An Economic Explication of the Prohibition of *Gharar* in Classical Islamic Jurisprudence

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Abstract

The forbidden *bay'u al-gharar* can best be translated as "trading in risk". In the face of risk, any trade would involve some degree of trading in risk, and thus jurists disagree over whether a specific contract is forbidden or not based on their varying assessments of whether the amount of risk is substantial or small. Moreover, the prohibition is often overruled in cases where clear economic benefit can only be served by a contract which includes substantial trading in risk.

We show that "trading in risk" is generally inefficient relative to other forms of risk sharing. Hence if a contract can attain its economic aim of increasing economic efficiency through either form of risk transfer, the prohibition of trading in risk should be applicable. Cases where such a prohibition is moot because the risk trading instrument is not used do not affect this general conclusion. In cases where trading in risk is integral to the contract, but where the contract is important to meet economic needs (e.g. *salam* and *'istiṣnā'*), the analysis is still useful in two regards: (i) we can consider whether or not there is a risk sharing mechanism which can reduce part of the inherent trading in risk (e.g. financial vs. mutual insurance), and (ii) we should consider such alternatives if secondary tools for managing the resulting risk are sought.

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1 Introduction

Gharar was forbidden in commutative financial contracts in a number of *Ḥadīths*. Perhaps the strongest textual prohibition of *gharar* is the one narrated by Muslim (#2783; Sakhr (1995)), 'Aḥmad (#7104, ibid.), 'Abū Dāwūd (#2932, ibid.), Al-Tirmidhī (#1151, ibid.), Al-Nasā'ī (#4442, ibid.), Al-Darāmī (#2441, ibid.) and 'Ibn Mājah (#2185, ibid.) on the authority of 'Abū Hurayra (translation of the version in Muslim) "that the Prophet (pbuh) prohibited the pebble sale and the *gharar* sale" (*nahā (ṣA'ws) 'an bay'i al-ḥaṣāti wa bay'i al-gharar*). There are a number of other *Ḥādīths* which forbid *gharar*, either by name, or by specifying one or more of its instances (e.g. selling "the birds in the sky or the fish in the water", "the catch of the diver", "unborn calf in its mother's womb", "the sperm and/or unfertilized eggs of camels", etc.). Jurists have sought many complete definitions of the term, some of which we shall discuss in detail. However, true to their juristic training, the nature of *gharar* and the criteria and reasons for its prohibition can best be inferred from the many examples provided by jurists.

In a recent paper, Al-Suwailem (1999) attempted to formalize the concept of *gharar* as the degree of zero-sumness of an exchange game. The rhetoric used by jurists does indeed suggest that the potential for poorly anticipated losses, and potential dispute, is the primary reason for the prohibition of *gharar* sales. However, as we shall see later in this section, such a definition falls significantly short of explaining the economic content of the prohibition of *gharar* in classical jurisprudence. Indeed, we shall see that there are many examples of pure zero-sum games which are not forbidden based on *gharar*, and other contracts which are forbidden because of *gharar*, but which are not near-zero-sum. In this regard, the jurists' rhetoric tying *gharar* to losses is similar to the rhetoric of some jurists who asserted that the prohibition of *ribā* was based on the exploitative nature of the latter. This was the opinion of 'Ibn Kaysān, that "the reason (*al-maqṣūd*) for the prohibition of *Ribā* is kindness towards people" (i.e. by not charging an increase). This opinion was reported and debunked in Al-Nawawī (n.d., vol.9: *"far' fī madhāhib al-'ulamā' fī bayān 'illat al-ribā fī al-'ajnās al-'arba'ah"*), since its logic would extend incorrectly to profit-making and the explicit permission of trading different genuses in different quantities, as well as trading non-fungibles (e.g. camels) in different quantities. Following along similar lines, we can reject the argument that the prohibition of *gharar* is intended to achieve the objective of eliminating zero-sumness from exchange is insufficient, since *gharar* is forbidden in some positive-sum contracts, and some zero-sum contracts are not forbidden based on *gharar*.

In what follows, we shall show that the term *gharar* can best be translated as "risk", and thus, the prohibition of *bay'u al-gharar* will best be translated as "trading in risk". Indeed, the presence of risk (i.e. randomness, and thus the possibility of loss relative to some benchmark) is essential for the contract to be forbidden based on *gharar*, as we shall see in the jurists' definitions listed below. However, I shall argue in the remainder of this introductory section, based on the arguments of classical jurists, that the injunction against "trading in risk" can be explained based on more subtle considerations of economic inefficiency arising from the mispricing of risk.

The remainder of this section will attempt to clarify the notion of *gharar*, and justify the translation of *gharar* as "risk" and the forbidden *bay'u al-gharar* as "trading in risk". The importance in classical jurisprudence of highlighting the type of *gharar* which leads to legal dispute will be used further to support the arguments given in later sections. Thus, trading in risk will be shown to be inherently inefficient. However, as we will see in Section 2, trading in certain amounts of risk can be allowed as an exception if the risk is minor (*gharar yasīr*), or if the economic need for the contract embodying the risk is substantial (e.g. in the *salam* contract). This explains the differences in opinion among classical as well as contemporary jurists regarding which types of *gharar* are considered minor and which are considered major.

1.1 What is the forbidden *bay'u al-gharar*?

The cost-benefit analysis involved in considering the prohibition of *bay'u al-gharar* can best be understood by quoting from some classical sources of jurisprudence. The first quote we present sets the stage by formulating "the sale of *gharar*" as a sale in which *gharar* was the dominant force (*ghalaba 'alayhi*) Al-Bājī Al-'Andalusī (n.d., vol.5, *ḥāshiyah*, under *bay'u al-gharar*) states:

> His (pbuh) prohibition of *bay'u al-gharar* render such a sale defective. The meaning of *bay'u al-gharar*, and Allāh knows best, refers to sales in which *gharar* was the major component (*ghalaba 'alayhi*) until the sale is justifiably described as *bay'u al-gharar*. This is the type of sale which is unanimously forbidden. On the other hand, minor (*yasīr*) *gharar* does not render a sales contract defective, since no contract can be entirely free of *gharar*. Thus, the [legal] scholars differ in determining which contracts are defective due to differences in opinion regarding the extent of *gharar* inherent in each: sic. whether it is substantial and invalidates the contract, or minor and retains the contract's validity".

Taqiyyudīn Al-Subkī explicitly summarizes the opinions of earlier jurists thus (**?**, vol. 9)) says:

> The scholars said that the criterion for invalidity of the contract based on *gharar* or its validity despite the existence of *gharar* is thus: if necessity dictates committing *gharar* which cannot be avoided without incurring an excessive cost, or if the *gharar* is trivial (*ḥaqīr*), the sale is rendered valid, otherwise it is rendered invalid.... Thus, the differences among scholars is based on this general principle, where some of them render a particular form of *gharar* minor (*yasīr*) and inconsequential, while others render the same form consequential, and Allāh knows best.

Perhaps the ultimate cost-benefit analysis with regards to permission or prohibition of a contract which embodies *gharar* is stated in the following passage from 'Ibn Taymiya (1998, vol. 4):

> In this regard, the corrupting factor in *gharar* is the fact that it leads to (*kawnuhu maṭiyyat*) dispute, hatred, and devouring others' wealth wrongfully. However, it is known that this corrupting factor would be overruled if it is opposed by a greater benefit (*al-maṣlaḥah al-rājiḥah*).

Thus, the very prohibition of *gharar* is understood by jurists to be based on a cost-benefit analysis. This view continues to be echoed in contemporary writings. Thus, Al-Ḍarīr (1997, pp.44-51) lists four necessary conditions for *gharar* to invalidate a contract:

1. It must be major. (We have already seen that the determination of whether *gharar* is considered major or minor is relativistic).

