Psychological Review
1989, Vol. 96, No. 2, 255–266

0033-295X/89/$00.75

Computational Approaches to Color Constancy: Adaptive and Ontogenetic Considerations

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Recent computational approaches to color constancy can be realized using a two-stage model of color vision. Adaptation at both the sensor stage and the second or reflectance channel stage is necessary to compute illuminant-invariant reflectance estimates. von Kries adaptation is shown to contribute significantly toward reducing, but not completely eliminating, the need for adaptation at the second stage. Examples of computations from the model using daylight illuminants, simulated natural reflectance functions, and putative human spectral sensitivity functions are shown. The ontogenetic plausibility of these models is also discussed. The role of the second-stage transformation in estimating surface reflectance is compared with the role of opponent color transformations in decorrelating primary receptor outputs.

Part of the adaptive value of trichromatic color vision derives from its ability to deliver information about differences in the material characteristics of objects and surfaces. Differences in the spectral reflectance characteristics of regions within an image can be indicative of material differences in those regions of the scene. These chromatic differences within an image may be useful for segregating or aggregating different image regions as a preliminary step toward representing surfaces and objects within the scene. It would also be useful if the representation of the reflectance properties of a given object remained invariant across changes in the illumination on the scene. This invariance would simplify the task of recognizing the same object or class of objects in different contexts or under different illuminants. However, the physical laws of surface illumination complicate this extraction of invariant reflectance information. The spectrum of the light that reaches the eye from a region in a scene is the product of the reflectance spectrum of the object and the spectral power distribution (SPD) of the illuminant. Changes in the illuminant SPD cause changes in the spectrum of light reaching the eye from the same surface in a scene. This inconstancy must somehow be overcome in order to realize the adaptive value of trichromatic color vision.

Several recent computational approaches to this problem have met with success (Brill, 1978; Buchsbaum, 1980; Maloney & Wandell, 1986). The success of these models depends on several important constraints on possible object spectral reflectance functions and illuminant spectral power distributions. It is assumed that object spectral reflectance functions and illuminant spectral power distributions can be modeled with a small amount of residual error using linear combinations of a small number of weighted basis functions. Maloney (1986) showed that a large sample of naturally occurring spectral reflectance functions can be described using weighted combinations of approximately two to four basis functions. Judd, MacAdam, and Wyszecki (1964) showed that different phases of daylight can be composed using weighted combinations of two to three basis functions. With these constraints on possible object reflectance functions and illuminant spectral power distributions, and with the additional assumption that the illuminant SPD is spatially uniform or only slowly varying across the scene, these models deliver potentially useful, illuminant-invariant representations of surface reflectance.

Of course, if the human visual system were to operate with the algorithms contained in these models, then one should expect errors (inconstancy) when surfaces and illuminants in the world violate the constraints detailed earlier. For example, an object with a spectral reflectance function that is not well-approximated by a linear model with only two or three parameters may not appear to have the same surface color under different illuminants. Alternatively, illuminants that require a large number of basis functions for their precise description may not be estimated accurately by these algorithms, with the result that errors are introduced into the computed spectral reflectance function for an object. Worthey (1985) and McCann, McKee, and Taylor (1976) have shown that there is a lack of constancy in the case of human color vision when three narrowband primaries are used for the illuminant. Such an illuminant clearly violates the constraint on possible illuminant spectral power distributions in the aforementioned computational models. It remains to be determined whether the human visual system exhibits color constancy for naturally reflecting objects illuminated by phases of daylight as these models imply and as Judd (1940) hypothesized.

Classical approaches to the problem of color constancy often relied on some type of adaptation within the visual system to remove the contribution of the illuminant. Worthey (1985) and Worthey and Brill (1986) argued that the type of receptor adaptation proposed by von Kries (Helson, Judd, & Warren, 1952) could contribute to constancy in the face of illuminant SPD

Computing time was supported by Core Grant NICHD-HD03352 to the Waisman Center. I thank Pat Klitzke for typing the manuscript.

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changes. D'Zmura and Lennie (1986) argued that receptor adaptation plays a role in achieving color constancy. They also argued that adaptation at a second, chromatically opponent site within the visual pathway may be necessary to achieve color constancy.

The goal of this article is to show that the computational model of color constancy proposed by Buchsbaum (1980) may be realized using a two-stage model of color vision. Adaptation at both of these stages is necessary to achieve color constancy. In particular, second-site adaptation is necessary to remove the residual effects of the illuminant SPD that remain following a first-site, von Kries-type receptor adaptation. Second-site opponent transformations are instrumental in estimating object spectral characteristics independently of the illuminant SPD. The models are discussed in terms of their ontogenetic plausibility and susceptibility to developmental perturbations, and in light of the function of opponent-color transformations as hypothesized by Buchsbaum and Gottschalk (1983).

Formulation of the Problem

A mathematical model is useful for understanding the computational problem involved in color constancy. I follow the scheme used by Maloney (1986) in describing object spectral reflectance functions and illuminant SPD's as linear models with a small number of basis functions. Consider first an object spectral reflectance function S(λ). We may describe this function with a linear model having *m* basis functions using Equation 1.

$$\mathrm{S}(\lambda) = \sum_{i=1}^{m} \sigma_i \mathrm{S}_i(\lambda) + e_s(\lambda). \tag{1}$$

The $\mathrm{S}_i(\lambda)$ are the basis functions, the σ_i are the weights on these functions, and $e_s(\lambda)$ is a residual error function. The σ_i weights are chosen to minimize the root mean square (RMS) error between S(λ), which is the true function, and the closest approximation to S(λ) achievable with *m* basis functions. Maloney (1986) showed that the residual error can be very small for a large collection of naturally reflecting surfaces when $m = 3$. Similarly, we may describe the illuminant SPD using a basis set of *n* functions.

$$\mathrm{L}(\lambda) = \sum_{j=1}^{n} w_j \mathrm{L}_j(\lambda) + e_\mathrm{L}(\lambda). \tag{2}$$

Again, Judd et al. (1964) showed that two to three basis functions are sufficient to capture most of the variance of daylight SPDs.

Next, consider the receptor spectral sensitivity functions that are used by a hypothetical visual system to sample the input to the eye. These receptor spectral sensitivity functions may be designated R(λ), G(λ), and B(λ), corresponding, respectively, to the long-, middle-, and short-wavelength-sensitive photoreceptor mechanisms in trichromatic human vision.

The response of one of the receptor types (e.g., the R receptors) to an input is then given by the integral

$$r = \int_{400}^{700} \mathrm{R}(\lambda)\mathrm{L}(\lambda)\mathrm{S}(\lambda)d\lambda. \tag{3}$$

Similarly, the responses for the G and B receptors are obtained by substituting the appropriate G(λ) and B(λ) spectral sensitivity functions in Equation 3. Thus, at each location on the retina, the visual system has available to it a triplet of measurements that may be represented as a vector $(\mathbf{r}\quad \mathbf{g}\quad \mathbf{b})^\mathrm{T}$, in which T represents the transpose of a vector. These three measurements serve as the primary information that is to be used to compute the spectral reflectance characteristics of the region of the scene from which the measurements were taken. The problem of color constancy may be stated with reference to Equations 1, 2, and 3: Using the vector $(\mathbf{r}\quad \mathbf{g}\quad \mathbf{b})^\mathrm{T}$ estimate the *m*-dimensional vector of weights in Equation 1, $(\sigma_1\quad \sigma_2\quad \sigma_3 \cdots \sigma_m)^\mathrm{T}$, that produced this triplet of receptor stimulations. Obviously, the error inherent in this estimate is determined in part by the 3 *df*s available in the measurement. I assume in the following, that the hypothetical visual system estimates all object spectral reflectance functions using three basis functions ($m = 3$). Again, to the extent that specific surfaces presented to such a visual system deviate significantly from such a three-dimensional model, one could expect errors in these estimates of surface reflectance functions. I will also assume that the hypothetical visual system models the illuminant SPD with $n = 3$ basis functions such as those used by Judd et al. (1964) to capture the phases of daylight.

