Conclusions

A RC fiber beam-column element is presented based on the flexibility method. Coupling between axial force and bending moments is introduced by the element subdivision into longitudinal fibers. The nonlinear nature of the element depends entirely on the constitutive behavior of the concrete and steel fibers. A new element state determination algorithm is proposed that also permits the straightforward application of element loads. Parameter studies of the effect of the number of fibers in a section and the number of sections in an element conclude the paper.

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Appendix II. Unit Conversions

Anglo-American units	SI metric units
1 inch (in)	0.0254 m
1 foot (ft)	0.305 m
1 ksi	6.89 MPa
1 kip	4.458 kN
1 kip-ft	1.36 kN-m

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Applicability of Pseudo-force method to Highly Nonlinear Dynamic Problems

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ABSTRACT

This paper deals with solution of nonlinear dynamic problems using Pseudo-force method. A solution algorithm involving Pseudo-force method is presented. The objective is to investigate the possibility of application of the method to highly non-linear dynamic problems to which the method has not been applied to by previous investigators, such as problems with combined material and geometric nonlinearities, and problems with frictional nonlinearities. It is shown that Pseudo-force method is applicable in the above mentioned dynamic problems, by comparing the results with analytical and experimental results.

INTRODUCTION

In most nonlinear dynamic problems the Newton-Raphson method (Belytschko and Hughes 1986; Clough and Wilson 1979; Modkar and Powell 1989; Stricklin and Haisler 1977) of solution is used. The alternative method of solution for nonlinear dynamic problems is the Pseudo-force method which involves representation of a nonlinear system by a linear one in which the nonlinear effects are represented by Pseudo-forces. The Pseudo-forces compensate for the difference in internal forces as obtained from the Pseudo-linear system and from the truly nonlinear system. The method corresponds to the initial stress method used in static nonlinear analyses. The method has been applied: (i) to dynamic analysis of shallow shells with geometric nonlinearities by stricklin and Haisler (1977); (ii) to hybrid frequency-time domain (HFTD) analysis of soil-structure interaction problems by Kawamoto (1983) and Darbre and Wolf (1988; 1990) in which uplift-contact nonlinearity due to rocking and elasto-plastic material nonlinearities were considered, separately; and (iii) to linear fluid - nonlinear dam structure systems by Fenves and Chavez (1990). However, the Pseudo-force method has not been applied to problems with combined material and geometric nonlinearities,and in problems with frictional nonlinearities.

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In this paper a solution algorithm involving Pseudo-force method is presented. The solution alorithm is applied to two highly nonlinear dynamic problems: (i) the first problem involving dynamic response of a single-degree-of-freedom (SDOF) with combined material and geometric nonlinearities under harmonic excitation; and (ii) the second problem involving transient response of a base isolated structure with sliding bearings i.e., frictional nonlinearities.

The first problem i.e., SDOF with material and geometric nonlinearities is considered because it has the necessary features for a good test problem like softening, negative stiffness and hardening. The solution to this problem is obtained using accurate multi-step predictor-corrector methods and Duffings method and compared with the solution of the Pseudo-force method for verification. The second problem i.e., frictional problem is chosen because it is highly nonlinear and the severity of the nonlinearity is compunded due to velocity dependence of the coefficient of friction, and due to biaxial interaction effects observed in experiments. Furthermore, experimental results exist for the base isolated structure with sliding bearings analyzed herein, thus facilitating verification of solution by the Pseudo-force method. It is shown that the Pseudo-force method is applicable to these highly nonlinear problems.

PSEUDOFORCE SOLUTION ALGORITHM

Consider the matrix equation of motion at time t:

$$\tilde{\mathbf{M}}\ddot{\mathbf{u}}_t + \tilde{\mathbf{C}}\dot{\mathbf{u}}_t + \tilde{\mathbf{K}}\bar{\mathbf{u}}_t + \mathbf{f}_t = \tilde{\mathbf{P}}_t \tag{1}$$

At time $t + \Delta t$

$$\tilde{\mathbf{M}}\ddot{\mathbf{u}}_{t+\Delta t} + \tilde{\mathbf{C}}\dot{\mathbf{u}}_{t+\Delta t} + \tilde{\mathbf{K}}\bar{\mathbf{u}}_{t+\Delta t} + \mathbf{f}_{t+\Delta t} = \tilde{\mathbf{P}}_{t+\Delta t} \tag{2}$$

