OWNERSHIP STRUCTURE AND EFFICIENCY IN LARGE ECONOMIES

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Abstract

The paper analyses the limit behavior of sequences of oligopolistic equilibria in which firms follow objectives consistent to their shareholders' interests. It is shown that the efficiency of the limit allocation depends on consumers' distributions of ownership. An exact characterization of the class of ownership structures that lead to Walrasian equilibrium allocations in the limit is provided.

—Preliminary and Incomplete—

1 Introduction

Perfectly competitive (or price taking) behavior is believed to arise – and is generally justified in the literature – when the number of economic agents that interact in the market is large, and each agent is small relative to the whole economy. There are, however, examples that show how monopoly profits and inefficient allocations can persist in equilibrium, even with an arbitrarily large number of small, competing agents. In an environment without uncertainty (or with uncertainty but a complete set of contingent securities) this happens if, as the economy grows larger, the sequence of its (oligopolistic) equilibria approach a *critical* equilibrium point of the limit economy (see Roberts [18]). The results of this paper point out

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yet another possible source of inefficiency in large economies: the firms' ownership structures. *If* firms follow an objective compatible with their shareholders' interests, the ownership structure plays an important role in achieving efficiency in the limit. By contrast, if firms maximize profits, the ownership structure is irrelevant for the limit behavior of the sequence of oligopolistic equilibria because a firm's profit function is independent of its ownership.

We say that a production plan chosen by a firm is compatible with that firm's shareholders' interests (given the production plans chosen by the rest of the firms) if no other production plan makes all shareholders better off (provided that the other firms do not change their plans). Such a production plan is therefore efficient (or Pareto undominated) from the point of view of the firm's shareholders and will thus be called S-efficient (with S standing for "shareholders"). We are interested in the strategic interaction of a large number of firms whose objective is compatible with their shareholders' interests in the sense of selecting S-efficient production plans. The Cournot-Nash equilibria of such game played by the firms must then have the property that every firm's equilibrium production plan is S-efficient given the production plans of the others. We therefore say that the equilibrium is Cournot S-efficient.

In this paper we study the limit behavior of Cournot S-efficient production allocations of a sequence of private ownership economies and show that, depending on the ownership structure, the equilibria may or may *not* approach a Walras equilibrium of the limit economy. We identify then the class of ownership structures that lead to a Walrasian equilibrium outcome in the limit. A necessary and sufficient condition for convergence to competitive equilibrium is that, for each firm, a significant set of its shareholders hold undiversified portfolios of shares. The result is fairly intuitive. If for almost all shareholders of a firm the distribution of their ownership is dispersed, the income effect of the firm's choices on their wealth must be negligible and thus the price effect, albeit becoming negligible itself, may still dominate the income effect. As a result, shareholders may disapprove the maximization of profits in arbitrarily large economies.

The paper is organized as follows. In section 2 we describe a general, private ownership production economy and construct a sequence of replicas such that the firms' ownerships do not become more spread with each replication (i.e., as the economy grows larger by replication, the number of shareholders each firm has and the ownership of each consumer stay the same). In section 3 we define a notion of convergence for sequences of Cournot S-efficient allocations of the replicas and show that the limits of such sequences are competitive equilibria of a properly defined continuum limit economy. In section 4 we point out to the importance of the ownership structure for the convergence to Walras equilibrium, by constructing an example of replicas, whose ownership structure is different than before and whose Cournot S-efficient allocations do not become competitive in the limit. We also identify the reason for this failure, which is that with the second, diffuse ownership structure, Cournot S-efficient allocations of the *continuum* limit economy are still not competitive. In section 5 we define a convergence topology on the set of private ownership economies and, finally, in section 6 we characterize the ownership structures for which any sequence of Cournot S-efficient allocations of a convergent sequence of increasingly larger, finite economies approaches a Walras equilibrium production plan of the limit economy.

2 Replica Economies

Consider an economy $\mathcal{E}$ with J firms and I consumers. For every $j \in \{1, .., J\}$ let Y_j be firm j's production set. Y_j is assumed to satisfy the following standard conditions:

$a)$ Y_j is closed, convex and contains the origin,

$b)$ $Y_j \cap \mathbb{R}^L_+ = \{0\}$ (i.e., Y_j excludes "free lunches").

For every $i \in \{1, .., I\}$, let $(\mathbb{R}^L_+, u^i, \omega^i, (\theta^i_j)_{j=1..J})$ be consumer i's characteristics, with $\mathbb{R}^L_+$ being the consumption set, $u^i : \mathbb{R}^L_+ \to \mathbb{R}$ an utility representation of his/her preferences, $\omega^i \in \mathbb{R}^L_+$ the endowment of goods and $\theta^i_j \in [0, 1]$ the endowment of shares in firm j's profits. The firms' ownership is normalized so that, for every $j \in \{1, .., J\}$, $\sum_{i \in I} \theta^i_j = 1$. Define S^j to be the set of firm j's shareholders, i.e., $S^j \overset{def}{=} \{i \in I : \theta^i_j > 0\}$, and let $Y \overset{def}{=} \prod_{j \in J} Y_j$. It is assumed that utility functions $(u^i)_{i \in I}$ are continuous, strictly quasi-concave and strictly increasing in each argument and that Y is endowed with the induced Euclidian topology of $\mathbb{R}^{LJ}$ and the associated Borel σ-algebra.

