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Rank Orderings of Photoreceptor Photon Catches from Natural Objects are Nearly Illuminant-Invariant

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To examine the potential contribution of first-stage photoreceptor adaptation to color constancy, photon catches from 337 natural objects illuminated with phases of daylight and tungsten light were calculated for a model human fovea. The rank ordering of these photon catches within each cone photoreceptor class was examined. When first-stage adaptation is modeled using multiplicative or subtractive mechanisms, or a monotonic nonlinearity, no reordering of the rank ordering of photon catches is possible across illuminant changes. The observed rank orderings remained nearly invariant across illuminant changes for all three photoreceptor classes, although there was some local shifting in the rank orderings, thus ruling out the ability of von Kries adaptation alone to produce perfect color constancy. This means that for objects with natural reflectance spectra, the ordinal relationships between the photon catches within a class of photoreceptors exhibit only minor changes for these illuminant shifts. This result may be attributable to the fact that approx. 95% of the variance in these reflectance spectra is captured by the first principal component; objects that produce relatively few absorptions under one phase of daylight illumination will also produce relatively few absorptions under another phase. A geometric formalism for understanding these relationships is presented, and limitations on this analysis are discussed.

Color Photoreceptors Reflectance Adaptation

INTRODUCTION

Photoreceptor or first-stage adaptation has been proposed as a mechanism for color constancy at least since the time of von Kries (MacAdam, 1970). Photoreceptor adaptation is supposed to offset partially the effects of a change in the spectral quality of the illuminant by adjusting the sensitivities of the photoreceptor classes. For example, if the illuminant contains predominantly short wavelength energy, then the sensitivities of the short wavelength sensitive (SWS) photoreceptors will decrease, thus reducing what would otherwise be a large response in the SWS photoreceptors to the unbalanced illuminant. A little reflection shows that first-stage photoreceptor adaptation as conceived here has certain limits in terms of its ability to compensate for the effects of an illuminant change. One of the obvious limits is that for a constant set of objects first-stage photoreceptor adaptation cannot change the ordinal relationships between the photon catches within a photoreceptor class. This places a potential limit on the contribution of first-stage photoreceptor adaptation to color constancy. This limit will be discussed in more detail below. It is only a potential limit because currently it is unknown how much reordering of these photon catches takes place when the illuminant changes. The purpose of this paper is to examine this question empirically using a set of natural spectra and a range of illuminants.

Uchikawa, Uchikawa and Boynton (1989) have shown empirically that adaptation alone produces only partial color constancy for human observers. There are at least two potential explanations for this result. First, adaptation may not have been complete in their experiment. Second, even if adaptation were complete, it may not be able to remove the effects of the illuminant completely. The first explanation says that complete adaptation may be able to produce color constancy. The second explanation says that there are limits on the degree of color constancy that adaptation can produce. These limits are a result of the interaction between spectral reflectance, illuminant spectra and the nature of first-stage photoreceptor adaptation. This paper is an examination of the second explanation.

Many computational models of color constancy have a first-stage rescaling procedure whose purpose is to adapt the photoreceptors to the prevailing illuminant (Hurlbert, 1986). There also may be other second-stage procedures in these models. The first-stage procedure

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matches the sensitivities of the photoreceptors to the space- or time-average illumination. Alternatively, this rescaling normalizes the responses of the three photoreceptor classes against the response to an average, gray reference surface (Hurlbert & Poggio, 1988) or to the brightest area in the image (Land & McCann, 1971). It is proposed generally that this rescaling occurs independently within each of the three photoreceptor classes. The color of an object is then given a triplet of lightness values after some thresholding to eliminate responses to gradual spatial changes in the intensity of the illuminant.

To understand the relationship between color constancy and the rank orderings of photon catches, it is necessary to examine the way in which adaptation has been modeled. Geisler (1981, 1983) and Hayhoe, Levin and Koshel (1992) have modeled adaptation in two ways. First, adaptation may result from a multiplicative gain change in the intensity vs response function of the photoreceptors. Second, adaptation may also shift the intensity vs response function of photoreceptors by a subtractive mechanism. Both of these mechanisms have in common the property that they do not alter the ordinal relationships within a photoreceptor class between responses to different regions in the image if the multiplicative or subtractive factor is applied uniformly over the image. It is easy to see why this must be so. Consider the photon absorptions within some short period of time from several different objects or surfaces under a given illuminant. These absorptions can be ordered from lowest to highest. Now multiplying all of these absorptions by a constant or subtracting a constant from all of these absorptions cannot reorder them. Multiplication by and subtraction of a constant do not change the ordinal relationships among a set of numbers. Any monotonic transformation of the photon absorptions also will leave the rank ordering of the absorptions unchanged. So in addition to multiplicative (divisive) and subtractive mechanisms, saturating nonlinearities like those observed in primate photoreceptors (Schnapf, Nunn, Meister & Baylor, 1990) also will preserve the rank ordering of the photon absorptions. The effects of noise, whether internal or external, will inevitably perturb the stability of the rank order, but these effects are not considered in this analysis.

It is possible then to propose the following limit on photoreceptor adaptation or rescaling algorithms for producing color constancy. *If the ordinal relationships among the photon catches within a class of photoreceptors change as the illuminant is changed, then first-stage photorecepter alone cannot undo that change by reordering the photon catches.* This potential constraint on the contribution of first-stage adaptation to color constancy leads naturally to an examination of photon catches from a set of surfaces or objects under at least two different illuminants and to a search for evidence of reordering among the photon catches. Evidence of such a reordering in even one of the photoreceptor classes would constitute a proof that first-stage photoreceptor adaptation alone could not produce color constancy for that set of objects under that illuminant change. To make the case more general, it would be preferable to have objects with natural spectral reflectance functions

