Expected Survival Time of an Exchange Rate Band with Intra marginal Interventions

by

YasarBarut

Department of Economics, Rice University, Houston, T.X. 77025. E-mail: ybarut@rice.edu, Fax: 713.285.5278

A bstract

The size of a speculative attack is calculated for an exchange rate target zone with intra marginal interventions. Using these values, we calculate the expected survival time of a target zone. We see that this yields more reasonable values for a unilateral target zone compared to a target zone model with marginal interventions. The expected survival time does not depend on the bandwidth and the speculative attacks can happen at any point inside the band with almost equal probability. We also complete some sensitivity analysis and apply our method to obtain the survival time of the "Swedish" target zone.

JEL: E52, F31, F33

Key words: Target zone, intramarginal interventions, mean reversion, speculative attacks, regime switching

1 Introduction

We calculate the expected survival time of a target zone when the interventions of the central bank are intra-marginal. The ... rst structural modeling of the exchange rate dynamics in a target zone by Krugman [14] assumed that the central bank intervenes the exchange rate only at the boundaries. However, subsequent tests of this model rejected its implications (e.g. Flood, Rose and Wathieson [8] and Lindberg and Sockerlind [17]). It has been observed that the intra-marginal interventions model discussed by Delgado and Droot and Obstfeld [11] ... to exchange rates dynamics in a target zone better. A liso the data released by the central banks (see, for example, Ciavazzi and Ciovannini [12] and Lindberg and Sockerlind [14]) indicate that the interventions are intra-marginal.

A central bank can defend a target zone incle...nitely, for example, by increasing the interest rate to such dently high values. If owever, typically a target zone is used to create a stable exchange rate which, in turn, contributes to the stability of internal macro variables. But a very high interest rate to defend the band contradicts the original purpose of implementing a target zone since it may include instability in the economy. So we expect that the central bank set aside a ... xed amount of reserves to defend the band. It is discussed by Krugman and R otemberg [15] even when the amount of reserves are known by the market, as long as this does not a ect the dynamics of the exchange rate inside the band, we still expect a credible target zone if the reserves are such dently high. If owever after some time the reserves will deplete and there will be a speculative attack on the exchange rate. The target zone will either be realigned or collapse to a free ‡ cating regime. In this paper, we assume the latter:

Previously Flood and 6 arber [9] Buiter [3], among others, calculated the expected survival time of axed exchange rate regime, which can be regarded as a narrow exchange rate band. If umas and Svenson [7] did similar calculations for a target zone with marginal interventions. They indicated that their calibrated calculations do not seem to ... the real life experience. The survival time in some cases for a reasonable amount of reserves is in order of centuries. In this paper we attempt to see if the survival time is more feasible if we use the intramarginal interventions model which ... to the real life target zone experience better. We observe that there is a substantial improvement in the expected survival time. We also observe that the survival time does not change substantially with the size of the band. The collapse can happen at any exchange rate away from the center of the band with almost equal chance. Hence a speculative attack can happen almost anywhere in the band with equal probability except at the boundaries where there is a higher chance of attack. This is of course contradictory to Krugman's model where the collapse can only happen at the boundaries but it is in accordance with the previous target zone experiences.

In the next section we discuss target zone models. In Section 3 we show how to calculate the expected survival time of a target zone with intra marginal interventions. We callibrate our approach to the parameters of the Swedish target zone experience in Section 4. We discuss possible extensions in the last section.

