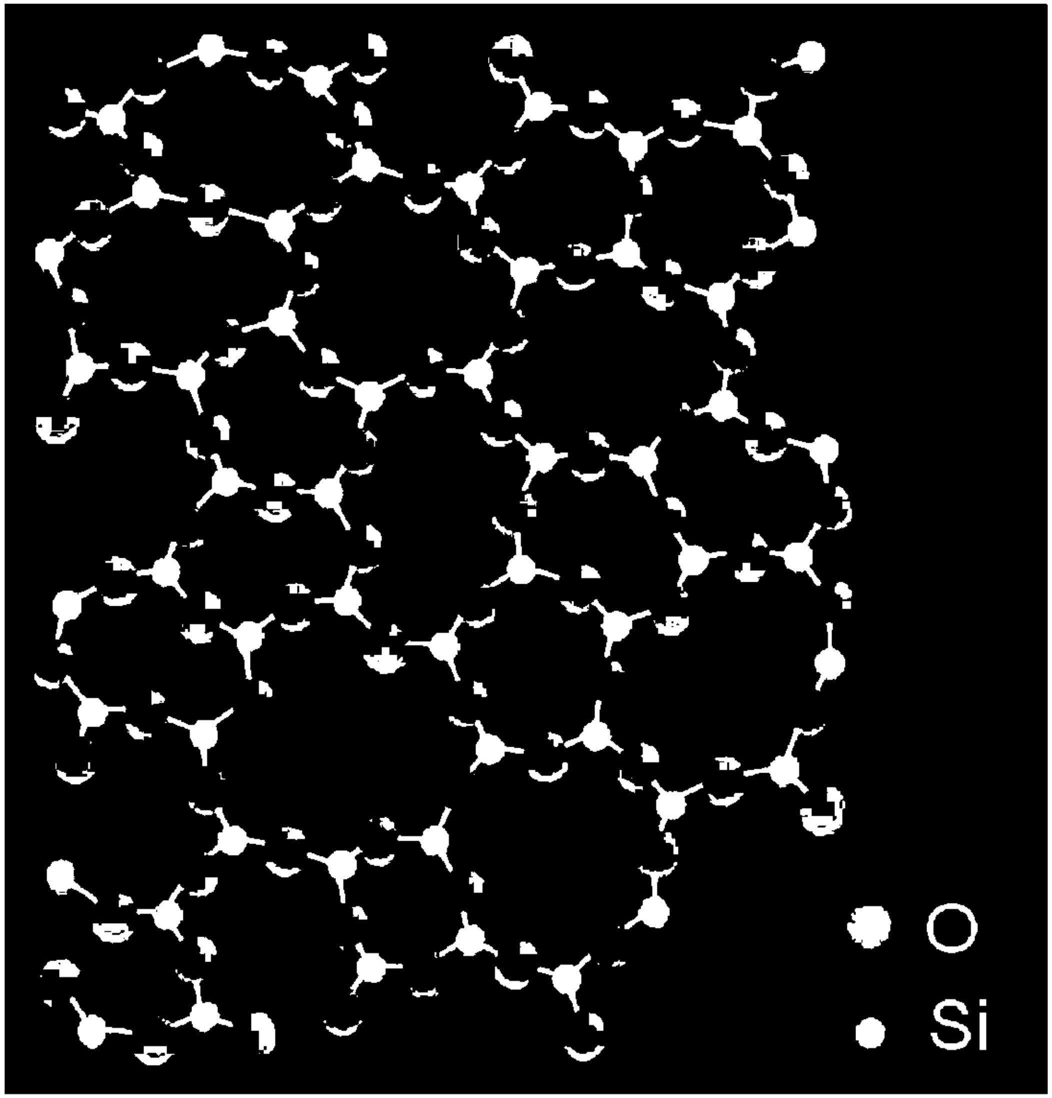
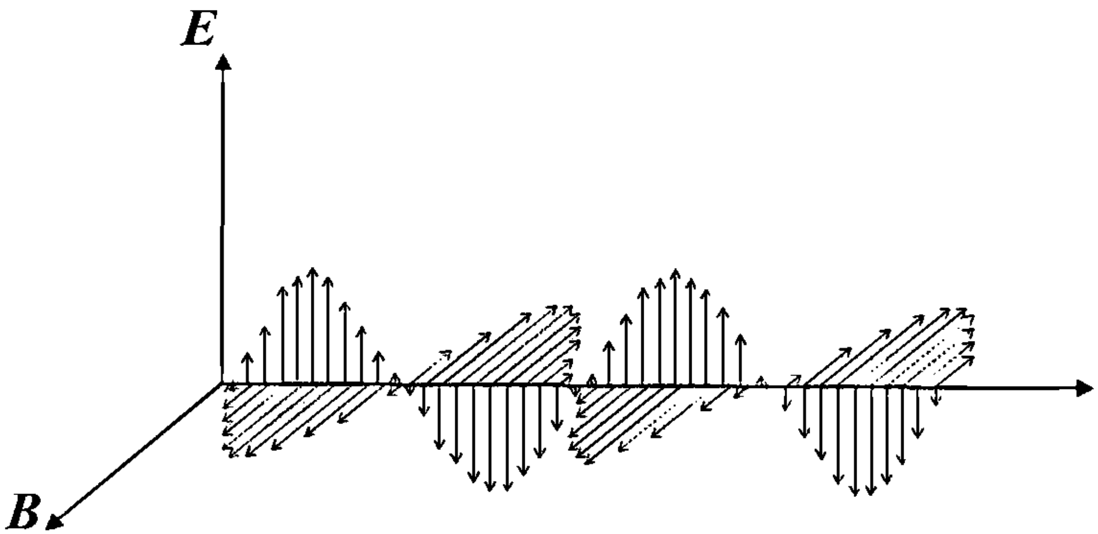
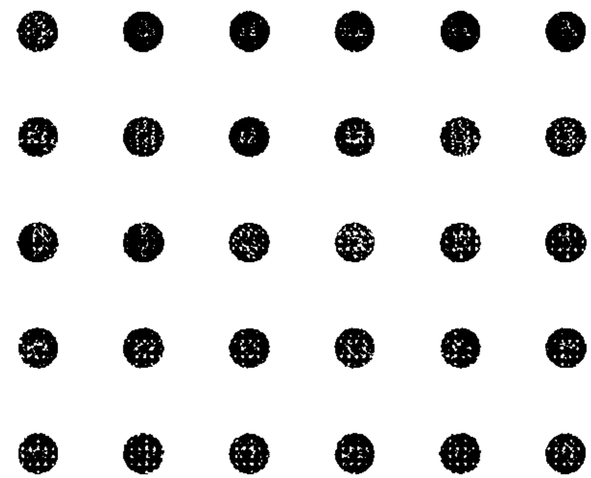
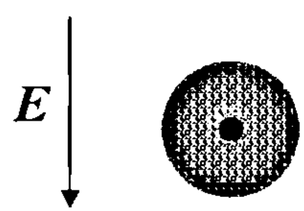
Light in a Dielectric


