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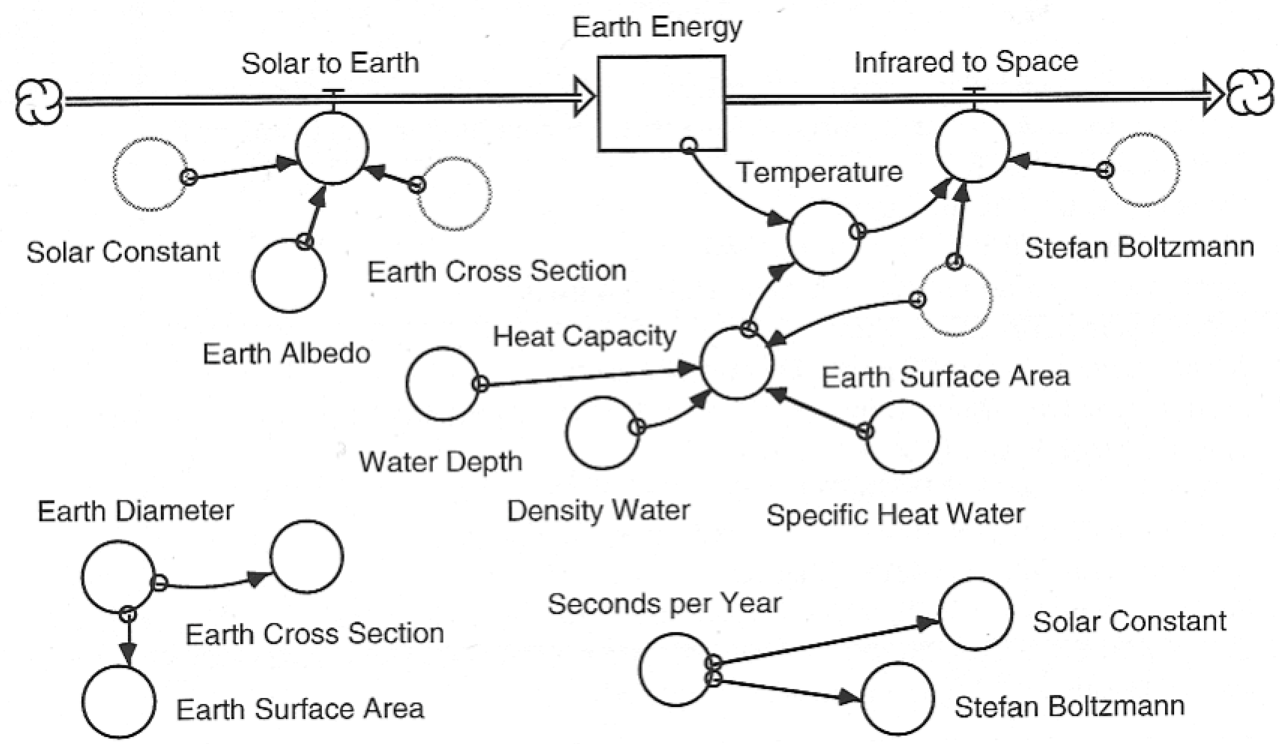
V

Building Working Models: The Earth Energy System

In Section III, we introduced the Earth energy system model and then modified the input side and discussed some changes to the output side. We now add several more elements to produce a working model of the Earth energy system. The physical principle invoked in this model is the conservation of energy. Radiant energy in the form of visible sunlight is absorbed by the Earth's surface; this energy warms the surface and the temperature increases. The Earth radiates energy into space in the infrared region of the electromagnetic spectrum; this loss of energy tends to cool the surface. The model for the Earth energy system conserves the energy flowing to and from the Earth and finds the temperature at which the energy flows are balanced.

We can identify two drivers in this model, "Solar Constant" and "Earth Albedo." Although we are treating both as constants this time, they could become variables.

"Solar Constant," the amount of solar radiation a square meter receives each year at the top of the Earth's atmosphere at the Earth's average distance from the Sun (see Glossary), could change to reflect changes in the Earth's orbit, and "Earth Albedo," the percentage of that radiation Earth reflects back into space, could change in response to global ice cover and global cloudiness. (Both ice and clouds reflect radiation.) Other converters have been added to the diagram to permit the computation of the needed parameters and variables. I have introduced two small separate subsystems



at the bottom of the system diagram. The one on the left uses "Earth Diameter" to compute the "Earth Cross Section" and "Earth Surface Area." These two parameters are ghosted into the main diagram. To the right, the converter "Seconds per Year" is used to convert "Solar Constant" and "Stefan Boltzmann" to annual values. (See equations 10, 11, and 13.)