2 Target I one II odels

2.1 Il arginal Interventions

In a target zone, the exchange rate of a country is restricted between two values, \underline{x} and \overline{x} , by the central bank of the country. The research on various aspects of a target zone took or after Krugman [14] modeled the exchange rate dynamics in a target zone. In his model, the logarithm of the exchange rate of home country x, the value of foreign money in terms of domestic money, is a function of its fundamental f and the expected rate of change in the exchange rate

$$x_t = f_t + {}^{\tiny{\text{\tiny R}}}E\left[c x_t \right] = c t \tag{1}$$

where ® is the semi-elasticity of the exchange rate with respect to the expected exchange rate depreciation. The fundamental is de...ned as

$$f_t = m_t + v_t (2)$$

If ere $m_t = ln(R_t + D_t)$ is the logarithm of the money stock, where R_t and D_t are the foreign reserves and the domestic credit, and the velocity v_t is an exagenous monetary shock. The money stock changes only at the boundary of the target zone

$$dm_t = d_{t,i} dl_{t}; (3)$$

where d_t and d_{lt} are positive only when $x = \underline{x}$ and $x = \overline{x}$ respectively¹. We will assume that commodity prices are ‡exible, purchasing power parity and uncovered interest rate parity hdd, and there is full capital mobility.

0 riginally, Krugman assumed that the interventions are made at the boundaries of the target zone (marginal interventions) and they are in...nitesimal. In this case we assume that v_t is a B rownian motion with a non-zero constant drift;

$$dV_{t} = {}^{1}dt + {}^{3}dB_{t}; \tag{4}$$

where B_t is the standard B rownian motion. The solution to this system is calculated by, among others, Krugman [14] for $^{-1} = 0$ and by D elgado and D umas [4] for the general case. The log of the exchange rate is given as a function of the fundamental;

$$X_{t} = f_{t} + ^{\otimes 1} + Ae^{1f_{t}} + Be^{2f_{t}}$$
 (5)

where

$$_{\text{$_{21;2}$}} = \frac{i^{-1\,\$} \, \cdot \cdot \, \, \boldsymbol{P}_{\overline{12\,\$2\,+\,\,3\!\!/\!\!4^2}}}{3\!\!/\!\!4^{2\,\$}} ;$$

¹S venssor[20] has a complete nontechnical treatise of target zone models.

The constants A and B; and the bounds on the fundamentals² can be obtained by using the boundary conditions

$$x(\overline{f}) = \overline{x}; \qquad x(\underline{f}) = \underline{x}; \qquad \frac{ck}{cf}(\overline{f}) = \frac{ck}{cf}(\underline{f}) = \emptyset :$$
 (6)

The last two equalities follow from the 'smooth pasting conditions. The explicit expressions for \overline{f} and \underline{f} can be found in D elgado and D umas [4]. The constants A and B are given in Froot and 0 bstfeld [11] or in Krugman and R otemberg [15]. The free ‡ cast solution to this system corresponds to "no bubble" case, i.e., $A = B = \emptyset$, and $x_t = f_t + {}^{\otimes 1}$:

2.2 Intra marginal Interventions

Using the data from EMS and the Mardic countries, Krugman's original target zone model has been tested. The model above implies that the distribution of the exchange rate within the band must be W-shaped. But the data shows that the distribution is hump-shaped. All so the nonlinear relation between the exchange rate and the fundamental cannot be shown to exist. It is also known that interventions made by the central banks are not only at the boundaries but also inside the target zone. For example, Lindberg and Soderlind [14] discuss the interventions made by the Riksbank and indicate that intra marginal intervention is a rule rather than an exception. Hence extensions to the original model seem necessary.

A mean reverting stochastic process for the fundamental would generate a hump-shaped exchange rate distribution. A Iso, the exchange rate function will be almost linear in fundamentals. L indoergand Soderlind [14], considering an intra marginal intervention policy, managed to obtain a hump-shaped distribution with some mass at the edges of the band. The overall ... t of their model to the data was quite good compared to the marginal interventions model.

Here, we will use an AR (1) process to model the exchange rate dynamics in the band. Equation (1) still demons the relation between the exchange rate and the fundamental. The fundamental is given by (2). The drift of the velocity is now zero, $dv_t = \frac{3}{4}dS_t$: Since the intervention policy is changed, the equation (3) will be dimerent. We assume that the monetary authority will intervene continuously with increasing size as the exchange rate moves away from some preferred level x_c^3 .

$$dm_t = i \% [f_t i f_c] dt + dl_t i dl_t$$
 (7)

½ is a policy parameter. d_t and d_t are same as in (3) and f_c corresponds to the preferred exchange rate level, $x_c = x_t(f_c)$.