Equation 3 may now be changed to reflect the fact that S(λ) and L(λ) are assumed to be completely described each by the three-dimensional vectors $(\sigma_1\quad \sigma_2\quad \sigma_3)^\mathrm{T}$ and $(\mathbf{w}_1\quad \mathbf{w}_2\quad \mathbf{w}_3)^\mathrm{T}$, respectively.

$$r = \int_{400}^{700} \mathrm{R}(\lambda)\Big[\sum_{j=1}^{3} w_j \mathrm{L}_j(\lambda) \sum_{i=1}^{3} \sigma_i \mathrm{S}_i(\lambda)\Big]d\lambda. \tag{4}$$

$$r = \sum_{j=1}^{3}\sum_{i=1}^{3} w_j\sigma_i \int_{400}^{700} \mathrm{R}(\lambda)\mathrm{L}_j(\lambda)\mathrm{S}_i(\lambda)d\lambda. \tag{5}$$

$$r = \sum_{j=1}^{3}\sum_{i=1}^{3} w_j\sigma_i \langle \mathrm{RL}_j\mathrm{S}_i \rangle. \tag{6}$$

In Equation 6, the angle bracket expression is used as a convenient symbol for the definite integral in Equation 5. Equation 6 may be extended to include the responses of the G and B receptors.

$$g = \sum_{j=1}^{3}\sum_{i=1}^{3} w_j\sigma_i \langle \mathrm{GL}_j\mathrm{S}_i \rangle. \tag{7}$$

$$b = \sum_{j=1}^{3}\sum_{i=1}^{3} w_j\sigma_i \langle \mathrm{BL}_j\mathrm{S}_i \rangle. \tag{8}$$

Equations 6, 7, and 8 may be written in matrix notation as

$$\begin{bmatrix} \mathbf{r} \\ \mathbf{g} \\ \mathbf{b} \end{bmatrix} = \sum_{j=1}^{3} w_j \begin{bmatrix} \langle \mathrm{RL}_j\mathrm{S}_1 \rangle & \langle \mathrm{RL}_j\mathrm{S}_2 \rangle & \langle \mathrm{RL}_j\mathrm{S}_3 \rangle \\ \langle \mathrm{GL}_j\mathrm{S}_1 \rangle & \langle \mathrm{GL}_j\mathrm{S}_2 \rangle & \langle \mathrm{GL}_j\mathrm{S}_3 \rangle \\ \langle \mathrm{BL}_j\mathrm{S}_1 \rangle & \langle \mathrm{BL}_j\mathrm{S}_2 \rangle & \langle \mathrm{BL}_j\mathrm{S}_3 \rangle \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}. \tag{9}$$

Notice in Equation 9 that the terms in the 3 × 3 matrix are constants that the hypothetical visual system uses to model illuminant SPDs and object spectral reflectance functions. We may express Equation 9 in symbolic form as

$$\mathbf{p} = \mathbf{E}\sigma, \tag{10}$$

where **p** is a 3 × 1 vector of receptor responses, **E** is a 3 × 3 illuminant matrix and σ is a 3×1 vector of reflectance weights. This formulation is identical to the one used by Maloney and Wandell (1986). The goal of the hypothetical visual system is to estimate σ from $\mathbf{p}$. As is evident in Equation 10, the solution to this problem is to find the inverse of the illuminant matrix $\mathbf{E}$. Then

$$\sigma = \mathbf{E}^{-1}\mathbf{p}. \tag{11}$$

The problem should now be clear from a computational point of view. Because the illuminant vector $(\mathbf{w}_1\ \ \mathbf{w}_2\ \ \mathbf{w}_3)^T$ is unknown, $\mathbf{E}$ is unknown, hence, $\mathbf{E}^{-1}$ is unknown. As Buchsbaum (1980) and Maloney (1985) have shown, some estimate of $(\mathbf{w}_1\ \ \mathbf{w}_2\ \ \mathbf{w}_3)^T$ must be obtained in order to solve this problem. In the following sections, I use Buchsbaum's method of estimating $(\mathbf{w}_1\ \ \mathbf{w}_2\ \ \mathbf{w}_3)^T$ by assuming that the average spectral reflectance function in the scene is equivalent to some fixed, standard reflectance function. I also show that such a process is connected to an adaptive reweighting of linear combinations of the outputs of the adapted primary receptor responses. Thus, second-site adaptation is necessary in this model to achieve color constancy because von Kries-type adaptation at the first stage in the system still leaves some error in the estimation of σ.

Examples of the Estimation of σ

In this section I give examples of how this problem may be solved using specific basis functions and receptor primaries. For convenience, Cohen's (1964) basis functions are used for describing object spectral reflectance functions and Judd et al.'s (1964) basis functions are used for describing phases of daylight. The Vos-Walraven (1971)[1] primaries are used for the spectral sensitivity functions of the R, G, and B sensors. Three different phases of daylight illumination with correlated color temperatures of 4,800 °K, 6,500 °K, and 10,000 °K were used. The w vectors corresponding to these phases of daylight are given in Table 3 of Judd et al. (1964) and are shown here in Table 1. These three vectors were used to compose the E matrices in Equation 10 corresponding to the three phases of daylight illumination. These matrices are also shown in Table 1. The constants in Equation 9 were determined using numerical integration at 10-nm intervals of the appropriate products of basis functions and receptor primaries.

Table 1
Illuminant Basis Weights and E Matrices

Illuminant and matrices
4,800 °K $\mathbf{w}^T = (1.00\ \ -1.14\ \ .677)$
$\mathbf{E} = \begin{bmatrix} 294.53 & -3.31 & 55.96 \\ 139.38 & -15.62 & 29.74 \\ 1.50 & -0.50 & -0.21 \end{bmatrix}.$
6,500 °K $\mathbf{w}^T = (1.00\ \ -0.293\ \ -0.689)$
$\mathbf{E} = \begin{bmatrix} 288.24 & -8.39 & 55.59 \\ 144.49 & -18.26 & 30.05 \\ 2.14 & -0.71 & -0.32 \end{bmatrix}.$
10,000 °K $\mathbf{w}^T = (1.00\ \ 1.005\ \ -0.378)$
$\mathbf{E} = \begin{bmatrix} 288.43 & -13.30 & 55.69 \\ 152.32 & -21.43 & 30.54 \\ 3.01 & -0.99 & -0.47 \end{bmatrix}.$

Table 2
Inverse Matrices for Each Illuminant

Illuminant and matrices
4,800 °K
$\mathbf{E}^{-1} = \begin{bmatrix} 0.00723 & -0.0114 & 0.308 \\ 0.0294 & -0.0580 & -0.382 \\ -0.0185 & 0.0568 & -1.64 \end{bmatrix}$
6,500 °K
$\mathbf{E}^{-1} = \begin{bmatrix} 0.0081 & -0.0125 & 0.226 \\ 0.0327 & -0.0626 & -0.1864 \\ -0.0188 & 0.0553 & -1.20 \end{bmatrix}$
10,000 °K
$\mathbf{E}^{-1} = \begin{bmatrix} 0.0087 & -0.0132 & 0.169 \\ 0.0352 & -0.0652 & -0.0702 \\ -0.0185 & 0.0527 & -0.892 \end{bmatrix}$

For illustrative purposes only, assume next that these three illuminant matrices were known. Their inverses are shown in Table 2. These $\mathbf{E}^{-1}$ matrices could be used in Equation 11 to solve for the reflectance vector σ given the three-dimensional receptor quantum catch vector $\mathbf{p}$. Examination of Table 2 shows the major problem confronting any model of color constancy: namely, the correct mapping between the receptor responses and the spectral reflectance vector σ clearly changes as a function of the illuminant. For example, assume that the $\mathbf{E}^{-1}$ matrix corresponding to the 4,800 °K illuminant were used to estimate σ when the receptor stimulations were actually produced by the 10,000 °K illuminant. In other words, assume that our hypothetical visual system always used the same $\mathbf{E}^{-1}$ matrix to estimate σ. Figure 1 shows an example of just how inaccurate such an approach would be. The true reflectance function is shown by the triangles, whereas the estimated function is shown by the squares. The vector used to compose the true function and the deliberately incorrect estimate of that vector are shown in the legend to Figure 1. The value of the RMS error from Equation 1 is .24. In other words, the reflectance estimate is off by an average of approximately 24% across wavelength. This error will be more meaningful when we compare it with another error later. It is clear that the assumption of a fixed illuminant, an obviously erroneous assumption, can lead to gross misestimates of the true reflectance function. Surely, by any color difference metric, these two surfaces would appear different, implying a clear lack of constancy.