All physical models work in time units of seconds, but it becomes awkward to express a year as 31,557,600 s when working with global models. (This number assumes 365.25 days/year to include leap years.) We can use other time units in our models, but we must carefully convert all physical parameters involving time to the new time units.

When a working part of the model is set aside from the main system diagram (the *main program*), we call it a *subroutine*. The output from the subroutine computation can be connected to the main program with a connector, but it is best to ghost it in. Similarly, parameters needed by the subroutine should be ghosted into it.

Below are listed all of the equations and constants used by the model; we will not go through all of them in detail as we did with the bathtub model. All physical parameters are expressed in SI units. (The SI stands for Système Internationale d'Unités, which is the internationally endorsed form of the metric system similar to the MKS [Meter-Kilogram-Second] system.)

The curly brackets, { }, have been used extensively in these equations to document the units involved and other modeler's comments.

We have chosen to store the "Earth Energy" in a one-meter layer of water covering the Earth's surface. This decision is frequently used by global modelers, and such models are called "swamp models" because to simplify the model they treat the Earth's surface as if it had the uniform conditions similar to the surface of a swamp. Equations 4, 9, 12, and 14 are involved

Earth Energy Model Equations

1. Earth_Energy(t) = Earth_Energy(t - dt) + (Solar_to_Earth - Infrared_to_Space) * dt
   INIT Earth_Energy = 0.0 {J, We do not know to put here yet. Let the model compute it for us.}
2. Solar_to_Earth = Solar_Constant {J/m2 yr} * (1-Earth_Albedo) * Earth_Cross_ Section {m2}
3. Infrared_to_Space = Earth_Surface_Area {m2} * Stefan_Boltzmann {J/m2 yr K4} * Temperature^4 {K4}
4. Density_Water = 1000. {kg/m3}
5. Earth_Albedo = 0.30 {30% as a fraction}
6. Earth_Cross_Section = PI*Earth_Diameter^2/4 {m2}
7. Earth_Diameter = 12742e3 {m}
8. Earth_Surface_Area = PI*Earth_Diameter^2 {m2}
9. Heat_Capacity = Water_Depth {m} * Earth_Surface_Area {m2} * Density_Water {kg/m3} * Specific_Heat_Water {J/kg K}
10. Seconds_per_Year = 3.15576E7 {s/yr}
11. Solar_Constant = 1368 {J/m2 s} * Seconds_per_Year {s/yr}
12. Specific_Heat_Water = 4218. {J/kg K}
13. Stefan_Boltzmann = 5.67E-8 {J/m2 s K4} * Seconds_per_Year {s/yr}
14. Temperature = Earth_Energy {J} / Heat_Capacity {J/K, 1st Law of Thermodynamics}
15. Water_Depth = 1.0 {m, temporary assumption}

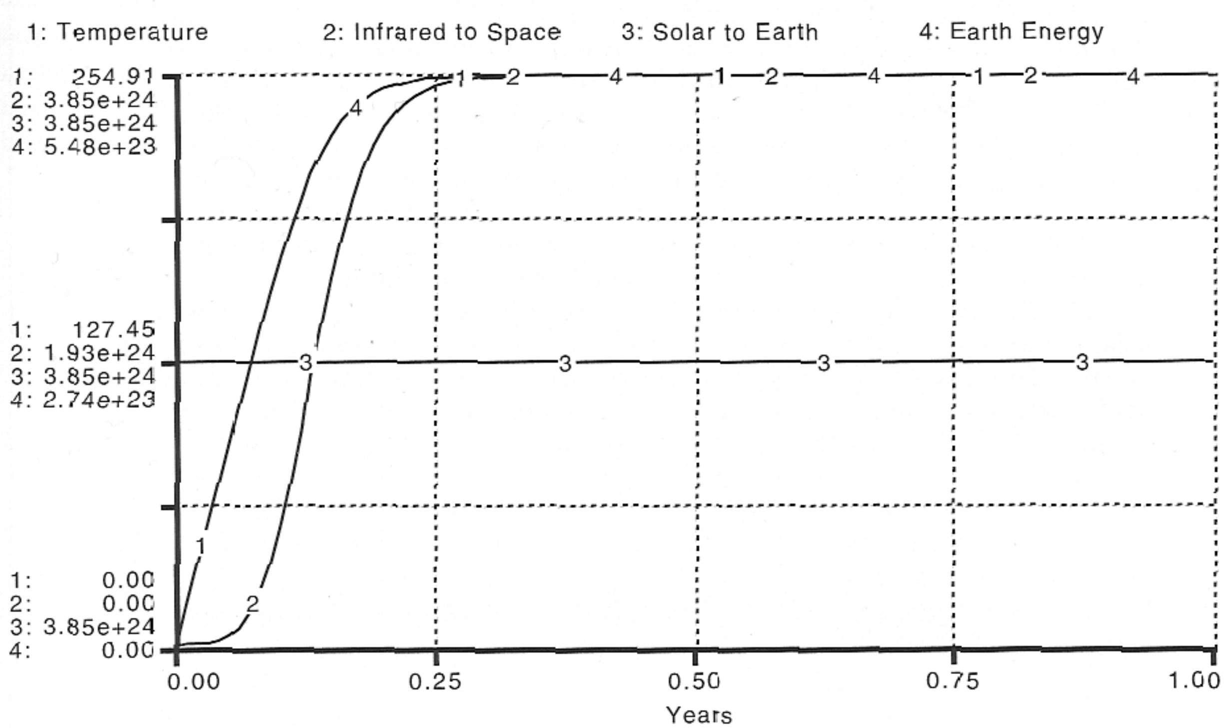
in the computation of the "Temperature." In Equation 9 the mass of the layer of water is computed and multiplied by the specific heat capacity of water to obtain the "Heat Capacity" of our swamp Earth. Equations 2 and 14 were discussed in Section III. Note that in Equation 1 we have set the initial value of the "Earth Energy" at zero; this is arbitrary, but allows us to watch the Earth warm up from absolute zero.