²D elgado and D umas [4] shows that there is a monotonic relation between the band on the fundamentals and the band on the exchange rate

³T his corresponds to supply shock interpretation of the mean reverting process as discussed by D elgado and D umas [4]

Delgado and Dumas [4] and Froot and Obstfeld [10] found the solution to this system as

$$x_{t} = \frac{f_{t} + \sqrt[8]{2}f_{C}}{1 + \sqrt[8]{2}} + AM \frac{\mu}{2\sqrt[8]{2}}; \frac{1}{2}; \frac{2}{t} + BM \frac{\mu}{2\sqrt[8]{2}}; \frac{3}{2}; \frac{2}{t}; \frac{2}{t};$$
 (8)

where $\hat{f}_t = p^p \mathbb{Z}(f_{c|i} \mid f_t) = \mathbb{Z}$ and \mathbb{Z} (\$\phi \phi \phi) is Krummer's function.

 $_{i}$ is the gamma function, $_{i}$ (z) = $_{0}^{R_{1}}$ $x^{z_{i}}$ 1 e^{i} x cx, z > 0. A and B are the constants of integration to be determined by the boundary conditions (4) and the condition $x_{c} = x_{t}(f_{c})$. The free ‡ cat solution in this case is $x_{t} = f_{t}$.

3 Speculative Attacks and the Collapse of a Target 1 one

In this paper, we will consider the collapse of a target zone into a free ‡ cat regime. It will be assumed that the reserves of the central bank are limited but still enough to support a credible target zone⁴. Since the central bank has to intervene the exchange rate with limited reserves, after a while the reserves will deplete and the target zone will lose its credibility. This, in turn, will cause a speculative attack. It is possible to calculate the minimum amount of reserves, which is equal to the amount of the speculative attack at the moment of collapse, necessary to defend the target zone. While Krugman [13] and Flood and Carber [9] did this for ... xed regimes, Krugman and Rotemberg [15] locked at the one sided target zone⁵. Finally, Dielgob and Diumas [5] obtained the level of reserves that triggers the speculative attack for the two sided target zone for the marginal interventions model. In this section, after we show the minimum reserves for the marginal intervention model, we will obtain the same for the model with intra-marginal intervention.

The expectation of the foreign reserves falling to zero or to some unacceptable high level it may trigger a buying attack or a selling attack. Since the reserve level in the marginal intervention case changes only at the boundaries, the speculative attacks will occur at the boundaries. Since it is possible to forecast the attack by observing the reserve level, at the time of the collapse there will not be any change in the exchange rate level otherwise there would be an arbitrage opportunity. If ence only the fundamental will change at the collapse. Just before the collapse, the fire \ddagger cast exchange rate corresponding to the fundamental \overline{f} is $\overline{f} + {}^{\otimes 1}$, right after the collapse the exchange rate will stay \overline{x} . If we assume the collapse is instantaneous the only change in the fundamental is the change

⁴Krugman and R otemberg [15] discusses the necessary amount of reserves to sustain a credible target zone and how that is related to 'smooth pasting condition.

⁵0 nesided zone has anly upper a lower bound and the central bank, in the case of marginal interventions, intervenes only at one end.

⁴⁰ therwise, it would not make sense to use a credible target zone model.

in the reserves. Therefore the size of the speculative attack is

$$\bar{a} = \bar{f} + {}^{\otimes 1}i \bar{x}$$

We can also calculate the size of the selling attack at the lower boundary as

$$\underline{a} = \underline{x}_{i} \,^{\otimes 1}_{i} \, \underline{f} : \tag{10}$$

Therefore whenever the reserves fall below \bar{a} and $x = \bar{x}$, there will be a buying attack and the target zone will collapse. Similarly, if the reserves are more than \dot{r}_i and $x = \bar{x}$, there will be a selling attack and the target zone will collapse.

