The Output Gap and Optimal Monetary Policy

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Abstract

In a standard New Keynesian model, a discretionary and myopic central bank concerned with changes in the output gap and inflation will implement a monetary policy that replicates the optimal, timeless perspective, precommitment policy. By stabilizing output gap changes, the central bank imparts inertia into output and inflation that is absent under pure discretion. It is shown that even a fully optimizing (i.e., non-myopic) central bank operating in a discretionary policy environment achieves better social outcomes if it focuses on inflation and output gap changes and not the output gap. An output gap targeting regime is found to always dominate inflation targeting and to dominate nominal income targeting except when inflation is primarily backward-looking or the social weight on output gap fluctuations is small. Because the output gap change is equal to output growth minus growth in potential, the analysis helps explain why recent authors have found that nominal income growth targeting may be superior to inflation targeting, nominal income growth depends, in part, on real output growth.

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1 Introduction

Recent work on the design of monetary policy reflects a general consensus on the appropriate objectives of monetary policy. As articulated by Svensson, "....there is considerable agreement among academics and central bankers that the appropriate loss function both involves stabilizing inflation around an inflation target and stabilizing the real economy, represented by the output gap" (Svensson 1999). Such a loss function forms a key component of "The Science of Monetary Policy" (Clarida, Galí, and Gertler 1999), and Woodford (1999a) has shown how it can be derived as an approximation to the utility of the representative agent.

Despite the apparent agreement among academics over the objectives of policy, it is not clear that inflation and output gap stabilization are the objectives actually pursued in the conduct of policy. In justifying interest rate increases during 2000, the press releases from the Federal Open Market Committee emphasized the growth in output relative to the growth in potential rather than the level of output relative to potential. For example, following rate increases during the first half of 2000, the FOMC stated that

> The Federal Open Market Committee voted today to raise its target for the federal funds rate by 25 basis points to 5-3/4 percent. .... The [Federal Open Market] Committee remains concerned that over time, increases in demand will continue to exceed the growth in potential supply. (Feb., 2, 2000)
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> The Federal Open Market Committee voted today to raise its target for the federal funds rate by 50 basis points to 6-1/2 percent. .... Increases in demand have remained in excess of even the rapid pace of productivity-driven gains in potential supply... (May 16, 2000)

Letting y_t denote log output and $\bar{y}_t$ log potential, the FOMC's press releases suggest it was responding to $(y_t - y_{t-1}) - (\bar{y}_t - \bar{y}_{t-1})$. Rather than the output gap, $x_t \equiv y_t - \bar{y}_t$, it appears the change in the gap, $x_t - x_{t-1}$, was the measure of real economic activity on which the Fed's attention was focused.

In remarks at the Wharton Public Policy Forum in April 22, 1999, Fed Governor Edward M. Gramlich also describes monetary policy in terms of a focus on demand growth relative to growth in potential output:

> "Solving a standard model of the macroeconomy, such a policy would effectively convert monetary policy into what might be called 'speed limit' form, where policy tries to ensure that aggregate demand grows at roughly the expected rate of increase of aggregate supply, which increase can be more easily predicted."
>
> ".. the monetary authority is happy with the cocktail party temperature at present but moves against anything that increases its warmth. Should demand growth threaten to outrun supply growth (the party to warm up), the seeds of accelerating inflation may be planted and monetary policy should curb the growth demand by raising interest rates."

The purpose of this paper is to examine what role demand growth relative to growth in potential, the change in the output gap, should play in the design of monetary policy. In a forward looking model, I show that a completely myopic central bank who acts with discretion to minimize a one period loss function in the variability of inflation and the change in the output gap will end up replicating the socially optimal policy outcomes of a central bank able to precommit. Pure discretion, in which the central bank minimizes the social loss function but is unable to precommit, leads to inefficient stabilization in the face of cost shocks (Woodford 1999). It is this inefficiency that is removed if the central bank myopically focuses on inflation and the change in the output gap, not on the output gap itself.

The reason for this surprising result can be traced to Woodford's demonstration that an optimal precommitment policy involves inertia when expectations are forward looking. By impacting inertia into policy actions, the central bank's current actions directly affect the public's expectations of future inflation. A central bank concerned only with social loss but operating under discretion will fail to introduce any inertia. When the central bank strives to stabilize the change in the output gap, however, the lagged output gap becomes an endogenous state variable. This introduces inertia into monetary policy, even under discretion. It the central bank places the same weight on stabilizing the change in the gap as society places on output gap stabilization, then the myopic central bank acting with discretion imparts exactly the optimal degree of inertia into its policy actions.

While the assumption of myopic behavior is not realistic, this result suggests, as do the FOMC's press releases, that there may be an important role for the change in the output gap in policy design. If potential output follows

a deterministic trend, the change in the output gap is equal to the growth rate of real output relative to trend. In this case, it may be that output growth relative to trend is the appropriate variable that a central should try to stabilize. If attempting to stabilize output growth improves macroeconomic outcomes, this may rationalize the recent support for nominal income growth targeting that Jensen (1999) and McCallum and Nelson (2000) report, since nominal income growth depends on real output growth. It may also account for the Fed's apparent focus on growth. At the very least, it suggests that a closer examination of the role of the output gap as a policy objective is called for.

Using a parameterized New Keynesian model, I evaluation outcomes under various central bank objectives when policy is conducted with discretion. These objectives are characterized as alternative targeting rules: inflation targeting, change in output gap targeting, output growth targeting, and nominal income targeting. Jensen (1999) shows that nominal income targeting often dominates inflation targeting. However, I find that a policy based on targeting the change in the output gap always dominates inflation targeting. And while optimal inflation targeting involves appointing a weight-conservative central banker who values inflation stability more highly than does society, society can do even better by appointing a liberal central banker who highly values stability in output gap changes.

The next section sets out the basic model and derives the fully optimal commitment and discretionary policies. The basic benchmark values of the model's parameters are discussed, and the asymptotic social loss function is evaluated under both precommitment and discretion. As Jensen (1999) and McCallum and Nelson (2000) also show, precommitment achieves a lower value of the loss function than does discretion.

Section 3 demonstrates that the precommitment equilibrium can be achieved under a central bank that myopically minimizes a loss function that depends on inflation and the change in the output gap. This result does not carry over to the case of a fully optimal discretionary central bank, but numerical simulations help define the parameters of the model that determine whether a gap change objective dominates pure discretion.

Section 4 introduces inflation persistence into the model. Previous research (Rudebusch 2000b) has shown that the presence of lagged inflation in the inflation adjustment equation can affect the ranking of alternative policy rules. I compare policies based on output gap changes to pure discretion as the coefficient on lagged inflation varies. If delegation also includes setting

the weight the central bank places on its output objective, as in Rogoff (1985) and Jensen (1999), a gap change objective assigned to a liberal central bank dominates assigning the social loss function to a conservative unless inflation is largely backward looking in nature.

Section 5 extends the model to allow for serially correlated cost shocks and stochastic fluctuations in potential output. This extended model is then used to compare a variety of alternative targeting regimes, including income growth targeting and nominal income growth targeting.

In concluding his recent analysis of nominal income targeting, Henrik Jensen notes that nominal income target "desires more serious attention in real-life policy design than has recently been the case (Jensen 2000, p. 30)." The results of this paper indicate that even better outcomes can be achieved if the central bank focuses on the change in the output gap rather than either the gap itself or nominal income growth.

2 The basic model under precommitment and discretion

The basic New Keynesian model consists of two relationships: an aggregate demand condition that links output and the real interest rate (an "expectational IS curve"), and an inflation adjustment equation. Clarida, Galí, Gertler (2000), Woodford (1999, 2000), McCallum and Nelson (1999), Svensson and Woodford (1999, 2000), among others, have popularized this simple model for use in monetary policy analysis. Its foundations are discussed in Walsh (1998). The appendix provides a more detailed derivation of the basic relationships.

The aggregate demand relationship is derived from the first order Euler condition for the representative household's optimal consumption choice problem. Assuming constant relative risk aversion and separability between consumption and leisure, the Euler condition can be approximated around the steady-state as

$$y_t = E_t y_{t+1} - \sigma \left(R_t - E_t \pi_{t+1} \right) + u_t \tag{1}$$

where y is output, π is the inflation rate, R is the nominal interest rate, and u is a stochastic disturbance. The parameter σ is equal to the steady-state ratio of consumption to output times the inverse of the household's elasticity of

intertemporal substitution. All variables are expressed as percent deviations around the steady-state. If output demand arises from consumption and government purchases, then u_t includes $g_t - E_t g_{t+1}$, where g is the percent deviation of government purchases around the steady-state.

The second component of the model is an inflation adjustment equation. Most recent analyses have employed the Calvo specification of staggered price adjustment, but Roberts (1995) shows that other basic models of price adjustment lead to a similar specification (see also Walsh 1998). With sticky prices, firms must base their pricing decisions on real marginal costs and their expectations of future price inflation. As a consequence, current inflation is given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t \tag{2}$$

where x is the output gap, defined as the difference between actual output and the flexible price equilibrium level of output.[1] The cost shock e_t is assumed to be a white noise process.[2]

The final aspect of the model specification is the social loss function. As is standard in this literature, this is taken to be a function of inflation and output gap variability:

$$L_t = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda x_{t+i}^2 \right] \tag{3}$$

This specification reflects the widespread agreement over the objectives of monetary policy alluded to by Svensson. Woodford (1999a) discusses the conditions under which equation (3) can be interpreted as an approximation to the utility of the representative agent.