This model produces the output plotted on the graph below.

The format for this graph is the same as the bathtub graphs. All of the vertical axis scales have been set to place the maximum value of each plotted variable at the top of the graph. This enables us to read the maximum values directly from the upper axis scales.

"Solar to Earth" (3) in the graph is a constant in this scenario; our only reason for plotting it was to obtain its value on the vertical axis scale. "Temperature" (1) and "Earth Energy" (4) are plotted exactly on top of each other. This always happens when two variables are *linearly* related and the plotting scales are normalized to their maximum values; Equation 14 gives the linear relationship between "Temperature" (1) and "Earth Energy" (4). "Infrared to Space" (2) follows a different curve because "Infrared to Space" is proportional to the fourth power of "Temperature," T^4 (Equation 3). When T is small relative to its maximum value, then T^4 is very, very small, as shown on the lower left corner of the graph; as the two variables approach their maximum values, "Infrared to Space" catches up with "Temperature," and they both slowly merge to their maximum values. This type of system behavior, in which output variables ultimately achieve a constant value and approach that value slowly, is called an "*asymptotic* approach to a *steady-state solution*."

Looking again at the output graph, we see that the model in the steady-state region predicts a temperature for the Earth of 255 K, or –18° C. This may seem low, but is actually a valid answer, since it represents a global average including the polar regions and the entire atmosphere, where temperature decreases approximately 7° C for every



kilometer increase in altitude. At the tropopause (the top of the lowest layer of the atmosphere, at about 12 km altitude) the temperature is −60° C! If the Earth's mean temperature were measured from space using infrared detectors, the value would be close to 255 K. The temperature computed using a planet's radiation balance is called the "effective planetary temperature"; it closely matches the planet's temperature measured from space. The effective planetary temperature is an important parameter for characterizing a planet's relationship to the Sun and its fundamental thermal condition. At this temperature the planet radiates into space exactly the same energy per day that it receives from the Sun. This is a very delicate balance, and any deviation will cause the planet to warm or cool. The effective planetary temperature and the black-body radiation law act together like an overall negative

BLACKBODY RADIATION

A new physical equation is introduced in Equation 3, on page 28, the Stefan-Boltzmann or "blackbody" radiation law. It is:

$$R_{bb} = \sigma T^4$$

The Stefan-Boltzmann constant, σ, is $\sigma = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$; its value is given in Equation 13 (where the units are also changed to the annual value). The Stefan-Boltzmann law computes the total power radiated per unit area, R_{bb}, from a perfect black material at a uniform temperature, T.

The name "blackbody radiator" seems a strange name to apply to our Earth, which we know from space photographs is predominantly blue, white, and green. The visible colors of the Earth, however, are the reflected light from the Sun, not the radiation produced by the Earth itself. We would need infrared eyes to see the Earth's own radiation, and we would see an entirely different Earth. When we look at an object that absorbs all the radiation that strikes it, it appears completely black. Physicists have proven that all materials radiate electromagnetic energy at each wavelength with exactly the same efficiency that they absorb radiation at the same wavelength (Kirchhoff's law). A "blackbody radiator," which is a perfect absorber, is, therefore, also a perfect radiator of electromagnetic radiation. It is the most efficient radiator possible; it emits the maximum radiation possible at a given temperature, and the distribution of that energy among the various wavelengths of the electromagnetic spectrum follows a specific law that depends upon the temperature.

