Gravity Waves

In a fluid in which the density changes with height, the force of gravity can create gravity waves if there is a vertical displacement of the fluid from equilibrium.

In the atmosphere the density of the air decreases approximately exponentially with altitude. There are many gravity waves in the atmosphere produced by mechanisms such as the wind blowing across a mountain range or the convective activity of thunderstorms.

The oceans have a constant density, but they have an upper surface where the density almost disappears. (The density of water is 1000 kg/m⁻³; the density of air at sea level is 1.2 kg/m⁻³.)



Figure 14.1 A layer of incompressible fluid of uniform density ρ and depth *H*. The layer is bounded below by a rigid surface and above by a free surface that has displacement η' from its mean elevation. The profile of horizontal motion for surface water waves is indicated.

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The arrows indicate the direction of motion of the water participating in the wave motion. The water must move upward in the positive phase of the wave, the upward pointing arrows. The water must move downward in the negative phase of the wave, downward arrows. Water must move horizontally from both sides toward the positive phase to fill the increased volume; this water leaves the negative phase decreasing the negative phase volume.

Assume that our wave is propagating to the right (the wave shape and arrows move to the right), and we consider the motion of a small parcel of water located on the far right hand side of the diagram above. The initial motion is up; then when the wave moves 0.5 time units to the right the water parcel motion is to the right. Another wave shift of 0.5 time units and the motion is down; then left, up, right, down, etc. Putting this together we get:

Or actually this.

As the surface wave moves across the water, the water moves in circles.
The water parcels near the surface exhibit larger motions than those deeper down. The horizontal arrows in the diagram below depict the diminishing amplitude of the motion with depth.



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Figure 14.4 Surface water waves (a) in the longwave limit, where they assume the form of *shallow water waves*, with horizontal wavelengths long compared to the fluid depth and horizontal motion that is invariant with elevation, and (b) in the shortwave limit, where they assume the form of *deep water waves*, with horizontal wavelengths short compared to the fluid depth and motion that decays exponentially away from the free surface as an edge wave.

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Figure 14.7 (a) Surface water waves produced by the initial disturbance in Fig. 14.6, with L >> H. Shallow water waves radiate *nondispersively* away from the initial disturbance: Wave components with different k propagate at identical phase speed $c = \sqrt{gH}$ (solid lines). The envelope of wave activity then propagates at the same speed $c_g = c$ (dashed line). Under these circumstances, the shape of the initial waveform is preserved, so wave activity (shaded) remains confined to the same range of x as initially. (b) As in part (a) but for L << H and a different x scale. Deep water waves radiate *dispersively* away from the initial disturbance: Wave components with different k propagate at different phase speeds $c = \sqrt{g/|k|}$ (solid line), so the initial waveform unravels into a series of oscillations. Occupying a progressively wider range of x, the envelope of wave activity (shaded) propagates at exactly half the median phase speed of individual components $c_g = c/2$ (dashed line). Individual crests and troughs therefore overrun the envelope, disappearing at its leading edge, to be replaced by new ones at its trailing edge.

Transitions and Energy

The deep water waves have the same amount of water above the average level as below; hence the average potential energy is zero. The energy in these waves is kinetic energy associated with the moving water. When these waves approach a coastline and the water depth becomes smaller, the motion becomes restricted by the bottom. Although some of the energy is dissipated by friction with the bottom, the bulk of the energy is converted into potential energy of the wave peaks and kinetic energy of the forward motion of the peaks, breakers.

Tsunamis travel as shallow water waves because of their very long wavelengths compared to the ocean depths in most cases. They preserve their shape without spreading out, and the entire depth of the ocean participates in the horizontal motion that shapes the wave. The energy is mostly kinetic in the moving water. When it approaches a coastline the back of the wave is in deeper water than the front and moves forward faster causing a growth in the wave height and pushing the wave forward. The leading negative phase will draw all of the coastal water into the following peak, which will rise as some of the kinetic energy is converted into potential energy. Because of the long wavelength it does not come onshore like a breaker or hurricane surge but a swift a steadily rising onshore rush of water.











NOAA ANIMATION SHOWS WORLDWIDE REACH OF INDIAN OCEAN TSUNAMI

The massive tsunami triggered by an undersea earthquake in the Indian Ocean literally rippled around the world. NOAA scientist Vasily Titov, using seismic data, rendered an animation showing how the tsunami waves propagated across the Earth. Some of the waves reached the United States and many other nations outside the Indian Ocean.



Go to animations.

http://www.noaanews.noaa.gov/video/tsunami-worldpropagation2004.mov

http://www.pmel.noaa.gov/tsunami/Mov/andr1.mov