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[1] Vos and Walraven's (1971) primaries extend to 680 nm. Smith and Pokorny's primaries (as cited in Ingling & Tsou, 1977) were used through appropriate extrapolation to extend the Vos–Walraven primaries to 700 nm.

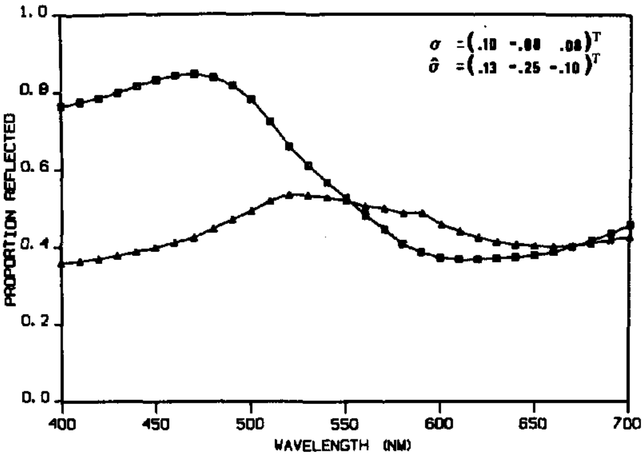

Figure 1. True reflectance function (triangles) and estimated reflectance function (squares). (The estimated function was obtained by using the reflectance-channel weights appropriate for a 4,800 °K illuminant with the receptor stimulations produced by a 10,000 °K illuminant. The weights in the legend for the true and estimated functions correspond to the weights on Cohen's (1964) first three basis functions. The purpose of this deliberately erroneous example is to illustrate the error in computed reflectance that results from a failure to adapt second-site gain factors to the illuminant when no von Kries scaling of receptor outputs occurs.)

A more systematic illustration of this problem is shown in Figure 2. In this figure, 25 different surfaces are represented by the solid symbols. These symbols correspond to the σ_2 and σ_3 values in various σ vectors all having $\sigma_1 = .10$. Also shown by the open symbols are the estimated values of σ_2 and σ_3 obtained by using the receptor stimulations appropriate to the 10,000 °K illuminant and the E^{-1} matrix appropriate to the 4,800 °K illuminant. It is obvious that the strategy of using a fixed set of weights to estimate σ would result in large, but systematic errors in estimating surface reflectance functions. This exercise illustrates the necessity of having a process in the hypothetical visual system that adjusts E^{-1} in order to estimate the reflectance vector correctly.

Table 2 also illustrates another characteristic of this type of model. The matrix $\mathbf{E}^{-1}$ produces three linear transformations on the receptor stimulation vector **p**; in other words, in the service of estimating σ with 3 *dfs*, three postreceptoral channels are created by selectively weighting the outputs of the primary receptors. I call these reflectance channels to denote their role in estimating object spectral reflectance characteristics. All three of these reflectance channels are chromatically opponent for the illumination conditions shown in Table 2. In particular, the R and G outputs are always of opposite sign in all three channels. This contrasts with the current notion in human color vision that there are three postreceptor channels, one of which adds the outputs of the R and G primaries in a nonopponent channel and two of which combine the outputs in a chromatically opponent manner, that is, R − G and B − (R + G) (e.g., Guth, Massof, & Benzschawel, 1980). Nonetheless, the interesting aspect of this general approach is that three postreceptor channels are necessary in using the receptor stimulations to estimate object reflectance characteristics—a task of obvious adaptive significance.

Given the variation exhibited by the $\mathbf{E}^{-1}$ matrices across illumination conditions, it is natural to inquire next about operations that might render the reflectance channel weights embodied in these $\mathbf{E}^{-1}$ matrices more nearly constant across illuminants. Such an operation would reduce the amount of second-site adaptation necessary to achieve color constancy. One such transformation that has often been proposed in connection with the problem of color constancy is a von Kries-type adaptation. One way to realize von Kries adaptation is to scale all local receptor outputs of a given class independently against the mean response of that class of receptors across the scene. This adaptation adjusts the sensitivity of a photoreceptor in order to make efficient use of its finite response range and to ensure that the inputs to the next stage fall within some expected range.

Scaling of local receptor responses by their respective means $\bar{r}$, $\bar{g}$, and $\bar{b}$ may be effected by premultiplying the receptor response vector **p** in Equation 11 by a diagonal matrix containing the inverses of the mean receptor responses in their appropriate positions. This is shown in Equation 12. The **V** matrix,

$$\sigma = \mathbf{V}\begin{bmatrix} 1/\bar{r} & 0 & 0 \\ 0 & 1/\bar{g} & 0 \\ 0 & 0 & 1/\bar{b} \end{bmatrix}\begin{bmatrix} r \\ g \\ b \end{bmatrix}, \qquad (12)$$

now symbolically reflects the fact that the inputs to the reflectance channels are now receptor responses scaled by their respective means rather than by the original, unscaled receptor responses. The weights in these channels must change to accom-

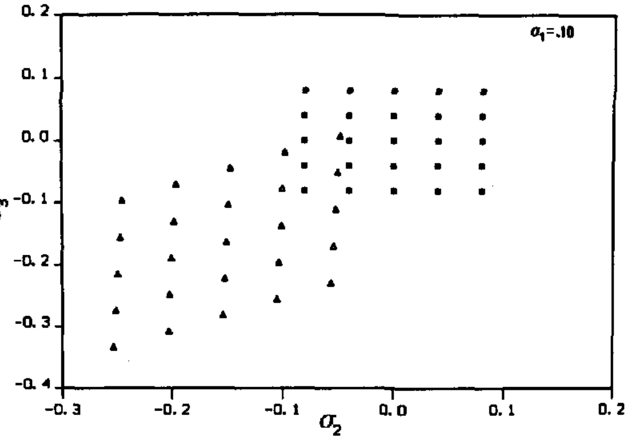

Figure 2. True (solid symbols) and estimated (open symbols) reflectance weights (σ_2 and σ_3) using reflectance-channel gain factors appropriate for a 4,800 °K illuminant and receptor stimulations produced by a 10,000 °K illuminant. (No von Kries adaptation was used. The σ_1 value for all surfaces is shown in the legend. Rank ordering by rows and columns of the estimated points is identical to the rank ordering by rows and columns of the true points: The upper left open point is an estimate of the upper left solid point. Failure to allow von Kries adaptation of receptor outputs and failure to adapt second-site gain factors to the illuminant result in gross misestimates of surface reflectance. True and estimated functions shown in Figure 1 correspond to the upper left solid and open symbols, respectively.)