There are many examples of blackbody radiators in our everyday environment; solar radiation is blackbody radiation. Because of the high temperature of the apparent visible solar surface (approximately 6,000 K), solar radiation occurs primarily in the visible light wavelengths. The incandescent light bulb is another example; the temperature of the filament in the light bulb is approximately 2,800 K, and its light is yellowish white. (When you use a dimmer on an incandescent light bulb you lower the temperature of the filament; the light bulb produces less light, and the light becomes yellow to red as it dims.) White lightning has a temperature about 30,000 K, and is bluish white. The Earth's effective temperature is around 255 K, and its radiation is in the infrared part of the electromagnetic spectrum, at wavelengths much too long to be seen with our eyes.

LINEARITY

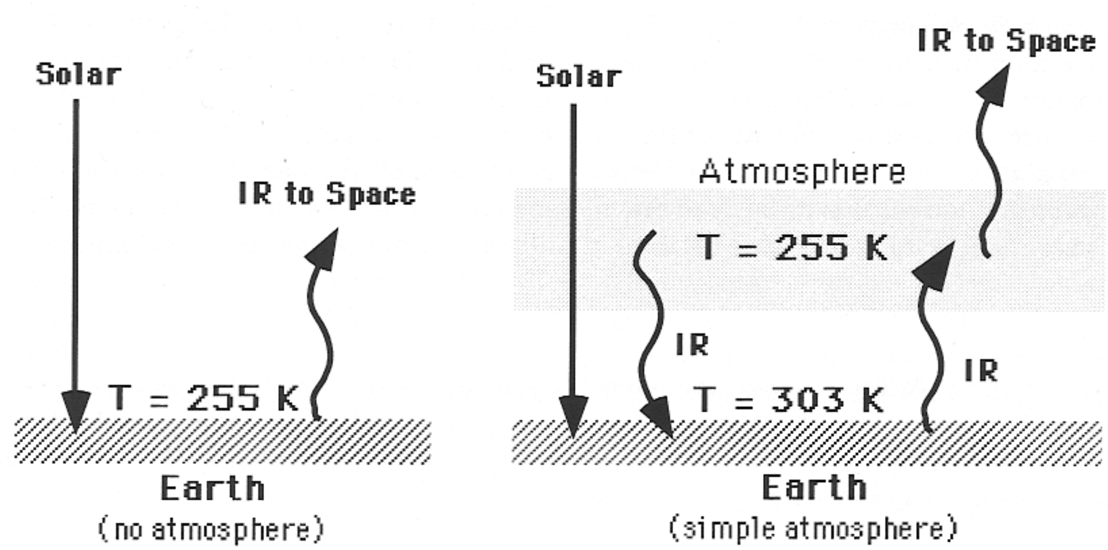
The terms *linear* and *linearly related* are frequently used in describing a system's behavior. Basically they mean that one system variable, when plotted on a graph as a function of another system variable, will plot as a *straight line*. This linear relationship has a specific meaning in mathematics. If one variable, y, is a function of another variable, x, they are linearly related if the algebraic expression describing their relationship can be written: $y = ax + b$, where a and b are constants. The important aspect of this equation is that x has the power one (*i.e.*, x^1). Again, the word linear is used because when y is plotted as a function of x, the result is a straight line. In Section IV, "Bathtub" is a linear function of time while the fill valve is on.

If the algebraic relationship were $y = ax^2 + bx + c$ (c is another constant), we would call it a *quadratic* relationship.

feedback process for the planet's climate. Because of the blackbody radiation law (radiated power σT^4), a small increase in planetary temperature will produce a proportionately much larger increase in outward infrared radiation, which will cool the planet. Similarly a small decrease in planetary temperature will cause a decrease in energy radiated, which will warm the planet.

To obtain a temperature for the Earth's surface that more closely matches the actual surface conditions, we need to modify the model to include an atmosphere, so that *greenhouse warming* of the surface is incorporated. The figure below illustrates the principles involved in the greenhouse effect. On the left is the situation that we modeled, the Earth without an atmosphere; the incoming solar energy is exactly balanced at 255 K by the outgoing infrared radiation to space.