[1] This simple inflation adjustment equation has been criticized on several grounds. Estrella and Fuhrer (2000) argue it implies implausible inflation dynamics, while Fuhrer (1997) and Rudebusch (2000b) find that lagged inflation is much more important than the forward looking expectational variable implied by theory. On this last point, Galí and Gertler (1999) argue that the poor empirical performance of equations such as (2) arises from the use of the output gap in place of the theoretically correct real marginal cost. In section 4 below, equation (2) is modified to include a lagged inflation term to deal with Estrella and Fuhrer's critique.

[2] This assumption is modified in section 5.

2.1 Precommitment

A central bank that is able to precommitment to a policy rule chooses a path for current and future inflation and the output gap to minimize the social loss function (3) subject to the inflation adjustment equation (2). Letting ψ_{t+i} denote the Lagrangian multiplier associated with the period $t+i$ inflation adjustment equation, the central bank's problem is to minimize

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{1}{2} \left(\pi_{t+i}^2 + \lambda x_{t+i}^2 \right) + \psi_{t+i} \left(\pi_{t+i} - \beta E_t \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i} \right) \right]$$

The first order conditions for this problem are

$$\pi_t + \psi_t = 0 \tag{4}$$

$$\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1} = 0 \qquad i \geq 1 \tag{5}$$

$$\lambda x_{t+i} - \kappa \psi_{t+i} = 0 \qquad i \geq 0 \tag{6}$$

Equations (4) and (5) reveal the dynamic inconsistency that characterizes the optimal precommitment policy. At time t, the central bank sets $\pi_t = -\psi_t$ and promises to set $\pi_{t+1} = -\left(\psi_{t+1} - \psi_t \right)$. But when period $t+1$ arrives, a central bank that reoptimizes will again obtain $\pi_{t+1} = -\psi_{t+1}$ as its optimal setting for inflation, since the first order condition (4) updated to $t+1$ will reappear. Defining policy under commitment as the solution to (4)–(6) implies a choice for π_{t+1}, π_{t+2}, ... that the central bank knows it will not wish to implement. As McCallum and Nelson (2000) note, this "behavior seems highly implausible..."

An alternative definition of an optimal precommitment policy requires the central bank to implement conditions (5) and (6) for all periods, including the current period. Woodford (1999b) has labeled this the "timeless perspective" approach to precommitment. One can think of such a policy as having been chosen in the distant past, and the current values of the inflation rate and output gap are the values chosen from that earlier perspective to satisfy the two conditions (5) and (6). McCallum and Nelson (2000) provide further discussion of the timeless perspective and argue that this approach agrees with the one commonly used in many studies of precommitment policies.

There is a third approach to defining a commitment policy that warrants mention, since it represents the natural extension of the approach used in the non-forward looking models employed in the traditional Barro and Gordon (1983) literature. In the model consisting of equations (1) and (2), the

only state variable is the current cost-push shock realization e_t. The logic employed in the Barro-Gordon literature defined commitment policies as the choice of a rule expressing the policy instrument as a function of the current state. In the present case, it would correspond to the choice of a rule of the form $x_t = be_t$ that minimizes the loss function subject to equation (2). Woodford (1999a) shows, however, that such a policy is suboptimal. A fully optimal precommitment policy will display inertia.

The definition of the optimal precommitment policy used in this paper is that of the timeless perspective approach. Combining (5) and (6), under the optimal precommitment policy inflation and the output gap satisfy

$$\pi_{t+i} = -\left(\frac{\lambda}{\kappa}\right)(x_{t+i} - x_{t+i-1}) \tag{7}$$

for all $i \geq 0$. The evolution of the output gap and inflation are governed then by equations (2) and (7). Combining these equations, the system can be written in state-space form as[3]

$$\begin{bmatrix} e_{t+1} \\ x_t \\ E_t\pi_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -\frac{\kappa}{\lambda} \\ -\frac{1}{\beta} & -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} e_t \\ x_{t-1} \\ \pi_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

The impact of a cost shock on inflation and the output gap under optimal precommitment can be obtained by numerically calibrating the system in (8) and solving it. Three unknown parameters appear in (8): β, κ, and λ. The discount factor, β, is set equal to 0.99, appropriate for interpreting the time interval as one quarter. A weight on output fluctuations of $\lambda = 0.25$ is used. This value is also used by Jensen (1999) and McCallum and Nelson (2000). McCallum and Nelson (2000) characterize the empirical evidence as consistent with a value for the impact of the output gap on inflation (κ) in the range [0.01, 0.05]. Roberts (1995) reports higher values. Following Jensen, I set $\kappa = 0.05$ as the baseline value. This parameter captures both the impact of a change in real marginal cost on inflation and the co-movement of real marginal cost and the output gap.

[3]It will be convenient include the first row in (8) and to let ε_{t+1} denote the innovation to the cost shock in period $t+1$ since the model will be extended below to allow $e_{t+1} = \gamma_e e_t + \varepsilon_{t+1}$ with $0 < \gamma_e < 1$. In this case, the first row of the 3×3 matrix on the right of (8) becomes $[\gamma_e \ 0 \ 0]$.

Figure 1 shows the response of inflation and the output gap to a transitory cost push shock. Despite the fact that the shock itself has no persistence, the output gap displays strong, positive serial correlation. By keeping output below potential (a negative output gap) for several periods into the future after a cost shock that increases inflation, the central bank is able to lower expectations of future inflation. A fall in $E_t\pi_{t+1}$ at the time of the positive inflation shock improves the trade-off between inflation and output gap stabilization faced by the central bank.

2.2 Optimal discretion

In contrast to the case of precommitment, a central bank that operates in a discretionary policy regime takes expectations as given. The central bank may recognize that expectations of future inflation depend, through the public's process for forming expectations, on the current state. But in the present model, the state is simply the exogenous shock e_t. Thus, the central bank in a discretionary environment can not affect the public's expectations of future inflation and so treats these as given in deciding on optimal policy for period t. The central bank sets policy at time t taking $E_t\pi_{t+1}$ as given, and the policy problem is reduced to the simple single period problem of minimizing $\pi_t^2 + \lambda x_t^2$ subject to (2) with expectations given.

Letting φ denote the Lagrangian multiplier attached to (2), the first order conditions under discretion are

$$\pi_t + \varphi_t = 0 \tag{9}$$

$$\lambda x_t - \kappa\varphi_t = 0 \tag{10}$$

Combining these two equation produces

$$\pi_t = -\left(\frac{\lambda}{\kappa}\right) x_t \tag{11}$$

In a discretionary policy regime with the central bank acting to stabilize inflation and the output gap., the equilibrium inflation and output gap are determined by equations (2) and (11). Figure 2 shows the impulse response of inflation and the output gap to a cost shock. The figure, which should be compared with figure 1, reveals that both macro variables return to baseline just one period after a positive inflation shock under a discretionary policy

regime. None of the persistence generated by the optimal precommitment policy occurs under discretion. A temporary cost shock moves the output gap below zero and inflation above zero, but only for a single period.

Table 1 compares the asymptotic social loss under commitment and discretion for the baseline parameter values and for larger and smaller values of λ.[4] Table 2 reports the standard deviations of inflation and the output gap under precommitment and pure discretion. Under discretion, the output-inflation trade-off is less advantageous. In response to a cost shock, the central bank allows inflation to fluctuation more, and the output gap less, than would be done under an optimal precommitment policy. As a consequence, the gain from moving from a discretionary policy regime to a commitment regime is greatest when inflation stabilization is relatively more important (i.e., as λ becomes smaller).

Table 1: Asymptotic Loss (social loss x 10^2)

	λ		
	0.1	0.25	0.5
Commitment	1.939	2.055	2.116
Discretion	2.195	2.228	2.239
% loss from discretion	13.21%	8.39%	5.78%

[4]The asymptotic loss is calculated as $\left[\sigma_\pi^2 + \lambda\sigma_x^2\right]/(1-\beta)$, where σ_π^2 and σ_x^2 are the asymptotic variances of inflation and the output gap. In all the models considered in this paper, the linear rational expectations solutions take the form $Z_t = MZ_{t-1} + v_t$ where v_t is a vector of mean zero, serially uncorrelated innovations. The variance covariance matrix of Z, denoted by Σ_{ZZ}, is obtained from

$$vec(\Sigma_{ZZ}) = [I - (M \otimes M)]^{-1} vec(\Sigma_{vv})$$

where Σ_{vv} is the variance-covariance matrix of v and $vec(X)$ is the vector of stacked columns of a matrix X. The unconditional variances of inflation and the output gap can then be found as $C\Sigma_{ZZ}C'$ for a suitably defined matrix C.

Table 2: Standard Deviations

	λ		
	0.1	0.25	0.5
Precommitment			
σ_π	1.335	1.396	1.427
σ_x	1.252	0.655	0.399
Pure discretion			
σ_π	1.463	1.485	1.493
σ_x	0.732	0.297	0.149

The relative variability of inflation and the output gap under the two different policy regimes suggest why there may be a gain under discretion from delegating policy to a Rogoff weight-conservative central bank. By placing greater weight on stabilizing inflation, such a central bank delivers more stable inflation at the cost of greater output gap volatility. This tends, however, to move the equilibrium closer to what would be achieved under a precommitment policy.

Because the central bank faces a less advantageous inflation – output gap trade off under discretion, there is a cost relative to commitment. This cost does not arise from the traditional inflation bias that was the focus of the Barro-Gordon literature. Instead, it arises from a stabilization distortion introduced by discretion.

