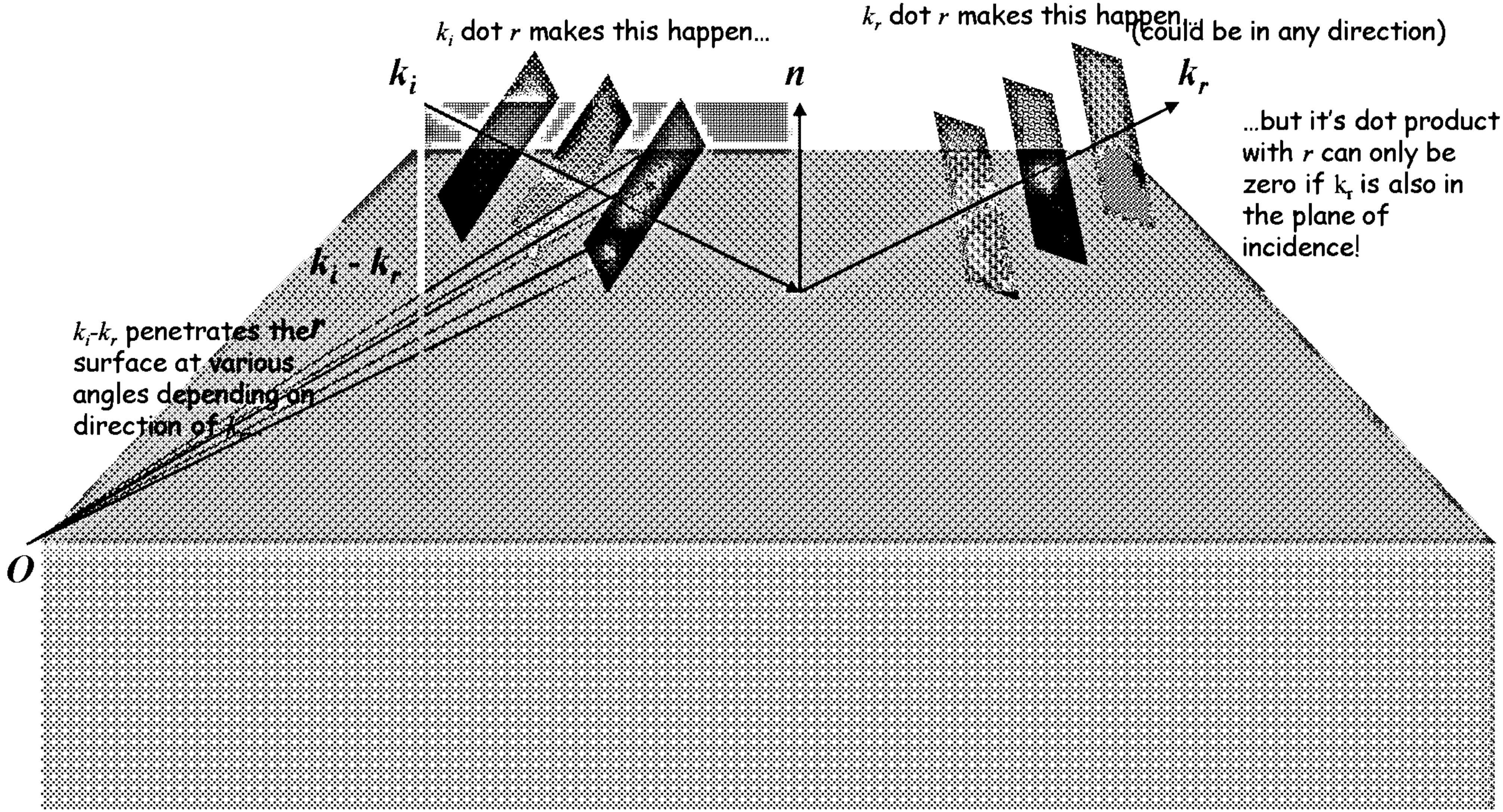
To remain constant at a certain place:

$$\omega_i = \omega_r = \omega_t$$

incident, reflected, and refracted all at same frequency.

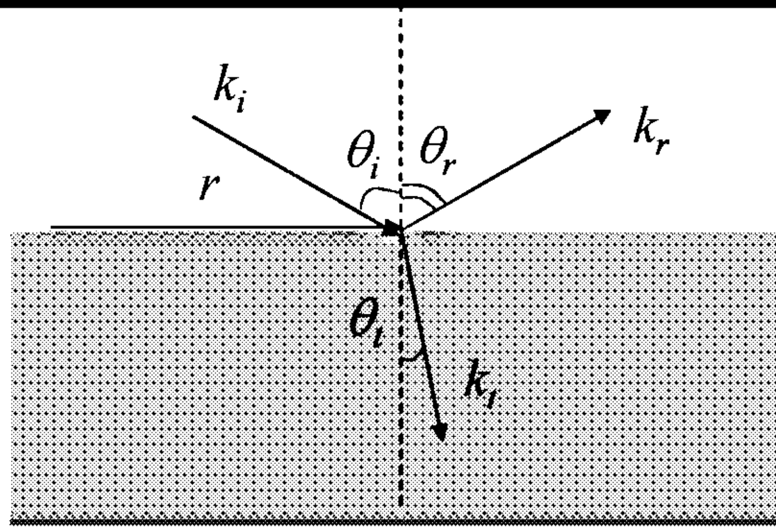
To remain constant at a certain time:

$$\vec{k}_i \cdot \vec{r} = \vec{k}_r \cdot \vec{r} = \vec{k}_t \cdot \vec{r}$$



$$\left(\vec{k}_i - \vec{k}_r\right) \cdot \vec{r} = 0$$

k_i, k_r, k_t are all co-planar



$$k_i r\cos\left(\tfrac{\pi}{2}-\theta_i\right)=k_r r\cos\left(\tfrac{\pi}{2}-\theta_r\right)$$

Same medium, same velocity, same wavelength, same wavenumber, so:

$$\boxed{\theta_i=\theta_r}$$ Law of Reflection

$$k_i r\cos\left(\tfrac{\pi}{2}-\theta_i\right)=k_t r\cos\left(\tfrac{\pi}{2}-\theta_t\right)$$

$$\frac{2\pi n_i}{\lambda_o}\sin(\theta_i)=\frac{2\pi n_t}{\lambda_o}\sin(\theta_t)$$

$$\boxed{n_i\sin\theta_i=n_t\sin\theta_t}$$ Snell's Law