Suppose the atmosphere lets all solar radiation pass through, totally absorbs all infrared radiation, and has a constant temperature. What is the temperature of such an atmosphere? The answer has to be 255 K, because the Earth ultimately must reradiate to space all of the energy received from the Sun, and our model has computed 255 K as the temperature required to do the job. In a simple model with the atmosphere, it is the atmosphere, not the



Earth's surface, that radiates into space. Therefore it is the atmosphere that has the 255 K temperature. But the temperature of the surface under this atmosphere is no longer 255 K. Why? Because it is now receiving both solar and atmospheric radiation, and in our simple scenario these two radiation sources are equal. This is depicted on the right side of the figure on page 31. (The atmosphere must radiate as much radiation downward as it does upward, because the molecules are emitting radiation in all directions.) Now the Earth's surface must acquire a temperature sufficient to radiate twice as much energy as before in order to reach a steady state. Our blackbody equation tells us that to achieve twice the radiation power the temperature must increase by the factor $\sqrt[4]{2}$.* Our new value for the surface temperature with an atmosphere is 303 K = 30° C, which is about right for June in Houston, but too high for a global average surface temperature. The reason our simple model gave us a temperature too high was that we assumed a totally opaque atmosphere in the infrared, where it is actually partially transparent. However, by adding greenhouse gases such as carbon dioxide to the atmosphere we decrease the atmospheric transparency in the infrared and make it more opaque.

Let's turn our attention to time constants. We can make a reasonable guess at the warming time constant for the "one-meter swamp Earth" from the graph; approximately three months were required for the Earth to reach the steady-state temperature. But the change is slow during the final two months. If we extend the straight line portion (roughly the first month) of the "Temperature" curve (1), which is also the "Earth_Energy" curve (4), from time = 0.0 until it crosses the top line representing the steady-state solution, we find a time of approximately 1.5 months. Our guess is that the model Earth warming time constant is between 1.5 and 3 months.

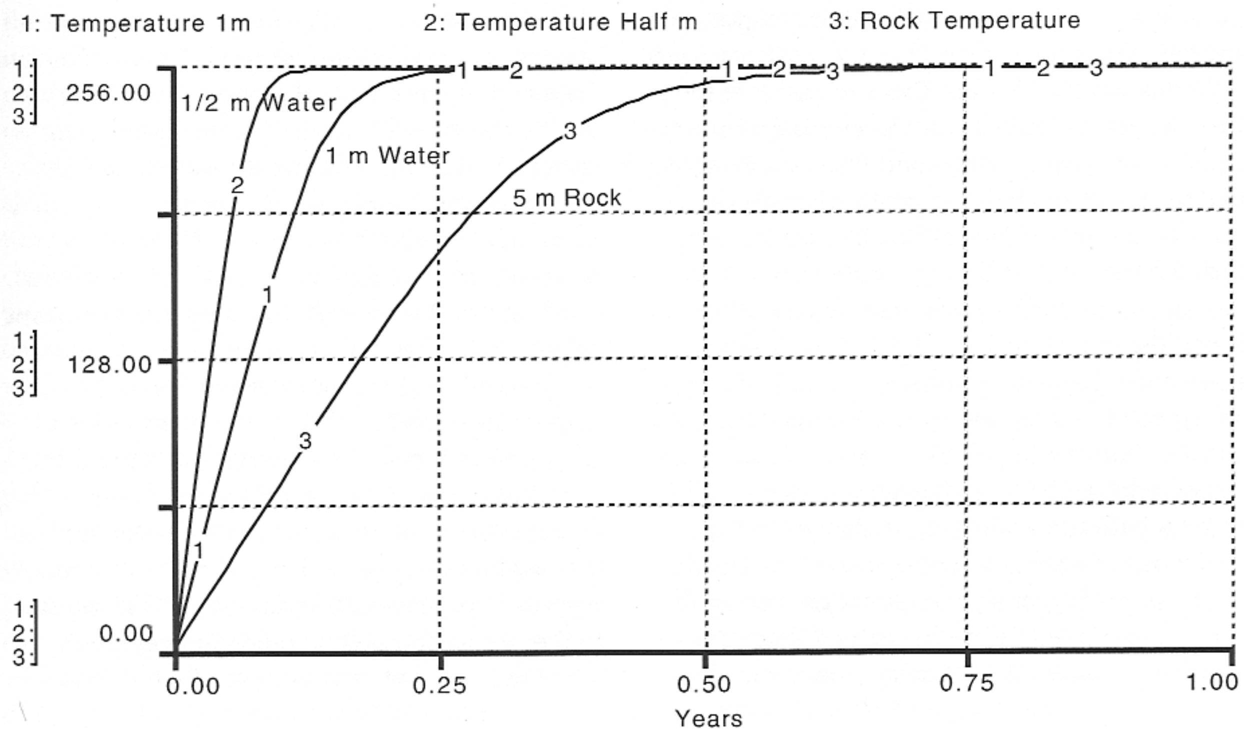
$$
\begin{aligned}
T_w &= \text{"Earth Energy"} / \text{"Solar to Earth"} \\
&= 5.48\text{e}{+}23 / 3.85\text{e}{+}24 = 0.142 \text{ years} \\
&= 1.7 \text{ months}
\end{aligned}
$$

We compute the warming time constant, T_w, by dividing the reservoir "Earth Energy" (use the steady-state value) by flow "Solar to Earth"; this computation yields 1.7 months, an answer close to that found from extending the straight line portion of the graph. Notice that the cooling time constant, T_c = "Earth Energy" / "Infrared to Space," has exactly the same value as the warming time constant; this is necessary for the system to remain in the steady state.