3 Discretion and the change in the output gap

The Barro-Gordon literature on the average inflation bias that could arise under discretion provided numerous possible solutions, including delegation to a conservative central bank (Rogoff 1985), incentive schemes (Walsh 1995), and inflation targets (Svensson 1997). Less well understood is how the gains of commitment in forward looking models might be obtained if the central bank must operate with discretion.

The discussion following Table 2 suggested there may be a gain from delegating to a Rogoff conservative central banker, one who places less weight on output gap fluctuations than society does, and Clarida, Galí, and Gertler (1999) show that, when cost shocks are serially correlated, the optimal simple

rule (without inertia) can be achieved under discretion if policy is conducted by a conservative central bank. Walsh (1999) finds a similar result in an open economy model. In general, however, the recent literature has typically assumed the central bank can commit to a policy rule, and optimal rules or rules constrained to take simple forms (such as the Taylor rule) are evaluated. An exception is Jensen (1999) who considers the optimal assignment of a nominal income growth objective to the central bank (in addition to inflation and output gap objectives). He numerically calculates the optimal weights on nominal growth and inflation objectives that society should assign to a central bank operating under discretion. Thus, rather than assume the central bank can commit to a simple rule, Jensen evaluates how changing the objectives of the central bank might affect output and inflation. This approach parallels that used to develop solutions to the traditional average inflation bias arising under discretion (e.g., Rogoff 1985, Walsh 1995, and Svensson 1997). Specifically, Jensen studies the effect of altering the weight on the inflation objective (ala Rogoff's conservative central banker) and of introducing nominal income growth targeting as a separate objective in the loss function.

In Woodford's original discussion of interest rate inertia, he argued that empirical evidence of inertial interest rate behavior reflected the attempt by central banks to influence forward-looking expectations. By committing itself to a rule that induces inertial behavior in the nominal interest rate, current changes in policy generate changes in expected future inflation rate and inflation. This allows the central bank to influence expected future inflation, improving its trade-off between inflation and output gap variability. Since nominal income growth is equal to $\pi_t + y_t - y_{t-1}$, nominal income targeting implicitly introduces y_{t-1} into the state vector and generates some persistence even under a regime of pure discretion. This accounts for the good performance of nominal income growth targeting that Jensen finds.

As argued in Walsh (1995), however, the appropriate starting point is to derive the optimal objectives of the central bank and to then evaluate how these might be implemented through, for example, inflation targeting (Svensson 1997) or nominal income targeting (Jensen 1999). While the use of the Barro-Gordon model, or other backward looking models, suggested that simply ensuring the central bank focuses on inflation and the output gap was sufficient to replicate the optimal commitment policy, this is no longer true when agents are forward-looking. Instead, it can be shown in a special case that the central bank should focus on inflation and the change in the output

gap. This motivates the closer examination given in the following sections to making the change in the output gap part of the central bank's loss function.

3.1 Myopic discretion

Consider the case of a myopic central bank, concerned only with minimizing its current period loss function, taking private sector expectations as given. Such a central bank ignores the intertemporal aspects of the policy problem for two reasons. First, because it is operating under discretion, it treats expectations of future inflation and output as given. Second, because it is assumed to act myopically, it ignores the impact its current policy choice may have on future states. To analyze how society would wish such a central bank to act, assume the central bank's loss function can differ from society's loss function given by (3). This simply reflects that fact that societies frequently assign goals to governmental policy making institutions, but these goals can differ from "social welfare" itself. Specifically, the central bank's loss function is modified to take the form

$$\sum \beta^i \left[\frac{1}{2} \left(\pi_{t+i}^2 + \lambda x_{t+i}^2 \right) + T(\pi_{t+i}, x_{t+i}; s_{t+i}) \right] \tag{12}$$

where $s_t = \{\pi_t, x_t, e_t, s_{t-1}\}$ is the history of the economy up to date t.

A completely myopic central bank acting under pure discretion solves a single period problem in which it minimizes $\frac{1}{2}(\pi_t^2 + \lambda x_t^2) + T(\pi_t, x_t; s_t)$, subject to (2), taking the current state s_t and expectations as given. The first order conditions are

$$\pi_t + T_\pi + \phi_t = 0 \tag{13}$$

$$\lambda x_t + T_x - \kappa \phi_t = 0 \tag{14}$$

where ϕ_t is the Lagrangian multiplier on the inflation adjustment equation (2) that constrains the joint behavior of inflation and the output gap.

Substituting (14) into (13),

$$\pi_t = -\left(\frac{\lambda}{\kappa} \right) x_t - \left(\frac{1}{\kappa} \right) T_x - T_\pi \tag{15}$$

Comparing (15) with (7) yields the following:

Proposition 1 If $T_x = -\lambda x_{t-1}$ and $T_\pi = 0$, myopic discretion replicates outcomes under the optimal precommitment policy.

Proof. Under precommitment, the equilibrium processes for π_t and x_t are given by the rational expectations solution to (2) and (7). Under discretion, the equilibrium processes for π_t and x_t are given by the rational expectations solution to (2) and (15), The outcomes are the same if and only if (7) and (15) are the same, which occurs when $T_\pi = 0$ and $T_x = -\lambda x_{t-1}$. ■

The main result, though, is stated in the next proposition.

Proposition 2 A myopic central bank operating under discretion will achieve the optimal precommitment policy outcome if its loss function is

$$\frac{1}{2}\left[\pi_t^2 + \lambda(x_t - x_{t-1})^2\right]$$

Proof. Expanding this loss function,

$$\begin{aligned}\frac{1}{2}\left[\pi_t^2 + \lambda(x_t - x_{t-1})^2\right] &= \frac{1}{2}\left[\pi_t^2 + \lambda x_t^2 - 2\lambda x_{t-1}x_t + \lambda x_{t-1}^2)\right] \\ &= \frac{1}{2}\left[\pi_t^2 + \lambda x_t^2\right] - \lambda x_{t-1}x_t + \frac{1}{2}\lambda x_{t-1}^2 \\ &= \frac{1}{2}\left[\pi_t^2 + \lambda x_t^2\right] + T\end{aligned}$$

where $T = -\lambda x_{t-1}x_t + \frac{1}{2}\lambda x_{t-1}^2$. Since $T_\pi = 0$ and $T_x = -\lambda x_{t-1}$, Proposition 1 applies. ■

Proposition 2 implies that a completely myopic central bank acting under discretion to minimize fluctuations in inflation and the change in the output gap will produce the same equilibrium outcomes as would occur under a central bank that is able to implement the socially optimal precommitment policy. Proposition 2 follows immediately when it is recognized that the relationship between inflation and the change in the output gap implied by the optimal precommitment policy and given in equation (7) is identical to the first order condition for a discretionary central bank with a loss function equal to $\pi_t^2 + \lambda\left(x_t - x_{t-1}\right)$.

If potential output follows a deterministic time trend, then $x_t - x_{t-1}$ is equal to output growth relative to trend.[5] It follows that

[5]Suppose $\bar{y}_t = \bar{y}_0 + \delta t$. Then, $x_t - x_{t-1} = (y_t - y_{t-1}) - (\bar{y}_t - \bar{y}_{t-1}) = y_t - y_{t-1} - \delta$, where $y_t - y_{t-1}$ is the growth rate of real output.

Proposition 3 When potential output follows a deterministic trend, a myopic central bank operating under discretion will achieve the optimal precommitment policy outcome if its loss function is a function of inflation variability and the variability of the growth rate of real output relative to trend growth, given by

$$\frac{1}{2}\left[\pi_t^2 + \lambda(y_t - y_{t-1} - \delta)^2\right]$$

where δ is the trend growth rate of potential output.

Recent work using forward-looking models has shown that focusing on the output gap, while eliminating the average inflation bias, still results in suboptimal stabilization policies. Proposition 2 shows that when private agents base their price setting behavior on forward-looking expectations, myopic central banks should define their goals in terms of inflation and the change in the output gap. When potential output follows a deterministic trend, the translates into a focus on inflation and the growth rate of output.

If the central bank is concerned with changes in the output gap, a natural inertia is introduced into the policy process in a way that mimics the optimal precommitment solution. A positive inflation shock is met with a real contraction that lowers the output gap. If policy actions are completely temporary, as they are under pure discretion based on the social loss function, the change in the output gap in the period following the shock will be positive as output rebounds from the temporary contraction induced by the shock. A central bank that is concerned with stabilizing the change in the gap will continue to maintain a contractionary policy to dampen this increase in the gap.

Vestin (2000) has used a forward looking model of the form given in equations (1) and (2) to study price level targeting under discretion. Previously, Svensson (1999b) had shown that price level targeting had desirable properties in a model with a Lucas-type aggregate supply function. Vestin reaches similar conclusions. Some intuition for these results can be obtaining by noting that the first order condition under precommitment, equation (7) would also arise if a myopic central bank operated with discretion to minimize a loss function that depends on output gap variability and price level variability. In this case, the central bank's first order condition would simply be $p_t = -\left(\frac{\lambda}{\kappa}\right) x_t$. Taking first differences yields (7).