In the case of intra marginal interventions, the reserves change at any point inside the band. Therefore, the reserves can fall below (or raise above) a critical value at any exchange rate. If, at time t, the fundamental is f_t and the exchange rate is f_t , then the free ‡ cat fundamental is f_t . Since after the collapse there is no expected change in the exchange rate, the change in the fundamental will be f_t , f_t , which is the critical value to trigger a speculative attack at that exchange rate. If ence the reserves at the time of an attack are given by

$$w_{t}(f) = \frac{{}^{\otimes} \frac{1}{2} f_{t}}{1 + {}^{\otimes} \frac{1}{2}} \left[A M \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A M = \frac{1}{2 {}^{\otimes} \frac{1}{2}} \left[\frac{1}{2} \right] A$$

Ifm < 1, there is a selling attack.

The adlapse time of a target zone with the marginal interventions is obtained by D umas and Svensson [7]. During the operation of a target zone it must be true that

$$\underline{a} \cdot \ m_t \cdot \ t^*_i \ \overline{a} \quad \ \ and \quad \ \underline{f} \cdot \ f_t \cdot \ \overline{f} :$$

This implies that the velocity $v_t = f_{t,i}$ m_t must satisfy

$$\underline{f}_i$$
 (f_i \underline{a}) v_t \overline{f}_i \underline{a}

The collapse will occur when v_t hits one of these bounds. Hence, Delgado and Svensson [7] indicated, the collapse time is equivalent to the ... rst passage time of a B rownian motion. The survival expected time T, conditional upon initial levels r_0 and f_0 , is given by

$$E[T j f_0; r_0] = (v_0; (\underline{f}; (\underline{f}; \underline{a}))) \Phi(\overline{f}; \underline{a}; v_0) = \frac{1}{4}^2; for^{-1} = 0;$$

$$(12)$$

$$E[Tjf_{0};r_{0}] = \frac{i V_{0} + (f^{\dagger}_{i} \overline{a})}{1} + \frac{(\overline{f}_{i} \underline{a}_{i} V_{0})}{1} + \frac{1}{1} \frac{1}{1} \frac{e^{i \mu(V_{0}_{i}} (f_{i} \overline{a}))}{1} for^{1} \bullet 0;$$

$$(13)$$

where $v_0 = f_0$; r_0 and $\mu = 2^1 = 3/2$.

D umas and Svensson [7] applied this formula to calculate the expected survival time in the Swedish case, where the exchange rate band is §1:5 percent. They assumed that the standard deviation $\frac{3}{4}$ and the drift are both 10 percent per year, which they admit to be quite large. They found out that a reserve level equal to 2.7 times the speculative attack size (r_i $\underline{a} = 100$ log percent) brings about an expected survival time of 10-20 years. When they change 1 to zero they obtained 120 years. A is they pointed out, the recent Finnish and Swedish experience contrasted to these results.

In the intra-marginal interventions model, while the fundamental still is bounded by \underline{f} and \overline{f} , the bound on reserves depends on the fundamental

$$f_{t,i} \times (f_t) = w(f_t) \cdot m_t \cdot f_i \cdot w(f_t) = f_i \cdot (f_{t,i} \times (f_t))$$
: (14)

This gives us a very complicated...rst time passage problem for a stochastic process, one in which the barriers depend on the process itself.

4 Computational Results and Discussions

0 btaining an explicit solution to this problem proved to be, if not impossible, very di¢ oult. To obtain the results, we use II onte Carlo simulation of 15000 observations. The time increment in the discretization of the model is set to 300 periods per year⁷.

