1. **Answer: 31**

   The angle sum of a polygon with n sides is $180(n-2)$. The total sum is then $180(n_1-2)+180(n_2-2)+\cdots+180(n_7-2) = 180(n_1+n_2+\cdots n_7) - 7\cdot 2\cdot 180 = 180\cdot 17$. Dividing through by 180 gives $n_1+n_2+\cdots+n_7-14 = 17$, so the total number of sides $14+17=31$.

2. **Answer: $\frac{\sqrt{3}}{5}$**

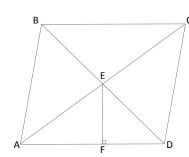
   Rank the shaded triangles by their area, largest to smallest. The largest shaded triangle has an area of $\frac{\sqrt{3}}{16}$. There are three of them, call them the first set. The i^{th} set of triangles has base $\frac{1}{4}$ that of the $(i-1)^{th}$ set, so the area of the i^{th} set is $\frac{1}{16}$ that of the of the $(i-1)^{th}$ set. So the total shaded area becomes an infinite geometric series:

$$\frac{\sqrt{3}}{16}\cdot 3\cdot\left(\frac{1}{1-\frac{1}{16}}\right) = \frac{\sqrt{3}}{5}.$$

3. **Answer: $2\cos(36)$**

   Consider the triangle formed by the diagonal and two sides of the pentagon. The interior angle of the pentagon is $108°$, so the other two angles are both $36°$. By the law of sines, $\frac{\sin(36)}{a} = \frac{\sin(108)}{d}$. Thus, $\frac{d}{a} = \frac{\sin(108)}{\sin(36)} = \frac{\sin(72)}{\sin(36)} = \frac{\sin((2)(36))}{\sin(36)} = 2\cos(36)$.

4. **Answer: 20**



   By the Pythogorean theorem, $ED = \sqrt{5}$. Since $ABCD$ is a rhombus, $AE \perp ED$. So triangle $\triangle FDE \sim \triangle EDA$. Thus we obtain the following ratio:

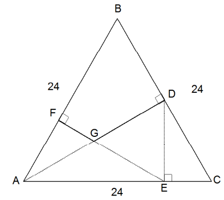
$$\frac{DF}{ED} = \frac{EF}{AE}$$
$$\frac{1}{\sqrt{5}} = \frac{2}{AE}.$$

   So $AE = 2\sqrt{5}$. Thus, the area is $\frac{1}{2}(2\times AE)(2\times DE) = \frac{1}{2}(4\sqrt{5})(2\sqrt{5}) = 20$.

5. **Answer: $265 - 132\sqrt{3}$**

   Denote r_1 the inner radius and r_2 the outer radius. Then the inner lane has distance $2\pi r_1$ and outer lane $2\pi r_2$. But since Sammy will only be racing for $2\pi r_1$, there is $2\pi(r_2-r_1)$ distance along the outer lane which he will skip. Let W denote Willy's starting position, S Sammy's starting position, and O the origin of the circular race track. Let θ be the angle between WO and SO. Then $2\pi(r_2-r_1) = \frac{\theta}{360}(2\pi r_2)$. Plugging in $r_1 = 11$ and $r_2 = 12$ and solving for θ, we get $\theta = 30°$. Using coordinate geometry, $W = (11,0)$ and $S = (12\cos 30, 12\sin 30) = (6\sqrt{3}, 6)$. Thus, $(WS)^2 = (11-6\sqrt{3})^2 + 36 = 265 - 132\sqrt{3}$.