Table 3
Inverse von Kries Scaled Matrices for Each Illuminant

4,800 °K

$$V = \begin{bmatrix} 0.213 & -0.159 & 0.0463 \\ 0.865 & -0.807 & -0.0575 \\ -0.544 & 0.791 & -0.247 \end{bmatrix}$$

6,500 °K

$$V = \begin{bmatrix} 0.233 & -0.181 & 0.048 \\ 0.942 & -0.904 & -0.040 \\ -0.542 & 0.799 & -0.257 \end{bmatrix}$$

10,000 °K

$$V = \begin{bmatrix} 0.250 & -0.201 & 0.0509 \\ 1.01 & -0.994 & -0.0212 \\ -0.534 & 0.803 & -0.269 \end{bmatrix}$$

modate this rescaling. The new V matrix is formally defined as E^{-1} multiplied by the inverse of the diagonal matrix containing the inverses of the mean receptor responses. This multiplication by the inverse of the von Kries scaling matrix is necessary to preserve the equality shown originally in Equation 11. The new matrix V is then defined as

$$V = E^{-1} \begin{bmatrix} \bar{r} & 0 & 0 \\ 0 & \bar{g} & 0 \\ 0 & 0 & \bar{b} \end{bmatrix}. \qquad (13)$$

Now, *for illustrative purposes only, assume that the mean receptor responses for each illuminant corresponded to those obtained by illuminating some standard reflectance surface.* This could obtain, under the present circumstances, for example, if the space average spectral reflectance function in the scene corresponded to some scaled version of Cohen's (1964) first basis vector. Table 3 shows the V matrices that would be obtained if the mean receptor responses corresponded to those obtained by illuminating Cohen's first basis function, this function having a weight of .10. This gives a relatively flat reflectance function with an average reflectance across wavelength of approximately .40. The particular scalar used is not critical. The fact that there is still residual variation in the three V matrices after von Kries adaptation means that there must also be adaptation at the second sites in order to estimate the vector with no error. D'Zmura and Lennie (1986) explicitly recognized this fact in their proposed model. The magnitude of this effect is illustrated in Table 3.

Although some second-site adaptation is necessary, as shown in Table 3, the contribution of von Kries-type adaptation should not be overlooked (e.g., Breneman, 1987; Hallett, Jepson, & Gershon, 1988; Worthey, 1985). To illustrate this, we may use the previous deliberately erroneous strategy of holding the V weights fixed and appropriate for one illuminant while introducing the receptor stimulations appropriate for another illuminant. In this example, $V_{4,800\,°K}$ was used while the receptor stimulations and mean receptor responses appropriate for the 10,000 °K illuminant served as the input to these inappropriately weighted reflectance channels. Figure 3 shows the results of this exercise. The true vector was the same as that used in Figure 1. The RMS error is now .024. In other words, reflectance is estimated with an average error of only 2.4% across wavelength. von Kries adaptation alone has reduced the error by a factor of 100. It is obvious in comparing Figures 1 and 3 that von Kries adaptation significantly reduces the error between the estimated and true reflectance functions even when no second-site adaptation is allowed.

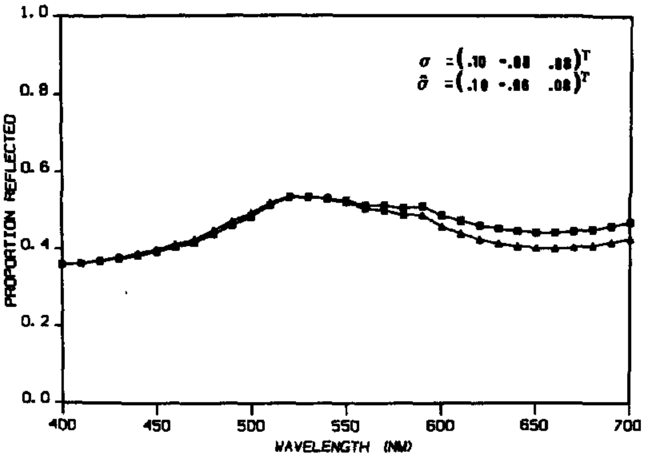

Figure 3. True reflectance function (triangles) and estimated reflectance function (squares) with von Kries scaling of receptor outputs. (This adaptation at the first site reduces the error in computed reflectance considerably even when no second-site adaptation is permitted.)

This is clearly illustrated in Figure 4. The true and estimated values of σ_2 and σ_3 are illustrated by the closed and open symbols, respectively. Comparing Figure 4 with Figure 2, we now

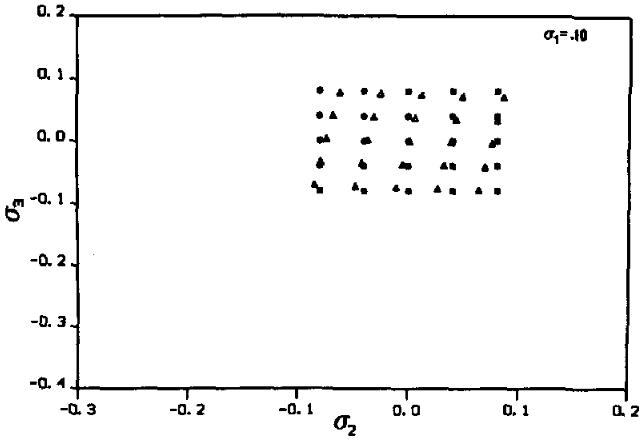

Figure 4. True (solid symbols) and estimated (open symbols) reflectance weights (σ_2 and σ_3) using reflectance-channel gain factors appropriate for a 4,800 °K illuminant and receptor stimulations produced by a 10,000 °K illuminant and using von Kries scaling of receptor outputs. (The true and estimated functions shown in Figure 3 correspond to the upper left solid and open symbols, respectively.)

see that as long as von Kries scaling is allowed at the first stage in the system, a fixed set of second stage, V, weights produces estimates of reflectance that are only slightly erroneous. The estimated reflectance function shown in Figure 3 actually corresponds to the largest error from the 25 surfaces illustrated in Figure 4. von Kries scaling reduces the amount of second-site adaptation necessary to achieve color constancy.

It is important to realize exactly what von Kries scaling accomplishes in terms of color constancy. To illustrate this, the receptor responses to a scene with 765 surfaces composed from Cohen's (1964) basis functions were scaled against the mean response for each class of receptor using the 4,800 °K and 10,000 °K phases of daylight. The average percentage difference in these scaled (adapted) responses across the 765 surfaces under the two illuminants was .03% for the long-wavelength mechanism, .03% for the medium-wavelength mechanism, and .02% for the short-wavelength mechanism. For 95% of these surfaces, the scaled response under one illuminant was within approximately ±3.4% of the scaled response under the other illuminant. Thus, for a given surface in the world, the scaled output of the first stage in the system is approximately constant given only the variation available from different phases of daylight as the illuminant and 3 *dfs* in the surface reflectance function.

This property of the scaled receptor responses is predictable from an examination of the response matrices in Table 1 in conjunction with Equation 10. For any illuminant and for any receptor class, the receptor response is given as a linear sum of three components corresponding to the three reflectance basis functions. This response (sum) is dominated by the component that corresponds to the first reflectance basis function S_1. The contribution of this component of the response is always a minimum of three times as great as the contribution from any other component. Therefore, as the response to a particular surface increases or decreases by some factor due to an illuminant change, the mean response for that class of receptors will change by approximately the same factor. Scaling by the mean will cancel the effects of the illuminant change.

Hallett et al. (1988) recently argued that the outputs of the cones are essentially color constant given only natural surfaces and high-intensity daylight illuminants. They argued that the residual variation in von Kries scaled receptor responses after an illumination change is within the noise level of individual cones, implying that another stage of adaptation is unnecessary to achieve color constancy. The earlier analysis supports this conjecture.