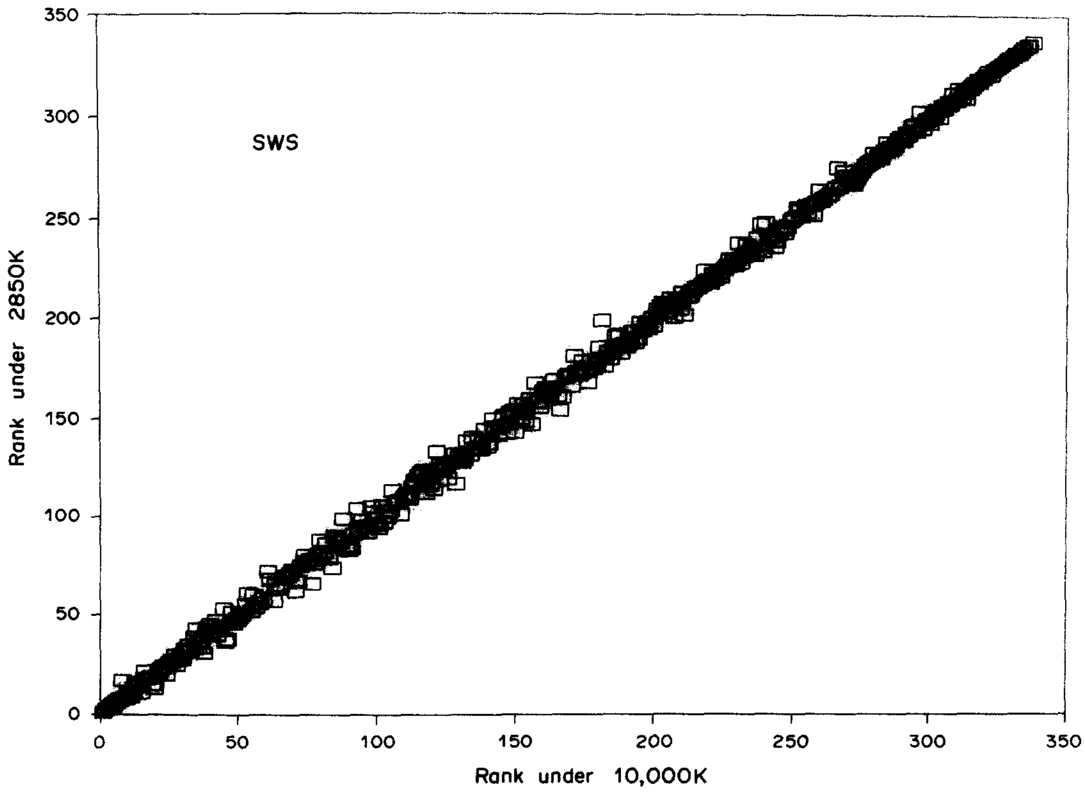

FIGURE 1. Scatterplot of the ranks of the photon catches for the SWS photoreceptors for all 337 objects. The rank of the photon catch for each object under the 10,000 K illuminant is indicated on the x-axis and its corresponding rank under the 2850 K illuminant is indicated on the y-axis.

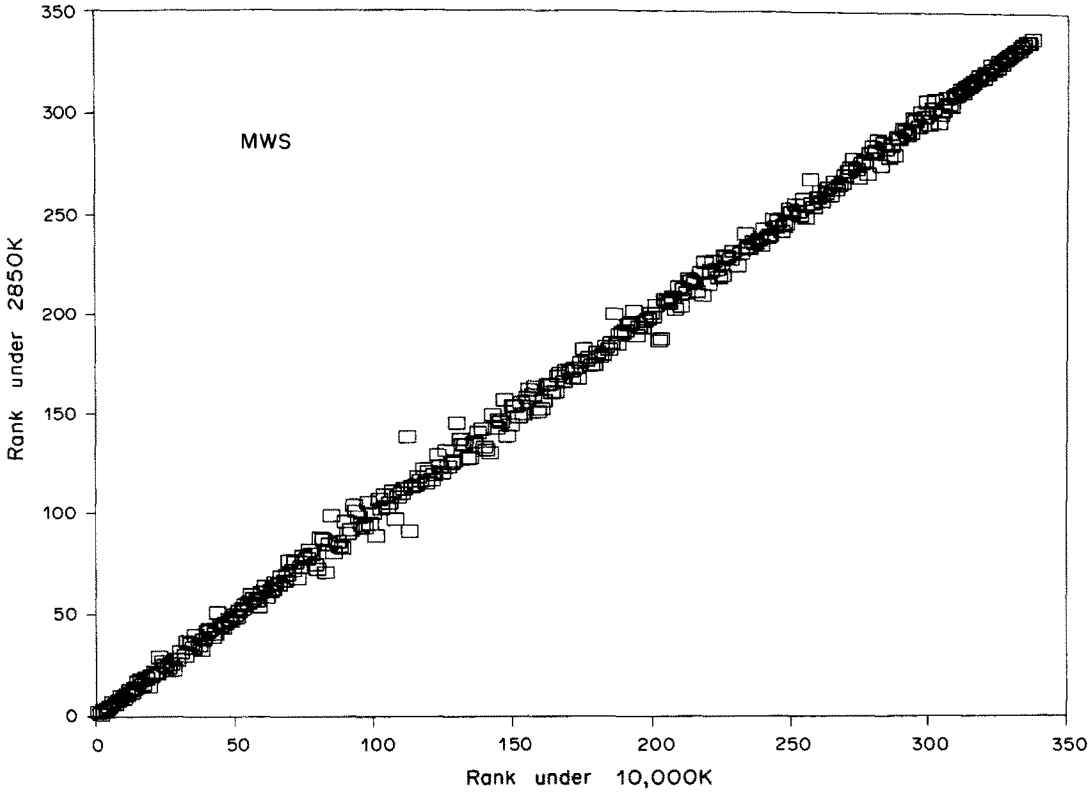

FIGURE 2. Same as in Fig. 1 for the MWS photoreceptors.

and natural illuminants. This was done using a set of spectral reflectance functions for natural objects published by Krinov (1947). The photon catches were calculated using different phases of daylight illumination or with a tungsten illuminant.

Color constancy surely involves more than simply preserving the ordinal relationships between photon catches from a constant set of objects under different illuminants. The claim is not that preservation of the rank ordering is a sufficient condition for color constancy. Rather, the claim is that it is a necessary condition if first-stage photoreceptor adaptation alone is supposed to produce color constancy. If this condition is violated, then first-stage photoreceptor adaptation alone cannot produce color constancy. What follows is an empirical test of this condition.

METHODS

Photoreceptor photon absorptions in the fovea of a model human eye were calculated using a large collection of natural objects and formations illuminated by phases of daylight and a tungsten illuminant. 337 reflectance functions from the set published by Krinov (1947) were used. Krinov published data on 370 reflectance functions covering the visible and infrared regions of the spectrum, but not all of these contained complete data in the visible region. Krinov divided these formations and objects into the following groups: forests and shrubs; grasses; mosses and lichens; field and garden crops; outcrops and soils; roads; water surfaces; water bodies and snow; and finally, buildings and building materials. It should be noted that some of these spectra represent optical integrations of two or more distinct materials. Krinov measured these reflectance functions from 1935 to 1938 in various regions of what was then the Sovient Union at various times of the year. These regions extended from Khibina in the north to Merv in the south and from Leningrad in the west to Mary in the east; an area bounded by latitudes 35–65° and longitudes 30–65°. Krinov tabled these data as the reflectance at wavelengths between 400 and 650 nm at 10 nm intervals. Krinov reported the average error in reflectance to be approx. 6% of its mean value.

