What does this do to a plane wave in the dielectric? To answer that first simplify the wave equation. Rather than keep all that detail, just write it like this:

$$\nabla^2\vec{\mathbf{E}} = \left(\frac{\tilde{n}}{c}\right)^2 \frac{\partial^2\vec{\mathbf{E}}}{\partial t^2}$$

The n with a tilde is the complex index of refraction. It can be any function of frequency and it can be complex, so it can describe light in a dielectric or light in any material. You could write it like this:

$$\tilde{n} = n_R + n_I$$

Now plug in our standard 1D plane wave: $\vec{\mathbf{E}} = \vec{\mathbf{E}}_o e^{j(kz-\omega t)}\hat{i}$

$$-k^2\vec{\mathbf{E}}_o e^{j(kz-\omega t)}\hat{i} = -\left(\frac{\tilde{n}}{c}\right)^2 \omega^2\vec{\mathbf{E}}_o e^{j(kz-\omega t)}\hat{i}$$

Cancel common terms and you get

$$k^2 = \left(\frac{\tilde{n}}{c}\right)^2 \omega^2$$

..which tells you the new dispersion relation:

$$\omega = \left(\frac{c}{\tilde{n}}\right) k$$

To see what happens to a plane wave based on the dispersion relation, use it to write the spatial frequency (k) in terms of the temporal frequency (ω):

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_o e^{j(kz-\omega t)} \hat{i} \longrightarrow \vec{\mathbf{E}} = \vec{\mathbf{E}}_o e^{j\left(\left(\frac{\tilde{n}}{c}\right)\omega z-\omega t\right)} \hat{i}$$

Now write out the complex index of refraction as its real and imaginary components…

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_o e^{j\left(\left(\frac{n_R + j n_I}{c}\right)\omega z - \omega t\right)}\hat{i}$$

…and distribute the imaginary part…

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_o e^{j\frac{n_R}{c}\omega z} e^{-\frac{n_I}{c}\omega z} e^{-j\omega t}\hat{i}$$

…and switch the spatial parts back to k…

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_o e^{-n_I k z} e^{j(n_R k z - \omega t)}\hat{i}$$

Since the E amplitude exponentially decays with distance, this tells us that inside the material, the imaginary part of the index of refraction causes absorption (loss) of the light.

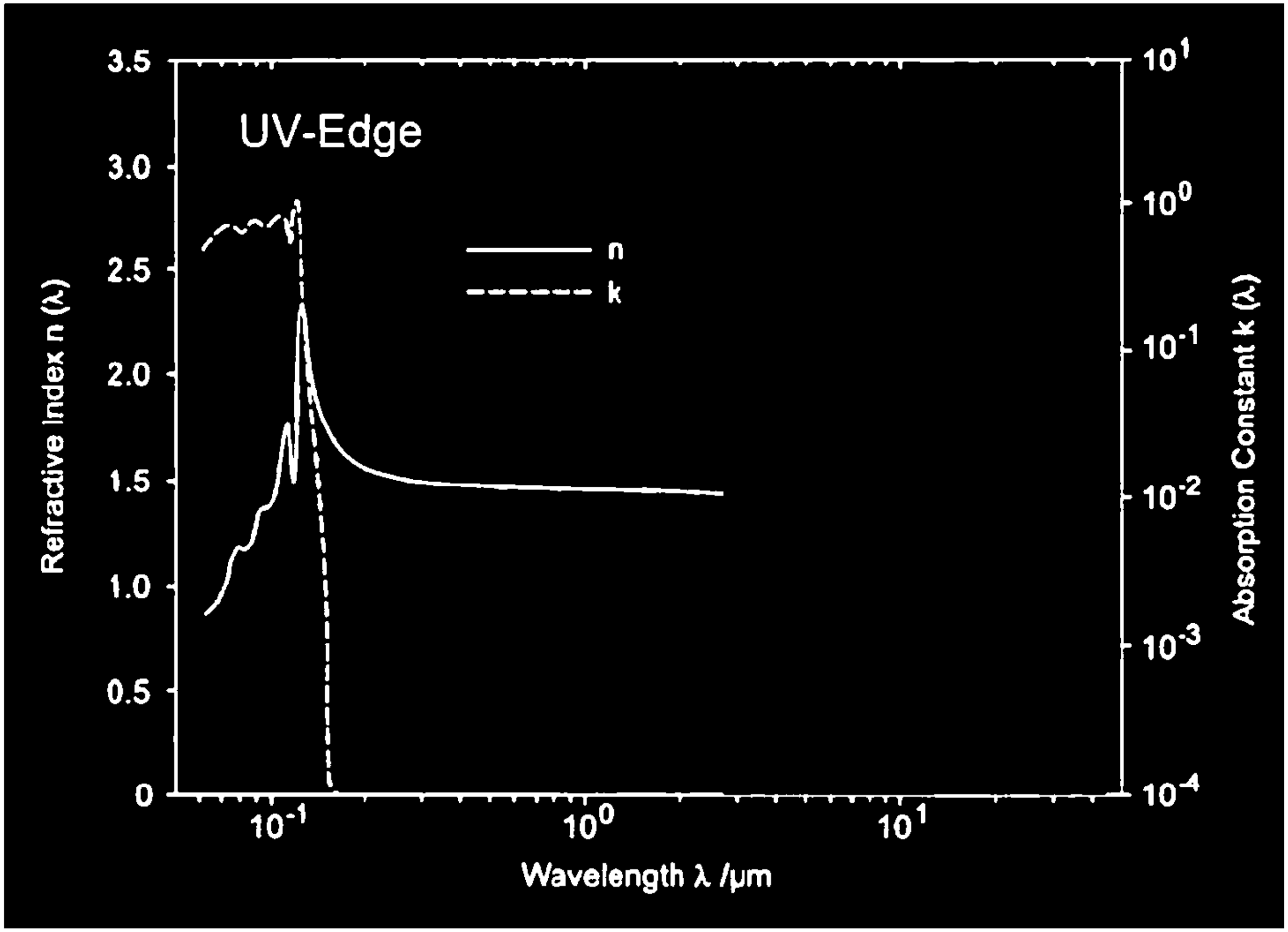
The presence of n_R tells us that the wave travels at a reduced velocity, since the spatial frequency has been modified from k in vacuum to $n_R k$ in the material:

$$v = \frac{\text{temporal frequency}}{\text{spatial frequency}} = \frac{\omega}{n_R k} = \frac{c}{n_R}$$