Let $(A, \mathcal{A}, \mu)$ be an arbitrary probability space, representing the space of "clones' names" in a certain replica of $\mathcal{E}$. We will be interested in two types of such spaces. The first one, used to construct finite replicas, is the space $(\{1, 2, ..., N\}, \mathcal{P}(\{1, 2, ..., N\}), \#)$ where $N \in \mathbb{N}$ is an arbitrary natural number and $\#$ denotes the normalized counting measure. The second probability space, used to construct a limiting, continuum replica economy, is $([0, 1], \mathcal{B}([0, 1]), \lambda)$ where $\mathcal{B}([0, 1])$ is the Borel σ-algebra of $[0, 1]$ and λ is the Lebesgue measure.

A replica economy in which the clones' names lie in A is denoted by $\mathcal{E}^A_r$. The production sector of $\mathcal{E}^A_r$ consists of firms indexed by $(j, a) \in J \times A$; firm (j, a)'s production set is Y_j. The set of firms of type j, $\{(j, a) \mid a \in A\}$, is alternatively called the *industry* j. The consumption sector of $\mathcal{E}^A_r$ consists of traders/consumers indexed by $(i, a) \in I \times A$; consumer (i, a) has preferences represented by the utility function u^i and is endowed with the bundle of goods ω^i and θ^i_j shares in the firm (j, a)'s profits.

If $(A, \mathcal{A}, \mu) = (\{1, 2, ..., N\}, \mathcal{P}(\{1, 2, ..., N\}), \#)$, the replica economy $\mathcal{E}^A_r$ will alternatively be denoted by $\mathcal{E}^N_r$ and called the N-th replica of $\mathcal{E}$. If $(A, \mathcal{A}, \mu) = ([0, 1], \mathcal{B}([0, 1]), \lambda)$, we will alternatively use the notation $\mathcal{E}^\infty_r$ for $\mathcal{E}^A_r$ and call $\mathcal{E}^\infty_r$ the continuum replica of $\mathcal{E}$.

Definition 2.1 *a) A feasible production plan for the replica economy* $\mathcal{E}^A_r$ *is any* μ*-integrable vector function* $y^A : (A, \mathcal{A}, \mu) \to Y$. $y^A_j(a) \in Y_j$ *denotes firm* (j, a)*'s production plan. The set of all feasible production plans is denoted by* Y^A.

b) A consumption allocation for $\mathcal{E}^A_r$ *is any* μ*-integrable function* $x^A : (A, \mathcal{A}, \mu) \to (\mathbb{R}^L_+)^I$, *with* $x^A_i(a)$ *denoting consumer* (i, a)*'s allocation.*

c) The consumption allocation x^A together with the production plan y^A is called feasible if and only if:

$$\int_A \sum_{i=1}^{I} x_i^A(a)d\mu(a) = \sum_{i=1}^{I} \omega^i + \int_A \sum_{j=1}^{J} y_j^A(a)d\mu(a).$$

For every feasible production plan y^A of the replica economy $\mathcal{E}_r^A$, consider the associated pure-exchange replica $\mathcal{E}_r^A\left(y^A\right)$ in which consumer (i,a)'s intermediate endowment is $\widetilde{\omega}^{(i,a)} \overset{def}{=} \omega^i + \sum_{j=1}^{J} \theta_j^i y_j^A(a)$. Let $D^{(i,a)}(p, y^A) \in \mathbb{R}_+^L$ be consumer (i,a)'s demand at prices $p \in \mathbb{R}^L$, in the economy $\mathcal{E}_r^A\left(y^A\right)$. Since consumer (i,a) owns shares only in firms (j,a) with $j \in J$, his/her demand depends on y^A only through $y^A(a)$. In addition, consumer (i,a) has the same characteristics as consumer i of $\mathcal{E}$ and thus, $D^{(i,a)}\left(p, y^A\right) = D^i\left(p, y^A(a)\right)$, where $D^i\left(\cdot, y^A(a)\right)$ represents consumer i's demand in the economy $\mathcal{E}\left(y^A(a)\right)$.

Definition 2.2 *A vector $p \in \Delta^{L-1}$ is called an equilibrium price for $\mathcal{E}_r^A\left(y^A\right)$ if and only if:*

$$\int_A \sum_{i=1}^{I} D^i\left(p, y^A(a)\right) d\mu(a) = \sum_{i=1}^{I} \omega^i + \int_A \sum_{j=1}^{J} y_j^A(a)d\mu(a). \tag{1}$$

Denote by $P^A\left(y^A\right)$ the set of all equilibrium price vectors of the pure-exchange replica $\mathcal{E}_r^A\left(y^A\right)$. The mapping $P^A : Y^A \rightrightarrows \Delta^{L-1}$ is called the pure-exchange equilibrium price correspondence of $\mathcal{E}_r^A$.