In incremental form

$$\tilde{\mathbf{M}}\Delta\ddot{\mathbf{u}}_{t+\Delta t} + \tilde{\mathbf{C}}\Delta\dot{\mathbf{u}}_{t+\Delta t} + \tilde{\mathbf{K}}\Delta\bar{\mathbf{u}}_{t+\Delta t} + \Delta\mathbf{f}_{t+\Delta t} = \tilde{\mathbf{P}}_{t+\Delta t} - (\tilde{\mathbf{M}}\ddot{\mathbf{u}}_t + \tilde{\mathbf{C}}\dot{\mathbf{u}}_t + \tilde{\mathbf{K}}\bar{\mathbf{u}}_t + \mathbf{f}_t) \tag{3}$$

in which, $\tilde{\mathbf{M}}$, $\tilde{\mathbf{C}}$, $\tilde{\mathbf{K}}$ and $\tilde{\mathbf{P}}$ represent the mass, damping, pseudo-linear stiffness and load matrices. Furthermore, $\bar{\mathbf{u}}_t$ = displacement vector; and $\mathbf{f}_t$ = Pseudo-force vector.

The incremental Pseudo-force vector $\Delta\mathbf{f}_{t+\Delta t}$ in Eq. 3 is unknown. This vector is brought on to the right hand side of Eq. 3. The solution algorithm developed involves the solution of equations of motion using Newmark's constant-average-acceleration method. Furthermore, a iterative procedure consisting of corrective Pseudo-forces is employed within each time step until equilibrium is achieved. The developed solution algorithm is as follows:

A. Initial Conditions:

1. Form the pseudo-stiffness matrix $\tilde{\mathbf{K}}$, mass matrix $\tilde{\mathbf{M}}$, and damping matrix $\tilde{\mathbf{C}}$. Initialize $\bar{\mathbf{u}}_0$, $\dot{\mathbf{u}}_0$ and $\ddot{\mathbf{u}}_0$.
2. Select time step Δt, set parameters $\delta = 0.25$ and $\theta = 0.5$, and calculate the integration constants:

$$a_1 = \frac{1}{\delta(\Delta t)^2};\quad a_2 = \frac{1}{\delta\Delta t};\quad a_3 = \frac{1}{2\delta};\quad a_4 = \frac{\theta}{\delta\Delta t};\quad a_5 = \frac{\theta}{\delta};\quad a_6 = \Delta t(\frac{\theta}{2\delta} - 1) \tag{4}$$

3. Form the effective stiffness matrix

$$\mathbf{K}^* = a_1\tilde{\mathbf{M}} + a_4\tilde{\mathbf{C}} + \tilde{\mathbf{K}} \tag{5}$$

4. Triangularize $\mathbf{K}^*$ using Gaussian elimination.

B. Iteration at each time step:

1. Assume the Pseudo-force vector $\Delta f^i_{t+\Delta t} = 0$ in iteration i = 1.
2. Calculate the effective load vector at time $t + \Delta t$:

$$\mathbf{P}^*_{t+\Delta t} = \Delta\tilde{\mathbf{P}}_{t+\Delta t} - \Delta f^i_{t+\Delta t} + \tilde{\mathbf{M}}(a_2\dot{\mathbf{u}}_t + a_3\ddot{\mathbf{u}}_t) + \tilde{\mathbf{c}}(a_5\dot{\mathbf{u}}_t + a_6\ddot{\mathbf{u}}_t) \tag{6}$$

$$\Delta\tilde{\mathbf{P}}_{t+\Delta t} = \tilde{\mathbf{P}}_{t+\Delta t} - (\tilde{\mathbf{M}}\ddot{\mathbf{u}}_t + \tilde{\mathbf{C}}\dot{\mathbf{u}}_t + \tilde{\mathbf{K}}\bar{\mathbf{u}}_t + \mathbf{f}_t) \tag{7}$$

3. Solve for displacements at time $t + \Delta t$:

$$\mathbf{K}^*\Delta\mathbf{u}^i_{t+\Delta t} = \mathbf{P}^*_{t+\Delta t} \tag{8}$$

