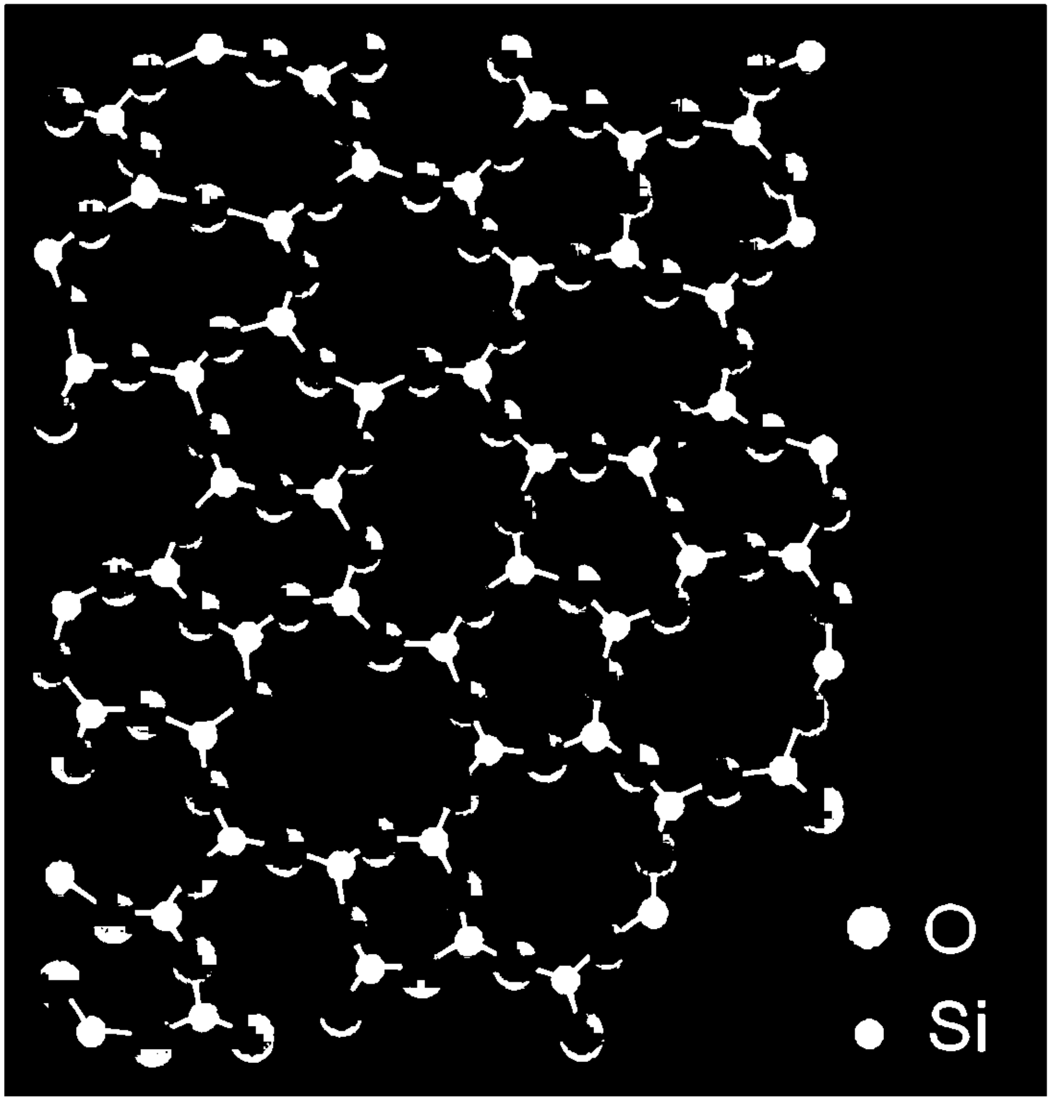
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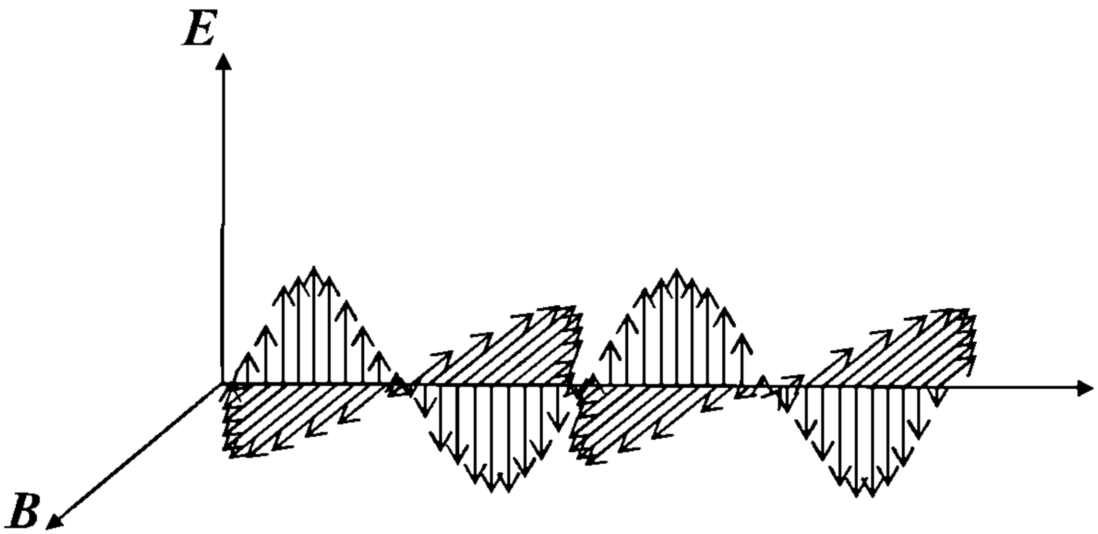