3.2 Output gap changes and optimal discretion

The previous subsection considered the policy choice of a myopic central bank. A concern for output gap changes leads a myopic policy maker to achieve the optimal precommitment outcomes. While the assumption of a myopic central bank is unreasonable, the surprising result that such a central bank could deliver the optimal precommitment policy suggests the role of stabilizing output gap changes as a policy objective warrants further study. An obvious question is whether similar gains can be achieved with an output gap change objective if the central bank is not myopic but instead acts to optimize fully under discretion. In this subsection, this issue is addressed. In general, no analytic results are available, so numerical methods are employed.

When the central bank operates under discretion to minimize the loss function that depends on inflation and output gap change variability, its decision problem at time t can be written as

$$\min \frac{1}{2} E_t \sum \beta^i \left[\pi_{t+i}^2 + \lambda (x_{t+i} - x_{t+i-1})^2 \right]$$

subject to $\pi_t - \beta E_t \pi_{t+1} - \kappa x_t - \varepsilon_t = 0$. McCallum and Nelson (2000) discuss two different definitions of optimal discretionary policy in this environment. Under the first, the central bank treats future expectations of both inflation and the output gap as exogenous when it chooses current inflation and output (subject to the inflation adjustment relationship). Alternatively, the central bank may take as given the process through which private agents form their expectations. In this latter case, for example, the central bank recognize that expectational terms such as $E_t \pi_{t+1}$ will depend on the state variables at time t and that these state variables may be affected by policy actions at time t or earlier.

These two definitions of an optimal discretionary policy were equivalent in the context of the model of the previous section. This was because the state vector under discretion consisted solely of the serially uncorrelated disturbance e_t.[6] Expectations of future inflation were functions of the exogenous process e_t and independent of current discretionary policy actions.

The two definitions differ when the inflation adjustment equation is modified to include some weight on lagged inflation, for example, as will be the case in the model of section 4. This modification is common in the literature

[6] Recall that γ_e was equal to 0 in the baseline parameter set.

and is normally justified on the grounds that a specification that incorporated π_{t-1} does a better job in matching the dynamic behavior of actual inflation. When lagged inflation enters the inflation adjustment equation, the state vector includes both e_t and π_{t-1}. Expectations of future inflation will now depend on π_t. Policy actions that affect current inflation will also affect $E_t\pi_{t+1}$, and the central bank will take this dependency into account under the second definition of an optimal discretionary policy. The central bank would ignore this dependency under the first definition in which it treats terms such as $E_t\pi_{t+1}$ as given. As McCallum and Nelson (2000) note, Clarida, Galí, and Gertler (1999) and Jensen (1999) assume the central bank does recognize the link between the state and expectations (i.e. they employ the second definition of discretionary policy – see the discussion in Clarida, Galí, and Gertler 1999, page 63, footnote 73 of NBER version).

The two alternative definitions of discretionary policy also differ once we assume the central bank's loss function involves the change in the output gap. In essentially choosing x_t to affect $x_t - x_{t-1}$, the central bank's policy choice will be a function of x_{t-1}. This introduces the lagged output gap as a state variable even though the underlying disturbances are serially uncorrelated and there are no other lagged endogenous state variables. Because the lagged output gap will be an endogenous state variable when it appears in the central bank's loss function, private agents will base their forecasts of future values of x_{t+i} and π_{t+i} on x_{t-1} and e_t. Following McCallum and Nelson and Jensen, it is assumed the central bank recognizes this dependence when it operates with discretion.

Assume the equilibrium solutions for the output gap and inflation as a function of the state variables take the form

$$x_t = a_x x_{t-1} + b_x e_t$$

and

$$\pi_t = a_\pi x_{t-1} + b_\pi e_t$$

Let φ_t denote the Lagrangian multiplier associated with the constraint given by the inflation adjustment equation (2). The first order conditions for optimal discretionary policy with an output gap change objective are

$$\pi_t + \varphi_t = 0 \tag{16}$$

and

$$\lambda(x_t - x_{t-1}) + \left[\frac{\beta\lambda(a_x - 1)^2}{1 - \beta a_x^2}\right] x_t = -\left(\beta a_\pi + \kappa\right) E_t \sum_{i=0}^{\infty} \beta^i a_x^i \pi_{t+i} \tag{17}$$

Equation (7) continues to give the relationship between the output gap and inflation consistent with the optimal precommitment policy. Comparing this condition with equations (16) and (17) reveals that simply replacing the output gap with the change in the output gap in the central bank's loss function does not yield a policy under discretion that replicates the optimal precommitment policy. It thus becomes an empirical issue whether discretion with an output gap change objective or discretion with an output gap objective yields better outcomes. Optimal discretionary policy with an output gap change objective will impart some persistence to output, unlike pure discretion, but it will no longer exactly replicate the optimal precommitment policy outcomes.

The first step in evaluating the consequences of assigning stabilizing the change in the output gap as an objective to a central bank that optimally implements discretionary policy is to express the model in state space form. The model consists of equations (1) and (2). For simplicity, the disturbance to the aggregate demand relationship (1) is set equal to zero; as is well know, this shock poses no issues of policy design and the nominal interest rate can be used to neutralize its affect on both the gap and inflation. In this case, the model can be written as

$$\begin{bmatrix} e_{t+1} \\ x_t \\ E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = A \begin{bmatrix} e_t \\ x_{t-1} \\ x_t \\ \pi_t \end{bmatrix} + BR_t + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{\sigma}{\beta} & 0 & \left(1 + \frac{\sigma\kappa}{\beta}\right) & -\frac{\sigma}{\beta} \\ -\frac{1}{\beta} & 0 & -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ \sigma \\ 0 \end{bmatrix}$$

Define $X_{1t} = [e_t, x_t]'$, $X_{2t} = [x_t, \pi_t]'$, $\chi_{t+1} = [\varepsilon_{t+1}, 0, 0, 0]'$, and let $Z_t = [X_{1t}, X_{2t}]'$. Then the system can be written compactly as

$$E_t Z_{t+1} = AZ_t + BR_t + \chi_{t+1} \tag{18}$$

The policy instrument R_t is set to minimize an objective function expressed as

$$L_k = E_t \sum \beta^i Z'_{t+i} Q_k Z_{t+i} \tag{19}$$

where Q_k depends on the specification of the single period loss function under policy regime k. Under pure discretion, denoted PD, this is simply $\pi_t^2 + \lambda x_t^2$ so

$$Q_{PD} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

With an output gap change objective, denoted by GC,

$$Q_{GC} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda & -\lambda & 0 \\ 0 & -\lambda & \lambda & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Under optimal discretionary policy regime $k = PD$, GC, the solution to the problem of minimizing (19) subject to (18) takes the form

$$R_t = -F_k X_{1t}$$

$$X_{1t} = M_{1k} X_{1t-1} + M_{2t} \chi_t$$

and

$$X_{2t} = M_{3k} X_{1t}$$

Details of the solution procedures are provided in Söderlind (1999) and in Jensen (1999).[7]

Table 3 presents the asymptotic loss obtained under the optimal discretionary policy with the central bank minimizing the social loss function (PD) and the optimal discretionary policy with an output gap change objective (GC), where the loss is expressed relative to the outcome under the optimal precommitment policy. Results are reported for various values of the policy preference parameter λ and the output gap elasticity of inflation κ.

For the benchmark parameter values ($\beta = 0.99$, $\lambda = 0.25$, $\kappa = 0.05$), social loss is lower in a discretionary policy environment when the central bank is assigned an objective involving the change in the output gap than when the central bank acts to minimize social loss. While the loss is not reduced to what could be achieved under precommitment, shifting to a gap change objective cuts the loss due to discretion by almost 30%. This gain arises

[7] Numerical calculations were carried using the MATLAB programs of Paul Söderlind.

from the persistence introduced by the change in the gap objective. Figure 3, which should be compared to Figures 1 and 2 shows that an output gap change objective generates persistence in the face of a temporary cost shock, but that the output gap is much more variable than under the optimal precommitment policy. This suggests that the advantages of GC over PD will fall if society places greater weight on output gap stabilization (i.e., a larger λ). This is verified in Table 3, which shows that the relative performance of pure discretion improves, for given κ (the output gap elasticity of inflation), as λ increases. Only for very small values of κ, however, does pure discretion dominate discretion with an output gap change objective.

The greater output gap variability under the GC policy also suggests that, in contrast to the case under pure discretion, policy under an GC objective might be improved if a weight-liberal central bank conducts policy – that is, a central bank who places relatively less weight on its inflation objectives. Such a central bank will produce greater stability in the change in the output gap and generate policy responses that would be closer to those called for under the optimal precommitment policy. This intuition will be verified in the next section.

Table 3: Loss relative to precommitment: PD and GC

	λ		
	0.1	0.25	0.5
$\kappa = 0.01$	2.14%, 4.73%	1.03%, 4.09%	0.49%, 3.57%
$\kappa = 0.05$	13.20%, 6.29%	8.42%, 6.13%	5.81%, 5.81%
$\kappa = 0.1$	23.49%, 6.16%	16.35%, 6.35%	11.87%, 6.24%

One interesting implication of Table 3 is that under pure discretion the loss relative to optimal precommitment varies more as the parameter κ varies than it does when there is an output gap change objective. The GC policy appears more robust with respect to uncertainty about the slope of the short-run output–inflation trade off.