2. The potentially affected contract must be a commutative financial contract. (This is a point to which we shall turn shortly: namely, that *gharar* – according to the majority of jurists[1] – does not affect gifts and other non-commutative contracts).

3. The *gharar* must affect the principal components of the contract (e.g. the price and object of sale, language of the contract, etc.). Thus, the sale of a pregnant cow is valid, even though the status of the fetus calf may not be known.

4. That there is no need met by the contract containing *gharar* which cannot be met otherwise.

1.2 How to translate "*bay'u al-gharar*"

While the translation of any term is not as important as the full understanding of its meaning, it is still very important to find appropriate translations for major juristic, if that is possible. For instance, El-Gamal (2000) has shown that the common translation of *ribā* as "interest" was defective since there is "*ribā* without interest" (e.g. in *ribā al-faḍl* which involves increase without deferment and *ribā al-nasā'* which involves deferment without increase), as well as "interest without *ribā*" (e.g. implicit rate of return in cost-plus sales with deferred payments). The other common translation of *ribā* as "usury" was also shown to be defective, since it elicits the image of exorbitant and/or compounded interest.

In the case of *gharar*, however, I would like to argue that a good translation already exists in the term "risk". The term risk (Italian: *risco*, French: *risque*) is derived from the latin roots *re* = back and *secare* = cut, thus reflecting the potential for a sailor to have his ship cut by hitting a rock. In other words, "risk" means "danger of loss". This is precisely the meaning of the Arabic term

[1] Al-Qarāfī (n.d., vol.1) reports that Al-Shāfi'i extended the prohibition to gifts, charity, etc.

gharar. The literal meaning of the term *gharar* according to Qāḍī ʻIyāḍ (c.f. Al-Qarāfī (n.d., vol.3, p.266)) is: "that which has a pleasant appearance and a hated essence", c.f. . The origin of the term is the three-letter past tense verb *gharra*, meaning "to deceive". Thus, the Encyclopedia of Jurisprudence (vol.21, CDROM version, Cairo: Harf, 1998) states that *tadlīs* = cheating (in trade) and *ghabn* = fraud and deception are among the categories of *gharar*. The type of uncertainty regarding future events which constitutes *gharar* may be one-sided or two-sided, and it may be intentional or unintentional. However, in all of the definitions that follow, one thing is common: the incorporation of risk.

Juristic definitions of *gharar* are varied, and may often seem contradictory. Thus, Al-Qarāfī (n.d., vol.3, p.265) states: "the origin of *gharar* is that which is not known to occur or not (e.g. birds in the sky or fish in the water). On the other hand, that whose existence is known, but whose characteristics are unknown (e.g. when a seller sells that which is hidden in his sleeve), it is called *majhūl* (unknown). Thus, the definitions of *gharar* and ignorance are each more general in some respects and less general in other respects... This is the reason for the [legal] scholars' differences over the natures of *gharar* and *jahālah* (ignorance)". We may contrast this definition with those collected in Al-Zuḥaylī (1997, vol.5, pp.2408-3411):

1. Al-Sarakhsī of the Ḥanafī school defined *gharar* thus: "*gharar* is that whose consequences are hidden".

2. Al-Shīraāzī of the Shāfiʻī school said: "*gharar* is that whose nature and consequences are hidden".

3. Al-'Isnawī of the Shāfiʻīs school said: "*gharar* is that which admits two possibilities, with the less desireable one being more likely."

4. 'Ibn Taymiya of the Ḥanbalī school said: "*gharar* is that whose consequences are unknown". His student 'Ibn Al-Qayyim said: "it is that which is undeliverable, whether it exists or not."

5. 'Ibn Ḥazm of the Ẓāhirī school said: "*gharar* is where the buyer does not know what he bought, or the seller does not know what he sold."

6. Dr. Al-Zuhayly's summary definition is thus: "*gharar* sale is any contract which incorporates a risk which affects one or more of the parties, and may result in loss of property."

7. A more explicit definition by Professor Muṣṭafā Al-Zarqā' is the following: "*gharar* is the sale of probable items whose existence or characteristics are not certain, due to the risky nature which makes the trade similar to gambling."

That final definition by Professor Al-Zarqā' seems to be the most appropriate one to use. It subsumes all the other definitions as special cases, and makes explicit the fact that the payoffs from the *gharar* exchange are rendered risky by the probable nature of some of its cornerstones (in terms of existence or characteristics). The analogy to gambling is quite appropriate, for an extreme "*bay'u al-gharar*" interpreted as "sale of risk" is indeed the direct case of gambling where a known price is paid for a probability distribution of possible payoffs. In this case, there is no economic content to the transaction, and it is purely a sale of risk. Therefore, as jurists consider more interesting contracts (e.g. *bay'u al-salam*, where the object of sale does not exist at the time of the contract), the economic efficiency gains from the contract is implicitly compared to the efficiency losses caused by the "trading in risk".

1.3 *Gharar* vs. "zero-sumness"

Consider the following hypothetical situation: Two traders leave on a ship from Madras, India; headed to Jeddah, Saudi Arabia. One trader is carrying a certain measure of spices, while another is carrying a certain amount of silk. They both wish to trade their respective merchandise in Jeddah, and use the proceeds to purchase dates, which they will bring back to Yemen. Aboard the ship, they both get ample opportunity to examine each others' merchandise. While aboard the ship, each trader thinks that the market price of the other's merchandise in Jeddah will be higher than the price of his own merchandise. They begin to bargain, and both agree with full consent to exchange their respective merchandise. The contract is concluded with their full consent, and they exchange the traded goods. This is a valid and final sales contract.

Notice, however, that neither party had any use for the merchandise itself. The trade was purely predicated on their respective perceptions of the market price for their respective merchandise in a different market. Once they arrive to Jeddah, they will each attempt to sell the merchandise they now have and use the proceeds to purchase dates. It is obvious that the exchange resulted in a zero-sum payoff structure. Compared to the amounts of dates they would each acquire had they not engaged in that barter aboard the boat, one trader in fact "wins" a certain measure of dates, while the other "loses" the exact same measure. They each bore an initial risk associated with the future market price of their merchandise in Jeddah at the date of their arrival, and agreed to exchange their respective risks. The net payoffs from the trade in such implied risks (embodied in the physical merchandise) were indeed zero-sum. However, there was no *gharar* involved in this transaction.

There are many economic reasons one can give for why this trade may still be advantageous. In fact, the traders' differing estimates of the future market prices in Jeddah may have been a function of their private information of market conditions. In this case, trading aboard the boat helps aggregate such private information in an efficiency enhancing manner. Alternatively, the differing estimates may have been a function of the respective traders' comparative advantages in selling certain types of merchandise. In this case, the trade would take place because each trader is better at finding a higher-price buyer for the other's merchandise. The net result would be allocation of the merchandise to those willing to pay higher prices, and thus who have higher valuations of the merchandise. This too is efficiency enhancing.

One can continue to think-up multiple scenarios where the ex-post seemingly zero-sum game may in fact be efficiency enhancing. However, more importantly, if we take as given the permission "and Allāh had permitted trade" as efficiency enhancing, the question is why certain trades (specifically *ribā* and *gharar*) are forbidden. In a standard trade like the one we considered in the current example, there is clearly a basic economic activity dominating the implicit trading in risk. In this case, the risk component does not dominate the transaction, and the sale is not characterized as "trading in risk". To make the case painfully clear, we can compare this situation with a pure gamble: One person pays another $1 to flip a coin. The first person loses his $1 if the coin comes up tails and regains his $1 plus a second $1 if it comes up heads. In this case, there is no economic activity involved, and hence no room for efficiency gains from trade. Moreover, we shall show below that in such situations (as well as in trades where the "trading in risk" dominates the exchange), the traded risk cannot be efficiently priced due to boundedly rational behavior. In this respect, it is well known that in societies where individuals are allowed to gamble, we observe individuals who gamble, and simultaneously purchase insurance coverage.[2] Such clearly inefficient behavior has been explained in behavioral economics and psychology by the boundedly rational behavior of humans in the face of risk. We shall turn to the relevant assumptions and conclusions in Sections 3 and 4.