It is also important to realize, however, that several conditions must be true before a single stage of adaptation becomes sufficient for color constancy. First, the scene must be sufficiently rich in spectrally different surfaces. *Sufficiently rich* in this context means that the space average reflectance in a scene must be approximately constant across scenes. This is clearly a statistical constraint on the number and diversity of surfaces within a scene. Violations of this constraint will lead to the necessity of a second stage of adaptation for achieving color constancy. The second condition that must be true in order for von Kries scaling to yield color constancy to within the noise level of the photoreceptors is that some process must scale the responses of local photoreceptors of a given class according to the global average response of that class of photoreceptors. This requires one of two types of processes within the visual system. Either there must be large scale integration of information across space (extensive, within receptor-type lateral interactions), there must be extensive eye movements across the scene coupled with a long time-constant of integration in the photoreceptors or both processes must be operative. Land (1986) has argued that large-scale spatial interactions are involved in color vision. Most quantitative tests of color constancy (e.g., McCann et al., 1976) have allowed eye movements so it is difficult to determine the extent of color constancy in the absence of such eye movements. Nonetheless, deviations from either of the conditions imposed earlier would necessitate the operation of a second stage of adaptation to achieve color constancy.

To summarize the results to this point, we may observe that

1. Three reflectance channels are created that use the receptor stimulations to estimate the three-dimensional reflectance vector σ.
2. von Kries-type rescaling of the receptor responses can contribute significantly to eliminating or minimizing the changes in second-stage, reflectance channel weights dependent on illumination changes.
3. Some second-site adaptation (i.e., readjustment of reflectance channel weights) may still be necessary under some conditions to estimate the reflectance vector σ accurately.

Second-Site Reflectance Channel Adaptation

In this section, I draw a connection between the process of estimating the illuminant and adjusting the reflectance channel weights. Recall that the **E** matrix depends only on the illuminant vector $(\mathbf{w}_1 \quad \mathbf{w}_2 \quad \mathbf{w}_3)^T$ for a fixed set of basis functions and receptor primaries. This, in turn, implies that the matrix $\mathbf{E}^{-1}$ depends only on this same vector. The **V** matrix in Equation 13, thus, depends only on the illuminant vector $(\mathbf{w}_1 \quad \mathbf{w}_2 \quad \mathbf{w}_3)^T$ and the mean values used to scale the responses of the receptors. If the average reflectance function is assumed to be constant across scenes, then the vector of mean receptor responses depends only on the illuminant. In other words, the correct second-site weights embodied in the **V** matrix should adjust to the *illuminant by adjusting to the mean receptor responses.*

Because the **V** matrix depends on the illuminant through the vector of mean receptor responses, the process of estimating the illuminant is functionally equivalent to adapting the reflectance channels so that their outputs serve as accurate estimates of the reflectance vector σ. How can the **V** matrix be estimated when the only information available to the hypothetical visual system consists of a set of three-dimensional response vectors corresponding to various regions in the scene? As several previous researchers have indicated, there is information in this set of receptor stimulation vectors (Buchsbaum, 1980; D'Zmura & Lennie, 1986; Maloney & Wandell, 1986). Namely, assuming a spatially uniform illuminant and a sufficiently spectrally rich scene, the mean receptor stimulation across the scene contains information about the illuminant. In particular, if the space average reflectance function in a scene always corresponded to some standard reflectance function, then the vector $\bar{\mathbf{p}} = (\bar{\mathbf{r}} \quad \bar{\mathbf{g}} \quad \bar{\mathbf{b}})^T$ would contain enough information to reconstruct the illuminant with 3 *dfs*.

This property of the mean response vector can be demonstrated by examining Equation 9. Consider the first column of the 3×3 matrix on the right side of this equation. We may reexpress this equation following the summation operation in the following form:

$$\begin{bmatrix} \mathbf{r} \\ \mathbf{g} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} [w_1\langle RL_1S_1\rangle + w_2\langle RL_2S_1\rangle + w_3\langle RL_3S_1\rangle] & k_{R2} & k_{R3} \\ [w_1\langle GL_1S_1\rangle + w_2\langle GL_2S_1\rangle + w_3\langle GL_3S_1\rangle] & k_{G2} & k_{G3} \\ [w_1\langle BL_1S_1\rangle + w_2\langle BL_2S_1\rangle + w_3\langle BL_3S_1\rangle] & k_{B2} & k_{B3} \end{bmatrix} \times \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}. \quad (14)$$

The remaining six numbers in this matrix have been represented in unexpanded form as various constants (k), inasmuch as they are not necessary for the present argument. Examining the left-most column of this $\mathbf{E}$ matrix reveals the fact that these three numbers (sums) represent the responses of the three primaries to the $S_1(\lambda)$ basis function alone illuminated by the phase of daylight described by the illuminant vector $(\mathbf{w}_1 \;\; \mathbf{w}_2 \;\; \mathbf{w}_3)^T$. Now assume that the average reflectance function in the scene corresponds to $\bar{\sigma}_1 S_1(\lambda)$; that is, the average reflectance function in the scene is simply a scaled version of the basis function $S_1(\lambda)$. The average reflectance function may be represented by the vector $(\bar{\sigma}_1 \;\; 0 \;\; 0)^T$. There is thus some $\bar{\mathbf{p}}$ vector $(\bar{\mathbf{r}} \;\; \bar{\mathbf{g}} \;\; \bar{\mathbf{b}})^T$ that corresponds to the average reflectance function in the scene. When the average reflectance vector is substituted on the right in Equation 14, we are left with the following:

$$\begin{bmatrix} \bar{\mathbf{r}} \\ \bar{\mathbf{g}} \\ \bar{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \bar{\sigma}_1[w_1\langle RL_1S_1\rangle + w_2\langle RL_2S_1\rangle + w_3\langle RL_3S_1\rangle] \\ \bar{\sigma}_1[w_1\langle GL_1S_1\rangle + w_2\langle GL_2S_1\rangle + w_3\langle GL_3S_1\rangle] \\ \bar{\sigma}_1[w_1\langle BL_1S_1\rangle + w_2\langle BL_2S_1\rangle + w_3\langle BL_3S_1\rangle] \end{bmatrix}. \quad (15)$$

Now, Equation 15 shows that the average receptor response vector $\bar{\mathbf{p}}$ in the scene contains unambiguous information about the illuminant *if the average reflectance function in the scene is assumed to be equivalent to a scaled version of a standard reflectance function.* In this case, the argument is simplified by assuming that the average reflectance vector contains zeroes in the positions corresponding to the second and third basis functions, although this is not necessary because there will always be only three unknowns on the right in Equation 15. This same argument was advanced by Buchsbaum (1980) in his formulation of the problem. This assumption allows us to substitute a value for $\bar{\sigma}_1$ in Equation 15. Given an empirical estimate of the average receptor response vector $(\bar{\mathbf{r}} \;\; \bar{\mathbf{g}} \;\; \bar{\mathbf{b}})$, then Equation 15 reduces to a set of 3 linear equations in three unknowns. The first column of the matrix $\mathbf{E}$ must then be $(1/\bar{\sigma}_1)\cdot\bar{\mathbf{p}}$. The solution to this set of equations results in an estimate of $(\mathbf{w}_1 \;\; \mathbf{w}_2 \;\; \mathbf{w}_3)^T$. Once this illuminant vector is estimated, the matrix $\mathbf{E}$ can be completely constituted and inverted to obtain an estimate of $\mathbf{E}^{-1}$. This matrix, in turn, may be multiplied by the inverse of the diagonal von Kries scaling matrix. This yields the desired $\mathbf{V}$ matrix as shown in Equation 13a. Thus, given any arbitrary quantum catch vector $(\mathbf{r} \;\; \mathbf{g} \;\; \mathbf{b})^T$, the reflectance vector $(\sigma_1 \;\; \sigma_2 \;\; \sigma_3)^T$, may be computed as