4 Endogenous persistence

The forward looking model employed in the previous sections has been criticized for failing to match the short-run dynamics exhibited by inflation (Estrella and Fuhrer 1999). Specifically, inflation seems to respond sluggishly

and to display significant persistence in the face of shocks, while (2) allows current inflation to be a jump variable that can respond immediately to any disturbance. Equation (2) therefore would be unlikely to display the inertial behavior of inflation that is observed in the data (Nelson 1998). This section modifies the inflation adjustment equation to incorporate endogenous persistence by including the lagged inflation rate in (2). This results in a specification for inflation adjustment that more closely matches that used in recent empirical investigations, and is a modification that seems to be necessary if model simulations are to match the time series properties of actual inflation.[8]

The inflation adjustment equation (2) is altered, therefore, to incorporate a direct effect of lagged inflation on current inflation. In this case, equation (2) is replaced with

$$\pi_t = (1-\phi)\beta E_t \pi_{t+1} + \phi \pi_{t-1} + \kappa x_t + e_t \tag{20}$$

where $\phi \in [0,1]$ measures the importance of backward looking inertia in the inflation process.

The choice of ϕ can be critical in assessing outcomes under alternative policies. In a backward looking model (i.e., $\phi = 1$), Ball (1999) found evidence that nominal income growth targeting could produce disastrous results. McCallum (1997), however, showed that this was no longer the case when expectations played a role. Rudebusch (2000b) reached similar conclusions in his analysis of nominal income targeting, finding that it performed poorly for high values of ϕ.

The appropriate value of ϕ has been the source of controversy in the literature. Rudebusch (2000b) estimates an equation that takes the basic form of (20) and concludes that, for the U.S., ϕ is about 0.7. That is, he finds that most weight is placed on the lagged inflation term. This is consistent with Fuhrer (1997) who reports estimates of ϕ close to 1. Galí and Gertler (1999) argue that the coefficient on lagged inflation rate is small when a measure of marginal cost is used in place of the output gap, however. Much of the recent theoretical literature has adopted a value of $\phi = 0$, with only forward looking expectations entering. This was the form used in equation (2) and employed in the previous sections of this paper. Jensen (1999) sets $\phi = .3$ in his analysis of nominal income targeting, arguing that

[8] Galí and Gertler (1999) argue that the lagged inflation rate is not necessary when a measure of marginal cost is used in place of the output gap, however.

for policy evaluation it is appropriate to emphasize the role of forward looking expectations. McCallum and Nelson (2000) set $\phi = .5$.[9] I follow Jensen in adopting a value of 0.3 as a baseline. However, in this section, I evaluate output gap growth and pure discretion policies for values of ϕ ranging from zero to one. Baseline values of all the parameters are given in Table 4.

Table 4: Baseline parameter values

σ	λ	κ	ϕ	σ_e
1.5	0.25	0.05	0.3	0.015

When $\phi \neq 0$, the lagged inflation rate becomes an endogenous state variable. To solve the model and derive the optimal discretionary policies, the model is again written in state space form. This yields

$$E_t\bar{Z}_{t+1} \equiv \begin{bmatrix} e_{t+1} \\ x_t \\ \pi_t \\ E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \bar{A} \begin{bmatrix} e_t \\ x_{t-1} \\ \pi_{t-1} \\ x_t \\ \pi_t \end{bmatrix} + \bar{B}R_t + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \equiv \bar{A}\bar{Z}_t + \bar{B}R_t + \bar{\chi}_{t+1} \tag{21}$$

[9] The specifications in both Jensen and in McCallum and Nelson differ slightly from that used in equation (20). Jensen's inflation equation is (using my notation)

$$\pi_t = \beta(1-\phi)E_t\pi_{t+1} + \phi\pi_{t-1} + (1-\phi)\kappa x_t + e_t$$

while McCallum and Nelson assume

$$\pi_t = \beta(1-\phi)E_t\pi_{t+1} + \beta\phi\pi_{t-1} + \kappa x_t + e_t$$

Jensen's specification can be written as

$$\pi_t = (1-\phi)\pi_t^* + \phi\pi_{t-1} + e_t$$

where $\pi_t^* = \beta E_t\pi_{t+1} + \kappa x_t$. This specification can be obtained from the model of Galí and Gertler (1999), where ϕ is the fraction of "rule of thumb" price setters. Note that in this formulation, the output gap has no impact on inflation as $\phi \to 1$. Inflation is then just an exogenous random walk process, and the standard backward looking Phillips curve is not obtained in the limit as all price setters follow the rule of thumb behavior.

where

$$\bar{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \frac{\sigma}{\beta(1-\phi)} & 0 & \frac{\sigma\phi}{\beta(1-\phi)} & \left(1+\frac{\sigma\kappa}{\beta(1-\phi)}\right) & -\frac{\sigma}{\beta(1-\phi)} \\ -\frac{1}{\beta(1-\phi)} & 0 & -\frac{\phi}{\beta(1-\phi)} & -\frac{\kappa}{\beta(1-\phi)} & \frac{1}{\beta(1-\phi)} \end{bmatrix}, \qquad \bar{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sigma \\ 0 \end{bmatrix}$$

The loss functions again take the form $L_k = E_t \sum \beta^i \bar{Z}'_{t+i} \bar{Q}_k \bar{Z}_{t+i}$ for $k = PD,\ GC$. The Q matrices for this version of the model are given in the appendix. As in the previous subsection, the optimal discretionary policy is derived for each loss function. The equilibrium solutions for the output gap and inflation are then used to evaluate the asymptotic social loss.

Figure 4 plots the gain over pure discretion from assigning an output gap growth objective as a function of the coefficient on lagged inflation in the inflation adjustment equation (ϕ). The solid line shows the percentage gain as a function of ϕ when the central bank puts a weight λ on its output objective, the appropriate weight from the social loss function. For all values of $\phi < 0.7$, society gains from assigning an output gap change objective to the central bank. The gain increases as ϕ rises until it peaks at $\phi = 0.5$. It then declines. When inflation is predominately backward looking, $\phi > 0.7$, pure discretion designed to minimize social loss based on the output gap measure leads to a smaller asymptotic loss. This result is not surprising. The presence of forward looking expectations imparts persistence under a commitment policy that is missing under pure discretion. The GC policy imparts greater persistence in a way that captures the persistence under commitment. When inflation is completely backward looking, however, the distinction between optimal commitment and optimal discretion disappears. There can be no gain from distorting the central bank's loss function. When inflation is forward looking however, the potential for a gain exists.

So far, only one aspect of policy delegation has been considered – the definition of the appropriate output variable in the central bank's loss function. Policy also depends on the relative weight assigned to the bank's inflation and output objectives, and this may differ from the value of λ that appears in the social loss. Alternative policy regimes can be characterized by the objectives assigned to the central bank and the weights attached to each objective. Alternative regimes defined it this way will be called targeting regimes.

Definition 4 A targeting regime is defined by a) the variables in the central

bank's loss function (the objectives), and b) the weights assigned to these objectives. Policy is implemented under discretion to minimize the expected discounted value of the loss function.

An inflation targeting regime, for instance, will be defined by the assignment of the loss function $\pi_t^2 + \lambda_{IT} x_t^2$ to the central bank, where the weight λ_{IT} is chosen optimally to minimize the asymptotic social loss function. Similarly, an output gap change targeting regime is one in which the central bank's loss function is $\pi_t^2 + \lambda_{GCT} \left(x_t - x_{t-1}\right)^2$ with λ_{GCT} chosen to minimize asymptotic social loss.

A gird search is conducted over values of λ_k to obtain the optimal weight to assign the central bank. The dashed line in Figure 4 shows the percent gain of shifting from an inflation targeting regime to an output gap change regime when the optimal weight is used. For all ϕ, $\lambda_{IT} < \lambda < \lambda_{GCT}$; that is, under inflation targeting it is optimal to delegate to a conservative central bank, while with an output gap change objective, it is optimal to delegate to a liberal central bank. The results when the two targeting regimes are compared are qualitatively similar to the gain that was found when the central bank used a weight equal to that in the social loss function (the solid line in the figure). Unless inflation is predominately a backward looking process, a central bank that is concerned with changes in the output gap outperforms an inflation targeting bank, and a liberal central bank with a gap change objective outperforms a conservative central bank that minimizes the social loss function.

5 Model extensions and other targeting regimes

5.1 Serially correlated cost shocks

The previous section introduced persistence through the inclusion of lagged inflation in the inflation adjustment equation. An alternative specification is to return to the basic form of the inflation adjustment equation given by equation (2), that is, with $\phi = 0$, and allow the cost shock to be serially correlated. Clarida, Galí, and Gertler (1999) show that when the social loss function (3) is assigned, there is no role for a conservative central bank when the cost shock is serially uncorrelated. That is, the optimal value of λ_{IT} in this case is just λ. However, when e_t follows the $AR(1)$ process

$$e_t = \gamma_e e_t + \varepsilon_t \tag{22}$$

and $\gamma_e > 0$, there are gains from delegating to a conservative central bank.

Table 5 shows the optimal values of λ_{IT} and λ_{GCT} and the associated asymptotic social loss as a function of γ_e. Serially correlated cost shocks reduce the optimal value of λ_{IT}, making a conservative inflation targeter desirable. In contrast, increased cost shock persistence makes it optimal to delegate to a more liberal central bank under an GCT regime. As the table shows, however, delegation to a liberal central bank assigned inflation and output gap change objectives dominates delegation to a conservative central bank assigned inflation and output gap objectives regardless of the value of γ_e.

