

# Real Oscillators

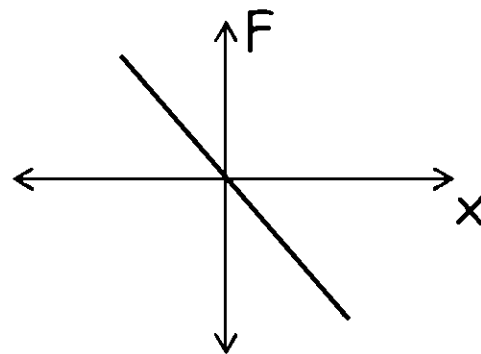
... constant forces  $\rightarrow$  integrate EOM  $\rightarrow$  parabolic trajectories.

... linear restoring force  $\rightarrow$  guess EOM solution  $\rightarrow$  SHM

... nonlinear restoring forces  $\rightarrow$  ?

linear spring

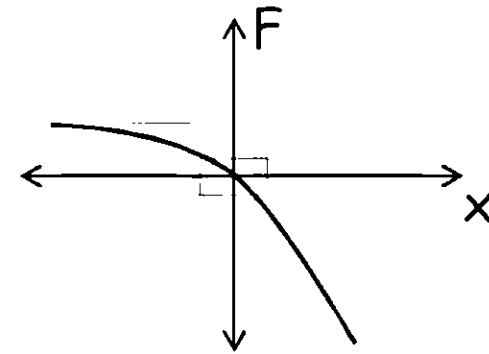
$$F = -kx$$



$$-kx = m\ddot{x}$$

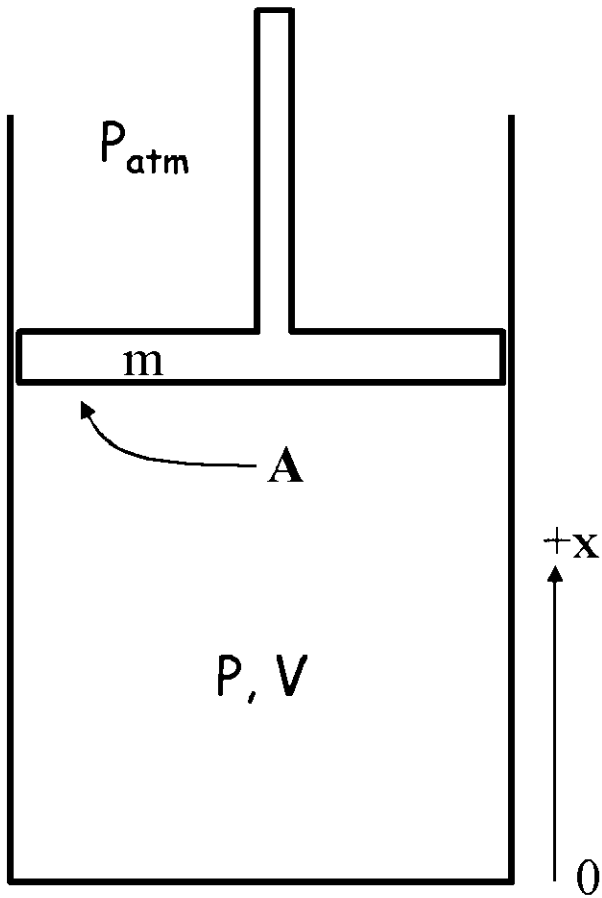
nonlinear spring?

$$F = c(1 - e^x)$$



$$c(1 - e^x) = m\ddot{x}$$

The spring of air :