The phases of daylight were constructed from basis vectors published by Judd, MacAdam and Wyszecki (1964). The mean and first two basis vectors were used to construct daylight illuminant spectral power distributions with correlated color temperatures of 4800, 6500 and 10,000 K. These three basis functions capture almost all of the variance in phases of daylight illumination. These different phases of daylight correspond to the distributions that result from the sun being at various elevations in the sky or alternatively, they correspond to the illumination from the sun at various times of the day. The tungsten illuminant was taken from Wyszecki and Stiles [1982, Fig. 5 (1.2.2), p. 18] and had a correlated color temperature of 2850 K. This illuminant was used to extend the range of correlated color temperatures. The intensities of all the illuminants were fixed arbitrarily to produce a retinal illuminance of 6400 td from the first object in Krinov's set through a 2 mm pupil in a model, human eye. This means of course that other objects produced different retinal illuminances

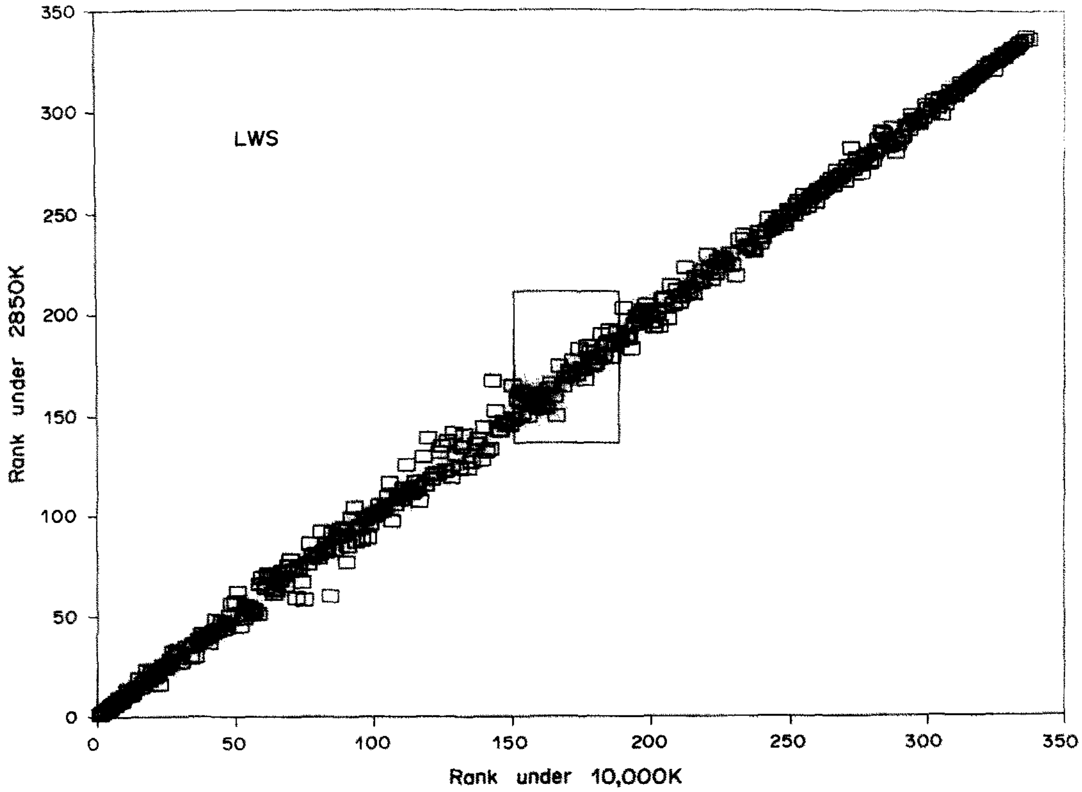

FIGURE 3. Same as in Fig. 1 for the LWS photoreceptors.

given the fixed intensity illumination. The 6400 td retinal illuminance was chosen as representative based on photometric measurements from several common objects at noon on a sunny day.

In order to calculate the average photon absorption per photoreceptor, I used the foveal receptor lattice dimensions and photoreceptor characteristics detailed in Geisler (1989). The effective aperture of a foveal cone

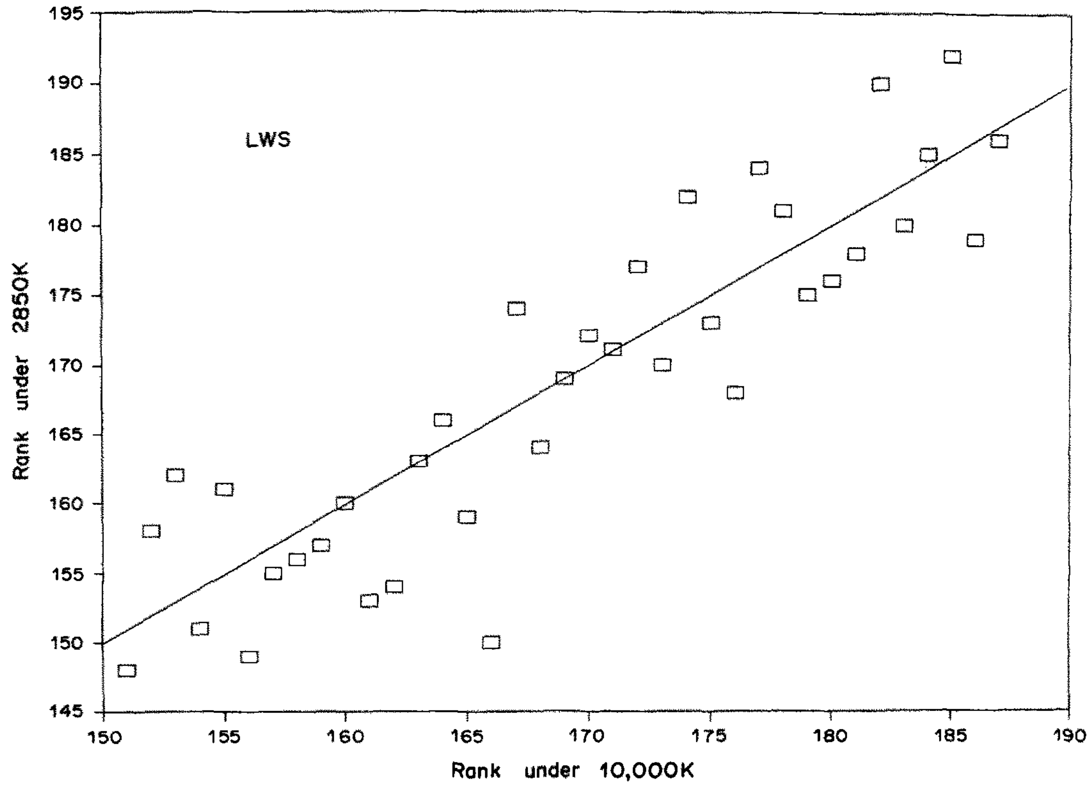

FIGURE 4. Detail showing the information in the box in Fig. 3. The middle 11% of the ranks for the photon absorptions under the 10,000 K illuminant are shown here. Notice that the rank ordering is not perfect. There is some shifting, but the shifting is local within the overall ranking shown in Fig. 3.