Our "one-meter swamp" Earth model would be totally useless for studying daily changes in the Earth system because the water averages out all thermal changes occurring in periods of less than a month. But it could be modified to explore seasonal or longer changes in the Earth system because the water could respond to changes occurring over periods longer than 1.7 months.

Why did we use one meter of water? It was an arbitrary choice. One of the really satisfying things about creating a working model is that it can easily be modified to try other ideas. We can, for instance, change the depth of the water or change the water to rock and see what happens. If we adapt our "swamp Earth" to create two new Earth models, one with a half-meter layer of water and the other with five meters of rock, and run all three models, they all reach the same final temperature: 255 K.

*Let R_1 and R_2 be the blackbody radiant powers emitted at temperatures T_1 and T_2; then we have $R_1 = \sigma T_1^4$ and $R_2 = \sigma T_2^4$ when we apply the blackbody radiation law. If we now divide the second equation by the first we get $R_2 / R_1 = (\sigma T_2^4)/(\sigma T_1^4)$, which simplifies to give us $R_2 / R_1 = (T_2 / T_1)^4$ or $T_2/T_1 = \sqrt[4]{R_2/R_1}$. For the simple greenhouse model considered here, R_2 is $2R_1$; hence $T_2 / T_1 = \sqrt[4]{2}$.



This is not surprising, because it is a steady-state model and we have not changed the two drivers, "Solar Constant" and "Albedo." The difference is in how long it takes the models to reach this temperature. The time constant for a half-meter of water is exactly half that for one meter of water; there is half as much water to heat up, so it can reach any particular temperature twice as fast. The time constant for five meters of rock is approximately four months. If we had used an Earth-sized rock in the model, we would still have ended up with a temperature of 255 K, but it would have taken a long time to reach a steady-state solution (which is one reason for using a meter of water). Another interesting comparison is to find the thicknesses of various substances that have the same warming time constant; the values in the box that follows will probably surprise you.

Now let's ask the question in a different way. Suppose we wanted a model with a surface that would respond to the daily heating of the Sun. How thick should we make the surface? We know that a meter of water has a time constant of 0.142 years; therefore, a centimeter of water will have a time constant of 0.00142 years, or 12.4 hours, still a little too long to closely follow daily solar heating. The daily thermal variations between night and day do not penetrate more than a few centimeters into the surface of this planet. And seasonal variations, summer to winter, all occur in the upper few meters.

In the oceans, currents and waves keep the upper hundred meters of the water mixed; the ocean temperature does not change significantly with the seasons. The atmosphere, however, responds to changes on all of these time scales. One centimeter of water has the same time constant as 45 meters of air, and seasonal

Thicknesses for Equivalent Warming Time Constants

1 meter of water = 2 meters of rock = 4,500 meters of air

variations are felt through the entire troposphere, 12 kilometers thick.

We saw in the swamp Earth model that the time constants for heating and cooling were equal in the steady-state solution. Now imagine that we have a switch to turn the Sun on and off. Our model output tells us that when we turn the Sun on, the Earth's temperature increases with a time constant of 1.7 months. When the Sun is turned off, the Earth's temperature must decrease at the same rate. The Earth has stored the solar energy, and cannot release it faster than is allowed by the time constant. In the swamp model there is always a delay of 1.7 months between a solar input change and the readjustment of temperature to the new steady state. This delay, or shift in the output variable (temperature) relative to the driver (solar input) is called a *phase shift*. We know from experience that the hottest month of the year is usually August in the northern latitudes but that the largest daily input of solar energy occurs at solstice on June 22, when the Northern Hemisphere is at its maximum tilt toward the Sun. Similarly, the hottest part of a summer day may occur several hours after noon. These phase shifts occur in many variables in many systems, and can usually be traced to some time constant related to a reservoir and a flow.