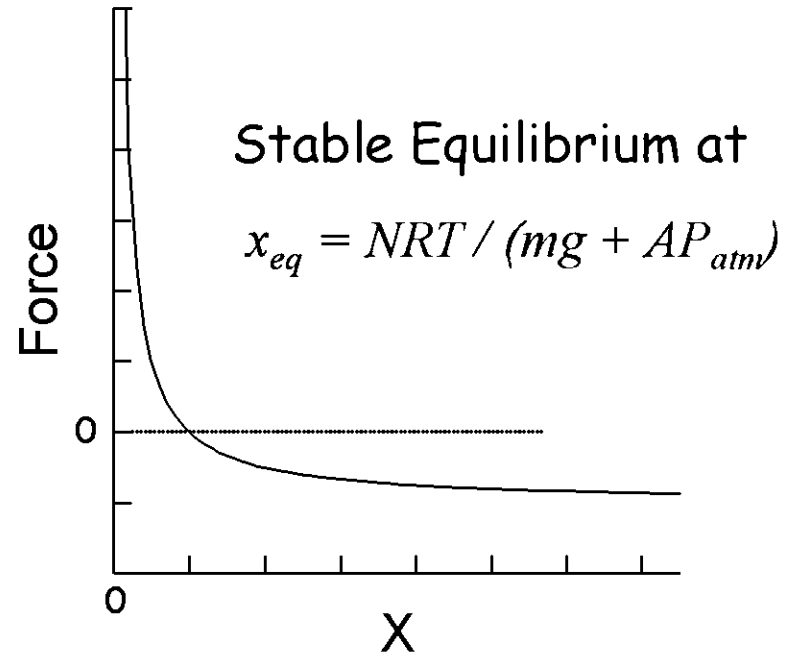
$$\Sigma F = -mg - AP_{atm} + AP = m\ddot{x}$$

use Ideal Gas Law:  $PV=NRT$

$$-mg - AP_{atm} + A \frac{NRT}{V} = m\ddot{x}$$

chamber volume:  $V=Ax$

$$\boxed{-mg - AP_{atm} + \frac{NRT}{x} = m\ddot{x}} \quad \text{EOM}$$

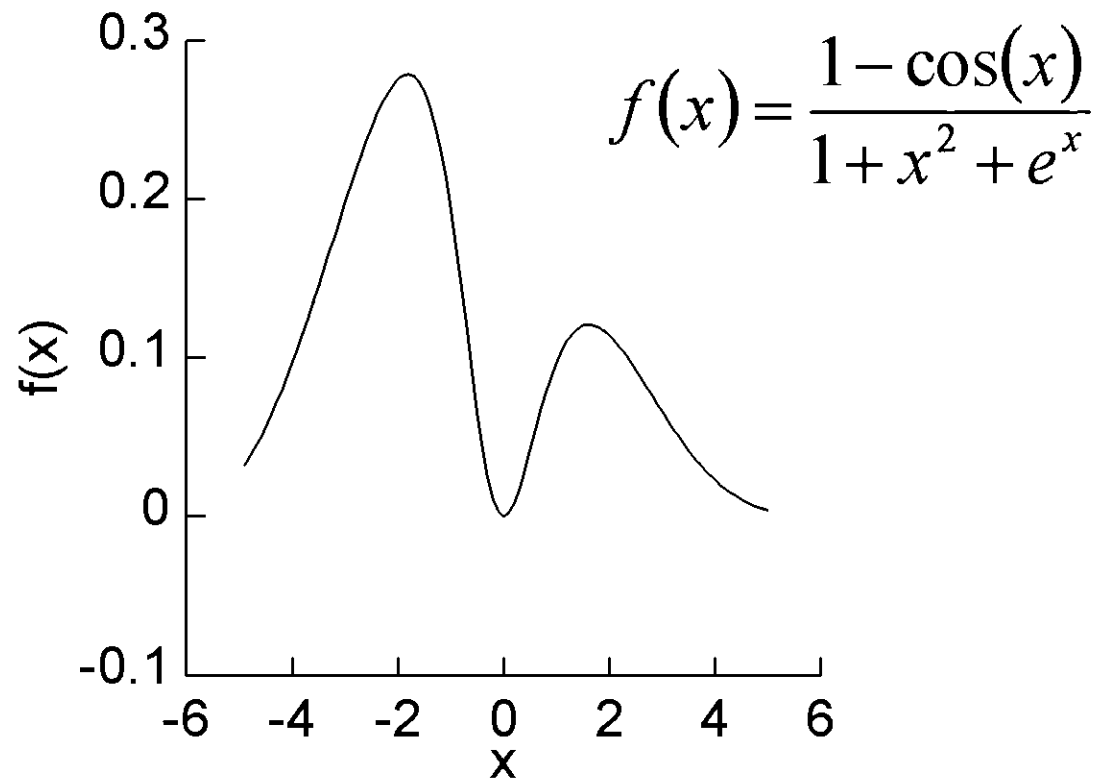


Taylor Series Expansions:

$$f(x) = \sum_{n=0}^{\infty} \frac{\frac{d^n f}{dx^n}(a)}{n!} (x-a)^n$$

Turns a function into a polynomial near  $x = a$

Example:



Expand  $NRT/x$  around  $x_{eq}$ :

$$\left[ -mg - AP_{atm} + \frac{NRT}{x_{eq}} - \frac{NRT}{x_{eq}^2} (x - x_{eq}) + \frac{NRT}{x_{eq}^3} (x - x_{eq})^2 - \dots \right] = m\ddot{x}$$

$$\left[ 0 - \frac{NRT}{x_{eq}^2} (x - x_{eq}) + \frac{NRT}{x_{eq}^3} (x - x_{eq})^2 - \dots \right] = m\ddot{x}$$

Is it safe to linearize it? Better check a unitless ratio. How about:

$$\left( \frac{x - x_{eq}}{x_{eq}} \right)$$

(Yes, excellent choice Dr. Hafner!)

$$\frac{NRT}{x_{eq}} \left[ 0 - \left( \frac{x - x_{eq}}{x_{eq}} \right) + \left( \frac{x - x_{eq}}{x_{eq}} \right)^2 - \dots \right] = m\ddot{x}$$

Displacement 5% of  $x_{eq}$ :      0      .05      .0025      ....

$$-\frac{NRT}{2x_{eq}} (x - x_{eq}) \approx m\ddot{x}$$

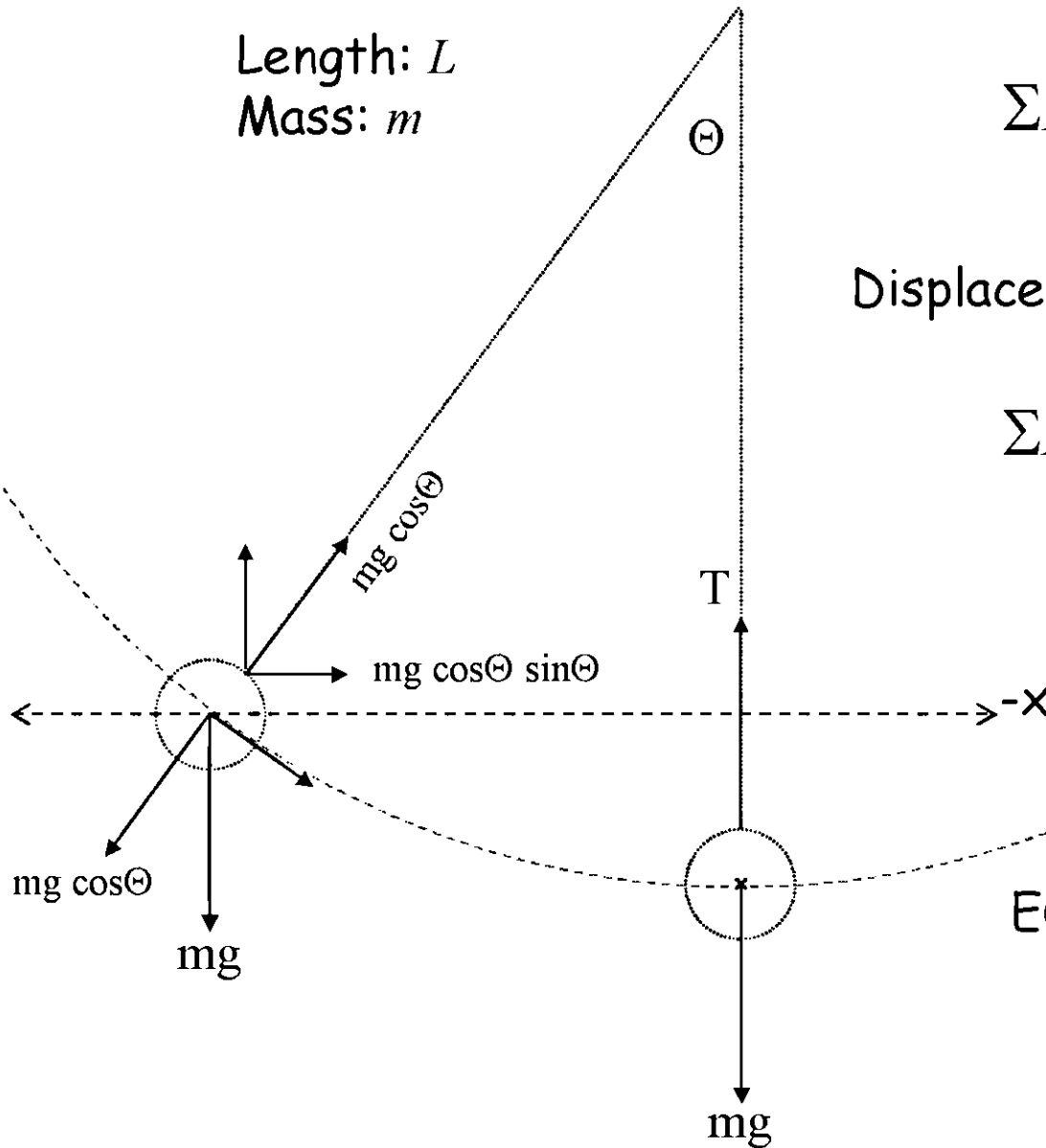
Perhaps you would prefer....