The argument above shows that certain zero-sum exchanges are not forbidden on the basis of *gharar*. On the other hand, certain types of contracts are indeed forbidden on the basis of *gharar* without having a zero-sum component. For instance, one of the ten classical forms of sales forbidden due to *gharar* is "two sales in one contract" (*bay'atayni fī bay'ah*), according to 'Ibn Juzayy (1998, p.192). This prohibition is based on the *Ḥadīth* narrated in *Al-Muwaṭṭa'*, Al-Tirmidhī (who rendered it a *Ḥadīth Ḥasan Ṣaḥīḥ*, Al-Nasā'ī, and 'Abu-Dāwūd on the authority of 'Abu Hurayra that the Prophet (pbuh) forbade two sales in one sale, c.f. Al-Jazarī (n.d., vol.1, p.446). This includes the situation where a seller offers two prices (one cash-and-carry and one deferred; or two "cash-and-carry prices", one price stated in barley and the other in wheat, etc.) for the same item. The buyer is offered the choice of either price, and whichever one he chooses is binding on the seller. In this case, it is clearly possible that both prices are selected such that both the buyer and the seller are better-off (i.e. the buyer would conduct the sale at either price, but selects the one better for him). Other examples of *gharar* sales are explicitly forbidden in *Ḥadīth*, e.g. pebble-sales, the sale of camel-sperm and unfertilized she-camel-eggs, the unborn calf in its mother's womb, etc. In all such cases, it is easy to see that it is not zero-sumness which is being forbidden, but rather "excessive risk" attached to the object of sale. In the example of "two sales in one", the ambiguity of the offer (due to the imbedded option to the buyer) leads to uncertainty regarding the price in the contract. Some Ḥanafī and Mālikī ju-

[2] A common contemporary example can be seen in the recent trend for individuals to acquire a second mortgage on their home, use the money for day-trading, and purchase private mortgage insurance!

rists have taken the opinion that by accepting one of the two offers, the buyer removes such uncertainty and the sale is rendered valid. However, many S̲h̲āfi'īs and the Ḥanbalīs ruled that this contract is invalid, c.f. Al-S̲h̲irbīnī (1994, vol.2, p.31), 'Ibn Qudāmah (n.d., vol.4, p.234). Thus, the "trading in risk" which is forbidden in the *Ḥadīt̲h̲* may refer to riskiness implicit in the contract language, uncertainty regarding the parties of the contract, or riskiness associated with the object of sale and price. We shall discuss those three categories of sources of *g̲harar* below, showing that they result in market inefficiencies which require an overriding increase in efficiency to render the contract permissible.

2 The injunction against "trading in risk"

In this section we address the injunction against trading in risk (used henceforth as our translation of *bay'u al-g̲harar*) and its economic efficiency. In this regards, we begin with the following statement regarding the legal reason for the prohibition, c.f. Al-K̲h̲arsh̲ī ((n.d., vol.5, under "forbidden sales"):

> ... and know that there have been differences [among legal scholars] with regards to the legal reason (*'illah*) [for the prohibition of] *g̲harar*. Thus, some said that [the prohibition is due to] the unjustified devouring of people's property, others said that it is due to the potential for disputation, and a third group said that it is based on the inability to deliver the promised goods.

Thus, we find that the "riskiness" by itself is not sufficient to justify the invalidating injunction against such trades. In fact, it is the potential for losses based on this riskiness, and the resulting legal disputes which give cause to issuing such an injunction.

In what follows, we shall argue on economic-legal grounds that it is economically efficient to issue an injunction against trading in risk, with allowance for exceptions where elimination of a particular contract (e.g. *salam*) would cause a more severe efficiency loss. As we have seen in the previous section, this is the state of affairs in the prohibition of *bay'u al-g̲harar* in Islamic jurisprudence. While the efficiency analysis we perform here is traditional in the Chicago-type school of law and economics, it may be seen by some as alien to Islamic jurisprudence. However, this very concept is at the heart of the literature on the Objectives of Islamic Law (*maqāṣid al-s̲h̲arī'ah*). Indeed, in what is perhaps the best known text on the subject, Al-S̲h̲āṭbī (1997, p.256) explicitly analyses the issuance of injunctions in cost-benefit terms:

> "Of course, the Law may contain injunctions which result in hardship for those who have to abide by it. However, the hardship itself is never the intention of the LawGiver. Indeed, the intention of the LawGiver must be to bring a [larger] benefit or prevent a [larger] loss. For instance, [harsh] punishments for various transgressions are intended to give significant disincentives for the transgressors

> and others from ever repeating that transgression... In this regard, one would never accuse a [good] physician of giving a bitter medicine [to heal] or cutting a limb [to preserve the patient's life and the health of the rest of his body] simply to cause hardship. Indeed, [this is an apt analogy since] Allāh is the Greatest Physician."

Of course, this is not surprising since the Qur'anic prohibition of gambling (the extreme form of trading in risk) is itself stated in terms of a cost-benefit analysis:

> "They ask you [O Muhammad] about wine and gambling. Say there is great sin (harm, *'itẖm*) in them, and some benefit to mankind. However, their sin (harm, *'itẖm*) outweighs their benefits." [2:219]

In this regard, we notice that Islamic Law and Jurisprudence does not always resort to injunctions to alleviate the possibility of "wrongful devouring of people's wealth". In this regard, many options are given to buyers to return defective merchandise (*ḵhiyār al-'ayb*), or simply to inspect the merchandise (*ḵhiyār al-ru'yah*) if its presence at the contract session is not possible or if the inspection requires a prolonged period of time. In such cases, the problem is solved through compensating both parties by returning the merchandise and the price to their original owners. Discussions of jurists are instructive in the case where the object of a sale were to perish partially or totally. After the conclusion of the sale, but prior to delivery, the buyer and seller are given different options and responsibilities for compensation depending on whether the perishing was caused by an act of nature, one of the trading parties, or a third party. Books of jurisprudence are full of similar examples where injunctions are not issued against sales in which the object may cease to be deliverable (or returnable). In such cases, the various schools of jurisprudence use their respective methods of reasoning from Legal Texts to arrive at appropriate compensation schemes.

It would be natural to study in the case of *bay'u al-g̱harar* the economic wisdom behind the Legislator's usage of an injunction against it (in the *Ḥadītẖ* cited in the introduction) instead of designing a compensation scheme. For instance, it would in principle be possible to sell the fish in the sea, and stipulate lengthy conditions of how the seller would compensate the buyer if the fish proves difficult or impossible to catch. Similarly, one could have sold a calf in its mother's womb with the Law stating that if the calf is still-born or born with a serious birth defect, then the seller would have to replace it with another type of calf of similar age. It would be useful to specify the economic grounds upon which one can distinguish between the cases where injunctions are issued to avoid "devouring people's wealth unjustly", and the cases where the contracts are permitted under the general "and Allāh has permitted trade and forbidden *ribā*" [2:257], with compensations determined ex post.