Table 5: Optimal Policy Weights and Loss Functions[10]

		Commitment		Inflation Targeting		Output Gap Growth Targeting	
		λ	L^c	λ_{IT}	Social loss	λ_{GCT}	Social loss
γ_e	0	0.25	2.055	0.25	2.228	0.65	2.113
	0.3	0.25	4.253	0.20	4.905	0.95	4.435
	0.6	0.25	15.349	0.10	20.108	1.65	16.273

5.2 Shifts in potential output and demand shocks

When potential output follows a deterministic trend, the change in the output gap is just real output growth relative to trend. In this case, the previous results under the GC and GCT regimes are equivalent to output growth (relative to trend) targeting regimes. When potential output is subject to stochastic shocks, output growth policies and policies that focus on the change in the gap will differ. Since policy objectives expressed in terms of inflation and output growth may be more transparent to the public than ones expressed in terms of the change in the gap, this subsection compares the two policies when potential output follows a persistent $AR(1)$ process.

In addition, Jensen (1999) recently reports that nominal income growth targeting may be superior to inflation targeting or to pure discretion. The intuition for this result is that nominal income growth targeting imparts an inertia to policy that is absent under pure discretion, and this inertia allows a

[10] Social loss is times 10^2.

nominal income growth targeting regime to achieve outcomes that are closer to the case under precommitment. Since this is the same rationale behind the superior performance of a policy based on output growth, it is of interest to compare nominal income growth and real income growth policies.

5.2.1 The modified model

In the previous sections, the basic model could be kept quite simple since only the output gap and inflation were relevant and only cost shocks generated a policy trade off that posed interesting issues of policy design. Under nominal income targeting or output growth targeting, however, shocks to potential output will induce policy responses. Thus, to compare outcomes under different delegation schemes, the model needs to be enriched to incorporate other possible disturbances that may affect the economy differently under alternative policy regimes.

Two changes are made to the model of section 4. First, a backward looking element in the form of lagged output is added to the aggregate demand relationship. Expressed in terms of the output gap, this yields

$$x_t = \theta x_{t-1} + (1-\theta)E_t x_{t+1} - \sigma(R_t - E_t\pi_{t+1}) + \mu_t \tag{23}$$

where

$$\mu_t = u_t - \bar{y}_t + \theta\bar{y}_{t-1} + (1-\theta)E_t\bar{y}_{t+1}$$

The demand shock u_t is assumed to be serially correlated and follows the $AR(1)$ process

$$u_t = \gamma_u u_{t-1} + \eta_t \tag{24}$$

Second, potential real output is assumed to follow an $AR(1)$ process:

$$\bar{y}_t = \bar{\gamma}\bar{y}_{t-1} + \xi_t \tag{25}$$

The innovation processes η_t and ξ_t are assumed to be white noise, zero mean processes that are mutually uncorrelated and uncorrelated with the cost shock innovation ε_t. The model consistent of equations (20), (22), (23), (25), (24), and (25). This makes the model almost identical to the one employed by Jensen (1999).[11]

[11] As noted earlier, Jensen's specification of the inflation adjustment equation with lagged inflation differs slightly from the one used here.

Noting that $E_t\bar{y}_{t+1} = \bar{\gamma}\bar{y}_t$, μ_t can be written as $\mu_t = u_t - (1-(1-\theta)\bar{\gamma})\,\bar{y}_t + \theta\bar{y}_{t-1}$. The state-space form of the entire model is then

$$\begin{bmatrix} u_{t+1} \\ \bar{y}_{t+1} \\ \bar{y}_t \\ e_{t+1} \\ x_t \\ \pi_t \\ E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \hat{A} \begin{bmatrix} u_t \\ y_t^t \\ \bar{y}_{t-1} \\ e_t \\ x_{t-1} \\ \pi_{t-1} \\ x_t \\ \pi_t \end{bmatrix} + \hat{B}R_t + \begin{bmatrix} \eta_{t+1} \\ \xi_{t+1} \\ 0 \\ \varepsilon_{t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where

$$A = \begin{bmatrix} \gamma_u & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{\gamma} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_e & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{1}{1-\theta} & \frac{1-(1-\theta)\gamma_u}{1-\theta} & -\frac{\theta}{1-\theta} & \frac{\sigma}{\beta(1-\phi)} & -\frac{\theta}{1-\theta} & \frac{\sigma\phi}{\beta(1-\phi)} & \left(\frac{1}{1-\theta} + \frac{\sigma\kappa}{\beta(1-\phi)}\right) & -\frac{\sigma}{\beta(1-\phi)} \\ 0 & 0 & 0 & -\frac{1}{\beta(1-\phi)} & 0 & -\frac{\phi}{\beta(1-\phi)} & -\frac{\kappa}{\beta(1-\phi)} & \frac{1}{\beta(1-\phi)} \end{bmatrix}$$

and $B' = [0\ 0\ 0\ 0\ 0\ 0\ \frac{\sigma}{1-\theta}\ 0]'$. Define $\hat{X}_{1t} = [u_t,\ \bar{y}_t,\ \bar{y}_{t-1},\ e_t,\ x_{t-1},\ \pi_{t-1}]'$, $\hat{X}_{2t} = [x_t,\ \pi_t]'$, and $\hat{\chi}_{t+1} = [\eta_{t+1},\ \xi_{t+1},\ 0,\ \varepsilon_{t+1},\ 0,\ 0,\ 0]'$. Then the system can be written as

$$E_t\hat{Z}_{t+1} = \hat{A}\hat{Z}_t + \hat{B}R_t + \hat{\chi}_{t+1} \tag{26}$$

where

$$\hat{Z}_t \equiv \begin{bmatrix} \hat{X}_{1t} \\ \hat{X}_{2t} \end{bmatrix}$$

The new parameters appearing it this extended model are the serially correlation coefficients γ_u and $\bar{\gamma}$, the weight on the lagged output gap in the aggregate demand equation, θ, and the variances of the innovations to demand and potential output. None of these parameters affects policy choice or the social loss under the policies considered earlier. These policies, and the social loss function, involved only the output gap and inflation. The stochastic process followed by potential output did affect equilibrium output but not the output gap or inflation. The structure of the aggregate demand

relationship did affect the rule for the nominal interest rate needed to achieve given values of the output gap and inflation, but it did not alter the equilibrium for either the gap or for inflation. This separation will no longer be true for some of the policy structures to be considered below, so we now need to parameterize the complete model.

Benchmark values are listed in Table 6. The values are those used by Jensen (1999).

Table 6: Baseline parameter values for extended model

σ	λ	κ	ϕ	θ	
1.5	0.25	0.05	0.3	0.5	
σ_e	σ_u	σ_y	γ_e	γ_u	γ_y
0.015	0.015	0.005	0	0.3	0.97

5.2.2 Policy regimes and loss functions

A total of seven alternative policy regimes are considered. These differ from one another in terms of the loss function the central bank is assumed to minimize. All seven regimes assume that the central bank operates with discretion. Four of the regimes, pure discretion, inflation targeting, output gap change, and output gap change targeting, have already been defined. The three new regimes are output growth targeting and two versions of nominal income growth targeting. The regimes and their single period loss functions are described in Table 7.

Table 7: Alternative policy regimes

Regime name		Loss function
Pure discretion	PD	$\pi_t^2 + \lambda x_t^2$
Inflation targeting	IT	$\pi_t^2 + \lambda_{IT}^* x_t^2$
Output gap growth	GC	$\pi_t^2 + \lambda \left(x_t - x_{t-1}\right)^2$
Change in gap growth targeting	GCT	$\pi_t^2 + \lambda_{GCT}^* \left(x_t - x_{t-1}\right)^2$
Output growth targeting	OGT	$\pi_t^2 + \lambda_{OGT}^* \left(y_t - y_{t-1}\right)^2$
Nominal income growth targeting	NIT	$\pi_t^2 + \lambda_{NIT}^* \left(\pi_t + y_t - y_{t-1}\right)^2$
Modified nominal income targeting	MNIT	$\pi_t^2 + \lambda x_t^2 + \lambda_{MNIT}^* \left(\pi_t + y_t - y_{t-1}\right)^2$

The nominal income targeting regime, NIT, is defined in a manner consistent with the other targeting regimes – that is, the central bank's objective contains inflation variability and nominal income growth variability, with the weight on nominal income growth chosen optimally. For the baseline parameter values, NIT performs poorly (social loss is almost 30% higher than under precommitment for the baseline parameter values and it does worse than even pure discretion). The modified nominal income targeting regime assumes the central bank is concerned with social loss $(\pi_t^2 + \lambda x_t^2)$ and in addition with nominal income growth variability. This specification is more similar to Jensen's definition of nominal income growth targeting, although it still differs from his. Jensen assumes the central bank's loss function is $(1+f)\pi_t^2 + \lambda x_t^2 + \lambda_{MNIT}(\pi_t + y_t - y_{t-1})^2$ where both f and λ_{MNIT} can be chosen optimally. To maintain closer comparibility with the other regimes which have only one free parameter, I set $f = 0$.

As before, each of the loss functions can be expressed as

$$E_t \sum_{i=0}^{\infty} \beta^i \hat{Z}'_{t+i} \hat{Q}_k \hat{Z}_{t+i}$$

for a suitably defined matrix $\hat{Q}_k$. The $\hat{Q}_k$ matrices corresponding to the different regimes are given in the appendix.

5.2.3 Evaluation

Each of the seven alternative policy regimes is evaluated for the baseline parameters and for several permutations from these baseline values. Results are reported in Table 8 which gives the asymptotic social loss under each regime. For comparison, the loss under the optimal precommitment policy (denoted PC) is also shown. For each column, the social loss under the regime yielding the lowest loss appears in bold.