4. Update the state of motion at time $t + \Delta t$:

$$\ddot{\mathbf{u}}_{t+\Delta t} = \ddot{\mathbf{u}}_t + a_1\Delta\bar{\mathbf{u}}^i_{t+\Delta t} - a_2\dot{\mathbf{u}}_t - a_3\ddot{\mathbf{u}}_t;\quad \dot{\mathbf{u}}_{t+\Delta t} = \dot{\mathbf{u}}_t + a_4\Delta\bar{\mathbf{u}}^i_{t+\Delta t} - a_5\dot{\mathbf{u}}_t - a_6\ddot{\mathbf{u}}_t;\quad \bar{\mathbf{u}}_{t+\Delta t} = \bar{\mathbf{u}}_t + \Delta\bar{\mathbf{u}}^i_{t+\Delta t} \tag{9}$$

5. Compute the state of motion at each element, compute the nonlinear forces from the force-displacement relationships, and compute the Pseudo-forces.
6. Compute the resultant Pseudo-force vector $\Delta f^{i+1}_{t+\Delta t}$.
7. Compute

$$Error = \frac{\| \Delta f^{i+1}_{t+\Delta t} - \Delta f^i_{t+\Delta t} \|}{Ref.\,\text{Max}.Force} \tag{10}$$

Where $\| . \|$ is the euclidean norm

8. If Error $\geq$ tolerance, further iteration is needed, iterate starting from step B-1 and use $\Delta f^{i+1}_{t+\Delta t}$ as the Pseudo-force vector and the state of motion at time t, $\bar{\mathbf{u}}_t$, $\dot{\mathbf{u}}_t$ and $\ddot{\mathbf{u}}_t$.
9. If Error $\leq$ tolerance, no further iteration is needed, update the Pseudo-force vector $\mathbf{f}_{t+\Delta t} = \mathbf{f}_t + \Delta\mathbf{f}^{i+1}_{t+\Delta t}$, go to step B-1.

SDOF SYSTEM WITH MATERIAL AND GEOMETRIC NONLINEARITY

The applicability of the described solution algorithm to a SDOF system with material and geometric nonlinearities is investigated. The SDOF system considered is shown in Fig.1(a). The elasto-plastic nonlinear springs represent material nonlinearity and the precompressed vertical bar yields the geometric nonlinearity. The SDOF system is governed by the following equation of motion (for no damping):

$$m\ddot{u} + f(u) = P\sin\Omega t \tag{11}$$

where: m = mass of the oscillator; u = displacement; P = peak value of forcing function; Ω = frequency of the forcing function; and the restoring force $f(u)$ of the system (see Fig. 1(b)) can be represented by the following equations:

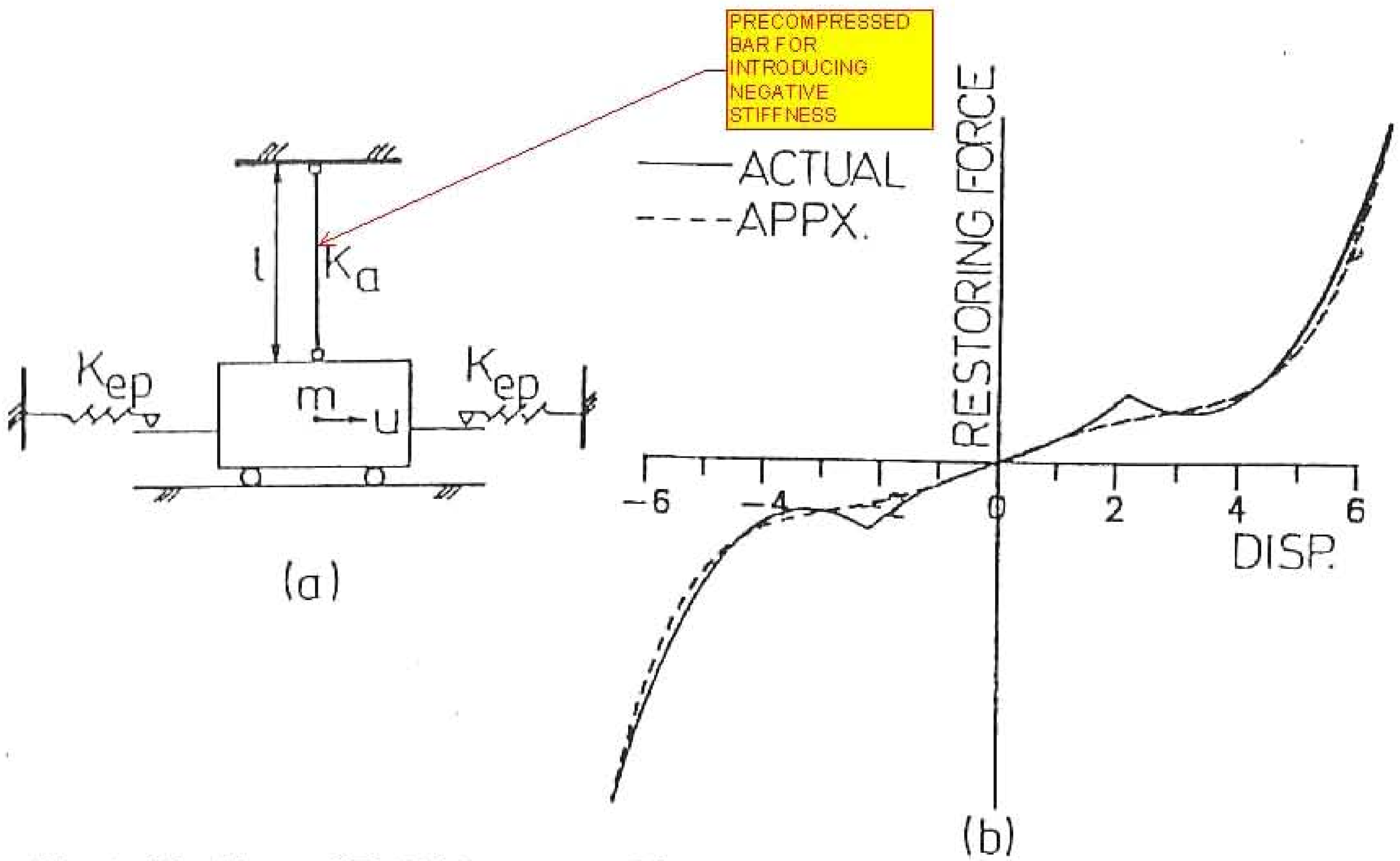

Fig. 1. Nonlinear SDOF System and its Restoring Force Characteristic.

$$f(u) = 2*K_{ep}*u + [(\sqrt{l^2+u^2}-l) - \Delta]*K_a*\frac{u}{\sqrt{l^2+u^2}} \qquad u \le 2.2 \qquad (12)$$

$$f(u) = 2*K_{ep}*2.2 + [(\sqrt{l^2+u^2}-l) - \Delta]*K_a*\frac{u}{\sqrt{l^2+u^2}} \qquad u > 2.2 \qquad (13)$$

where: l, Δ are the length and the precompressed deformation of the vertical bar, respectively; K_{ep} = initial stiffness of the nonlinear elasto-plastic spring; K_a = axial linear stiffness of the vertical bar.

A continuous restoring function approximation, shown in Fig. 1(b) in dashed line, is chosen for solution by Gear's predictor-corrector method (Gear 1971) and Duffing's method (Stoker 1950). The continuous restoring function can be represented by the following equation:

$$f(u) = K_1(u - \alpha u^3 + \beta u^5) \qquad (14)$$

where: K_1 = initial linear stiffness; and α, β = constants. The solution of the SDOF system with above continuous restoring force by Duffing's method, with $u = U \sin \Omega t$, in consistent units, is as follows:

$$\frac{\Omega^2}{\omega_1^2} = 1 - \frac{P}{\omega_1^2 U} - \frac{3}{4}\alpha U^2 + \frac{5}{8}\beta U^4 \qquad (15)$$

where: U = peak amplitude; and $\omega_1^2 = K_1/m$.

The response curves, for various levels of forcing function, obtained using Duffing's method, Gear's method, and Pseudoforce method are shown in Fig. 2. Since transient response stays in the system forever without damping a small amount of damping ζ = 0.005 was used for numerical simulation. The results from Duffing's method and Gear's method shown in Fig. 2(a) and 2(b) match very well in all regions excepting the region of superharmonics, as expected. The results from Pseudoforce method, shown in Fig. 2(c) and 2(d): (i) converge over all regions; (ii) match well with the Duffing's solution, excepting for deformations close to U = 2.2 where the restoring force characteristics differ (see Fig. 1(b)); and (iii) reproduce the superharmonic oscillations observed in solution by Gear's method. However the Pseudoforce method failed to converge for near static application of the forcing function, since snap through occurs, which is consistent with the conclusions of the previous investigators (Stricklin et al. 1977).