$$\sigma = \mathbf{E}^{-1}\mathbf{K}^{-1}\mathbf{Kp}, \quad (16)$$

with $\mathbf{K}$ representing the diagonal von Kries scaling matrix and $\mathbf{K}^{-1}$ its inverse. Mathematically, it is obvious that the two middle matrices $\mathbf{K}^{-1}$ and $\mathbf{K}$ are unnecessary because $\mathbf{K}^{-1}\mathbf{K} = \mathbf{I}$ and $\mathbf{E}^{-1}\mathbf{Ip} = \mathbf{E}^{-1}\mathbf{p}$, where $\mathbf{I}$ is a 3×3 identity matrix. However, from the perspective of a two-stage model of color vision, we may rewrite Equation 16 as

$$\sigma = (\mathbf{E}^{-1}\mathbf{K}^{-1})(\mathbf{Kp}). \quad (17)$$

Now it is apparent that the first stage of processing corresponds to the capture of quanta by the receptors and von Kries adaptation ($\mathbf{Kp}$), whereas the second stage corresponds to an adaptive reweighting of three linear combinations of the outputs of the first stage ($\mathbf{E}^{-1}\mathbf{K}^{-1}$). This adaptive reweighting of the second site depends on the adaptive state of the first stage. The aforementioned analysis suggests that under certain conditions, the output of the first stage ($\mathbf{Kp}$) may be approximately color constant implying, in turn, that changes in the second stage ($\mathbf{E}^{-1}\mathbf{K}^{-1}$) may be unnecessary given the limitations imposed by noise at the first level in the system (Hallet et al., 1988).

The critical step that allows the computation to proceed is the assumption noted by Buchsbaum (1980) that the mean receptor response across the entire scene was produced by some fixed, standard reflectance function. This assumption implies that a hypothetical visual system using this scheme would assign the vector of mean receptor responses to an external surface with invariant reflectance characteristics. Indeed, the setting of the gain factors in the reflectance channels may be conceived of as an adaptation of the von Kries-scaled receptor responses (output of Stage 1) to this fixed, reference surface. This implies that there is an additional constraint on the accuracy of the reflectance computations in Buchsbaum's model. Namely, the average reflectance function in the scene is independent of the illuminant and is, in fact, constant across scenes. It is important to make this constraint explicit. Laboratory experiments might reasonably test this model by introducing an artificial correlation between the space-average reflectance function in the scene and the spectral power distribution of the illuminant. Such a correlation would clearly violate this constraint, and we should expect errors (inconstancy) from such a model.

As Buchsbaum (1980) has noted, this method of estimating σ is sensitive to discrepancies between the actual average reflectance function in a scene and the assumed reference reflectance function. Buchsbaum (1980) has quantified errors resulting from such discrepancies (see Figures 2 and 3 from Buchsbaum, 1980). The scheme is fairly robust in the sense that relatively small errors are introduced even when the average reflectance function deviates significantly from the system's reference reflectance function. If one were willing to tolerate an iterative algorithm, these errors could be reduced. How could this be accomplished? Consider the vector of estimated reflectance weights for a region in the scene $(\hat{\sigma}_1 \;\; \hat{\sigma}_2 \;\; \hat{\sigma}_3)^T$. This vector at each location could be used in conjunction with Equation 10 to produce a vector of predicted quantum catches at each location, $(\hat{\mathbf{r}} \;\; \hat{\mathbf{g}} \;\; \hat{\mathbf{b}})^T$. The difference between this predicted vector and the actual quantum catch vector at each location could be minimized by iterating the estimate of the illuminant vector

$(w_1 \ \ w_2 \ \ w_3)^T$ (see Maloney, 1985, p. 104). In this sense, the assumption of a standard average reflectance function serves only to guide the initial estimate of **w**. However, because iterative algorithms are costly in terms of time, in a real-time system there may be a tradeoff between the accuracy of the final σ estimate and the time involved to compute this vector.

Ontogenetic Considerations

The models of color constancy proposed by Buchsbaum (1980) and Maloney (1985) were clearly meant to apply to human color vision because explicit comparisons are made between various aspects of the models and psychophysical evidence. In this section, these models are examined in regard to their ontogenetic plausibility. A related issue concerns the susceptibility of these computational models to developmental perturbations—changes induced in the computation of surface reflectance because of age differences in various factors related to color vision. The purpose of this section is not to confront these computational models with current data on the development of color vision because such data are insufficiently precise at present to allow this. Rather, the purpose of this section is to deduce various ontogenetic consequences from these models in order to understand their properties more fully.

First, consider the question of ontogenetic plausibility: Could the process of ontogenetic development plausibly lead to the end point of mature color vision represented by these computational models of color constancy? An answer to this question requires an examination of the processes involved in computing color-constant reflectance descriptors. As interpreted earlier, there are two major processes involved. First, quanta are captured by the photoreceptors and some type of von Kries scaling or first-stage adaptation must take place. Second, reflectance channels must be created by adjusting second-site weights on the basis of the mean receptor responses. Considering the first process, there is evidence of first-stage adaptation in the young visual system (Dannemiller, 1985; Dannemiller & Banks, 1983; Hansen & Fulton, 1981, 1985; Pulos, Teller, & Buck, 1980). Furthermore, the young visual system is probably trichromatic by at least 3 months of age (Teller & Bornstein, 1987). So the neural substrate for the first stage is probably present from very early in life. Much less is known about second-stage processes in the visual system, although Brown, Liman, and Teller (1986) reported evidence of red/green chromatic opponency in 3-month-olds. Even lacking this evidence, however, there is no reason, in principle, that second-stage, adaptable channels could not be programmed innately or develop postnatally in the pathway from ganglion cells to lateral geniculate nucleus (LGN) to visual cortex.

A far more difficult problem arises, however, when we consider the fact that there are fully 27 constants that are necessary for the computations in this class of models as outlined earlier. These constants represent the integrals of the products of illuminant basis function (3), with surface reflectance basis functions (3), with spectral sensitivity functions (3). Because these constants all involve the spectral sensitivities of the receptor mechanisms, any change in spectral sensitivity would require a corresponding change in these constants. Now, there are at least two specific problems that would arise ontogenetically regarding these constants. First, one might argue that these constants are programmed innately into the visual system. We will set aside the question of where in the structure of the mature system these constants reside. They must, nevertheless, reside somewhere if the computations are to proceed (D'Zmura & Lennie, 1986; Maloney, personal communication, early in 1988). It is not enough simply to solve for the illuminant vector **w** in these models. Knowledge of this vector must be used in conjunction with these constants to transform the receptor responses into illuminant-invariant reflectance estimates. Thus, simply knowing the illuminant vector **w** is of little use if it is not used to construct the E^{-1} matrix that allows reflectance estimates to be computed. Suppose that the constants associated with the mature state were innately programmed into the structure of the color vision system. This preprogramming, in effect, assumes that the spectral sensitivity functions do not change from birth. For if they were to change postnatally, then errors would be introduced into the computation of surface reflectance. I will give examples of these errors later.

Second, a more adaptive developmental strategy would be to allow the 27 constants to be recalibrated as spectral sensitivity changed throughout the course of development. However, this would require the visual system to estimate its own spectral sensitivity functions and to use these estimates to adjust its own constants. Could the visual system discover its own spectral sensitivity functions? Or perhaps a less demanding question would be, How could a visual system even know that its constants were in need of recalibration because of some developmental change in spectral sensitivity? All of these questions would seem to require some way of knowing that the organism's current spectral reflectance computations were in error. But this would require independent knowledge of the surface reflectance in order to determine the error, and this is completely implausible. Allowing recalibration by exposing the visual system to a constant scene with the only changes occurring across the course of a day as the phase of daylight changes, might at first seem plausible. However, this strategy would require that the system be credited with knowledge that the reflectance properties of the scene were constant despite changing receptor stimulations. This is precisely the problem of color constancy. Such a recalibration strategy begs the question. Building in a recalibration process, whereas attractive from an ontogenetic perspective, can probably be ruled out on logical grounds.