In this regard, we return to Al-Ḵharsẖī's statement about the potential for dispute. Indeed, the jurists often refer to ignorance and risk which lead to disputes (*al-mu'addiyah lil-nizā'*) in the context of invalidating various contracts. Clearly, there is still potential for legal dispute in a variety of contract where no such injunction was issued (e.g. a hidden defect, the object perishing or becoming defective, etc.). In such cases, legal dispute would be resolved by a judge or jurist based on the compensation scheme developed within their school of jurisprudence. However, if the contract is not particularly important (e.g. if another contract, which does not contain the ignorance or risk can meet the same economic need), the calculation of the proper compensation is difficult (the ignorance or risk is sufficiently significant), then it would seem best to facilitate the court's work by simply invalidating the contract. Moreover, even if the associated economic inefficiency and legal costs are small, an alternative contract form which attains the economic objectives of the defective one may be available. In such cases, it is best to avoid unnecessary trading in risk. In the model constructed below, we shall show in a highly stylized fashion how trading in risk (insurance) can often be replaced by risk sharing which results in a more efficient outcome.

3 Efficiency losses induced by trading in risk

In this section, we shall discuss the types of risk and uncertainty which may be implicitly traded in a contract. Those types of risk and uncertainty may be classified into three categories according to the source of risk (assuming that the parties of the contract are known):

1. Ambiguity in the contract language may lead to uncertainty regarding the nature of the object of sale or price. An example of this type of forbidden *gharar* is the offer: "I sell you the item hidden in my sleeve", c.f. 'Ibn Juzayy (1998, p.192). Another example (ibid.) is an unspecified or uncertain term of deferment (e.g. "with a price of $x deferred until so-and-so returns"). Other forms of ambiguity in the contract language (e.g. in the "pebble sale", ibid.) may be caused by offer and acceptance being inferred from physical events (the falling or throwing of a pebble) rather than clear legal language of offer and acceptance.

2. The object of sale may be known, but its delivery may be doubtful (e.g. the sale of a run-away animal, fish in the sea, birds in the sky, etc.).

3. The object of sale may itself contain risk or uncertainty. The extreme cases of such uncertainty may pertain to items which may not exist or may cease to exist (e.g. the sale of an unborn calf in its mother's womb). However, the risk may be minor regarding the existence of the object of sale (or price), but its characteristics may be uncertain, hence the application of the prohibition of *gharar* sales to portable items which are very far away (ibid). In such cases, the cost of transportation may be small relative to the efficiency loss which would result from uncertainty, thus rendering the contract defective.

The second and third categories are closely related in the sense that they pertain to a probabilistic environment which affects significantly the characteristics of the traded items (price and object of sale) – and in the extreme, the very existence of such items. This the classical economic notion of "risk" embodied in the traded goods, and it is our main object of investigation in this paper. However, there is also the type of risk and uncertainty which arises from ambiguity in the contract language.

We begin by addressing the latter type, which is easier to cover. The first thing we notice is that riskiness which arises from the ambiguity in contract language is easy to remove without affecting the underlying economic activity. In this regard, while ambiguity (e.g. and unspecified term of deferment, or an unspecified object of sale, etc.) can lead to costly legal disputes due to divergences between the subjective beliefs of the individuals who engaged in this transaction, there is no clear economic benefit from including such ambiguity. Thus, the inefficiency of such contract language is obvious. Consequently, we notice that jurists who argue for overriding certain prohibitions of *gharar* sales due to economic need rarely – if ever – question those cases of extremely ambiguous language.[3]

More problematic are the cases where riskiness pertains to the object of sale. Perhaps the most obvious exception is the permissibility of *salam* sales (Islamic futures where the price is fully paid in advance[4] and delivery of the object of sale is deferred) and *'istiṣnā'* (commission to manufacture, with advance payment of part of the price). In such cases, the object of sale most often does not exist, and therefore, its future existence is only probable. However, the riskiness of the contract is justified by jurists due to the economic benefit (efficiency gains) which would be lost if those two contracts were not available. The seller in such contracts receives the price and uses them to acquire or produce the object of sale, which would not exist otherwise. When both parties engage in this contract, the price paid to the seller must be lower than the discounted marginal utility the buyer will derive from the object of sale, and larger than the discounted costs of its production. Therefore, such exchange is inherently efficiency enhancing to an extent that it judged to exceed the inherent efficiency losses due to the implicit trading in risk. A puzzle to which we must turn later is this: why is this contract permitted, while conventional modern forwards (where both the price payment and the goods delivery are deferred) is forbidden.[5]

3.1 Behavioral assumptions: Prospect Theory

Expected utility maximization was suggested as a model of decision making under uncertainty at least as early as Bernoulli (1738). Modern axiomatizations of

[3]Program sales, or *bay'u al-raqm*, is a special case where jurists disagree. However, in this case, they seem to disagree over whether or not selling by the label sufficiently describes the sold items. Hence, it is not an issue of overriding the prohibition of ambiguity over the object of sale, it is an issue of whether or not such ambiguity exists.

[4]Hence the name *salam* which literally means advance payment.

[5]While a class of researchers who work in the field labeled "Islamic financial engineering" attempt to synthesize such forward contracts, the classical prohibition of such sales – called *al-bay' al-muḍāf* – has not been overturned by contemporary jurists, who recognize the strict conditions associated with *salam* based on a valid *Ḥadīth*, c.f. M. Taqi Usmani (1998, pp.187-189).

this theory were provided by von Neumann and Morgenstern (1944) and Savage (1954). Expected utility theory postulates that a prospect $(x_1, p_1; \ldots; x_n, p_n)$, where outcome x_i would occur with probability p_i is valued by:

$$U(x_1, p_1; \ldots; x_n, p_n) = \sum_{i=1}^{n} p_i u(x_i).$$

The function $u(x)$ represents utility of outcome x, and a tradition starting with Bernoulli (1738) has been to assume that $u(.)$ is concave, simultaneously modeling diminishing marginal utility and risk-aversion.

Shortly after the axiomatization of expected utility theory, a large number of violations of this theory's predictions were found in empirical as well as experimental research. Kahmenan and Tversky (1979) summarize some of those violations, and attributed them to three characteristics of the expected utility theory:

1. Linearity in probabilities: $U(x_1, p_1; \ldots; x_n, p_n) = \sum_{i=1}^{n} p_i u(x_i)$.
2. "Asset integration": a subject using expected utility, and starting from a wealth level w will accept a prospect $(x_1, p_1; \ldots; x_n, p_n)$ if and only if $U(w + x_1, p_1; \ldots; w + x_n, p_n) > u(w)$.
3. Risk aversion: the utility function $u(.)$ is concave.

Kahmenan and Tversky (1979) enumerated violations of expected utility theory which were observed in previous experiments starting with Allais (1953), and reported new results which show similar violations. The types of violation they addressed were grouped into the following categories (Kahmenan and Tversky (1979, pp.265,268,269,271):

1. **The certainty effect:** Individuals are often observed overweighting outcomes considered certain relative to those which are probable.
2. **The reflection effect:** While individuals exhibit risk averse behavior over potential gains, they show risk seeking behavior over potential losses.
3. **Loss aversion:** The disutility of a loss is larger in absolute terms than the utility of a gain of equal magnitude.
4. **The isolation effect:** When faced with a pair of prospects, subjects seem to decompose them into their common and different components, and base their choice on the components that are different, thus "isolating" the common components as irrelevant for decision. Experiments in which a single problem is decomposed in different ways we thus observed to lead to different choice patterns.