BASE ISOLATED STRUCTURE WITH SLIDING BEARINGS

The applicability of the described solution algorithm to a frictional problem is investigated herein by analyzing a single story base isolated structure with a sliding isolation system tested by Hisano et al. (1988). The analytical models necessary for analyzing the base isolated structure being considered i.e., the analytical models for the sliding bearings, springs and the superstructure, have been described in detail by Nagarajaiah et al. (1991a; 1991B). The analytical model represents the superstructure with three degrees of freedom per floor, two lateral and one torsional degree of freedom, and the isolation system is modeled by representing the force-displacement characteristics of each sliding bearing and spring explicitly. Furthermore, the model for the sliding bearing accounts for the variation of coefficient of friction with velocity and for the biaxial interaction effects. The analytical models described by Nagarajaiah et al. (1991a; 1991b) and the solution algorithm, involving pseudoforce method, presented in this paper have been implemented in a computer program 3D-BASIS. The computer program 3D-BASIS is used for analysis of the base isolated structure.

The results of the analysis are compared with experimental results from bidirectional shake table tests on the same sliding isolated model by Hisano et al. (1988). The tested model was a 1/8 scale single story steel structure, 120 in (3048 mm) long and 90 in (2286 mm) wide, on a sliding isolation system consisting of 9 sliding bearings with 4 rubber springs. The model weighed 10.1 tons (101 kN), with 8.05 tons (80.5 kN) of superstructure weight and 2.05 tons (20.5 kN) of base weight. The radius of gyration was r = 0.29 L. The model had symmetric stiffness and mass properties. For the scaled superstructure the lateral period was 0.11 sec (corresponding to 0.3 sec in prototype) and the torsional period was 0.07 sec (0.2 sec in prototype). The damping ratio measured in the superstructure was 1%. For the isolation system the lateral period was 0.35 sec (1.0 sec in prototype) and the torsional period was 0.208 sec (0.588 sec in prototype). The diameter of the sliding bearings were between 2.75 in (69.85 mm) and 1.4 in (35.56 mm). The measured coefficient of friction varied with velocity between f_{max} = 0.1 and 0.2.

The model structure was excited by time scaled accelerations of 1940 El Centro NS and EW components. The peak table acceleration in both the directions was scaled up by a factor of 1.5. Fig. 3 shows the measured and simulated frame acceleration and the base displacement in the NS direction, the displacement orbit of the center of mass of the base, and the force displacement loops of one of the sliding bearings (the experimental force-displacement loops were not available).