First, we look at the 'Swedish' exchange rate band of §1:5 percent. The parameters are given by L indberg and Soderlind [14] as @=0:353571, %=0:0312163, %=3:684211, $x_c=\frac{1}{3}0:63\%$, $f^{\dagger}=0:045445$, $f^{\dagger}=\frac{1}{3}0:031636$ and $f_c=\frac{1}{3}0:016530$. We choose in the maximum size of reserves to trigger a collapse, as 2.5 times the initial reserves. The maximum size of the speculative attack is $f^{\dagger}=\frac{1}{3}0$. Figure 1 shows the expected survival time versus the reserves whose unit is the maximum size of the speculative attack. 2.7 times the speculative attack in the marginal attack case corresponds to 4.5 in the graph. For that we obtain 30 years of survival time. If the reserves are no more than 3 times the maximum size of a speculative attack, the survival time is about 15 years. Considering the fact that the Swedish target zone lasted about 10 years (from 0 ctober 1982 to Fall 1992) in the longest case these values are comparable.

In the next step, we look at the exect of the bandwidth. We plot a symmetric target zone, $(x_i \ x_c = x_{ci} \ \underline{x})$, for dimerent values of bandwidth 2k. We assumed that $x_c = 1$, $^{\circ} = 1:35$, $\frac{1}{2} = 3:68$, $\frac{1}{4} = 1:13$ and $f_c = 1$. First, using (8) we solve for f^1 numerically for each k. Then we set the reserve m_1 to be the largest value of the dimerence f^1 ; $\frac{1}{2}$ which corresponds to the largest value of $\frac{1}{2}$ since f^1 ; $\frac{1}{2}$ is monotone in $\frac{1}{2}$. We noticed that while the bandwidth changes between 0.02 to 0.14, the survival time stays around 3.5 years. When

 $^{^{7}}$ W eexperimented with changing the number of sample paths and discretization size for the ... rst simulation below. Increasing both do not make a quantative diagram errore in the result but it increases the simulation time substantially. The code is written in Causs. We can supply it upon request



 $m_1 = 4.5$ (f_1^{\dagger}) 1:17) the survival time is ... xed about 30 years. In the marginal intervention model, one can see from equations (12) and (13) that the expected survival time increases with f_1^{\dagger} or f_2 which increases with f_2^{\dagger} and f_2^{\dagger} are very low. In the intermedial properties are very low. In the intermedial properti

We also look at the erect of the starting point of the exchange rate inside the target zone. Dumas and Svensson [7] found that for the marginal intervention target zone model, when the reserves have reached minimum, it would still take an expected time of 30 years for the target zone to collapse if the fundamental and the exchange rate start at the strong end of the band. Even with $^1=10\%$, it is about 10 years. Of course, this has implications for the realignments of the target zone since after a realignment, it will matter where to locate the exchange rate in the new zone because this will erect drastically the time of the next realignment. We have run the Swedish target zone with $m_0=4(f^0_{\ i}, \ x)$, we see that the survival time decreases monotonically from 27 years to 20 years as the initial value of the exchange rate in the band dranges from the lover boundary to higher boundary. If we choose $m_0=f^0_{\ i}, \ x$ then it changes from 3.5 years to 9 months. Hence the survival time is still sensitive to the initial location of the exchange rate in the band at though the relation is weaker nowsince a substantial amount of reserves are used inside the band

A consequence of intra-marginal intervention is the possibility of collapse of a target zone when the exchange rate is inside the band. This is of course more in accordance with the real life experience compared to the marginal intervention model, where a collapse can only occur at the boundaries. If ext we look at

^{*}The other factor for the size of the bandwidth is the monetary independence, see Svensson [20] for an excellent discussion, also Svensson [18] and Svensson [19] for the relation between the interest rate di¤ erentials and the bandwidth.

If though we assumed that after the collapse, the exchange rate dynamics become free ‡ cating if the re alignments are decided enobgenously (rather than given as exceptous shocks as in the realignment literature) the location of the exchange rate in the newband will be important for the timing of the next realignment.

10 B entida and Svensson [2] and W erner [21] assume that the exchange rate position within the band is unchanged. D umas, Jennergen and W aslund [4]...xes the fundamental after the jump and calculates the new position of the exchange rate from the model. B all and R oma [1] set the starting value as the mid-point of the newband. B entida and Svensson [2] assumes that it starts from the strongend of the target zone. B ut, the realignment process in each of these papers is exceptous.