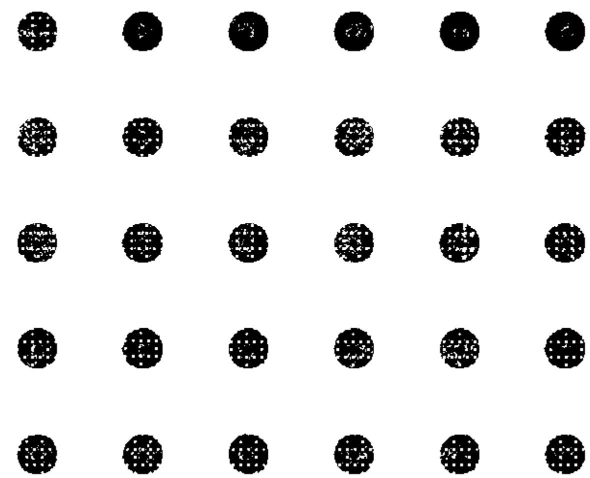
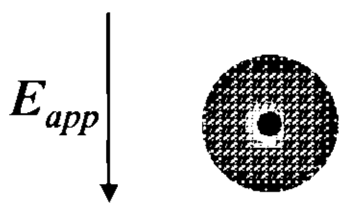
# Light in a Dielectric