The model of behavior they present to accommodate all of those violations of expected utility maximization postulates that individuals evaluate a prospect $(x_1, p_1; \ldots; x_n, p_n)$ by evaluating the objective function:

$$V(x_1, p_1; \ldots; x_n, p_n) = \sum_{i=1}^{n} \pi(p_i)\nu(x_i).$$

The x_i's are considered to be "prospects", i.e. deviations from a reference point or perceived status quo (assumption R0). The value function ν which leads to behavior consistent with the experimental evidence is shown to satisfy the two characteristics:

V1. The value function is steeper for losses than it is for gains:

$$\nu(x) < -\nu(-x).$$

V2'.

$$d\nu(x)/d(x)\big|_y < d\nu(x)/dx\big|_{-y}.$$

A typical value function satisfying the generalized prospect theory postulates which can explain expected utility violations, as well as discounting anomalies, is shown in the diagram on the left of the Figure 1. The functional form and parameters used in this figure are those estimated by Tversky and Kahneman (1992) and used in our numerical example later in this paper:

$$\nu(x) = \begin{cases} x^{\alpha} & \text{if} \quad x \geq 0 \\ -\lambda\ (-x)^{\alpha} & \text{if} \quad x < 0 \end{cases}$$

where they estimate $\alpha = 0.88$ and $\lambda = 2.25$. The two diagrams in Figure 1 show how prospects of changes in consumption from some fixed level are evaluated by a standard concave utility function (right), and a prospect theoretic value function (left). The prospect theoretic value function treats increases and decreases in consumption relative to a reference point asymmetrically, while the standard utility function has the same slope from the right and left. We shall see later that recent analysis of consumption behavior of school teachers supports the prospect theoretic model over expected utility theory.

The second component of prospect theory for decision making under risk is the weighting function $\pi(p)$. In order for prospect theory to accommodate the experimental evidence, the function $\pi(p)$ must satisfy the following conditions:

1. Discontinuity at 0: $\pi(0) = 0$, and $\pi(\epsilon) > \epsilon$ for very small ϵ (overweighting of small probabilities).

2. Sub-additivity for small values of p, i.e. $\pi(\epsilon + \delta) < \pi(\epsilon) + \pi(\delta)$.

3. Additivity or super-additivity for intermediate values of p, but another discontinuity at $p = 1$, where $\lim_{p \uparrow 1} \pi(p) < 1$, while $\pi(p) = 1$.

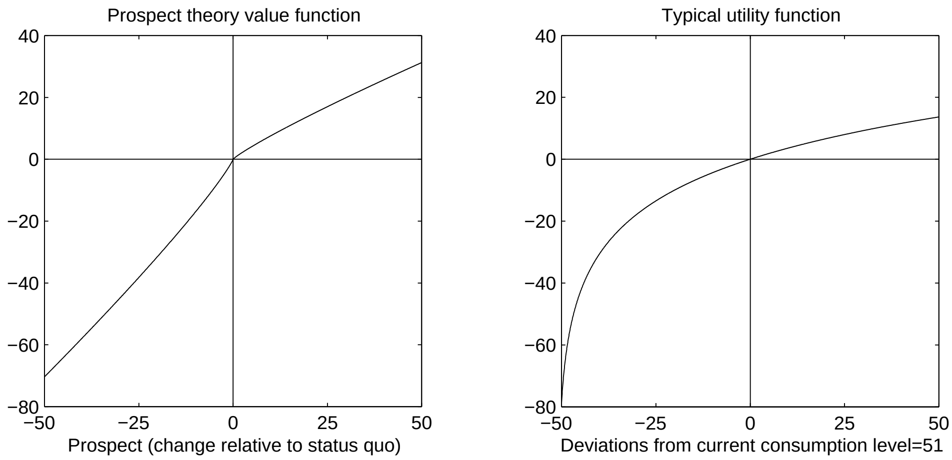

Figure 1: Traditional utility vs. prospect theory

Since estimation of a discontinuous function is very difficult, Tversky and Kahneman (1992) experimented with the smooth approximations to π and obtained good fitting estimates using the specification:

$$\pi(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}},$$

with $\gamma = 0.61$.

3.2 Evidence in favor of prospect theory

The experimental and empirical evidence in support of prospect theory has been mounting over the past two decades. While the theory's grounding in experimental evidence may lead some to dismiss its applicability for modeling real economic behavior, recent empirical evidence has suggested that the theory is quite appropriate for modeling decision making in the face of risk. Most notable in this regard are the recent empirical solutions of two major puzzles in finance and industrial organization using prospect theory:

- **The equity premium puzzle:** It is well known that stocks and other equity-based financial securities are riskier than U.S. government bonds. With risk averse agents buying and selling those securities, the expected returns on the riskier stocks must be higher than those for the less risky bonds. In a seminal paper, Mehra and Prescott (1985) showed that the historical difference between the rates of returns for stocks and bonds (the "equity premium") was too large to explain by any reasonable degree of risk aversion. Despite numerous attempts to explain this puzzle, none of the traditional explanations have been successful, see Kocherlakota (1996) for a critical survey of the various attempts to explain away this anomaly.

The puzzle is explained thus by Shiller (1999, forthcoming):

> "... the equity premium of US stocks over short-term government bonds has averaged 6.1% a year for the United States for 1926 to 1992, and so one naturally wonders why people invest at all in debt if it is so outperformed by stocks."

He further argues that the attempts to explain the puzzle by utilizing the increased riskiness of stocks relative to bonds are not satisfactory in principle since:

> "Most investors ought to be investing over decades since most of us expect to live for many decades, and to spend the twilight of their lives living off savings. Over long periods of time, it has actually been long-term bonds (payoffs of which are fixed in nominal terms), not the stocks, that have been more risky in real terms, since the consumer price index has been ... very variable over long intervals of time ..."

The best explanation to date which agrees with experimental evidence on individual choice in portfolio selection experiments seems to be a combination of loss aversion and myopia (excessive discounting of the distant future), consistent with the model adapted in Chapter 2. Benartzi and Thaler (1995) conduct a simulation analysis based on stock and bond returns assuming that all agents in the economy evaluate investment decision based on the prospects (i.e. ignoring wealth effects). The agents are assumed to exhibit prospect theoretic preferences with the value function $\nu(.)$ and the probability payoff function $\pi(.)$ shown above. Individuals are assumed to maximize those prospect theoretic preferences with a finite "evaluation period", or time horizon. They find that investment in bonds is consistent with evaluation periods of one year or less, while longer evaluation horizons would still induce individuals to invest exclusively in stocks. Allowing individuals to construct portfolios of both stocks and bonds, and assuming that they have a one-year evaluation period, the optimal portfolio seems to be roughly between 30 and 55 percent in stocks, which they report to be consistent with common portfolio choices of individuals in the market, c.f. Benartzi and Thaler (1995, pp.84-5). This explanation has three components which are consistent with the behavioral assumptions discussed in detail in Chapter 2: (1) loss aversion, (2) overweighting small probabilities, and (3) an extreme form of excessive discounting of the distant future relative to the near future. The result is a mis-pricing of the risk differential between stocks and bonds. Specifically, individuals pay too much for the differential in risk between stocks and bonds.

- **The risk-return paradox:** In a study of eighty-five industries, Bowman (1980) found an anomalous negative relationship between risk and return. One of his explanations is that managers are not risk averse, and may indeed be risk loving. In a more detailed study Fiegenbaum and Thomas

(1988), whose findings were confirmed in Fiegenbaum (1990), managers of organizations falling short of their target levels exhibited risk loving behavior, while managers of those above their target levels exhibited risk averse behavior. This suggests that the business's target level plays the role as a status-quo with respect to which prospects are calculated. The incentive scheme facing managers, which severely discourages falling below business targets, may induce this type of behavior. Thus, if managers' compensation can be approximated by a simple one-zero function based on meeting their targets or failing to do so, managers will attempt to increase their chances breaking even or better by undertaking more risk when behind and less while ahead of their targets.