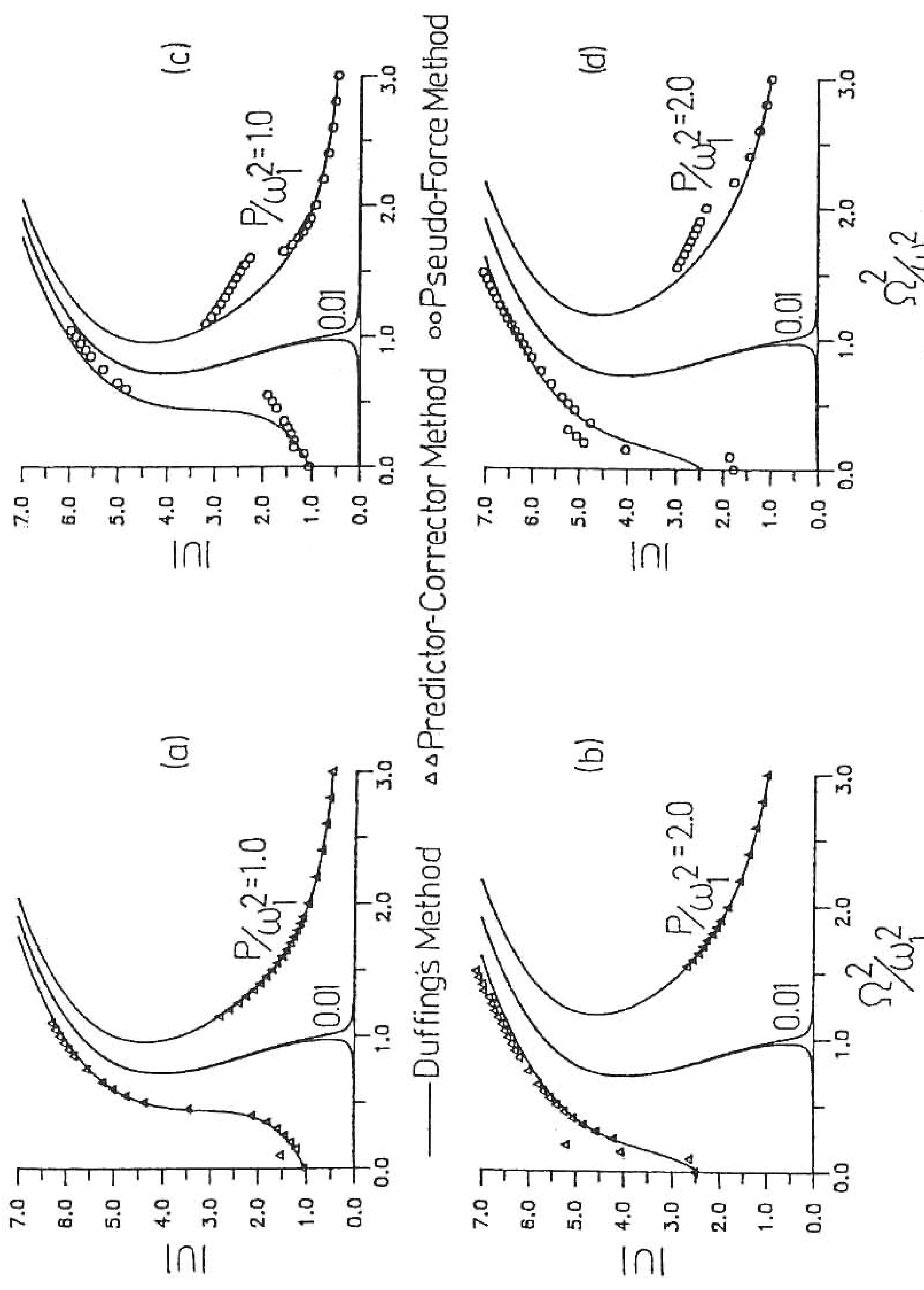

Fig. 2. Response Curves for the Nonlinear SDOF System.