The time constant of a system plays an important role in how the system responds to system drivers with different time periods. Consider the situation in which the system driver is changing much faster than the time constant of the system. The system in this case responds by reducing or damping its response to the driver; the system tends to average the effect that the driver tries to produce. A meter

GLOBAL WARMING CONTROVERSY

In our model and discussions we have included only the science that relates to radiation laws, and we found a direct relationship between the energy received by the Earth's surface and the resulting steady-state temperature. For the real Earth, processes like ocean currents or volcanic eruptions influence the measured global average temperature. Over the longer time scales (longer than decades) radiation processes will have the dominant influence.

At least part of the global warming controversy is semantics. Warming to the scientist means increasing the radiant energy to the Earth's surface; warming to many others means increasing the temperature. Consider a pot of water on the stove. As heat is added the temperature rises, but when the water starts boiling the temperature remains constant. Would you say that you were no longer warming the pot? The scientists would say that the warming is continuing as before because heat is still being added to the pot; others might respond that you are no longer warming the pot because the temperature is no longer rising. Scientists look at the Earth and say with certainty that increasing greenhouse gases cause global warming because they know that additional greenhouse gases increase the infrared radiant energy from the atmosphere to the Earth's surface. In fact, there is as much scientific certainty in this conclusion as there is in the law of gravity. Scientists are less concerned about the year-to-year changes in global average temperature because these variations are normal and expected. The use of global annual average temperatures as proof of global warming is fraught with problems because they are difficult to measure and show a lot of natural variability and because it takes a long time to demonstrate an unambiguous temperature increase using rigorous statistical techniques.

thickness of water will exhibit a small temperature change in response to daily cycles in solar radiation; yet the water will achieve an average temperature corresponding to the average solar input over a season. If the system driver changes much more slowly than the system time constant, the system will follow the changes in the driver in a continuously evolving steady state. The average temperature of a one-meter layer of water will gradually change with the seasonal changes in solar radiation.

In systems capable of natural oscillations, a special response can occur when the driver period matches the system time constant. In this case the system's response is greater than it would be to the same driver operating at slower periods. This response is often called a *resonance response*. We can illustrate all three responses with a glass half filled with water. The glass of water is the system, your hand the driver. To determine the system time constant, push the glass quickly to one side. The water sloshes back and forth with a certain period, which is the system time constant. When we move the glass across the table more slowly than the time constant, the water follows along with little sloshing. If we wiggle the glass rapidly back and forth at periods faster than the time constant, we can create lots of small waves in the glass, but the average height of the water in the glass is unchanged. (You need to move your hand fairly fast to make sure that you are faster than the sloshing time constant.) Finally, when we move the glass back and forth at a period close to the system time constant, we observe the sloshing amplitude grow and eventually the water sloshes out of the glass.

In a large complex system like the Earth, there are many subsystems and components with many different time constants. The output from such a system has a lot of natural variability; for a given set of drivers the system will approach a steady state, but superposed on it will be natural *fluctuations*. The system components with time constants matching driver periodicities will exhibit the largest regular responses; the other system components produce apparently random variations sometimes referred to as *noise*. It is only the average that is steady in the steady state of a complex system. One needs to look no further than the weather to find a perfect example of large fluctuations superposed upon a steady state. In fact, the weather fluctuations are so large that defining their averages becomes very difficult.

Exercises

1. The steady-state solution to the Earth energy problem corresponds to the condition in which the incoming solar radiation is exactly balanced by (equal to) the outgoing infrared radiation. We can write this steady-state solution as an algebraic expression using previously defined parameters.

$$S\,(1-A)\,\pi\, r^2 = \sigma\, T^4\, 4\pi\, r^2$$

This equation simplifies to

$$S\,(1-A) = 4\,\sigma\, T^4$$

The temperature in this equation is called the effective planetary temperature; for the Earth this is T_E, where $T_E = 255$ K. We now form a difference equation from the steady-state solution by allowing S and A to be variables. (You may think of the difference equation as the time derivative of the equation multiplied by the time difference, dt.)

$$dS\,(1-A) - S\,dA = 4\,\sigma\, 4T^3\, dT$$

We divide this equation by the previous equation (left-hand side by left-hand side and right-hand side by right-hand side), and rearrange the resulting equation.

$$\frac{dT}{T} = \frac{1}{4}\,\frac{dS}{S} - \frac{1}{4}\,\frac{A}{(1-A)}\,\frac{dA}{A}$$

This form of the difference equation expresses the fractional change in the Earth's steady-state temperature as a function of the fractional change in the solar constant and the fractional change in the Earth's albedo. Equations written in this form are useful for sensitivity studies. Note that the solar constant variation term is positive, corresponding to an increase in temperature associated with an increase in solar constant. The Earth albedo variation term is negative because increases in albedo produce decreases in the Earth's temperature.