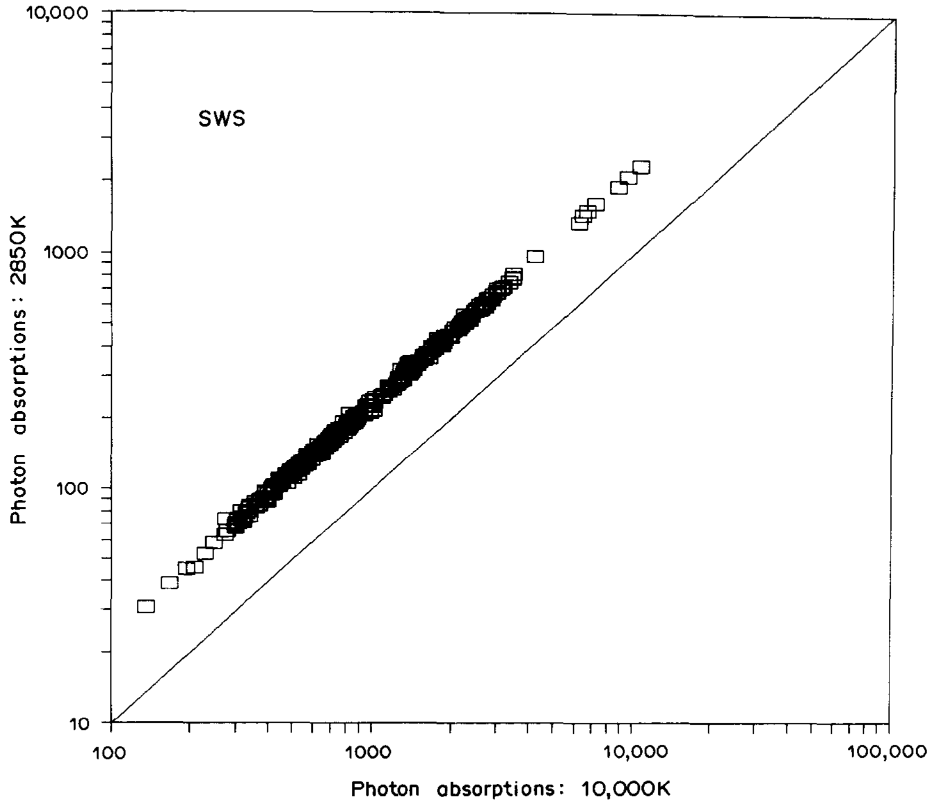

FIGURE 5. Calculated mean number of photons absorbed per SWS photoreceptor per 100 msec under the 10,000 K illuminant (x-axis) and the 2850 K illuminant (y-axis). The absorptions from all 337 spectra are plotted on double logarithmic axes.

was taken as 28 sec of arc (Miller & Bernard, 1983). The spectral transmittance of the ocular media (macular and lens pigments) was taken from Wyszecki and Stiles (1982) and the photoreceptor spectral sensitivity functions of Smith and Pokorny (1972, 1975) scaled to a peak absorptance of 0.5 were used. I calculated the absolute average number of photons absorbed per photoreceptor during a 100 msec period under all conditions. Details of the calculations are given in Dannemiller (1992).

Realistic values were assumed for the parameters of the eye, photoreceptors and illuminant intensity. These parameters, however, act only as scalars in determining the average photon absorption, so changes in these values would leave the rank ordering unchanged. The absolute number of photons absorbed is important in one respect; the variance in the number of photons absorbed by a photoreceptor during some period of time is directly proportional to the average absorption because of the random nature of light emission. Any analysis that considers the effects of photon noise must specify the actual values of these parameters.

RESULTS AND DISCUSSION

The rank order of the photon absorption within each photoreceptor class for each of the 337 objects was first calculated under all four illuminants. These rank orders were then compared within each photoreceptor class separately for pairs of illuminants. Figures 1, 2 and 3 show scatterplots of the ranks for the SWS, MWS and LWS photoreceptors for the most distinct illuminant substitution. In each plot the rank of the photon catch from each object under the 10,000 K illuminant is shown on the x-axis, and the corresponding rank of the photon catch for the same object under the 2850 K illuminant is shown on the y-axis. This is the extreme illuminant shift; the other shifts produced similar results. Recall, that if the rank ordering changes across an illuminant substitution, then in principle first-stage photoreceptor adaptation alone could not produce color constancy. The scatterplots show that in all cases, the rank order correlation is very high. The ordinal relationships between the photon catches from this set of objects under different illuminants are preserved very well within all three photoreceptor classes.

The largest shift in rank—26 positions from a total of 337 positions—occurred in the MWS photoreceptors. The average absolute values of the shifts in rank were 3.66, 3.10 and 2.94 positions for the LWS, MWS and SWS photoreceptors, respectively. The middle 11% of the distribution in terms of rank under the 10,000 K illuminant is replotted in Fig. 4. This is a magnified view of the box shown in Fig. 3. This detail shows that the rank ordering is not perfectly stable, but rather that there is some shifting of position. This shifting tends to be local, however; objects that are near the bottom of the rank ordering under the 10,000 K illuminant remain near the bottom of the rank ordering under the 2850 K illuminant. The rank order correlation within this middle set of 37 spectra is 0.86. These data show that von Kries adaptation alone could not produce perfect color constancy, although the degree to which the rank ordering was perturbed under an illuminant shift would depend on the range and distribution of the photon absorptions,

