

1. **Answer: 1681**

There are 41 words in the problem statement. Since 41 is itself a prime, the answer is  $41^2 = 1681$ .

2. **Answer:  $\frac{100}{x-1}\%$** 

After yesterday, the fraction of the initial gold remaining is  $1 - \frac{1}{x} = \frac{x-1}{x}$ . Therefore, in order to reach the original amount of gold, we must multiply by  $\frac{x}{x-1} = 1 + \frac{1}{x-1}$ . Thus, the gold must be increased by  $\frac{100}{x-1}$  percent.

3. **Answer: (5, 0), (4, 1), (1, -2), and (2, -3)**

We factor the expression as follows:

$$\begin{aligned} ab + a - 3b - 3 &= 5 - 3 \\ (a - 3)(b + 1) &= 2 \end{aligned}$$

We can use a table to find appropriate values for  $a$  and  $b$ .

Thus, (5, 0), (4, 1), (1, -2), and (2, -3) are the desired solutions.

$a - 3$	$b + 1$	$a$	$b$
2	1	5	0
-2	-1	1	-2
1	2	4	1
-1	-2	2	3

4. **Answer:  $x = 1$** 

$$\begin{aligned} f(x) + xf\left(\frac{1}{x}\right) &= x \\ f\left(\frac{1}{x}\right) + \frac{1}{x}f(x) &= \frac{1}{x} \\ f\left(\frac{1}{x}\right) &= \frac{1}{x} - \frac{1}{x}f(x) \\ f(x) + x\left(\frac{1}{x} - \frac{1}{x}f(x)\right) &= x \\ x &= 1 \end{aligned}$$

5. **Answer:  $-\frac{5}{4}$** 

We complete the square:

$$\begin{aligned} 2x^2 + 2xy + 4y + 5y^2 - x &= (x^2 + 2xy + y^2) + (x^2 - x + \frac{1}{4}) + (4y^2 + 4y + 1) - (\frac{1}{4} + 1) \\ &= (x + y)^2 + (x - \frac{1}{2})^2 + (2y + 1)^2 - \frac{5}{4} \end{aligned}$$

Notice that  $x = \frac{1}{2}$  and  $y = -\frac{1}{2}$  would yield the minimum, which is  $-\frac{5}{4}$ .

6. **Answer: 506**

Let  $P_n$  be the value of the dollar in gold after the  $n^{\text{th}}$  bailout. Let  $s = \frac{1}{2}$ . Then after the  $n^{\text{th}}$  bailout, the dollar is a factor of  $(1 + s^{2^{n-1}})$  of its  $(n-1)^{\text{th}}$  value. Thus,

$$\begin{aligned} P_4 &= \frac{1}{980}(1+s)(1+s^2)(1+s^4)(1+s^8) \\ &= \frac{1}{980}(1+s+s^2+s^3)(1+s^4)(1+s^8) \\ &= \frac{1}{980}(1+s+s^2+s^3+s^4+s^5+s^6+s^7)(1+s^8) \\ &= \frac{1}{980}(1+s+s^2+s^3+s^4+s^5+s^6+s^7+s^8+s^9+s^{10}+s^{11}+s^{12}+s^{13}+s^{14}+s^{15}) \\ &= \frac{1}{980} \left( \frac{1-s^{16}}{1-s} \right). \end{aligned}$$

Plug in  $s = \frac{1}{2}$ , and we find that  $P_4 = \frac{1}{490} \left( 1 - \frac{1}{2^{16}} \right)$ . So  $b + c = 490 + 16 = 506$ .

7. **Answer: 32670**

Largest multiple of 60 below 2009 is 1980, so find the sum for  $k = 1$  to 1979, so that we have each value of  $\lfloor k/60 \rfloor$  exactly 60 times. This sum is therefore  $60(1 + 2 + \dots + 32) = 60(1 + 32) \frac{32}{2} = 31680$ . The remaining terms are all 33, and there are  $2009 - 1980 + 1 = 30$  of them, giving an answer of  $31680 + 30 \times 33 = 32670$ .

8. **Answer:  $\bar{1}0\bar{1}1$**

$$\begin{aligned} (1\bar{1}00)(\bar{1}1) &= \bar{1}1000 + 1\bar{1}00 = \bar{1}1\bar{1}00 \\ \bar{1}\bar{1}00 + 1\bar{1}1 &= \bar{1}0\bar{1}1 \end{aligned}$$

9. **Answer: -58**

Let the roots be  $r$ ,  $s$ , and  $t$ . Then they satisfy  $r + s + t = -a$ ,  $rs + st + rt = b$ , and  $rst = -c$ . So we have  $-(a + b + c + 1) = r + s + t - rs - rt - st + rst - 1 = (r-1)(s-1)(t-1) = 2009 = 7 * 7 * 41$ .

Thus the roots are 8, 8, and 42, and  $a = -(r + s + t) = -58$ .

10. **Answer: 20**

For convenience, set  $x = \sum_{n=1}^{\infty} \frac{\delta(n)}{n^2}$  and  $y = \sum_{n=0}^{\infty} \frac{(-1)^{n-1} \delta(n)}{n^2}$ .

The crucial observation is that  $\frac{1}{2}(x + y)$  and  $\frac{1}{2}(x - y)$  give the same summation as  $x$ , restricted to the terms with odd  $n$  and even  $n$  respectively. The latter summation is easily related to  $x$  using the fact that  $\delta(2n) = \delta(n)$  (since multiplying by 2 is simply appending a 0 in the binary expansion), as follows.

$$\begin{aligned} \frac{1}{2}(x - y) &= \sum_{\text{even } n \geq 2} \frac{\delta(n)}{n^2} \\ &= \sum_{n=1}^{\infty} \frac{\delta(2n)}{(2n)^2} \\ &= \sum_{n=1}^{\infty} \frac{\delta(n)}{4n^2} \\ &= \frac{1}{4}x. \end{aligned}$$

Thus we have  $\frac{1}{2}(x - y) = \frac{1}{4}x$ . It follows that  $x = 2y$ , so  $x/y = 2$ . Thus, the desired answer is 20.