Uncertainty, Risk, and "Expected" Profits
Or, “What can I really expect?”

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Note

In all numerical examples, please multiply each figure by $10^X$, where $X$ is whatever size it takes to get your attention.
An example

- You own a petroleum reservoir which contains exactly 1000 barrels.
- The marginal out-of-pocket cost of production is exactly $9.50/bbl.
- You forecast oil price to average $10/bbl. at your wellhead.
- What do you expect your net cash flow to be?
$500, because there are 1000 barrels at 50¢/bbl. margin.

Right?
WRONG!!
Correct answer

$1,000

Why?

- The price of oil is forecast to be normally distributed with a mean of $10 and an standard deviation of 1.8.
- You would not produce when the price is below your marginal cost.
“The flaw of averages”
With credit to Prof. Sam Savage, Stanford University

The value of a function evaluated at its average value ≠ The average value of the function*

• Unless the function is linear. (Jensen’s inequality)

P.S. And very little in this world is linear.
**A sobering example**

The State of the drunk at his AVERAGE position is **ALIVE**

But the AVERAGE State of the drunk is **DEAD**

Used by courtesy of Prof. Sam Savage, Stanford University
**A more interesting example**

- You operate a warehouse out of which you sell widgets at a margin of $2/widget.
- You can replenish your inventory every morning.
- Overnight charges for any inventory left over from the day’s activity = $1/widget.
- Average sales = 5 widgets/day.
- Therefore, you daily inventory 5 widgets.
- What is your expected daily profit?
## Sales Distribution!

<table>
<thead>
<tr>
<th>Sales</th>
<th>Cost</th>
<th>Margin</th>
<th>Profit</th>
</tr>
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<tbody>
<tr>
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</tr>
<tr>
<td>Average</td>
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<td>$5.91</td>
<td></td>
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</tbody>
</table>

Equally likely
The right input

Not

What are the average sales?

But

How are the sales distributed?
The right question

Not just
- What is the expected profit?

But
- What is my risk?
  - E.g., What is the probability of losing money?
Definitions

Uncertainty

Objective

Independent variables, the value of which are unknown.
  • E.g., future oil price

Risk

Subjective. In the eye of the beholder.

Whatever you worry might happen to you.
  • E.g., the probability of negative cash flow
Definitions, cont’d

Mean

- Average.
- Expected value.
- \[(\text{Each possible outcome}) \times (\text{its probability})\] (number of possible outcomes)

Mode

- The most likely outcome.
- Not very useful.
- Usually not even very \textit{likely}.
- E.g., the mode i.e., the most likely outcome for a roll of honest die is 7, but its probability is only 1 chance in 6.
  - Is it only a cruel coincidence that these are exactly the same odds as you have of dying in a game of Russian roulette?
What is the expected profit?
Another good question

How can I maximize my expected profit?
Expected profit vs. inventory level

$6.36
Another good question

Assume I am going to do this just once.

How can I minimize my risk?

But, first I must define what I mean by “risk.”
How can I minimize risk?

Cumulative risk profiles

Which inventory level is the most likely to lose money?
What is the optimum strategy?
What if you had multiple warehouses?

Expected profit

Risk of loss

5.91 6.18 6.27 6.36

5 6 7 8
What if you had multiple warehouses?

Risk of loss

Expected profit

Efficient frontier

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Portfolio Optimization

See:


Other input variables the E&C industry might enter as distributions

- Material costs
- Labor rates
- Delays, strikes
- Changes in legislative or regulatory environment
- Sub-contractor fees
- Currency exchange rates
Correlations among projects

- Negative correlations
  - Are preferable to
- Zero correlations
  - Which are preferable to
- Positive correlations
Distributions as input: Use what you already know about the uncertainties!

- Use the same distributions of the same parameter in all projects.
- Avoid “normal distributions” and “standard deviations,” etc.
- Triangular distribution may be the most useful, at least for starters.

E.g.
- <10% probability of <30
- Mean of 50
- <10% probability of >60

This illuminates the risks and avoids the flaw of averages.

Use historical data.
### Sales distribution!

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Summary

- Avoid the flaw of averages
- Input independent variables as distributions, not averages.
- Use the same distributions for each independent variable in all applications.
  - I.e., Manage your “distribution of distributions.”
    (Thanks again to Sam)
- The answer is not a “number,” but a distribution.
- Make decisions using risk/return trade-off.
- Seek negatively correlated projects.
- Optimize your portfolio of projects for maximum return at minimum risk.
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