Table 8: Alternative policy regimes

	(1) Baseline	(2) $\sigma_y = 0.01$	(3) $\kappa = 0.01$	(4) $\kappa = 0.1$	(5) $\lambda = 0.1$	(6) $\lambda = 0.5$	(7) $\phi = 0.6$
PC	4.315	4.315	5.335	3.466	3.783	4.744	14.376
PD	5.167	5.167	5.515	4.435	4.755	5.416	17.150
IT	5.114	5.114	5.505	4.360	4.718	5.302	15.597
GC	4.664	4.664	5.508	3.641	4.010	4.791	15.207
GCT	4.457	4.457	5.354	3.594	3.924	4.770	14.553
OGT	4.531	4.737	5.584	3.636	3.946	4.957	14.607
NIT	5.547	5.836	12.149	3.814	3.924	8.206	18.076
MNIT	4.731	4.817	5.503	3.621	3.846	5.202	14.438

With the baseline parameter values, targeting the change in the output gap (output gap change targeting) yields the lowest social loss of any of the discretionary regimes. It comes within about 3% of the precommitment loss (4.457 vs. 4.315). Output growth targeting is slightly worse (at 4.531) because shifts in potential output affect policy through their impact on output growth, although such shocks would not induce a response under an optimal precommitment policy. Still, targeting the growth rate of output is the second best discretionary regime and does significantly better than either pure discretion or inflation targeting. Both GCT and OGT are superior to both forms of nominal income growth targeting regime.

Column 2 of Table 8 shows the impact of doubling the variance of shocks to potential output. The first five regimes depend only on inflation and the output gap, so none of these are affected by this change. However, policy regimes based on output growth or nominal income growth are affected. Policy based on output growth remains superior to either of the nominal income based regimes in the face of this change.

I next consider alternative values of the output gap elasticity of inflation, κ. For both smaller values of this elasticity (col. 3) and larger values (col. 4), the GCT policy continues to yield the lowest social loss. The modified nominal income targeting regime is next best in both cases.

As we saw earlier, variations in the social weight λ on output gap stabilization can affect the relative performance of pure discretion and output gap change policies. Column 5 reports results for a smaller value of this weight, while column 6 does so for a larger value. The nominal income growth regimes do much better when λ is small, with $MNIT$ producing the smallest social

loss. GCT is next though (tied with NIT), while GCT is the best regime when λ is set to twice the baseline value.

Finally, the last column of Table 8 shows the impact of increasing the weight on lagged inflation in the inflation adjustment equation. As was the case when λ took on a small value, MIT is best with GCT second when ϕ is increased to 0.6.

To summarize, except for inflation processes that are primarily backward looking, or social loss functions that place little weight on output stabilization, the targeting regime based on inflation and the change in the output gap dominates the other regimes.

6 Conclusions

Previous work on monetary policy in forward looking New Keynesian models has focused on optimal simple rules under the assumption that the central bank is able to commit to a rule. In this paper, I have assumed that the relevant policy regime is one of discretion, and the problem faced in designing policy is to assign a loss function to the central bank. The approach is one used by Jensen to examine nominal income growth targeting and is consistent with the contracting approach employed by Persson and Tabellini (1993), Walsh (1995), and Svensson (1997), although that earlier literature was concerned mainly with the average inflation bias that could arise under discretion.

In a forward looking New Keynesian model it was shown that the optimal, timeless perspective precommitment policy could be achieved by a totally myopic, discretionary central bank if the bank was assigned an output gap change objective rather than an output gap objective. While virtually all the recent literature has assumed that a social loss function dependent on inflation and the output gap is the appropriate objective of policy, discretionary policy with such a social loss function imparts too little persistence to output and inflation. A policy aimed at stabilizing the change in the output gap (together with inflation) imparts the socially optimal degree of persistence when the central bank is myopic. Previous authors have often introduced ad hoc interest rate smoothing objectives in order to generate this greater policy persistence.

When cost shocks are serially correlated or there is endogenous persistence in the model (via the presence of lagged inflation in the inflation adjustment

equation), discretion with an output gap change objective no longer coincides with the optimal precommitment policy. Simulations suggested that delegation to a liberal central bank with a gap change objective dominates delegation to a conservative central bank with an output gap objective except when forward looking expectations are relatively unimportant.

Policy regimes based on the change in the gap were also compared to alternative targeting regimes such as inflation targeting and nominal income growth targeting. Except for cases involving a low social weight on output stabilization or a small role for forward looking expectations in the inflation process, output gap change targeting proved superior to other regimes. These findings may explain why the FOMC in its policy press releases appears to focus on the change in the output gap in justifying policy actions rather than on the level of the gap as the previous literature has implicitly assumed.

Appendix

A1. Model foundations

Equations (1) and (2) can be interpreted as governing the dynamic adjustment of the economy around the steady-state equilibrium. In this appendix, the general equilibirum model structure that leads to these equations is specified. For simplicity, the model ignores government.

Households

The preferences of the representative household are defined over a composite consumption good C_t, real money balances M_t/P_t, and leisure $1 - N_t$. Households maximize the expected present discounted value of utility:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \frac{\gamma}{1-b} \left(\frac{M_t}{P_t} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]$$

The composite consumption good consists of differentiate products produced by monopolistically competitive final goods producers (firms). There are a continuum of such firms of measure 1. C_t is defined as

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \qquad \theta > 0$$

Given prices p_{jt} for the final goods, the houshold's demand for good j is

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} C_t$$

where the aggregate price index P_t is defined as

$$P_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

The budget constraint of the household is, in real terms,

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + R_{t-1} \frac{B_{t-1}}{P_t} + \Pi_t$$

where M_t (B_t) is the household's nominal holdings of money (one period bonds). Bonds pay a gross nominal rate of interest given by R_t. Real profits received from firms are equal to Π_t.

In addition to the demand functions for the individual goods, the following first order conditions must hold in equilibrium:

$$C_t^{-\frac{1}{\sigma}} = \beta E_t \left(\frac{R_t P_t}{P_{t+1}} \right) C_{t+1}^{-\frac{1}{\sigma}} \tag{27}$$

$$\frac{\left(\frac{M_t}{P_t}\right)^{-b}}{C_t^{-\frac{1}{\sigma}}} = \frac{R_t - 1}{R_t} \tag{28}$$

$$\frac{\chi N_t^{\eta}}{C_t^{-\frac{1}{\sigma}}} = \frac{W_t}{P_t} \tag{29}$$

Firms

Following the literature on staggered price setting, we adopt a Calvo specification in which the probability a firm adjusts its price each period is given by $1 - \omega$. If firm j sets its price at time t, it will do so to maximize expected profits, subject to the production technology

$$c_{jt} = N_{jt}^{a}$$

where N_{jt} is employment by firm j in period t.

Let φ_t denote the firm's real marginal cost (equal to $W_t / a P_t N_t^{a-1}$). The firm's decision problem then involves picking p_{jt} to maximize

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt+i}}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left(\frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i}$$

where the discount factor $\Delta_{i,t+i}$ is given by $\beta^i (C_{t+i}/C_t)^{-\frac{1}{\sigma}}$. The first order condition is

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[(1-\theta) \left(\frac{1}{p_{jt}} \right) \left(\frac{p_{jt}}{P_{t+i}} \right)^{1-\theta} + \theta \varphi_{t+i} \left(\frac{1}{p_{jt}} \right) \left(\frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i} = 0$$

Since all firms adjusting in period t set the same price, let p_t^* be the optimally set price at time t. Then,

$$\left(\frac{p_t^*}{P_t} \right) = \left(\frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\varphi_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\theta} C_{t+i} \right]}{E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{P_{t+i}}{P_t} \right)^{\theta-1} C_{t+i} \right]} \tag{30}$$

The aggregate price index is

$$P_t^{1-\theta} = (1-\omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta} \tag{31}$$

Using the definition of $\Delta_{i,t+i}$, equation (30) becomes

$$\left(\frac{p_t^*}{P_t}\right) = \left(\frac{\theta}{\theta-1}\right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\frac{1}{\sigma}} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\frac{1}{\sigma}} \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}}$$

Equilibrium and the approximation

Equilibrium paths for output, consumption and prices are given by equations (27) (30), and (31). Because the nominal interest rate is treated as the monetary policy instrument, equation (28) simply determines the nominal quantity of money in equilibrium.

Let $\hat{x}_t$ denote the percent deviation of X around its flex-price equilibrium. Equations (27) and (29) can be approximated as

$$\hat{c}_t = E_t \hat{c}_{t+1} - \sigma \left(\hat{r}_t - E_t \hat{\pi}_{t+1}\right) \tag{32}$$

$$\eta \hat{n}_t + \frac{1}{\sigma} \hat{c}_t = \hat{w}_t - \hat{p}_t$$

Real marginal costs is then

$$(1-a+\eta)\hat{n}_t + \frac{1}{\sigma}\hat{c}_t = \left(\frac{1-a+\eta}{a} + \frac{1}{\sigma}\right)\hat{c}_t \equiv \gamma \hat{c}_t$$

Finally, the price adjustment equation (30) can be approximated as

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{c}_t \tag{33}$$

where real marginal cost is $\hat{\varphi}_t - \hat{p}_t = \gamma \hat{c}_t$, and

$$\kappa = \gamma \left(\frac{(1-\omega)\left[1-\omega\beta\right]}{\omega}\right)$$

Equations (32) and (33) are the basis for equations (1) and (2) of the text.