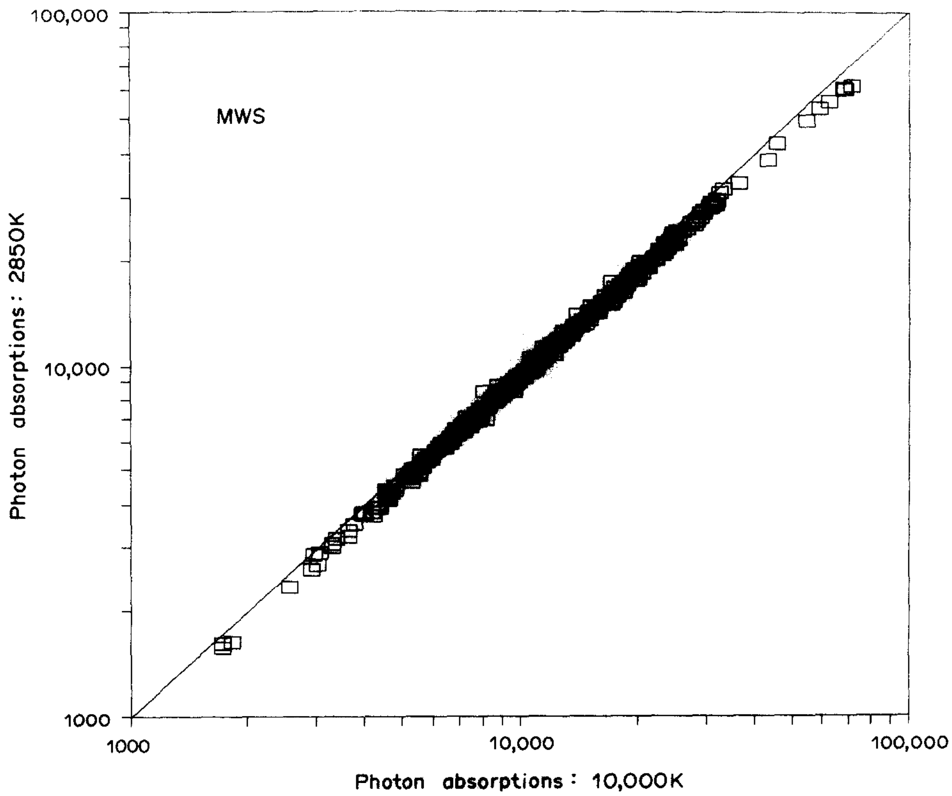

FIGURE 6. Same as in Fig. 5 for the MWS photoreceptors.

which, in turn, would depend on the composition of the scene. A scene composed only of the 37 spectra considered in Fig. 4 would be highly unusual because all of the regions in the image would produce very similar mean photon absorption levels in the LWS photoreceptors. Scenes with a more diverse composition would have more stable rank orders.

The calculated photon absorptions under the two extreme illuminants are shown in Figs 5, 6 and 7 for the SWS, MWS and LWS photoreceptors, respectively.

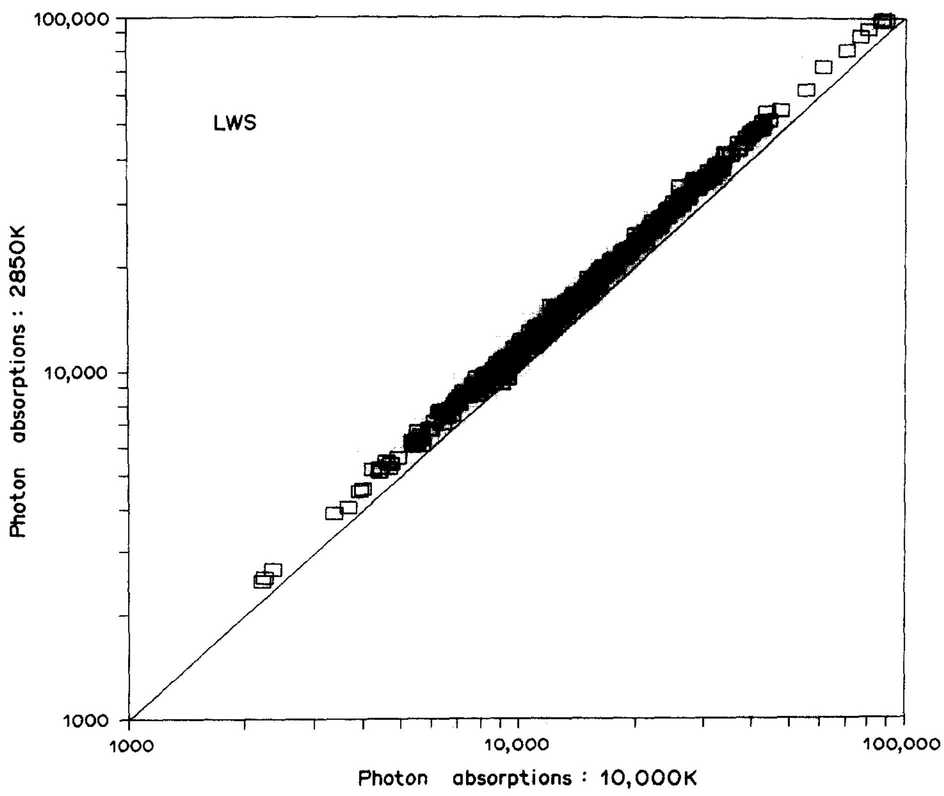

FIGURE 7. Same as in Fig. 5 for the LWS photoreceptors.

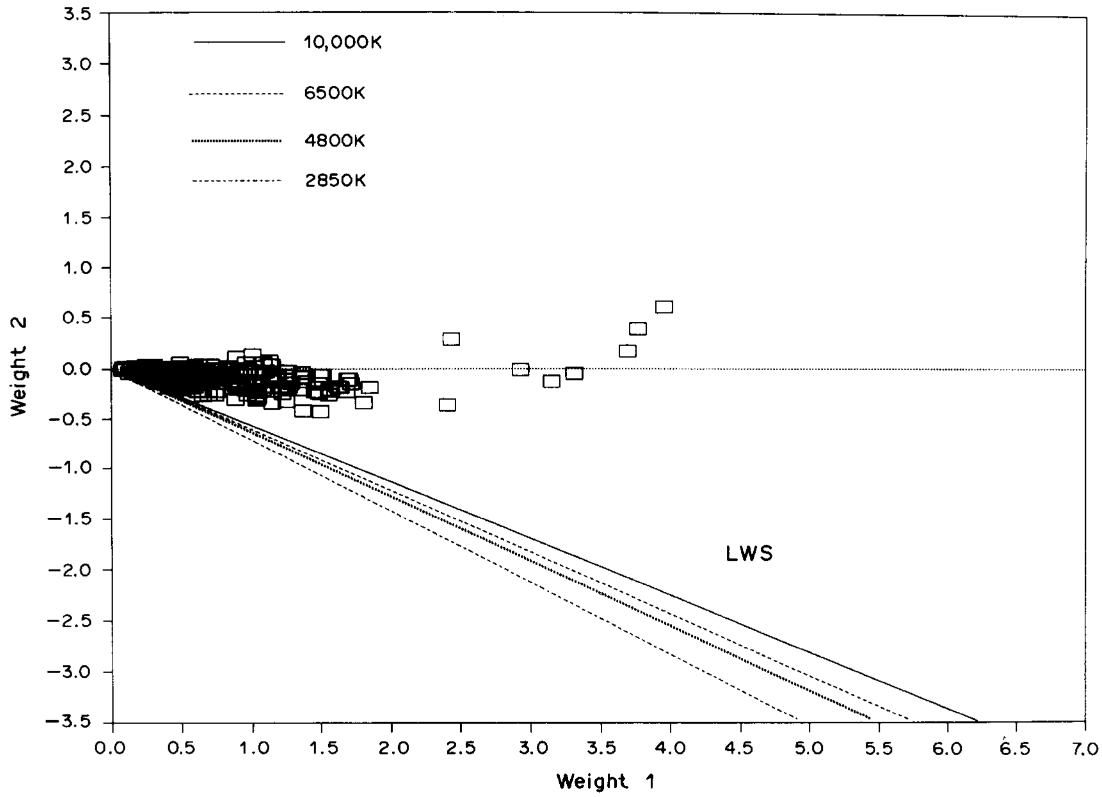

FIGURE 8. Principle component scores for the 337 spectra. The first principle component score is indicated on the x-axis (Weight 1), and the second principle component score is indicated on the y-axis (Weight 2). Each spectrum is represented as a vector with its tail at the origin and its head at the point x, y. Also shown are the lines that represent the four illuminants in conjunction with the LWS photoreceptors. The photon absorption is found by projecting the vector for a given spectrum onto the line for a given illuminant. See the text for further details.