Regardless of the reason, the behavior of managers is found by Fiegenbaum (1990) to be consistent with prospect theory: (i) managers of firms falling short of their target levels exhibit risk seeking behavior, (ii) managers of firms performing above their target levels exhibit risk averse behavior, (iii) the absolute median slope of the risk return association (negative for the first group and positive for the second) was found to be 3 times larger for the risk seeking group. Those three results exactly describe the three main characteristics of the $\nu(.)$ function described above: (i) concave above the status quo, (ii) convex below the status quo, and (iii) a kink at the status quo with the slope from below steeper than the slope from above.

In a later study, Gooding et al. (1996) question the traditional computation of a constant reference points for all industries near their median performance. They find strong evidence that reference points for each industry: (i) tend to lie above the industry median, (ii) vary across industries, and (iii) vary over time. Allowing for this heterogeneity and time variation, they find stronger evidence in the vast majority of industries for risk seeking behavior below firms' targets and risk averse behavior above them. This robust finding suggests a strong mis-pricing of risk across industries and over time. If the stock prices of firms reflected in part the risk-return tradeoff for that firm, then the reversal of managers' risk attitude around an arbitrary target point should result in a discontinuity in the stock's price relative to its return. While I am not aware of empirical findings in this regard, casual empiricism suggests that such points of discontinuity in the earnings to stock price relationship do not exist. In other words, the pricing of stocks does not seem to reflect this "anomalous" managerial attitude towards risk.

There are many other empirical phenomena which cannot be explained by usual expected utility maximization but can be explained using prospect theory. Camerer (1998) provides a mini-survey of some of those phenomena, including (see ibid. for references and more details):

- An observed strong negative correlation between daily wages and hours worked by New York City cab drivers cannot be explained using standard expected utility theory. Empirical analysis and interviews revealed that drivers have a one-day horizon and a target dollar amount for each day. Once they reach that target amount, they quit for the day.

- Price elasticities of demand which are larger after price increases than they are after price decreases cannot be explained with standard utility theory, but agree with prospect-theoretic loss aversion. Recent studies have shown that such asymmetric elasticities do in fact exist, with one study for orange juice estimating $\lambda = 2.4$.
- School teachers were observed to violate the consumption smoothing patterns predicted by standard life-cycle consumption models. In fact, teachers who knew their union-negotiated one-year ahead wages were found to react positively to unexpectedly high future wages, but not to react to unexpectedly low future wages. This was recently explained by a model of prospect-theoretic loss aversion relative to a reference point determined by past consumption patterns.
- State lotteries are more popular in states with larger populations (and thus larger jackpots), and more popular within each state the larger the jackpot. This was recently explained by the fact that lottery ticket buyers give more attention to the larger jackpots, ignoring the fact that the probability of winning is much smaller in those cases. This has been shown empirically to be very difficult to explain with expected utility maximizing buyers, but fits perfectly with the prospect theoretic value and probability weighting functions.

In summary, the evidence in favor of prospect-theoretic "bounded rationality" in the sense of violating the axioms of expected utility theory is not restricted to carefully crafted laboratory experiments. The myopic prospect-by-prospect evaluation procedures which people use, together with their non-linear weighting of probabilities, their asymmetric risk aversion/loving over gains and losses, and their loss aversion, seem to agree not only with empirical and experimental evidence, but also introspection. As we shall see in the following model, humans with this form of bounded rationality will reach inefficient risk-trading solutions instead of efficient risk sharing ones. This would suggest that the prohibition of risk trading is efficiency enhancing, unless there is no risk sharing contract which can meet the same economic needs. In latter cases, as we have seen above, the prohibition of *gharar trading* is overruled by such efficiency gains. We now turn to a model where both risk trading (insurance) and risk sharing are available.

4 Trading in risky assets: Model

Consider a situation where two agents S (seller) and B (buyer) are engaging in a commission to manufacture the commodity c. The buyer's valuation of the good is random, depending on a state of nature $\theta \in \{\theta_1, \theta_2\}$:

$$value_B(c) = \begin{cases} \beta_1 & \text{if} \quad \text{if } \theta = \theta_1 \text{ (probability p)} \\ \beta_2 & \text{if} \quad \text{if } \theta = \theta_2 \text{ (probability 1-p)} \end{cases}$$

The seller's cost of manufacturing c, $cost_S(c)$, is also random,

$$cost_S(c) = \begin{cases} \phi_1 & \text{if} \quad \text{if } \theta = \theta_1 \text{ (probability p)} \\ \phi_2 & \text{if} \quad \text{if } \theta = \theta_2 \text{ (probability 1-p)} \end{cases}$$

Assume further that the seller and buyer exist in a competitive market as price takers, where the price of the commodity is μ. To make trading in risk interesting, let $\phi_2 > \mu$ (i.e. the seller may incur a loss if nature chooses θ_2).

We consider four scenarios:

- **Scenario 1:** No trade takes place between B and S. Their utilities from meeting are $(0, 0)$ (the seller does not incur the cost, the buyer does not realize the value).
- **Scenario 2:** The buyer and seller trade at the market price μ. No further exchange takes place.
- **Scenario 3:** The buyer and seller trade at the market price, but they engage in "risk and return sharing". When they share risks and returns, the buyer and the seller negotiate the ratio at which they split the surplus for each θ. We consider the simplest sharing rule of 50-50 in both states of nature, which gives each trader $(0.5(\beta_1 - \alpha_1), 0.5; 0.5(\beta_2 - \alpha_2), 0.5)$. Notice that this "surplus sharing" agreement gives payoffs which are independent of the market price.
- **Scenario 4:** The seller and buyer trade at the market price μ, and one of them offers the other "insurance", or a guaranteed payoff. The party providing insurance will choose the smallest guaranteed level for the insured party such that the latter will prefer to accept insurance over his surplus under trade alone.

Further assume that the buyer and seller both have identical preferences over "prospects" or lotteries $(x, p; y, 1 - p)$ for $y \geq x \geq 0$ are given by the function:

$$\nu(x) = \begin{cases} x^\alpha & \text{if} \quad x \geq 0 \\ -\lambda \, (-x)^\alpha & \text{if} \quad x < 0, \end{cases}$$

and the probability weighting function is given by:

$$\pi(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}.$$

In the numerical example below, we shall use the estimated values of $\alpha = 0.88$, $\lambda = 2.25$, and $\gamma = 0.61$, as estimated by Tversky and Kahneman (1992), and used extensively in experimental as well as empirical research. We begin by illustrating the payoff structure to various strategies in a numerical example.

In the insurance scenarios, one party offers the other a lottery which makes his payoffs constant regardless of θ. It is well known (see previous section) that one of the primary motivations of prospect theory is that humans are observed to evaluate prospects differently depending on the manner in which they are presented to them. This is Kahmenan and Tversky (1979)'s isolation effect. We now make two crucial assumptions for our analysis:

A.1 *When a trader is presented a lottery by the other, they evaluate its effect on their net position (use asset integration) if the lottery results in "insurance against θ", i.e. θ-independent net payoffs. However, when presented by another lottery, they evaluate it separately.*

A.2 *When offering a lottery, the trader making the offer evaluates its net effect on his net position (use asset integration) if the other party accepts the lottery and all other things remain constant.*

To make the analysis tractable, we make our calculations for the following values:

- $\alpha_1 = 0$, $\alpha_2 = 11$.
- $\beta_1 = 10$, $\beta_2 = 15$.
- $p = 1 - p = 0.5$.
- $\mu = 10$.