Thus, if we wish to adhere to this class of models as explanations of human color constancy, we must consider the consequences of innately programming into the visual system the constants characterizing the mature state. As stated earlier, this leaves the organism susceptible to errors in the computation of surface reflectance when its own spectral sensitivity differs from that implied by the constants. I consider two such cases here. First, the young infant's spectral sensitivity functions for any or all of the photoreceptor classes might differ by a scalar multiple from the corresponding adult spectral sensitivity functions. Second, a difference in ocular transmissivity across the visible spectrum might exist between young and mature human beings. The problem in both of these cases is that these differences are unknown to the organism. Its computations therefore proceed as if its spectral sensitivity were mature. But, both of these potential differences alter receptor responses from the values

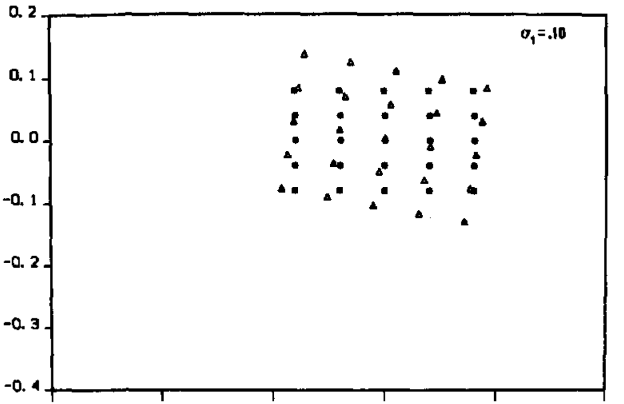

Figure 5. True (solid symbols) and estimated (open symbols) reflectance weights (σ_2 and σ_3). (Estimated points [open symbols] were derived by scaling the B[λ] spectral sensitivity function to .25 times its adult value and failing to take this reduction in sensitivity into account in estimating the illuminant. If such a developmental perturbation were to occur, the result would be a systematic misestimation of surface color.)

that would obtain for an adult viewing the same scene. We will evaluate the effects of these differences next.

First, consider the consequences of a scalar difference in spectral sensitivity. For illustrative purposes, suppose that the short-wavelength spectral sensitivity function of the young infant was 0.25 times that of the adult. The effect of this alteration on quantum catches is easy to assess. The r and g quantum catches are unaffected, whereas the b quantum catch becomes .25 its adult value. This means that the vector of mean receptor responses that is ultimately used to determine the illuminant weights $(w_1 \quad w_2 \quad w_3)^T$ is the same as the adult vector with the exception of the mean blue response. The mean blue response is now $0.25\ \bar{b}$. This altered mean response vector is used in conjunction with the adult constants to determine the illuminant weights. Here is where the error arises in this problem. The adult constants are used to determine the illuminant weights, but these constants do not reflect the scaled alteration of the B(λ) function. The illuminant is estimated incorrectly, leading inevitably to incorrect estimates of surface reflectance functions.

For this example, I assumed the aforementioned scalar deviation (.25) in B(λ) and used the 4,800 °K illuminant. von Kries adaptation was allowed to operate. The effects of erroneously estimating the illuminant are shown in Figure 5. The same 25 reflectance functions used in the previous Figures 2 and 4 are repeated here. The σ_1 value was .10. The true reflectance weights for σ_2 and σ_3 are shown by the solid symbols, whereas the estimated weights are shown by the open symbols. There are clearly errors, as there must be when the illuminant is estimated incorrectly. This means that a young human who was viewing these 25 surfaces would systematically misperceive their true colors. Of course, this assumes that the color differences represented by these errors are above threshold. Current data on chromatic discrimination early in life do not permit an empirical assessment of this assumption.

Next, consider the effects of developmental changes in ocular spectral transmissivity (Werner, 1982; Werner, Donnelly, & Kliegl, 1987). This change would be equivalent to imposing different preretinal filters on the visual input. In this case, the spectral sensitivities of all of the photoreceptor classes are altered to some extent, unlike the independent perturbation of the B(λ) function in the previous example. In this example, I used Werner's (1982) equation for age changes in optical density of the ocular media at 400 nm. The two ages that I considered were 3 months and 40 years. Norren and Vos's (1974) ocular transmission function was scaled appropriately at the two ages. Because the question of interest here concerns *differences* in ocular spectral transmissivity, the ratio of the 3-month-old function to the 40-year-old function was used to perturb the input to the 3-month-old's visual system in the model. The resulting function may be expressed as $F(\lambda) = 10^{.54D(\lambda)}$. D(λ) is the optical density spectrum given in Norren and Vos (1974).

Again, I used a 4,800 °K illuminant and simply perturbed the 3-month-old's quantum catches by including F(λ) in all of the integrals like the one shown in Equation 3. von Kries adaptation was allowed to occur. The mean receptor quantum catches were then used in conjunction with the adult constants to solve for the illuminant weights $(w_1 \quad w_2 \quad w_3)^T$. Here again is where the error is introduced. The illuminant is not estimated properly because the adult constants do not reflect the differences in ocular transmissivity. The results are shown in Figure 6. The conventions are the same as in Figure 5. Now we see that such a developmental difference results in negligible errors in the estimates of surface reflectance. This perturbation apparently is one that would not seriously affect the accurate perception of surface reflectance. It may also be concluded that individual differences in ocular transmissivity at a constant age do not have much of an effect on the estimation of σ.

What may we conclude about these errors that result from innately programming adult constants into the visual system? Clearly, the independent perturbation of spectral sensitivity functions results in larger errors than those produced by devel-

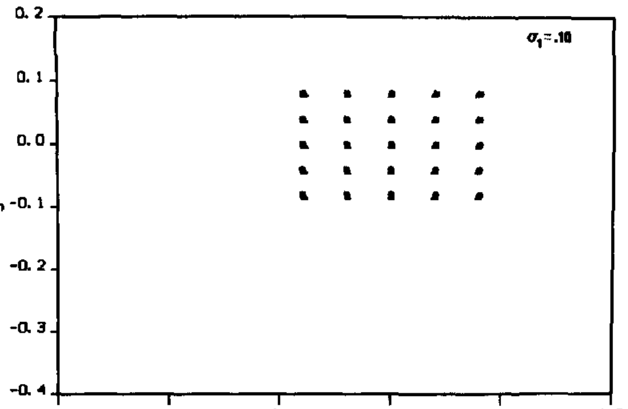

Figure 6. True (solid symbols) and estimated (open symbols) reflectance weights (σ_2 and σ_3). (Estimated points [open symbols] were derived by assuming a different ocular spectral transmission function than the one characterizing the adult state. Failure to take such a developmental perturbation into account produces only minor errors in the computation of surface color.)

opmental differences in ocular transmissivity. To some extent this is an unfair comparison because the simulation of the effects of ocular transmissivity was constrained by data, whereas the simulation of the effects of a scalar difference in spectral sensitivity was not. Nonetheless, we may inquire about the implications of these errors. To answer this question, one must have an understanding of the consequences of deficient or anomalous color vision. Consider dichromatic color vision that may be modeled in the present context as a scalar of zero applied to one of the spectral sensitivity functions. Clearly dichromatic individuals are at a disadvantage as far as the accurate perception of surface color, yet these individuals manage to survive and to reproduce. The consequences of less extreme perturbations in spectral sensitivity such as the ones illustrated earlier may thus be slight enough to allow equivalently successful adaptations to the environment given the minor errors in the perception of surface color across development and individuals that would result from innately specifying these reflectance channel weights. Alternatively, if one assumes that von Kries adaptation plays the major role in color constancy, then individual differences or ontogenetic perturbations in spectral sensitivity would have little effect on the perception of surface reflectance and its constancy across illuminants. Minor perturbations in spectral sensitivity will affect the mean response of a given class of photoreceptors in a manner similar to the way in which responses in local regions of the image are affected. Because the mean response is used to scale outputs in local regions, these minor differences in spectral sensitivity across individuals and within an individual at different points in development will tend to be removed at the first level in the system.