A2. The loss functions

For the model given by equation (21), the weighting matrices for the loss functions corresponding to pure discretion and an output gap growth objective are

$$Q_{PD} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_{GC} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & -\lambda & 0 \\ 0 & -\lambda & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The policy weighting matrices for the model given by equation (26) are

$$Q_{PD} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_{IT} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{IT} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_{GC} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & -\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_{GCT} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_{GCT} & 0 & -\lambda_{GCT} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda_{GCT} & 0 & \lambda_{GCT} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_{OGT} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_{OGT} & -\lambda_{OGT} & 0 & -\lambda_{OGT} & 0 & \lambda_{OGT} & 0 \\ 0 & -\lambda_{OGT} & \lambda_{OGT} & 0 & \lambda_{OGT} & 0 & -\lambda_{OGT} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_{OGT} & \lambda_{OGT} & 0 & \lambda_{OGT} & 0 & -\lambda_{OGT} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_{OGT} & -\lambda_{OGT} & 0 & -\lambda_{OGT} & 0 & \lambda_{OGT} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_{NIT} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_{NIT} & -\lambda_{NIT} & 0 & -\lambda_{NIT} & 0 & \lambda_{NIT} & \lambda_{NIT} \\ 0 & -\lambda_{NIT} & \lambda_{NIT} & 0 & \lambda_{NIT} & 0 & -\lambda_{NIT} & -\lambda_{NIT} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_{NIT} & \lambda_{NIT} & 0 & \lambda_{NIT} & 0 & -\lambda_{NIT} & -\lambda_{NIT} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_{NIT} & -\lambda_{NIT} & 0 & -\lambda_{NIT} & 0 & \lambda_{NIT} & \lambda_{NIT} \\ 0 & \lambda_{NIT} & -\lambda_{NIT} & 0 & -\lambda_{NIT} & 0 & \lambda_{NIT} & 1+\lambda_{NIT} \end{bmatrix}$$

$$Q_{MNIT} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_{MNIT} & -\lambda_{MNIT} & 0 & -\lambda_{MNIT} & 0 & \lambda_{MNIT} & \lambda_{MNIT} \\ 0 & -\lambda_{MNIT} & \lambda_{MNIT} & 0 & \lambda_{MNIT} & 0 & -\lambda_{MNIT} & -\lambda_{MNIT} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_{MNIT} & \lambda_{MNIT} & 0 & \lambda_{MNIT} & 0 & -\lambda_{MNIT} & -\lambda_{MNIT} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_{MNIT} & -\lambda_{MNIT} & 0 & -\lambda_{MNIT} & 0 & \lambda_{MNIT} + \lambda & \lambda_{MNIT} \\ 0 & \lambda_{MNIT} & -\lambda_{MNIT} & 0 & -\lambda_{MNIT} & 0 & \lambda_{MNIT} & 1 + \lambda_{MNIT} \end{bmatrix}$$

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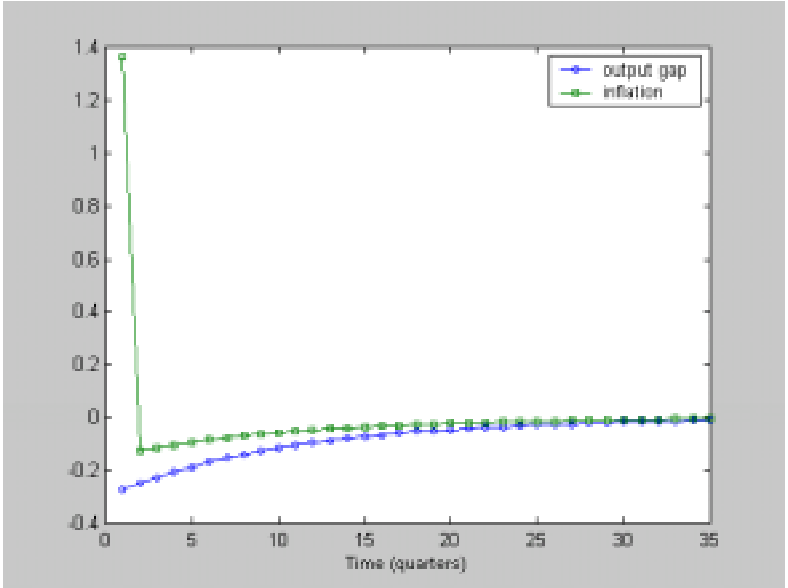

Figure 1: Responses to a Temporary Cost Push Shock under the Optimal Precommitment Policy

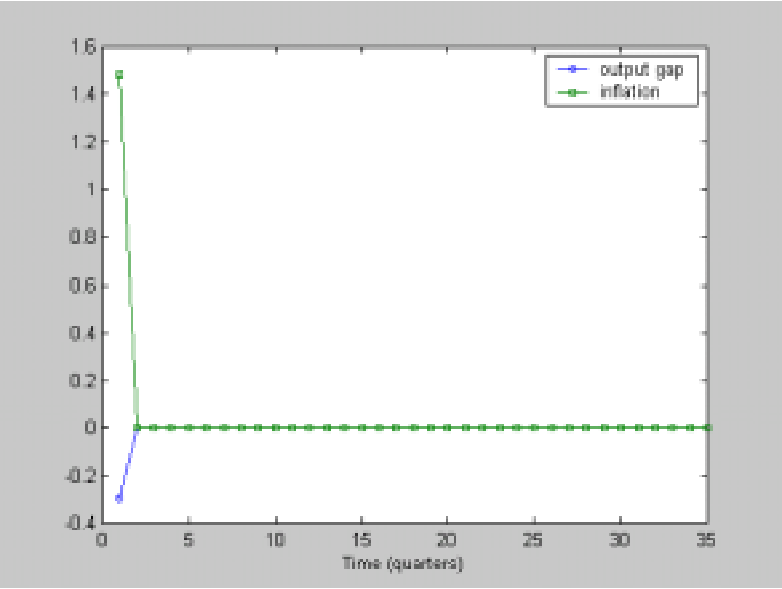

Figure 2: Responses to a Temporary Cost Push Shock under the Pure Discretionary Policy

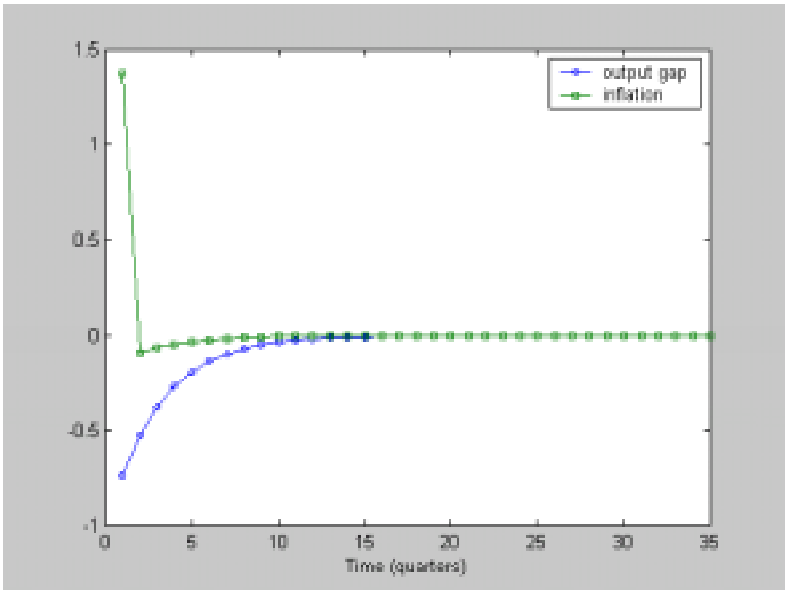

Figure 3: Responses to a Temporary Cost Push Shock under the *OGG* Policy

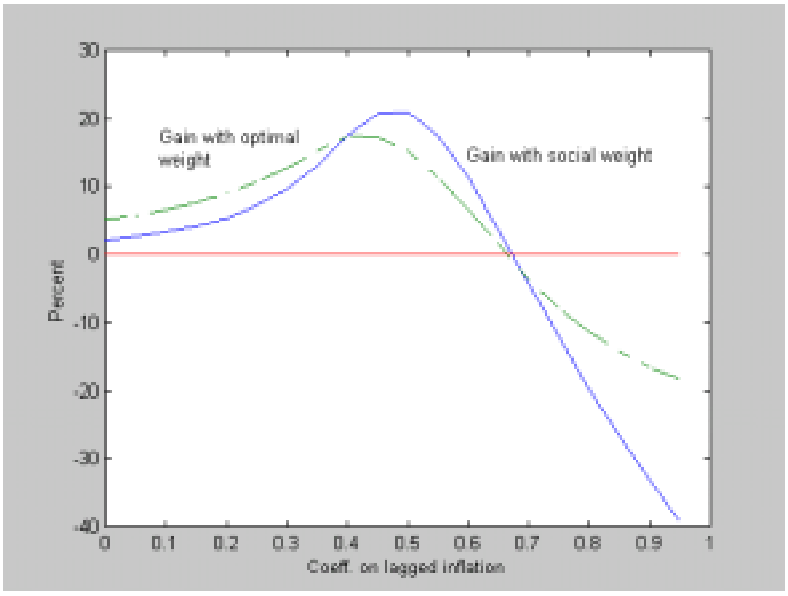

Figure 4: Percent gain from discretion with an output growth objective relative to discretion with a social loss function