

1. **Answer: 2**

Let the set of coins be $\{A, B, C, D, E, F, G, H\}$. First, weigh $\{A, B, C\}$ vs. $\{D, E, F\}$. If one group is lighter (say $\{A, B, C\}$), then use the second weighing to measure A vs B . If we find that one is lighter than the other, than it must be the counterfeit. If A vs B weigh the same, then we know that C must be the counterfeit. If both groups weighed the same, then measure G vs H . If G vs H weigh the same, then no counterfeit exists.

2. **Answer: 210**

The smallest prime number whose digits add up to greater than 10 is 29. Solving $x + (x + 1) = 29$ gives $x = 14$. The answer is $14 \times 15 = 210$.

3. **Answer: 49**

We note that to get a zero at the end of a number, we must multiply by 10. Since $5 \times 2 = 10$, and there are more factors of 2 in $200!$ than 5, it suffices to count how many 5's appear in the prime factorization of 200. Each of 5, 10, 15, \dots , 200 has a factor of 5 in it, which gives 40 factors. In addition, 25, 50, \dots , 200 gives a second factor of 5, so that is 8 additional factors of 5. 125 has a third factor of 5. So $200!$ has 49 factors of 5.

4. **Answer: 314159**

The number is prime.

5. **Answer: $14\sqrt{5}$**

We can use Heron's formula to calculate the area of the triangle. The semiperimeter equals $\frac{7+9+12}{2} = 14$. Applying Heron's formula

$$\text{Area} = \sqrt{14 * (14 - 7) * (14 - 9) * (14 - 12)} = 14\sqrt{5}$$

6. **Answer: 5**

Just make a table:

\$8	\$2	\$0.6
2	4	10
2	1	20
1	8	10
1	5	20
1	2	30

7. **Answer: $x = \frac{a \pm a\sqrt{5}}{2}$**

$$a^2 = x^2 - ax$$

$$0 = x^2 - ax - a^2$$

$$x = \frac{a \pm \sqrt{a^2 - 4(-a^2)}}{2}$$

$$x = \frac{a \pm |a|\sqrt{5}}{2} = \frac{a \pm a\sqrt{5}}{2} \text{ The } \pm \text{ takes care of the fact that we do not know the sign of } |a|.$$

8. **Answer: 180 meters**

The regular trains will intersect at (150, 0) meters at $t = 3$ seconds. Regardless of the location of the fly at any time, we know that the fly will be moving at 60 meters/second. Therefore, at $t = 3$ seconds the fly will have traveled a total of $3 * 60 = 180$ meters.

9. **Answer: $(n + 1)^4$**

Note that the digits of 14641_n in base n are the binomial coefficients $\binom{4}{i}$ so that $14641_n = \sum_{i=0}^4 \binom{4}{i} n^i = (n + 1)^4$.

10. **Answer:** $150\sqrt{3}$ meters²

Using all the fence available, he can create a regular hexagon with side length 10. Notice that a regular hexagon can be split up into 6 equilateral triangles, each with a side length of 10. The area of an equilateral triangle with side length 10 is $\frac{10^2\sqrt{3}}{4}$ and hence the area of the hexagon will be $\frac{10^2\sqrt{3}}{4} * 6 = 150\sqrt{3}$

11. **Answer:** 49

A locker will be open if it has an odd number of distinct divisors. For each divisor d of an integer x , the integer $\frac{x}{d}$ must also be a divisor of x . The only way for x to have an odd number of divisors is if $d = \frac{x}{d}$ which means that $x = d^2$ and hence x must be a perfect square. Thus, the open lockers are all the perfect squares, and the seventh perfect square is $7^2 = 49$.

12. **Answer:** 140 acres

Multiply 2240 pounds per long ton times 7000 grains per pound times 1 kernel per grain to get the number of kernels. Divide the number of kernels by 56 kernels per plant, and divide the result by 500 corn plants per rood. Then divide the number of roods by 4. The result is $\frac{2240*7000}{4*56*500} = 140$.