SO, a dielectric will slow light down and cause it to be absorbed. If we now think back to our mass-on-a-spring model, is it possible convert it to a real part and an imaginary part?

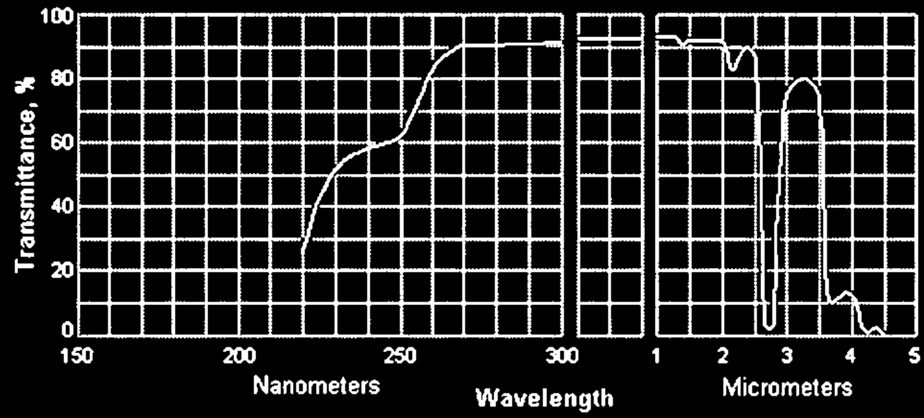
$$\tilde{n} = \sqrt{1 + \frac{Ne^2}{m\varepsilon_o\left(\omega_o^{\ 2} - \omega^2 - j\omega\gamma\right)}}$$

Sort of, with lots of algebra. The book and the next slide show you the result. At the resonance n_R changes a lot and n_I becomes very large – lots of absorption! For glass, this happens in the UV. (on that graph n_I is called k) glass

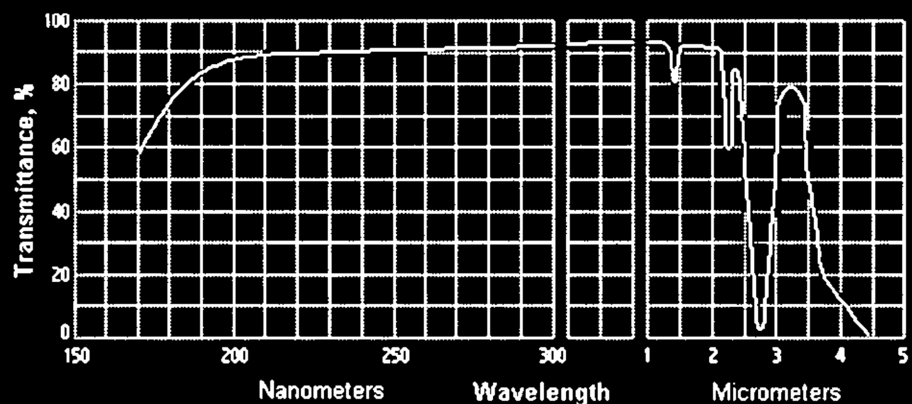


At visible wavelengths you can see that n_R gradually increases with frequency, which is why white light is separated into colors (explained next week!)

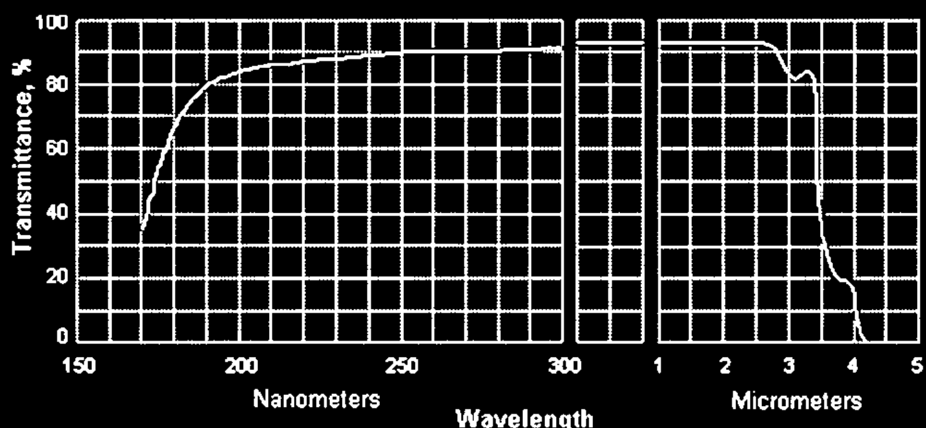
λ = 200 - 300 nm absorption peaks are actually due to impurities. When removed the peak for glass (due to Si and O) is seen at deep UV.



Metal impurity: 25 – 30 ppm
OH^- content: 400 – 500 ppm



Metal impurity: 5 ppm
OH^- content: >1000 ppm



Metal impurity: 0.3 ppm
OH^- content: 0.2 ppm