Both axes are logarithmic, so a mutiplicative (divisive) adaptation would correspond to a vertical or horizontal shift with no change in slope. The slopes of the least-squared error regressions lines through the data in Figs 5, 6 and 7 were 1.01, 1.00 and 0.99. This shows that the photon absorptions under the 2850 K illuminant could be transformed into the absorptions under the 10,000 K illuminant using a simple multiplicative scaling with very little residual error.

Why are the rank orderings of these photon catches so similar across illuminants? The answer may be found in the fact that for this set of reflectance spectra, the first principle component captures approx. 95% of the total variance (Maloney, 1986), and the first two principle components capture over 98% of the variance in these reflectance spectra. A geometric representation of this situation shows the consequence of this fact for the stability of the rank orderings. Figure 8 shows a plot of the first two principle component scores for all 337 spectra. The axes of the plot are orthogonal by definition. The principle component score reflects the amount of that component that would have to be used to approximate the actual reflectance spectrum. Each spectrum can be considered a vector with its tail at the origin and its head at the point representing the two principle component scores. As noted before, most of the variance is captured by the first principle component. This is indicated in Fig. 8 by the fact that the cloud of scores is elongated in the direction of the first principle component. The first principle component is also constrained to be positive. Using only two principle components simplifies the exposition, but there is some justification for this simplification because of the total amount of variance captured by these principle components.

Also plotted in Fig. 8 are the lines that represent the various illuminants in conjunction with the LWS photoreceptors. Each illuminant is represented by a point in this two-dimentional space; variations in the intensity of the illuminant simply move the point toward or away from the origin along the fixed line. The line for a given illuminant passes through the origin and a point determined by the inner products between the illuminant spectrum, the LWS photoreceptor spectral sensitivity function, the ocular transmission function and the first two principle components mentioned above. The abscissa of this point is determined by the inner product involving the first principle component, and the ordinate is determined by the inner product involving the second principle component. Each line is constrained to pass through the origin because a zero intensity illuminant will have an inner product of zero with both principle components. The photon absorption for a given spectrum is then given by the projection of the two-dimensional vector representing that spectrum on the line representing the illuminant. Variations in the illuminant cause these lines to rotate through this principle component space. Figures 9 and 10 show similar constructions for the MWS and SWS photoreceptors.

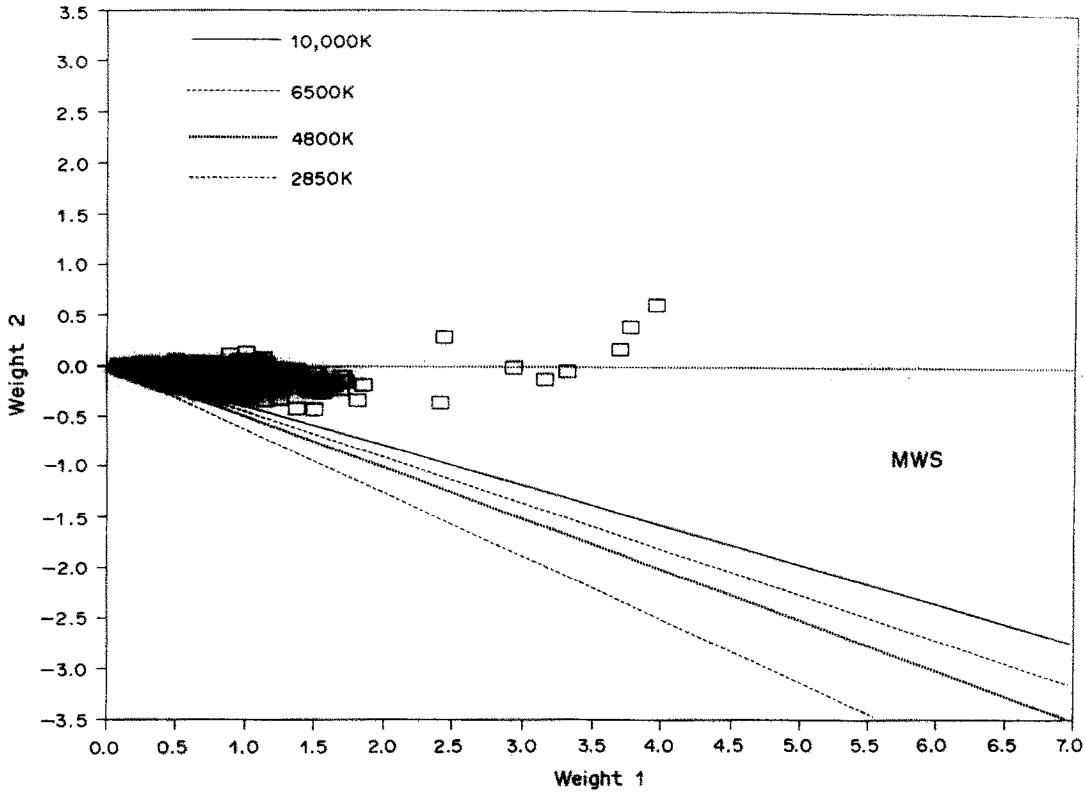

FIGURE 9. Same as Fig. 8 for the MWS photoreceptors.