Discussion

This analysis shows that a model proposed originally by Buchsbaum (1980) for extracting illuminant-independent reflectance information is compatible with a two-stage adaptation model of color vision. The first stage consists of a von Kries-type rescaling of the independent receptor responses. The second stage consists of an adjustment of the gain factors in reflectance channels to the mean illumination in the scene that depends on the illuminant and the average spectral reflectance function in the scene. This second stage of adaptation is formally equivalent to estimating the illuminant and is necessary because von Kries-type adaptation alone is only sufficient under some conditions to render the second-stage gain factors completely independent of the illuminant.

It is interesting to note that Worthey (1985) has analyzed the color constancy data of McCann et al. (1976) using a two-stage adaptation approach similar to the one explicated here. Worthey showed that the predictions of such a model with von Kries-type adaptation approximately describe the results of McCann et al. (1976) when the illuminant was shifted along a blue–yellow locus (i.e., a tritanopic confusion line). There were still, however, some discrepancies between the predictions and the actual data, and the present analysis offers one possible explanation for these discrepancies. Worthey assumed that the second-stage, opponent-channel gain factors were constant across illuminants. This is equivalent in the present context to assuming no second-site adaptation to the illuminant. Because second-site adaptation is known to occur in the human visual system (Pugh & Mollon, 1979) and because it is seen to be necessary to estimate reflectance accurately, the discrepancy between the predicted and observed results noted by Worthey may be attributable to the operation of second-site adaptation.

It is interesting to compare the function of reflectance channels as elaborated in the aforementioned model and the role of opponent color coding as described by Buchsbaum and Gottschalk (1983). Recall that the function of the second-stage reflectance channels as described earlier was to effect a three-dimensional estimate of an object's reflectance function given the receptor stimulation vector $(\mathbf{r} \quad \mathbf{g} \quad \mathbf{b})^T$. I noted that for the particular basis functions and receptor primaries used earlier, all three of the second-stage channels were chromatically opponent in the sense that they always involved a subtractive combination between the R and G primaries. Buchsbaum and Gottschalk (1983) showed that three postreceptor channels, two of which are chromatically opponent and one of which is nonopponent, would be predicted on the grounds that the visual system is attempting to make the most efficient use of its available channel capacity. Because of the significant degree of overlap between the $R(\lambda)$ and $G(\lambda)$ spectral sensitivity functions, the signals from these two receptor types are highly correlated. Indeed, simulations in the current work showed that the R and G responses were correlated at approximately the .975 level across a large sample ($n = 765$) of representative, simulated object reflectance functions illuminated by simulated daylight. Thus, on purely information theoretic grounds, as Buchsbaum and Gottschalk noted, it does not make sense to pass these signals untransformed to higher levels in the visual system. The linear transformation proposed by Buchsbaum and Gottschalk decorrelates the R, G, and B signals to use available channel capacity efficiently.

The reflectance channels as described and the second-stage channels as proposed by Buchsbaum and Gottschalk are similar in several respects. First, in both cases, the optimal weights for the second-stage channels must vary with the adaptive states of the first-stage photoreceptors. This must occur in the reflectance channel model because the accurate estimation of object reflectance functions depends on an estimation of the illuminant. The reason for the adjustment in the model proposed by Buchsbaum and Gottschalk (1983) is that the efficient use of channel capacity depends on the covariance matrix between the three classes of photoreceptor spectral sensitivity functions. These covariances, in turn, depend on the von Kries scale factors used to adjust the photoreceptor sensitivities.

A second important function of the second-stage channels in both models is to decorrelate the outputs of the first stage photoreceptors. The reason for this is obvious in the case of Buchsbaum and Gottschalk's (1983) model. The most efficient use of three postreceptor channels occurs when the signals in those channels are uncorrelated. The reason for the goal of decorrelating the outputs in the reflectance channel model is not as obvious. Consider what it is that the reflectance channels are doing. Given as input a triplet of scaled receptor stimulations, two of which are highly correlated (r and g), the reflectance channels use these three measurements to obtain an estimate of the object reflectance function that gave rise to those measurements in conjunction with a particular illuminant. Suppose that our hypothetical visual system were to use the untransformed, unscaled receptor stimulations to estimate reflectance. Such a strategy would only be accurate to the extent that two of the σ_i weights were also highly correlated for objects in the world. To the extent that the three σ_i weights vary independently of one another for objects in the world, then the use of the untransformed receptor stimulations is an inaccurate strategy. What transformation of the receptor stimulations would allow the most accurate predictions of σ from p? This transformation must take a triplet of correlated measurements and use them to predict a triplet of uncorrelated values. In other words, at the output of this transformation the signals in the three reflectance channels must vary orthogonally. By definition, the basis reflectance functions were chosen to be orthogonal, so the ultimate goal of the reflectance channel transformation is to use the receptor stimulations to span this orthogonal, three-dimensional basis space. Thus, both Buchsbaum and Gottschalk's model of second-stage channels and the current reflectance channel model share the important function of decorrelating the outputs of the receptor stimulations.

There is one difference between the two models, however. One additional criterion that the opponent transformation must satisfy in Buchsbaum and Gottschalk's (1983) model is that of distributing the signal energy among the three channels so as to minimize the error in the received signal. No such criterion was imposed on the second-stage weights in the reflectance channel model. This partitioning of signal energy depends on the statistical distribution of color signals. A color signal in this context is the function that results from multiplying an illuminant SPD with an object spectral reflectance function. In other words, a signal in this context contains information about a reflectance spectrum confounded with an illuminant SPD. Because empirical evidence (Guth et al., 1980; Ingling & Tsou, 1977) shows that the opponent-channel weights in human vision optimally distribute the signal energy according to Buchsbaum and Gottschalk's a priori prediction, an additional stage may actually intervene between the first-stage photoreceptor outputs and the reflectance channel transformation as described earlier. The goal of this intermediate transformation is not directly involved in estimating reflectance. Rather, its purpose is to maximize the reliable transfer of signal information between the photoreceptors and the reflectance channels. It is only necessary to invert this linear transformation at the output end of this intermediate stage in order to allow the reflectance channels to operate as described. The reflectance channels would then operate on these illuminant-dependent signals to extract illuminant-independent reflectance information. A tentative identification of these stages with levels in the visual system might then take the following form: The first stage encompasses the signal transformations at the photoreceptors; the second, chromatically opponent stage as delineated by Buchsbaum and Gottschalk may occur just prior to or at the level of the ganglion cells (DeMonasterio, Gouras, & Tolhurst, 1975); and the final reflectance-channel stage may occur in the visual cortical areas (Land, 1983; Livingstone & Hubel, 1984; Zeki, 1980). Such a scheme would allow cortically based reflectance channels to integrate information over a large spatial scale. The ganglion cell–LGN–cortex part of the pathway obeys the constraints on signal decorrelation, reliable signal transmission, and chromatically opponent processing outlined by Buchsbaum and Gottschalk, whereas the reflectance-channel transformation uses this information to compute illuminant-invariant estimates of surface reflectance.

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Received October 22, 1987
Revision received June 7, 1988
Accepted July 7, 1988 ■

Mineka Appointed Editor of *Journal of Abnormal Psychology,* 1990–1995

The Publications and Communications Board of the American Psychological Association announces the appointment of Susan Mineka, Northwestern University, as editor of the *Journal of Abnormal Psychology* for a 6-year term beginning in 1990. As of January 1, 1989, manuscripts should be directed to

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Squire Appointed Editor of *Behavioral Neuroscience,* 1990–1995

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