Figure 2: Probability of a collapse in a intra marginal intervention target zone

the distribution of the location of a speculative attack and the collapse. We plot the density, calculated using a kernel smoothing method, for a symmetric target zone of $\dot{x}=1:5\%$, $\dot{x}=1:131$, $\dot{x}=1:135$,

We also did some sensitivity analysis. For a symmetric target zone of x=1:5%, x=0:0.31, x=0:0.35, x=0:0.35

Figure 3: Expected Survival time for diagreent values of the drift of the velocity

5 Same Extensions

A Ithough the model above seems to capture the collapse of a target zone more realistically, it still needs to be improved. One necessary improvement is the addition of a drift term to the velocity since in most cases the economic conditions in the domestic country is warsening over time. If we assume that the velocity of the fundamental is given as in (4) then we expect to have a shorter expected survival time since now the reserves are also spent to counter the exect of the positive drift of the fundamental. In this case, the solution to (1) is given by

$$x_{t} = \frac{f_{t} + \sqrt[8]{h} f_{c} + \sqrt[8]{1}}{1 + \sqrt[8]{h}} + \text{AM} \quad \frac{\mu}{2^{\otimes} \cancel{h}}; \frac{1}{2}; \stackrel{?}{t} + \text{BM} \quad \frac{\mu}{1 + \sqrt[8]{h}}; \frac{3}{2}; \stackrel{?}{t}; \stackrel{?}{t}$$

where $f_t = (f_t)^{1/2} f_t$. Since the free ‡ cat solution is $x_t = f_t + e^{-1/2}$, the reserves in the point of attack will be

$$w_{t}(f) = \frac{{}^{\otimes} \cancel{!}_{1}(f_{t} + {}^{\otimes} 1_{i} \ f_{c})}{1 + {}^{\otimes} \cancel{!}_{2}} ; \text{ All } \frac{\mu}{2^{\otimes} \cancel{!}_{2}}; \frac{1}{2}; {}^{2}_{t} ; \text{ B M} \frac{\mu}{2^{\otimes} \cancel{!}_{2}}; \frac{3}{2}; {}^{2}_{t}; {}^{2}_{t};$$

We simulate the survival time for parameter values of x = 1:5%, x = 0:031, x = 3:68, x = 0:031, x = 0:031,

We can also use more general stochastic processes for the velocity and the fundamental since a more

general process may improve the ... t to the data. Let us assume that the velocity is an I to process

$$dV = {}^{1}(f_{t})dt + {}^{3}\!\!/_{t}(f_{t})dB_{t}$$

and the interventions follows more general mean reverting process $dm_t = \frac{1}{2} \frac{1}{2} (f_t) [f_t] f_c]dt + dl_{tj} dl_{t}$. If we assume that the exchange rate is twice dx erentiable function x(f) of the fundamental, I to's lemma will yield

E (ck)=ct =
$${}^{1}(f)x^{Q}(f) + \frac{1}{2}x^{Q}(f)x^{Q}(f)$$
:

A fler we plug for E (ck)=at from (1) we obtain the second order di¤erential equation

$$x(f) = f + {}^{\otimes}({}^{1}(f)_{i} {}^{1}(f))x^{0}(f) + {}^{\otimes}{}^{4}{}^{2}(f)x^{0}(f).$$
(15)

Under certain smoothness assumptions on 1 (f); 4 (f) and 4 (f), this equation has a unique solution x (f). Then the bounds on reserves are given by (14) as $f_{tj} x(f_t) \cdot m_t \cdot f_j$ ($f_{tj} x(f_t)$). Following the arguments of Section 3 we can ... and the bound on the reserves as

$$f_{t \mid } x(f_t) \cdot m(f_t) \cdot f_i (f_{t \mid } x(f_t))$$
:

We can always solve approximately (15) with power series. Then numerical methods of the previous section enable us to calculate the survival time.