$$-\frac{NRT}{m x_{eq}} (x - x_{eq}) \approx \ddot{(x - x_{eq})}$$

$$\text{SHM with } \omega_o = \frac{\sqrt{NRT/m}}{x_{eq}}$$

Simple Pendulum:

Length:  $L$   
Mass:  $m$



Stable Equilibrium:

$$\Sigma F_x = 0$$

$$\Sigma F_y = T - mg = 0$$

Displace by  $\Theta$ :

$$\Sigma F_x = -mg \cos(\Theta) \sin(\Theta)$$

$$= -mg \frac{\sqrt{L^2 - x^2}}{L} \frac{x}{L}$$

$$\text{EOM: } -mg \frac{\sqrt{L^2 - x^2}}{L} \frac{x}{L} = m\ddot{x}$$

*Expand it!*

$$-\frac{g}{L^2} x \sqrt{L^2 - x^2} = \ddot{x}$$

Derivatives:

$$f = x \sqrt{L^2 - x^2}$$

$$f' = \sqrt{L^2 - x^2} - x^2 (L^2 - x^2)^{-\frac{1}{2}}$$

$$f'' = -3x (L^2 - x^2)^{-\frac{1}{2}} - x^3 (L^2 - x^2)^{-\frac{3}{2}}$$

$$f''' = -3 (L^2 - x^2)^{-\frac{1}{2}} - 6x^2 (L^2 - x^2)^{-\frac{3}{2}} - 3x^4 (L^2 - x^2)^{-\frac{5}{2}}$$

$$-\frac{g}{L^2} \left[ 0 + Lx + 0 - \frac{3}{6L} x^3 \dots \right] = \ddot{x}$$

Now express as a unitless ratio of the dependent variable and some parameter of the system:

$$-g \left[ 0 + \left( \frac{x}{L} \right) + 0 - \frac{1}{2} \left( \frac{x}{L} \right)^3 \dots \right] = \ddot{x}$$

Displacement 5% of length:      0    .05    0    .0000625 ...

$$-\frac{g}{L} x \approx \ddot{x} \quad \text{SHM with } \omega_o = \sqrt{\frac{g}{L}}$$

The world is not linear. However, one can use a Taylor expansion to linearize an EOM by assuming only small perturbations around a point of stable equilibrium (which may not be the origin).