6. **Answer: $\frac{117\sqrt{3}}{2}$**



AD bisects BC since $\triangle ABC$ is equilateral, so $CD = 12$. $\triangle ACD$ is a 30-60-90 degree triangle, so $AD = 12\sqrt{3}$. Likewise, $\triangle DCE$ is also 30-60-90, so $EC = 6$ and $ED = 6\sqrt{3}$. So $AE = AC - EC = 24 - 6 = 18$. $\triangle EAF$ is also 30-60-90, so $AF = 9$ and $EF = 9\sqrt{3}$. Since $\angle AEG = \angle AEF = 30°$, $\angle GED = 60°$. Likewise, $\angle CDE = 30°$ implies $EDG = 60°$. So $\angle DGE$ must also be $60°$ and $\triangle GED$ is equilateral, so $EG = GD = ED = 6\sqrt{3}$. $FG = EF - EG = 9\sqrt{3} - 6\sqrt{3} = 3\sqrt{3}$. $FB = AB - AF = 24 - 9 = 15$ and $BD = BC - CD = 12$. So area of quadrilateral $BFGD$ is $area\triangle BFG + area\triangle BDG = \frac{1}{2}(3\sqrt{3})(15) + \frac{1}{2}(6\sqrt{3})(12) = \frac{117\sqrt{3}}{2}$.

7. **Answer:** $\mathbf{\sqrt{3} - \frac{3}{2}}$

   Let r be the radius of the three largest circles and s be the radius of the smallest circles. Consider the equilateral triangle $\triangle ABC$ formed by the centers of the three largest circles. This triangle has side length $2r$ and altitude $r\sqrt{3}$. Let O be the center of the smallest circle, and consider altitude AM, passing through O. $AO = r + s = \frac{2}{3}AM$, so the altitude is also $\frac{3}{2}(r + s)$. Equating these and solving for the ratio gives $\frac{s}{r} = \frac{2\sqrt{3}}{3} - 1$.

8. **Answer:** $\mathbf{\frac{3}{4}}$

   Suppose the first two points are separated by an angle α. Note that α is therefore randomly chosen between 0 and π. If we are to place the third point so that the three lie on the same semicircle, we have an arc of measure $2\pi - \alpha$ to choose from. The probability of this placement is therefore $1 - \frac{\alpha}{2\pi}$. This varies evenly from 1 at $\alpha = 0$ to $\frac{1}{2}$ at $\alpha = \pi$. The average is therefore $\frac{3}{4}$.

9. **Answer:** $\mathbf{\frac{10\pi - 12\sqrt{3}}{9}}$**, or** $\mathbf{\frac{10\pi}{9} - \frac{4\sqrt{3}}{3}}$

   $\angle XAY = 120°$, so the radius of circle A is $\frac{2\sqrt{3}}{3}$. $\angle XBY$ in $60°$, so the radius of circle B is 2. The area of the sector AXY is $\frac{1}{3}$ the area of circle A, so the area formed between segment XY and arc XY in circle A is the area of sector AXY inus the area of $\triangle AXY$.

$$\frac{1}{3}\pi\left(\frac{2\sqrt{3}}{3}\right)^2 - \frac{1}{2} \cdot 2 \cdot \frac{\sqrt{3}}{3} = \frac{4\pi - 3\sqrt{3}}{9}.$$

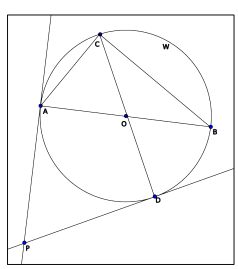
   Similarly, sector BXY is $\frac{1}{6}$ of the area of circle B, so the area formed between segment XY and arc XY in circle B is

$$\frac{1}{6}\pi(2)^2 - (2)^2\frac{\sqrt{3}}{4} = \frac{2\pi - 3\sqrt{3}}{3}.$$

   The total area shared by the two circles is then:

$$\frac{4\pi - 3\sqrt{3}}{9} + \frac{2\pi - 3\sqrt{3}}{3} = \frac{10\pi - 12\sqrt{3}}{9}$$

10. **Answer:** **50°**



   Note that $\angle ACB = 90°$, so AB must be the diameter of W. Then CO is the median from C to AB, where O is the origin of W, and CD passes through O. Then $CO = BO$ and $\angle BCD = \angle CBA = 25°$. We calculate $\angle COB = 180° - 2 \times 25° = 130°$. Then $\angle AOD = 130°$. Consider the quadrilateral $PDOA$. $\angle P = 360° - \angle BAD - \angle CDP - \angle AOD = 360° - 90° - 90° - 130° = 50°$.