We can now use this formalism to examine the conditions that will produce a change in the rank ordering of the photon absorptions contingent on an illuminant change. Figure 11 is a schematic showing a collection of reflectance spectra as an ellipsoidal cloud and three illuminants, I1, I2 and I3, as lines. Two objects, O1 and O2, are shown as squares. The absorptions produced from O1 and O2 under illuminant I1 are indicated by their projections on I1, and similarly, the absorptions produced by these two objects under

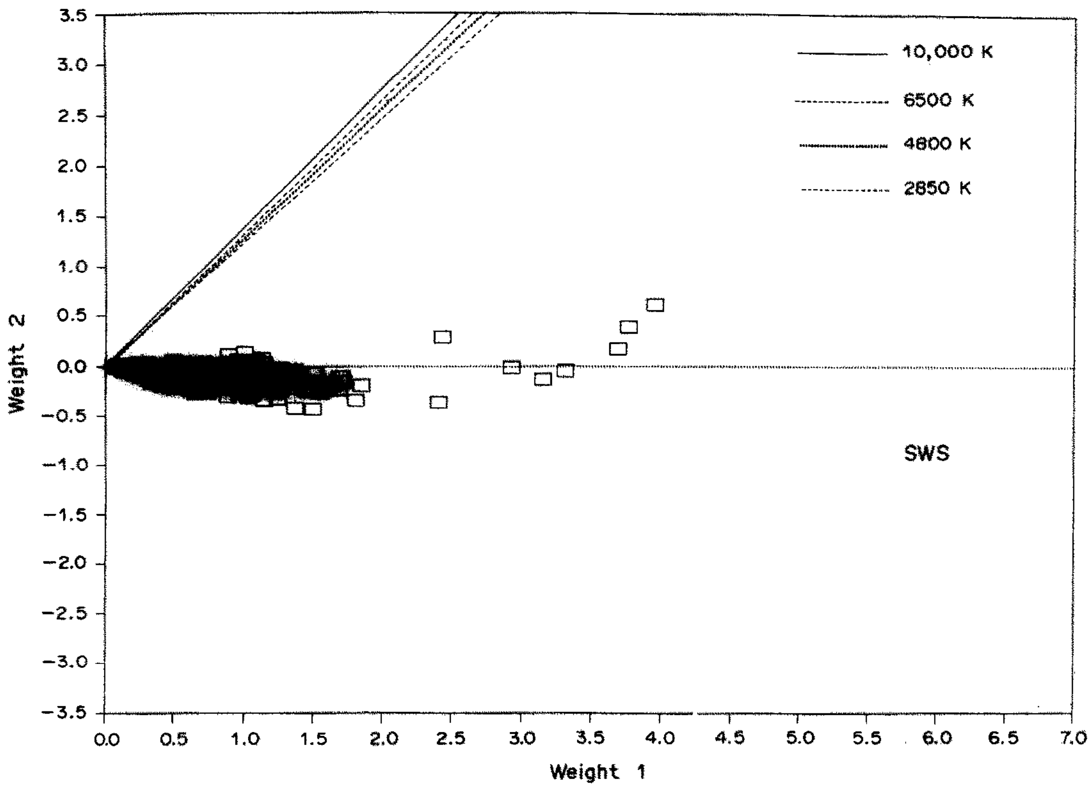

FIGURE 10. Same as Fig. 8 for the SWS photoreceptors.

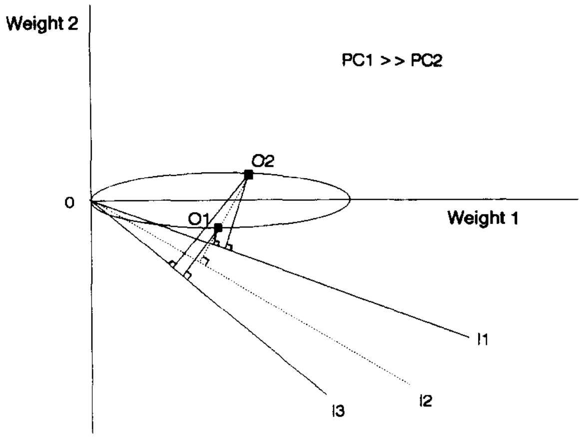

FIGURE 11. Schematic using the same conventions as Figs 8, 9 and 10. The collection of spectra are shown as an ellipsoidal cloud elongated in the direction of the first principle component; the first principle component accounts for much more of the variance than does the second principle component (PC1 $\gg$ PC2). Two spectra O1 and O2 are represented by the squares. Also shown are three illuminant lines, I1, I2 and I3. The projection of the vector representing a spectrum onto an illuminant line gives the number of photon absorptions produced by that spectrum under that illuminant. O2 produces more absorptions than O1 under I1, but the reverse is true under I3, so the rank order of these two spectra reversed under the shift from I1 to I3. This must mean that at some intermediate illuminant, I2, the absorptions from these two spectra were equal.

illuminant I3 are indicated by their projections on I3. Notice that O2 produces more absorptions than O1 under illuminant I1, but the reverse is true under illuminant I3. This means that the rank order of the absorptions from these two spectra has reversed under the shift from I1 to I3 and vice versa. Now if the rank order of these two objects has shifted, then this must mean that there is an intermediate illuminant under which the absorptions are equal. This intermediate illuminant is found by constructing a line through the origin and perpendicular to the line connecting the two spectra, O1 and O2. This illuminant is shown here as I2. The projections of O1 and O2 on I2 are equal; this means that these two spectra produce the same number of absorptions for this photoreceptor class under this illuminant. So any pair of spectra for which we can find an intermediate illuminant that lies between I1 and I3 will exhibit a change in the rank order of their photon absorptions as the illuminant shifts from I1 to I3.

Now we can examine the actual plots of the reflectance spectra and illuminants shown in Figs 8, 9 and 10 to determine the likelihood of finding shifts in the rank order of photon absorptions. There are several characteristics that work together to ensure that the shifts in the rank ordering are minimal and tend to be local within the ordering. First, notice that for all three photoreceptor classes, a large shift in the correlated color temperature of the illuminant causes only a small rotation of the illuminant line. The magnitude of this rotation will determine how large the shifts in photon absorptions will be. The bandwidths of the photoreceptor spectral absorption functions constrain these rotations (Barlow, 1982); in the extreme, photoreceptors with monochromatic absorption functions would produce no rotation, and hence the rank order could not change. But there must be a tradeoff between sensitivity and the tendency to allow rotations of the illuminant line; broader absorption bandwidths allow more photons to be caught at the expenses of allowing more shifts in the rank orderings of photon absorptions.

The other factor that determines how many and how large the rank order shifts will be is the distribution of the object spectra in the two-dimensional principle component space. Consider an extreme case in which all of the variation among the spectra was captured by one principle component only. This would correspond to all of the spectra plotting only to positions along the first principle component axis in plots like those shown in Figs 8, 9 and 10. There would be no variation along the second principle component axis. In this extreme case, that corresponds to all of the spectra being simply scaled versions of each other, no reordering of the photon absorptions would be possible no matter how large the rotation of the illuminant. Now consider what happens when there is some residual variation along the second principle component axis. It is this variation that leads to shifts in the rank ordering of the photon catches with illuminant shifts. It is this variation that rules out the ability of von Kries or other monotonic adaptation mechanisms alone to produce perfect color constancy. It is also this variation that leads to differences in the color of objects. But the failure of these mechanisms is one of degree. If most of the variance is in the first principle component scores, as it is for this collection of objects, then the reorderings of the photon catches will tend to be few and small in magnitude, especially when coupled with small rotations of the illuminant.