The sensitivity of the Earth's temperature to changes in the solar constant is measured by the increase in temperature produced by a 1% increase in the solar constant.

a. Find the sensitivity of the Earth's temperature to solar constant changes.
b. Find the sensitivity of the Earth's temperature to changes in the Earth's albedo (again, for a 1% change in albedo). Assume a steady-state albedo, A = 0.3 (30%).
c. Find the temperature sensitivities of Venus (A= 0.71 [71%]) and Mars (A= 0.17 [17%]) to albedo changes.

2. We can modify the first equation in Exercise 1 to include greenhouse warming of the surface at temperature T_S (=288 K) by an atmosphere at temperature T_A. See the figure for the greenhouse model on page 31.

$$S\,(1-A)\,\pi\, r^2 + a\,\sigma\, T_A^4\, 4\,\pi\, r^2 = \sigma\, T_S^4\, 4\pi\, r^2$$

This equation expresses the energy balance at the surface, which must occur for the steady-state solution. The new second term on the left-hand side of the equation represents the infrared radiation incident on the Earth's surface from the atmosphere. We have introduced a new parameter "a," which is the effective gray-body absorptivity for the atmosphere. (A gray body is similar to a blackbody but less efficient by the factor "a.") We have also used the property (known as Kirchhoff's law) that a material emits radiation at a given wavelength with the same efficiency with which it absorbs radiation at that same wavelength. The factor "a" can take values between 0.0 and 1.0; a = 1.0 corresponds to a blackbody, and a = 0.0 would be an atmosphere totally transparent to infrared radiation and also incapable of emitting infrared radiation. The factor "a" is directly related to the amount of greenhouse gases in the atmosphere.

The first equation in Exercise 1 is the equation that defines the effective planetary temperature T_E (=255 K); so, we may use this definition to replace the first term in the equation above with $\sigma\, T_E^4\, 4\pi\, r^2$. Our equation can now be expressed in a rather simple form.

$$T_E^4 + aT_A^4 = T_S^4$$

This is still the energy balance equation for the surface, but we are using temperature variables to simplify the form of the equation. We now want to write a similar equation for the energy balance that must occur in the atmosphere.

$$a\, T_S^4 = 2\, a\, T_A^4$$

The left-hand side of this equation is the radiation energy (per square meter) from the surface (at T_S) that is absorbed in the gray-body atmosphere (with efficiency a). The right-hand side is the total radiated energy (per square meter) emitted by the atmosphere; the factor 2 appears here because the atmosphere radiates equal amounts of energy upward into space and downward to the surface. You may want to think of the atmosphere as having two surfaces of equal area, one facing upward and one facing downward. Several algebraic steps were left out in developing this last equation; you should fill in the missing steps.

We can use this last equation to eliminate the atmospheric temperature T_A from our previous result. Fill in the missing steps.

$$T_E^4 \;=\; \left(1 - \frac{a}{2}\right) T_S^4$$

Following the methods that were used in Exercise 1, we now form a difference equation allowing T_S and "a" to be variables but keeping T_E constant. When the resulting equation is written in the fractional format for sensitivity analysis, we have the following result.

$$\frac{dT_S}{T_S} = \frac{1}{8} \frac{a}{(1-\frac{a}{2})} \frac{da}{a}$$

Specifying "a" for our model atmosphere is somewhat difficult, because the real atmosphere has many layers at many different temperatures, rather than the single one used here, and the Earth's atmosphere has clouds that come and go at several different levels. When an overall average for the outgoing radiation for the whole Earth is determined, we find that 7% of the outgoing radiation comes from the surface, with the balance of 93% from the atmosphere, including the clouds. To use this information we need to write an equation for the fraction of the outgoing radiation that originates within the atmosphere.

$$\frac{aT_A^4}{aT_A^4 + (1-a)\,T_S^4} = 0.93$$

The numerator, you will recognize, is the outward radiation (per square meter) from the atmosphere. The denominator is the total outward radiation; the second term is the surface radiant energy that did not get absorbed in the atmosphere. With the help of the third equation in this problem, solve the above equation for "a." (Hint: Form the ratio T_A^4 / T_S^4 in both equations.)

Using the value that you found for "a", find the sensitivity of the Earth's surface temperature to changes in the effective gray-body absorptivity (again use a 1% change in "a").

Compare your result for the sensitivity of the Earth's surface temperature to changes in the effective gray-body absorptivity to your results from Exercise 1 for the sensitivities to solar constant and albedo changes. Considering that "a" is the result of atmospheric greenhouse gases, comment upon the relationship of humankind's alteration of the global concentration of greenhouse gases to the natural global energy balance.