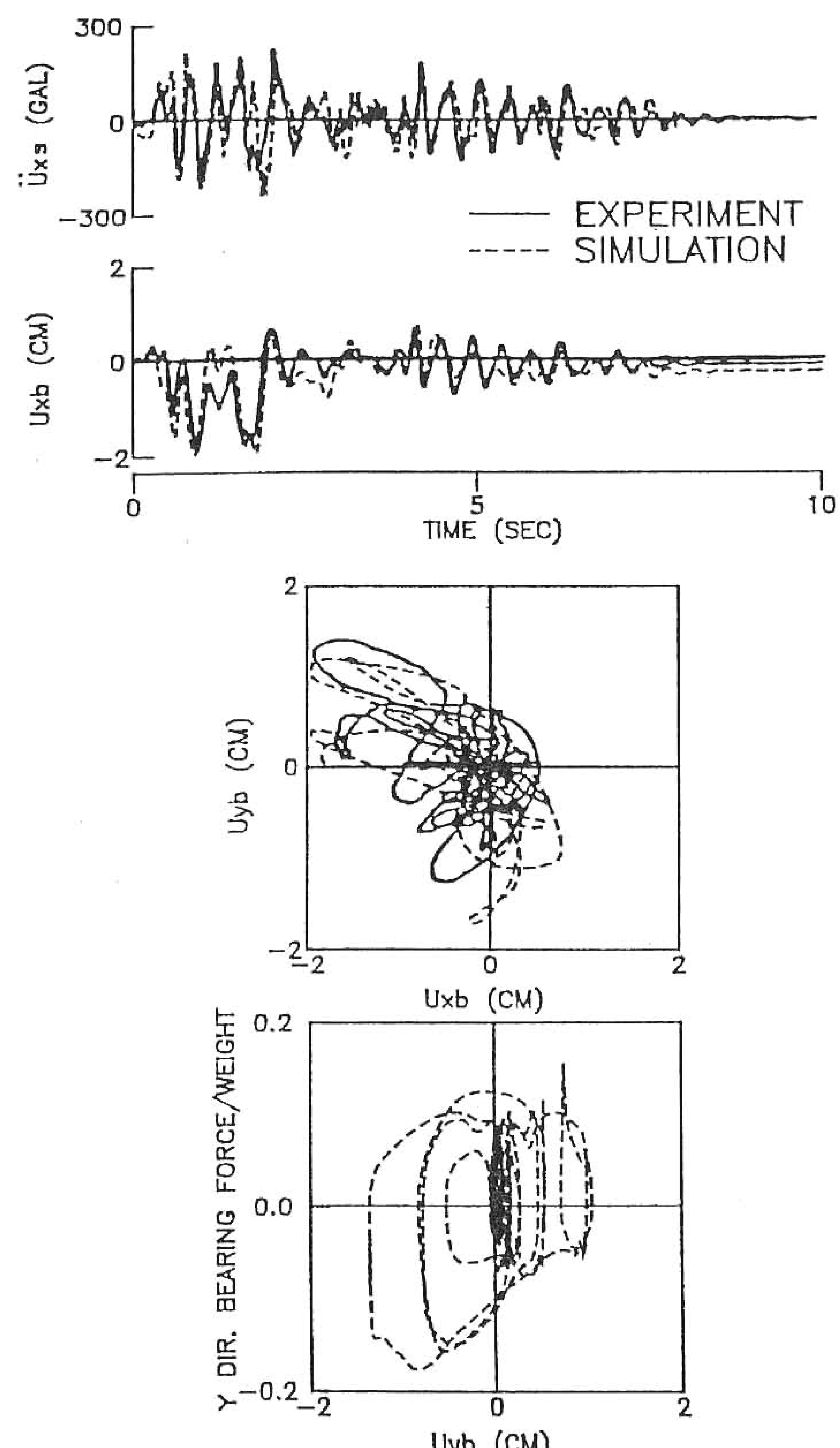

Fig. 3. Comparison of Simulated and Measured Response of a Sliding Base Isolated Structure with Frictional Nonlinearities.

The historical accelerogram of 1940 El Centro motion scaled appropriately was used as the excitation for the analytical simulation, as the achieved shake table acceleration time history was not available. Despite this a comparison between the measured and simulated results show good agreement, including major features of the displacement orbit. The high degree of nonlinearity, due to velocity dependence of the coefficient of friction, and the biaxial interaction effects, is evident in the Y direction force displacement loops shown in Fig. 4.

CONCLUSIONS

From the test problems presented it is evident that the Pseudo-force method is capable of handling highly nonlinear problems and yields satisfactory results. Hence the Pseudo-force method may be applicable to a larger class of nonlinear dynamic problems than the existing applications.

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NONLINEAR EARTHQUAKE ANALYSIS OF CONCRETE GRAVITY DAMS INCLUDING SLIDING

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Abstract

The sliding stability of a concrete gravity dam during a design earthquake must be investigated to asses the seismic safety of the dam. A new numerical procedure for solving the equations of motion, the Hybrid Frequency Time Domain procedure, is used to compute the earthquake response of concrete gravity dams including the nonlinear sliding behavior and the frequency-dependent response of the impounded water and the flexible foundation rock. As an example, the earthquake-induced sliding response of a typical concrete gravity dam is computed to illustrate the nonlinear behavior.

Introduction

The earthquake response of concrete gravity dams, including sliding at the base, depends on the dynamic characteristics of the dam, the impounded water, the foundation rock, and the ground motion. The effects of dam-water interaction as well as dam-foundation rock interaction are dependent on the excitation frequency. For assumed linear behavior of the system, a frequency domain solution of the equations of motion is most convenient (Fenves and Chopra, 1984). However, during a design-level earthquake the base shear force for the dam will exceed the shear strength of the interface between the dam and the foundation rock, causing sliding of the dam along this interface (Chopra and Zhang, 1991). Frequency domain solution procedures cannot be directly applied to computing the response of the dam including the nonlinear sliding behavior because

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