13. **Answer:** 4.2 revolutions

The wheel goes through three laps of $2\pi * 7 = 14\pi$ distance, for a total distance of $3 * 14\pi = 42\pi$. In one full turn, the wheel goes through a distance of $2\pi * 5 = 10\pi$ and thus after all three laps the wheel undergoes $\frac{42\pi}{10\pi} = 4.2$ revolutions.

14. **Answer:** $R\sqrt{3}$

Label the center of the circle O and the vertices of the triangle A, B, and C. Then we can find the length of segment \overline{AB} by using the law of cosines on triangle AOB. By symmetry, we can see that the angle AOB equals $\frac{360}{3} = 120$ degrees. Applying the law of cosines

$$(\text{Length of } \overline{AB})^2 = R^2 + R^2 - 2 * R * R * \text{Cos}(120) = 2R^2 - 2R^2 * (-\frac{1}{2}) = 3R^2$$

15. **Answer:** $x = -2, -1, -\frac{1}{2}, \frac{2}{3}$

Simple trial and error of the first few integers yields the roots $x = -2, -1$. From there, the remaining polynomial can be solved using the quadratic formula. The final factored form is

$$(2 + x)(1 + x)(6x^2 - x - 2) = (2 + x)(1 + x)(1 + 2x)(-2 + 3x)$$

16. **Answer:** 20

Each circle can intersect with any other circle in at most two distinct points. As long as the intersection points between every two circles are all unique, then we can calculate the greatest number of intersections possible starting with the case of two circles and working up to five circles:

2 circles \rightarrow 2 points max

3 circles \rightarrow $2 + 2 * 2 = 6$ points max

4 circles \rightarrow $6 + 2 * 3 = 12$ points max

5 circles \rightarrow $12 + 2 * 4 = 20$ points max

17. **Answer:** 18190₋₁₀, or 18190

As with positive base conversion, we proceed by iteratively dividing by -10 and calculating the positive remainder, which is a digit from 0 to 9. Thus, we arrive at $2010_{10} = 1 * (-10)^4 + 8 * (-10)^3 + 1 * (-10)^2 + 9 * (-10)^1 + 0 * (-10)^0$.

18. **Answer:** $\frac{133}{243}$

Consider the equivalent problem of subtracting the infinite series $\frac{64}{729} - \frac{128}{2187} + \frac{256}{6561} \dots$ from the infinite series $1 - \frac{2}{3} + \frac{4}{9} \dots$. Since the ratio between terms in both series is $-\frac{2}{3}$, we can use the formula for a converging geometric series:

$$\frac{1}{1 - (-\frac{2}{3})} - \frac{\frac{64}{729}}{1 - (-\frac{2}{3})} = \frac{3}{5} - \frac{64}{729} * \frac{3}{5} = \frac{3}{5} - \frac{64}{1215} = \frac{729-64}{1215} = \frac{665}{1215} = \frac{133}{243}$$

19. **Answer:** $12\sqrt{5}$

Note that $1010100_\phi - .010101_\phi = \phi^6 + \phi^4 + \phi^2 - \phi^{-2} - \phi^{-4} - \phi^{-6} = \phi^6 + \phi^4 + \phi^2 - (-\phi)^{-2} - (-\phi)^{-4} - (-\phi)^{-6}$. Thus, $1010100_\phi - .010101_\phi = \sqrt{5}(F(6) + F(4) + F(2)) = \sqrt{5}(8 + 3 + 1) = 12\sqrt{5}$.

20. **Answer:** 144

By the n th day $\frac{n^2+n}{2}$ markings have been made. The wall has volume $6 * 8 * .5 * 12^3 \text{ in}^3$, the tunnel has volume 288 in^3 , and the each mark removes volume 4 in^3 . Thus, $6 * 12^3 + 6 * 12$ marks are required. Hence, $n^2 + n = 12^4 + 12^2$, so $n = 12^2$.