The following table summarizes the prospects and utilities of various prospects facing the buyer and seller:

scenario		prospect, U_S	prospect, U_B
No exchange	prospect	$(0, 0.5; 0, 0.5)$	$(0, 0.5, 0, 0.5)$
	Utility	0	0
Trade at μ	prospect	$(10, 0.5; -1, 0.5)$	$(0, 0.5, 5, 0.5)$
	Utility	2.2444	1.7338
"risk sharing"	prospect	$(5, 0.5; 2, 0.5)$	$(5, 0.5; 2, 0.5)$
	Utility	2.8001	2.8001
S offers B "insurance $S \to B$ $(1.9, 0.5; -3.1, 0.5)$ evaluate overall positions	prospect	$(8.1, 0.5; 2.1, 0.5)$	$(1.9, 0.5, 1.9, 0.5)$
	Utility	3.7638	1.7592
B offers S "insurance" $B \to S$ $(-7.49, 0.5; 3.51, 0.5)$ evaluate overall positions	prospect	$(2.51, 0.5; 2.51, 0.5)$	$(7.49, 0.5, 1.49, 0.5)$
	Utility	2.2476	3.2972
Simultaneous "insurance" trade lotteries evaluate individually	prospect	$(-7.49, 0.5; 3.51, 0.5)$	$(1.9, 0.5; -3.1, 0.5)$
	Utility	-4.2973	-1.8215

We now consider the following game. After the exchange takes place, each of S and B simultaneously make "offers" to each other. The offer can take the form of "risk sharing" or "insurance". Each player considers the offer made by the other, and decides whether to accept or reject it. We have the following four cases:

1. If both parties offer each other "risk sharing", S compares his payoff of 2.2444 to to his payoff of 2.8001, and accepts the offer. Similarly, B compares his payoff of 1.7338 to his payoff of 2.8001 and accepts the offer. Risk sharing is implemented, and the payoffs are $(2.8001, 2.8001)$.

2. If B offers risk sharing to S, but S offers insurance to B, the two players play the following sub-game:

   Buyer:

Seller:		accept	reject
	accept	(3.3323,0.9786)	(2.8001,2.8001)
	reject	(3.7638,1.7592)	(2.2444,1.7338)

   The off-diagnoal utilities are obtained from the table of prospects and utilities, corresponding to the "risk sharing" and the S offers insurance to B. The (reject,reject) cell corresponds to trading with no risk sharing or trading. The (accept,accept) cell utilities are obtained as follows: The buyer adds his utility after risk sharing 2.8001 to the utility of the lottery $(1.9, 0.5; -3.1, 0.5)$, which is -1.8215, hence his payoff is 0.9786. The seller, on the other hand, incorporates the status quo under risk sharing $(5, 0.5; 2, 0.5)$ with the lottery he is offering, thus facing $(3.1, 0.5; 5.1, 0.5)$, thus receiving a utility of 3.3323.

   There are two pure strategy Nash equilibria to this game. We only consider pure strategy equilibria since mixed strategy equilibria will require more assumptions to be calculated in the absence of expected utility maximization (how do you weight probabilities with which you mix?). The two equilibria correspond to "risk sharing" and "S insures B. Since the purpose of our study is to analyze whether the availability for risk trading can be efficiency reducing, we consider the equilibrium in which the seller refuses the buyer's offer, while the buyer accepts the seller's. Of course, if the other equilibrium is selected, then the permissibility or prohibition of trading in risk would be a moot point in this case. Thus, we assume that the selected utilities are (3.7638,1.7592).

3. If S offers risk sharing to B, but B offers insurance to S. Now, the players face the following subgame:

Buyer:

Seller:	accept	reject
accept	(-1.4972,2.5203)	(2.2476,3.2972)
reject	(2.8001,2.8001)	(2.2444,1.7338)

Again, the off-diagonal payoffs correspond to the simple "risk sharing" and "B gives insurance to S" from the table. The (reject,reject) outcome is the straight-forward utility profile from trading. Now we turn to the (accept,accept) sell. The seller adds to his utility of "risk sharing", 2.8001, the utility of the lottery $(-7.49, 0.5; 3.51, 0.5)$, which is -4.2973. Therefore, seller's utility in the (accept,accept) sell is -1.4972. The buyer, on the other hand, considers his net prospect after risk sharing and the lottery transfer. His lottery becomes $(12.49, 0.5; -1.51, 0.5)$, which gives him utility 2.5203.

Again, this subgame has two pure Nash equilibria, one corresponding to (accept,reject) and the other corresponding to (reject,accept). the first corresponds to "B insures S" and the latter corresponds to "risk sharing". For the same reasons cited above, we consider the case where the insurance outcome is selected. Again, if the risk sharing outcome is selected in this case, then the issue of whether or not to allow risk trading would be moot. Therefore, we assume that they selected utilities of this case is (2.2476,3.2972).

4. If both traders offer each other insurance, they consider their payoffs in four cases, depending on whether each accepts or rejects the other's offer. In other words, if they reach the node where both offer insurance, they foresee playing a **sub-game:**

Buyer:

Seller:	accept insurance	reject insurance
accept insurance	(-2.0529,-0.0877)	(2.2476,3.2972)
reject insurance	(3.7638,1.7592)	(2.2444,1.7338)

The off-diagonal payoffs are the same ones we get in cases 2 and 3 above. If they both reject each others' offers, they get the utilities from simply trading, which are (2.2444,1.7338). If they each accept each other's payoffs, they recognize that neither one is receiving "insurance". Therefore, they evaluate the lotteries being traded separately. Their overall utilities in this case are obtained by adding the utility of simply trading to the utility of accepting the lottery offered them, which yields (-2.0529,-0.0877).

This sub-game has two pure strategy Nash equilibria (accept, reject) and (reject,accept). We assume that the two players reach a solution to this coordination problem. Thus, the payoffs will be either (2.2476,3.2972), or (3.7638,1.7592).

We now consider the overall game played by the two traders. The extensive form is shown below in Figure 2. Using the solution for all subgames as derived above, we get the following normal form of the overall game:

		Buyer:	
		offer risk sharing	offer insurance
Seller:	offer risk sharing	(2.8001,2.8001)	(2.2476,3.2972)
	offer insurance	(3.7638,1.7592)	(2.2476,3.2972) or (3.7638,1.7592)

Notice that regardless of the way the buyer and seller solve their subgame coordination problem in the (insurance, insurance) case, they will face an overall normal form of the game which makes offering insurance a weakly dominant strategy. In fact (offer insurance, offer insurance) is the unique Nash equilibrium of this game. However, the sum of the two utilities in this Nash equilibrium is either 5.5448 or 5.5230, compared to a sum of utilities of 5.6002 under risk sharing. Notice that had we selected the "risk sharing" equilibrium for either of the (offer insurance, offer risk sharing) or the (offer risk sharing, offer insurance) equilibria, we can get a "risk sharing" outcome of (2.8001,2.8001) as an equilibrium. In such cases, the availability of insurance is ignored, and therefore its prohibition is efficiency-neutral. However, to guard against the cases where the equilibrium selection mechanism in the subgames results in insurance usage, maximizing the sum of the utilities would require banning insurance or "trading in risk".

To illustrate the Hicks-Kaldor inefficency of the two possible insurance Nash equilibria which result in utility profiles (3.7638,1.7592) or (2.2476,3.2972), respectively, note that the traders can negotiate a risk sharing arrangement which beats both. For instance, giving the seller (7.5,0.5;2.5,0.5) and the buyer (2.5,0.5;1.5,0.5) will provide them with a utility profile of (3.7748,1.7699), while giving the buyer (6.75,0.5;2,0.5) and giving the seller (3.25,0.5;2,0.5) will result in a utility profile of (2.2530,3.3241). It is clear that those negotiated allocations of the surpluses (10,0.5;4,0.5) can improve in the Pareto sense over either rent maximizing insurance equilibrium. Thus, the availability of insurance may lead to inefficiency, and as long as the transactions costs involved in negotiating a risk-sharing formula are not too high, the prohibition of financial insurance (trading in risk) will be efficiency neutral at worst, and efficiency enhancing at best.