6 Candusian

It was shown by Lindberg and Soderlind [14] and Ball and Roma [1] that the target zone models with intramarginal interventions produce a better ... to the real life target zone experience. In this paper, we showed that it also yields a better result for the expected survival time than the target zone model with marginal interventions. In general, it is not possible to know the amount of reserves which a central bank is willing to spend to protect the zone. The amount of reserves can be thought as the commitment of the bank to the survival of the target zone. One important extension in this area would be to calculate the timing of a speculative attack to a realigning target zone, hence the calculation of the timing of realignment.

R eferences

- [1] Cli¤ ard A. Ball and Antonio Roma, A jump di¤ usion model for the european monetary system, Journal of International Money and Finance 12 (1993), 475-492.
- [2] Giuseppe Bertda and Lars E. Svensson, Stochastic devaluation risk and the and the emprical ... t of the target zone models, Review of Economic Studies 60 (1993), 69 -712.

- [3] Willem H. Buiter, Borrowing to defend the exchange rate and the timing and magnitude of speculative attack, Journal of International Economics 23 (1987), 221-239.
- [4] Francis D elgado and B ernard D umas, Target zones, broad and narrow, Exchange rate targets and currency bands (P aul Krugman and III arcus III iller, eds.), Cambridge U niversity P ress, 1992.
- [5] _____, III one tary contracting between central banks and the design of sustainable exchange rate zones, Journal of International Economics 34 (1993), 201–224.
- [4] Bernard Dumas, L. Peter Jennergren, and Bertil II aslund, Realigment risk and currency option priding in tager zones, European Economic Review 39 (1995), 1523-1544.
- [7] Bernard Dumas and Lars E.O. Svensson, Howlong do unilateral target zone last?, Journal of International Economics 36(1994), 46-481.
- [8] R. P. Flood, A. K. Rose, and D. J. M. athieson, A. n. emprical exploration of exchange rate target zones, Carnegie Rochester Series on Public Policy, vol. 35, 1991, pp. 7-66
- [9] Robert P. Flood and Peter III. Garber, Collapsing exchange rate regimes, Journal of International Economics 17 (1984), 1–13.
- [10] Kenneth A. Froot and M. aurice O bstfeld, Stochastic process switching. Some simple solutions, Econometrica 59 (1991), 241–250.
- [11] _____, Target zones, broad and narrow, Exchange rate targets and currency bands (Paul Krugman and Marcus Miller, eds.), Cambridge University Press, 1992.
- [12] F. Giavazzi and M. Giovannini, Limiting exchange rate ‡exibility. The european monetary system, The MIT Press, Cambridge 1989.
- [13] Paul Krugman, A model of balance of payments crises, Journal of M oney, Credit and B anking 11 (1979), 311-325.
- [14] _____, Targetzones and exchange rate dynamics, Quarterly Journal of Economics 106(1991), 669-682.
- [15] Paul Krugman and Julio Rotemberg Speculative attacks on target zones, Exchange rate targets and currency bands (Paul Krugman and Marcus Miller, eds.), Cambridge University Press, 1992.
- [14] H ans L indberg and Paul Soderlind, Intervention policy and mean reversion in exchange rate target zones: The swedish case, Scandinavian Journal of Economics 4 (1994), 499-513.
- [17] _____, Testing the basic target zone on swedish data 1982-1990, European Economic Review 38 (1994), 1441–1465.

- [18] Lars E. O. Svensson, Target zones and interest rate variability, Journal of International Economics 31 (1991), 27-54.
- [19] _____, The term structure of interest rate di¤ erentials in a target zone, Journal of III one tary Economics 28 (1991), 87-116
- [20] _____, A n interpretation of recent research on exchange rate target zones, Journal of Economic Perspectives 6(1992), 119-144.
- [21] A lejandroll . Werner, Exchange rate target zones, realignments and interest rates die erential: Theory and evidence, Journal of International Economics 39 (1995), 353-367.