O
Si







$$\vec{p} = -q\,\vec{\mathbf{r}}$$

dipole moment

Polarization

$$\vec{\mathbf{P}} = -Ne\,\vec{\mathbf{r}}$$

Like a mass on a spring:

$$-K\vec{\mathbf{r}} - m\gamma\frac{d\vec{\mathbf{r}}}{dt} - e\vec{\mathbf{E}}_{app} = m\frac{d^2\vec{\mathbf{r}}}{dt^2}$$

restoring force

damping force

ma

driving force

$$\vec{\mathbf{E}}_{app} = \vec{\mathbf{E}}_o e^{-j\omega t}$$

leads to solution:

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_o e^{-j\omega t}$$

$$\vec{\mathbf{r}} = \frac{-e\vec{\mathbf{E}}_{app}}{-m\omega^2 - jm\omega\gamma + K}$$

$$\vec{\mathbf{P}} = -Ne\left(\frac{-e^2}{-m\omega^2 - jm\omega\gamma + K}\right)\vec{\mathbf{E}}_{app}$$

Polarization of the material assuming the negative charge is a mass on a spring.

...but each atom also feels the fields from the other atoms:

$$\vec{\mathbf{P}} = \underbrace{\left(\frac{Ne^2}{-m\omega^2 - jm\omega\gamma + K}\right)}_{F}\left(\vec{\mathbf{E}}_{app} + \frac{\vec{\mathbf{P}}}{3\varepsilon_0}\right)$$

$$\vec{\mathbf{P}} = \left(\frac{F}{1 - F/3\varepsilon_0}\right)\vec{\mathbf{E}}_{app}$$

$$\vec{\mathbf{P}} = \left(\frac{\dfrac{Ne^2}{-m\omega^2 - jm\omega\gamma + K}}{1 - \dfrac{Ne^2}{-m\omega^2 - jm\omega\gamma + K} \Big/ 3\varepsilon_0} \right) \left(\vec{\mathbf{E}}_{app} \right)$$

$$\vec{\mathbf{P}} = \left(\frac{Ne^2/m}{-\omega^2 - j\omega\gamma + \underbrace{K/m - Ne^2/3m\varepsilon_0}_{\omega_0^{\;2}}} \right) \left(\vec{\mathbf{E}}_{app} \right)$$

Polarization goes through a resonance:

$$\vec{\mathbf{P}} = \left(\frac{Ne^2/m}{\omega_o^{\;2} - \omega^2 - j\omega\gamma} \right) \vec{\mathbf{E}}_{app} \qquad \omega_o = \sqrt{\frac{K}{m} - \frac{Ne^2}{3m\varepsilon_o}}$$

Maxwell's equations in a dielectric:

Gauss $$\nabla \cdot \vec{\mathbf{E}} = \frac{-\nabla \cdot \vec{\mathbf{P}}}{\varepsilon_o}$$

No Monopoles $$\nabla \cdot \vec{\mathbf{B}} = 0$$

Faraday $$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

Ampere-Maxwell $$\nabla \times \vec{\mathbf{B}} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \vec{\mathbf{E}} + \mu_o \frac{\partial \vec{\mathbf{P}}}{\partial t}$$

Manipulate into a wave equation (as before):

$$\nabla^2 \vec{\mathbf{E}} = \frac{1}{c^2} \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2} + \frac{1}{c^2 \varepsilon_o} \frac{\partial^2 \vec{\mathbf{P}}}{\partial t^2}$$

$$\nabla^2 \vec{\mathbf{E}} = \frac{1}{c^2}\left[1 + \frac{Ne^2}{m\varepsilon_o\left(\omega_o{}^2 - \omega^2 - j\omega\gamma\right)}\right]\frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

Dispersion!

Assuming a plane wave solution:

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_o e^{j(kz-\omega t)}$$

Gives the dispersion curve:

$$\omega = \frac{c}{\sqrt{1 + \frac{Ne^2}{m\varepsilon_o\left(\omega_o{}^2 - \omega^2 - j\omega\gamma\right)}}}k$$

therefore, complex wave number:

$$\tilde{k} = k_r + k_i$$

therefore, light is attenuated:

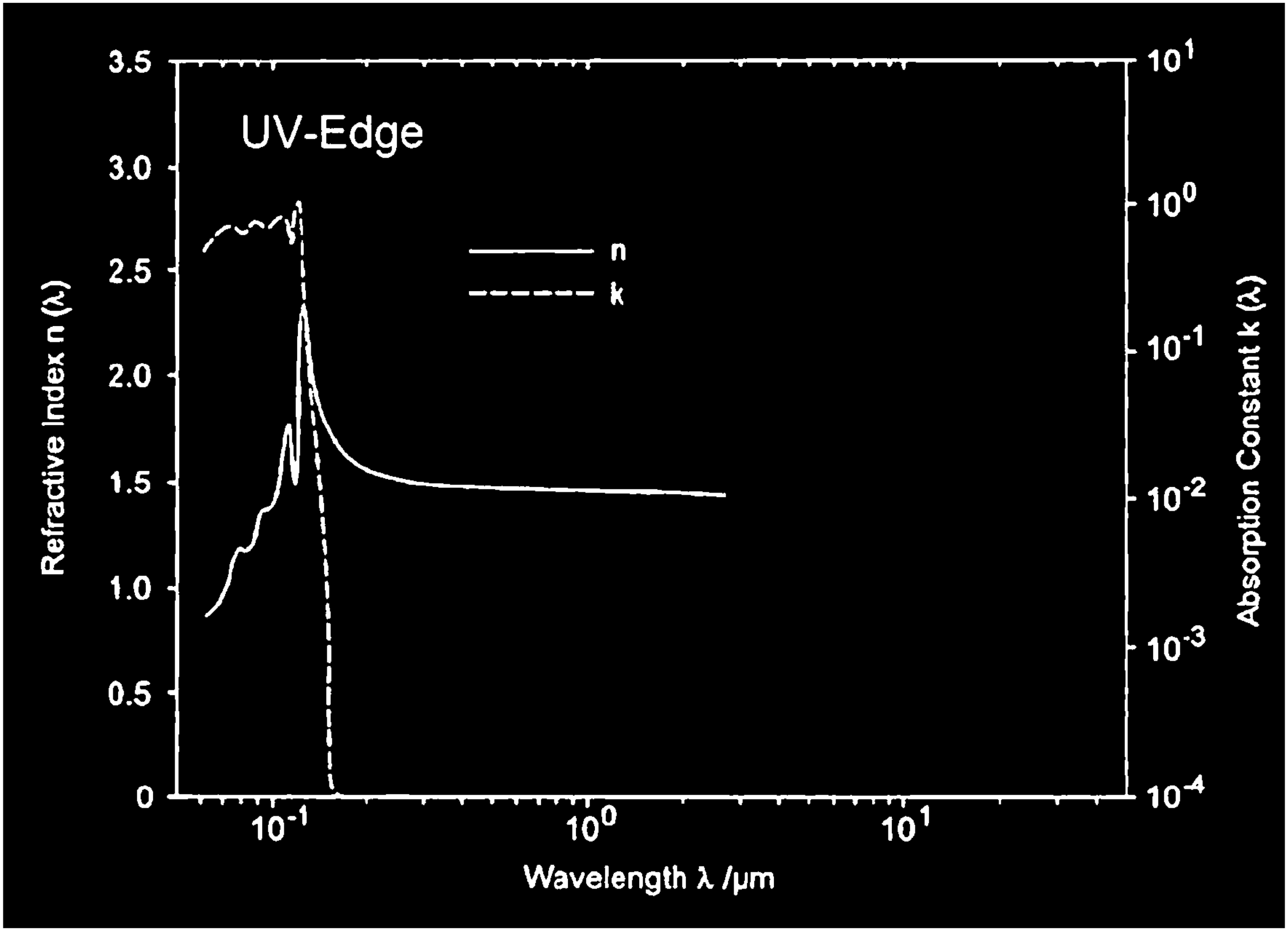
$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_o e^{-k_i z} e^{j(k_r z - \omega t)}$$

Absorption!

Usually described by the index of refraction:

$$\omega = \frac{c}{\tilde{n}} k$$

glass



UV-Edge
n
k
Refractive Index n (λ)
Absorption Constant k (λ)
Wavelength λ /μm
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}

Light in a dielectric (such as glass) travels at reduced phase velocity (c/n) and reduced wavelength (λ/n).