The general extensive form of the game we have thus analyzed is shown in Figure 2.

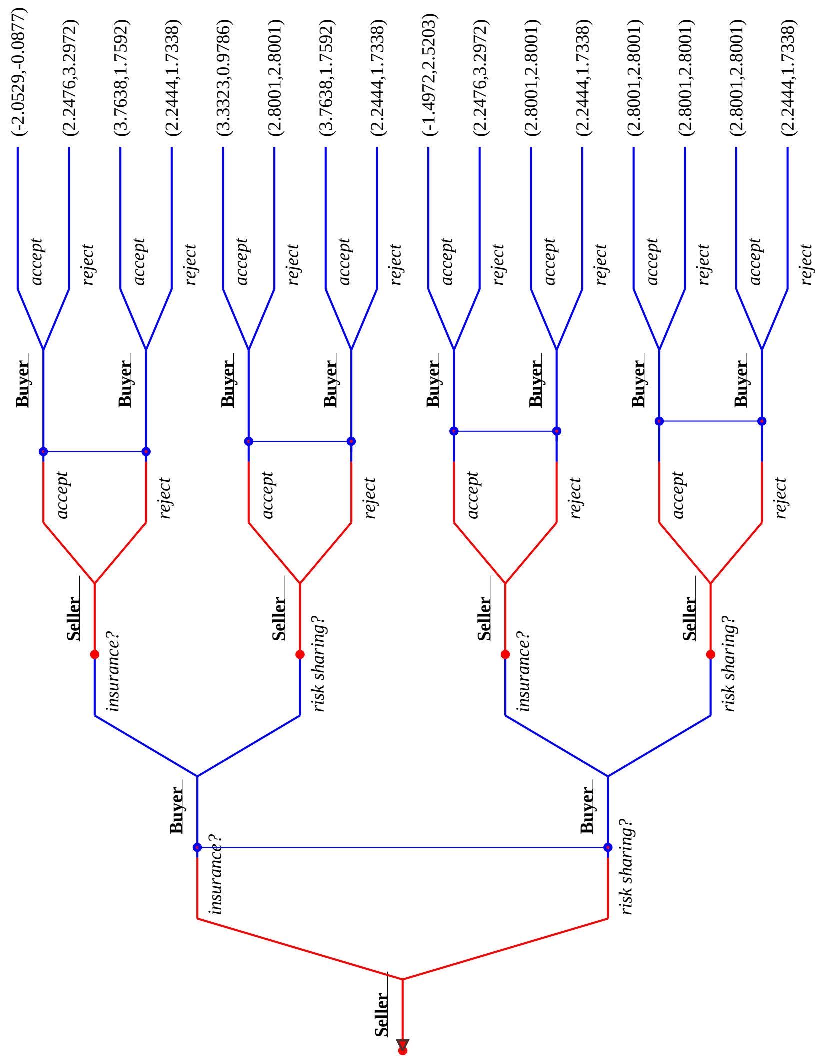

Figure 2: Extensive form of the game

The three pure Nash equilibrium outcomes of the overall extensive form are (2.2476,3.2972), (3.7638,1.7592) and (2.8001,2.8001). As we have seen in the detailed analysis, the efficient outcome of (2.8001,2.8001) is obtained only if it is selected in the (insurance,risk sharing) and (risk sharing,insruance) subgames.

In such cases, we may select the equilibrium in which trading in risk is not used. However, as we have seen, if the risk trading tool (insurance) is available, there are also equilibria in which it will be used. If such equilibria are selected (perhaps because of a first mover or historical bias), the permission of risk trading would lead to inefficiency.

The exact numerical values used in evaluating the model's node payoffs are not special: in general, either the insurance option will be used or it will not be used. If it is used, the party offering the insurance will extract the most rents from the party receiving it, granting them a utility slightly above the one they get without insurance. This option becomes very valuable for the party offering the insurance, giving them a much higher increase in utility. This makes the equilibrium selection in the (offer insurance, offer insurance) subgame a choice between one payoff which extremely favors one party or one which extremely favors the other. In either of those cases, the sum of the two utilities will be lower (with concave utilities over gains) than the sum of utilities under an intermediate risk sharing scenario which is mutually negotiated. If the insurance using equilibria are not selected in other sub-games, the issue of permitting or forbidding trading in risk becomes moot. Therefore, a social planner who wishes to maximize the sum of the utilities of the two traders would "forbid" trading in risk (or the offering of insurance), to ensure that the parties reach an "efficient" allocation of the surpluses in θ_1 and θ_2. Indeed, the negotiation skills necessary to achieve one of the two equilibria of the subgame can be put to better use to negotiate an efficient surplus sharing rule. Note that the notion of efficiency used in this analysis (Kaldor-Hicks) does not require that the parties in fact reach a risk sharing arrangement which dominates both insurance outcomes in the Pareto sense. In fact, there is no such risk sharing arrangement. However, the sum of the two utilities higher for most reasonable risk sharing formulas, which implies that if one of the two insurance equilibria were selected as the default, it would always be possible to negotiate a Pareto superior risk sharing arrangment.

5 Concluding remarks

We have shown that the permissibility of risk trading can be efficiency reducing if individuals suffer from the following prospect theoretic bounded rationality traits:

1. Loss aversion.

2. Evaluating the same prospect in different ways depending on how it is presented to them. In particular, humans suffering from this form of bounded rationality can be exploited by decomposing a net position into a random component plus "insurance".

3. Risk loving behavior below some benchmark "reference point", and risk averse behavior above that reference point. By manipulating the sequence

in which prospects are presented to an individual, they can be enticed into paying high rents for "insurance".

We did not extensively use the probability weighting function, but it does further contribute to enticing individuals to take risks with high potential returns and low probabilities of realizing such returns. This too would expose them to large risks which they can later overpay to insure against. This model of human decision making against risk has been shown very successful in explaining behavior in the laboratory as well as a variety of empirical applications.

Moreover, we have extensive evidence of financial market problems caused by mispricing of risk. For instance:

- Fox et al. (1996), showed that professional options traders suffer from the same prospect theoretic bounded rationality traits found in the laboratory.
- Fortune (1996) has shown that observed option prices give implied volatilities (using the celebrated Black-Scholes pricing formula) which are very poor predictors of the actual volatility of the underlying asset prices. Moreover, he showed that puts tend to show higher implied volatilities than calls on the same underlying asset.
- Johnston and McConnell (1989) explained the rise and dramatic fall of the GNMA CDR futures, one of the earliest interest rate derivatives using mortgage backed securities, by a fundamental mispricing of an embedded option.

Therefore, evidence contradicts the notion that private individuals may misprice risk due to their lack of training, but that professional institutions and traders in risk will in fact price risk appropriately. We have shown in our model that if a risk sharing alternative is available to trading in risk (there is little difference between selling insurance and selling options or futures, they are both state contingent lotteries), a prohibition of the latter is at worst efficiency neutral, and at best efficiency enhancing.

We have also shown that the prohibition of "*bay'u al-gharar*" is best translated as "trading in risk". This suggests that the informal qualitative determination jurists use in deciding whether the amount of *gharar* in any given contract may be substantially enhanced through an economic comparison of the efficiency gains provided by the given contract and the best efficiency gains one could obtain with an alternative which is void of that form of *gharar*.

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