There is some minor shifting in the rank ordering across illuminant substitutions. Perfect color constancy would be strictly impossible for this collection of objects if the only permissible transformation on the photon absorptions were one of rescaling within each photoreceptor class. Worthey and Brill (1986) reached the same conclusion from an analysis of the role of von Kries adaptation in color constancy. But human color vision is probably not perfectly color constant (Valberg & Lange-Malecki, 1990). If postreceptoral combinations between photoreceptor classes were permitted as in the linear model algorithms proposed by Maloney and Wandell (1986), Buchsbaum (1980) and Brill (1978), then the reordering of photon absorptions within photoreceptor classes is no longer an insoluble problem. Second-stage combinations across the three photoreceptor classes could reorder the photon catches.

There are several limitations of this analysis that must be acknowledged. Rescaling the photon catches by the average photon catch has implications for the time-course of the adaptation mechanism. Specifically, it implies that the time constant of this adaptation would have to be relatively slow. Eye movements will expose the photoreceptors to different parts of the image over time. In order for these photoreceptors to adapt perfectly to the illuminant, the time constant of this adaptation would have to be rather slow. Hayhoe *et al.* (1992) found psychophysical evidence of both a fast, multiplicative adaptation and a slow, subtractive adaptation in their study of the temporal properties of light adaptation. As they pointed out in their paper, the fast adaptation would work against adapting local regions of the retina to the illuminant, while the slow adaptation—a temporal filter with a long time constant—would work toward this purpose. Schnapf *et al.* (1990) reported the existence of a slow mechanism that desensitizes primate cones to bright lights; the full effects of this mechanism are not observed until approx. 1 sec after the onset of a bright adapting field.

Another limitation of this analysis is that it does not consider the effects of noise. In particular, the random nature of light emission will reduce the stability of these rank orderings. However, with some spatial pooling over the photoreceptors, the effects of this noise could be reduced because such pooling would effectively increase the signal to noise ratio. Finally, this analysis does not consider the complications introduced by shadows and gradients of illumination. The imposition of a shadow coincident with an illuminant shift would perturb the rank ordering of the photon absorptions because the objects that were now in the shadow would produce far fewer photon absorptions than they did under the original illuminant. It is of course true that within the shadow, objects would tend to maintain a stable rank ordering. So, a more realistic analysis should include the effects of shadows and illumination gradients.

These results place an important constraint on the types of procedures necessary to recover truly illuminant invariant estimates of an object's color. The results imply that the conditions in the daylight environment are such that first-stage photoreceptor adaptation coupled with eye movements could contribute substantially to human color constancy if rather slow adaptation mechanisms were available, although this would not produce perfect color constancy. It should prove possible to apply these calculations to other sets of objects and illuminants with known spectra and to the visual systems of other species to determine the extent to which first-stage photoreceptor adaptation alone could produce color constancy for that species. Extensive reordering of the photon catches across an illuminant change would limit the contribution that first-stage photoreceptor adaptation alone could make to color constancy. The contribution of second-stage mechanisms to color constancy would become progressively more important for species/environment combinations that violate this rank ordering constraint.

REFERENCES

Barlow, H. B. (1982). What causes trichromacy? A theoretical analysis using comb-filtered spectra. *Vision Research*, *22*, 635–643.

Brill, M. H. (1978). A device performing illuminant-invariant assessment of chromatic relations. *Journal of Theoretical Biology*, *71*, 473–478.

Buchsbaum, G. (1980). A spatial processor model for object colour perception. *Journal of The Franklin Institute*, *310*, 1–26.

Dannemiller, J. L. (1992). Spectral reflectance of natural objects: How many basis functions are necessary? *Journal of the Optical Society of America A*, *9*, 507–515.

Geisler, W. S. (1981). Effects of bleaching and backgrounds on the flash response of the cone system. *Journal of Physiology*, *312*, 413–43.

Geisler, W. S. (1983). Mechanisms of visual sensitivity: Backgrounds and early dark adaptation. *Vision Research*, *23*, 1423–1432.

Geisler, W. S. (1989). Sequential Ideal Observer analysis of visual discriminations. *Psychological Review*, *96*, 276–314.

Hayhoe, M. M., Levin, M. E. & Koshel, R. J. (1992). Subtractive processes in light adaptation. *Vision Research*, *32*, 323–333.

Hurlbert, A. (1986). Formal connections between lightness algorithms. *Journal of the Optical Society of America A*, *3*, 1684–1693.

Hurlbert, A. C. & Poggio, T. A. (1988). Synthesizing a color algorithm from examples. *Science*, *239*, 482–485.

Judd, D. B., MacAdam, D. L. & Wyszecki, G. (1964). Spectral distribution of typical daylight as a function of correlated color temperature. *Journal of the Optical Society of America*, *54*, 1031–1040.

Krinov, E. L. (1947). Spectral reflectance properties of natural formations. *National Research Council of Canada, Ottawa, TT-439.*

Land, E. H. & McCann, J. J. (1971). Lightness and retinex theory. *Journal of the Optical Society of America*, *61*, 1–11.

MacAdam, D. L. (1970). *Sources of color science*. Cambridge, Mass.: MIT Press.

Maloney, L. T. (1986). Evaluation of linear models of surface spectral reflectance with small numbers of parameters. *Journal of the Optical Society of America A*, *3*, 1673–1683.

Maloney, L. T. & Wandell, B. A. (1986). Color constancy: A method for recovering surface spectral reflectance. *Journal of the Optical Society of America A*, *3*, 29–33.

Miller, W. & Bernard, G. 1983). Averaging over the foveal receptor aperture curtails aliasing. *Vision Research*, *23*, 1365–1369.

Schnapf, J. L., Nunn, B. J., Meister, M. & Baylor, D. A. (1990). Visual transduction in cones of the monkey *Macaca fascicularis*. *Journal of Physiology*, *427*, 681–713.

Smith, V. C. & Pokorny, J. (1972). Spectral sensitivity of color-blind observers and the cone photopigments. *Vision Research*, *12*, 2059–2071.

Smith, V. C. & Pokorny, J. (1975). Spectral sensitivity of the foveal cone photopigments between 400 and 500 nm. *Vision Research*, *15*, 161–171.

Uchikawa, K., Uchikawa, H. & Boynton, R. M. (1989). Partial color constancy of isolated surface colors examined by a color-naming method. *Perception*, *18*, 83–91.

Valberg, A. & Lange-Malecki, B. (1990). "Colour constancy" in mondrian patterns: A partial cancellation of physical chromaticity shifts by simultaneous contrast. *Vision Research*, *30*, 371–380.

Worthy, J. A. & Brill, M. H. (1986). Heuristic analysis of von Kries color constancy. *Journal of the Optical Society of America A*, *3*, 1708–1712.

Wyszecki, G. & Stiles, W. S. (1982). *Color science*. New York: Wiley.

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