$$\vec{p} = q\vec{\mathbf{r}}$$

dipole moment

Like a mass on a spring:

$$-K\vec{\mathbf{r}} - m\gamma\frac{d\vec{\mathbf{r}}}{dt} - e\vec{\mathbf{E}}_{app} = m\frac{d^2\vec{\mathbf{r}}}{dt^2}$$

restoring force

damping force

ma

driving force

$$\vec{\mathbf{E}}_{app} = \vec{\mathbf{E}}_o e^{-j\omega t}$$

leads to solution:

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_o e^{-j\omega t}$$

$$\vec{\mathbf{r}} = \frac{-e\vec{\mathbf{E}}_{app}}{-m\omega^2 - jm\omega\gamma + K}$$

$$\vec{\mathbf{P}} = -Ne\left(\frac{-e^2}{-m\omega^2 - jm\omega\gamma + K}\right)\vec{\mathbf{E}}_{app}$$

Polarization of the material assuming the negative charge is a mass on a spring.

...but each atom also feels the fields from the other atoms:

$$\vec{\mathbf{P}} = \underbrace{\left(\frac{Ne^2}{-m\omega^2 - jm\omega\gamma + K}\right)}_{F}\left(\vec{\mathbf{E}}_{app} + \frac{\vec{\mathbf{P}}}{3\varepsilon_0}\right)$$

$$\vec{\mathbf{P}} = \left(\frac{F}{1 - F/3\varepsilon_0}\right)\vec{\mathbf{E}}_{app}$$

$$\vec{\mathbf{P}} = \left( \frac{\dfrac{Ne^2}{-m\omega^2 - jm\omega\gamma + K}}{1 - \dfrac{Ne^2}{-m\omega^2 - jm\omega\gamma + K} \Big/ 3\varepsilon_0} \right) \left( \vec{\mathbf{E}}_{app} \right)$$

$$\vec{\mathbf{P}} = \left( \frac{Ne^2/m}{-\omega^2 - j\omega\gamma + \underbrace{K/m - Ne^2/3m\varepsilon_0}_{\omega_0^{\,2}}} \right) \left( \vec{\mathbf{E}}_{app} \right)$$

Polarization response goes through a resonance:

$$\vec{\mathbf{P}} = \left( \frac{Ne^2/m}{\omega_o^{\,2} - \omega^2 - j\omega\gamma} \right) \vec{\mathbf{E}}_{app} \qquad \omega_o = \sqrt{\frac{K}{m} - \frac{Ne^2}{3m\varepsilon_o}}$$

(this is not surprising since we set it up like a mass on a spring)

Maxwell's equations in a dielectric:

Gauss $\nabla \cdot \vec{\mathbf{E}} = \dfrac{-\nabla \cdot \vec{\mathbf{P}}}{\varepsilon_o}$ ⟵

**Although there is no free charge, polarization of material affects fields and creates "bound charge"**

No Monopoles $\nabla \cdot \vec{\mathbf{B}} = 0$

Faraday $\nabla \times \vec{\mathbf{E}} = -\dfrac{\partial \vec{\mathbf{B}}}{\partial t}$

Ampere-Maxwell $\nabla \times \vec{\mathbf{B}} = \mu_0 \varepsilon_0 \dfrac{\partial}{\partial t} \vec{\mathbf{E}} + \mu_o \dfrac{\partial \vec{\mathbf{P}}}{\partial t}$

Manipulate into a wave equation as before. Take time derivative of Faraday, substitute Ampere-Maxwell. Result…

$$\nabla^2 \vec{\mathbf{E}} = \frac{1}{c^2} \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2} + \frac{1}{c^2 \varepsilon_o} \frac{\partial^2 \vec{\mathbf{P}}}{\partial t^2}$$

…a wave equation for E that is affected by polarization.

Substitute the dependence of the P field on the applied E field:

$$\nabla^2 \vec{\mathbf{E}} = \frac{1}{c^2}\frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2} + \frac{1}{c^2 \varepsilon_o}\left(\frac{Ne^2/m}{\omega_o^{\;2} - \omega^2 - j\omega\gamma}\right)\frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

Rearrange:

$$\nabla^2 \vec{\mathbf{E}} = \frac{1}{c^2}\left[1 + \frac{Ne^2}{m\varepsilon_o\left(\omega_o^{\;2} - \omega^2 - j\omega\gamma\right)}\right]\frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

This looks like the vacuum wave equation for light, except with a more complicated constant. Instead of travelling at $c$, light in a dielectric travels at $c$ divided by a term that a) depends on frequency, b) is complex, and c) goes through a resonance.