In the remaining of this section we investigate a few properties of the exchange equilibrium price correspondences P^A. We are interested in production profiles that are equilibrium compatible, in the sense that the exchange economy they induce has at least one equilibrium. Therefore, firms' action sets (or production possibilities) in every replica $\mathcal{E}_r^A$ need to be restricted so that to guarantee that the correspondence P^A is non-empty valued. We show that there exists a subset $\widehat{Y}$ of Y such that, for every A, P^A is non-empty valued on $\widehat{Y}^A \overset{def}{=} \left\{y : A \to \widehat{Y} \mid y \text{ is } \mu\text{-integrable}\right\}$.

Second, we show that correspondences P^A are "similar" in the sense that they induce the same correspondence on the set of distributions on $\widehat{Y}$. To prove that, we show that each set $P^A\left(y^A\right)$ depends on y^A only through the distribution it induces on $\widehat{Y}$.

To proceed, let $\widehat{Y} \overset{def}{=} \widehat{Y}_1 \times ... \times \widehat{Y}_J \subseteq Y$ be a subset of Y such that $P^\infty(y) \neq \emptyset$ for every Lebesgue integrable function $y : [0,1] \to \widehat{Y}$. Such non-empty subset does exist. For example, taking $\widehat{Y}_j = Y_j \cap \left\{z \in \mathbb{R}^L \mid z_l \geq -\min_{i \in I} \frac{\omega_l^i}{\sum_{j=1}^{J} \theta_j^i}\right\}$ guarantees that $P^\infty(y) \neq \emptyset$ for every Lebesgue integrable function $y : [0,1] \to \widehat{Y}$. However, $\widehat{Y}$ can, in general, be a strict superset of the one just constructed. In the sequel we are going to assume that $\widehat{Y} \supseteq \prod_{j=1}^{J}\left\{z \in \mathbb{R}^L \mid z_l \geq -\min_{i\in I} \frac{\omega_l^i}{\sum_{j=1}^{J}\theta_j^i}\right\}$ and thus $\widehat{Y} \supseteq \{0\}$. Let $\mathcal{M}\left(\widehat{Y}\right)$ denote the space of all probability measures on $\widehat{Y}$, endowed with the topology of weak convergence.

For every production plan $y^A : (A, \mathcal{A}, \mu) \to \widehat{Y}$ of the replica $\mathcal{E}_r^A$ define $\nu_{y^A} \in \mathcal{M}\left(\widehat{Y}\right)$ as the distribution induced by y^A on $\widehat{Y}$. Thus, $\nu_{y^A}(B) \stackrel{def}{=} \mu \circ \left(y^A\right)^{-1}(B)$ for every measurable subset B of $\widehat{Y}$. If $(A, \mathcal{A}, \mu) = (\{1, 2, ..., N\}, \mathcal{P}(\{1, 2, ..., N\}), \#)$ then

$$\nu_{y^N}(B) = \frac{1}{N}\#\left(\left\{n \in \{1, 2, ..., N\} \mid y^N(n) \in B\right\}\right);$$

if $(A, \mathcal{A}, \mu) = ([0, 1], \mathcal{B}([0, 1]), \lambda)$, then

$$\nu_y(B) = \lambda(\{t \in [0, 1] \mid y(t) \in B\}).$$

For every $\nu \in \mathcal{M}\left(\widehat{Y}\right)$ define

$$P(\nu) \stackrel{def}{=} \left\{p \in \Delta^{L-1} \mid \sum_{i=1}^{I}\int_{\widehat{Y}} D^i(p, z)\, d\nu(z) = \sum_{i=1}^{I}\omega^i + \sum_{j=1}^{J}\int_{\widehat{Y}_j} z_j d\nu(z)\right\},$$

where z_j is the projection of z on $\widehat{Y}_j$.

Proposition 2.3 *The correspondence $P : \mathcal{M}\left(\widehat{Y}\right) \rightrightarrows \Delta^{L-1}$ is non-empty valued and $P^A\left(y^A\right) = P\left(\nu_{y^A}\right)$ for every $y^A \in Y^A$. Moreover, if $\widehat{Y}$ is closed and convex, then P is compact-valued and upper hemi-continuous.*

Proof. Let $\nu \in \mathcal{M}\left(\widehat{Y}\right)$ arbitrary. According to Skorokhod theorem, there exists $y : [0, 1] \to \widehat{Y}$ measurable such that $\nu = \lambda \circ y^{-1}$.

From the construction of $\widehat{Y}$ we know that

$$\left\{p \in \Delta^{L-1} \mid \int_0^1 \sum_{i=1}^{I} D^i(p, y(t))\, d\lambda(t) = \sum_{i=1}^{I}\omega^i + \int_0^1 \sum_{j=1}^{J} y_j(t) d\lambda(t)\right\} \neq \emptyset$$

which, after a change of variables becomes

$$\left\{p \in \Delta^{L-1} \mid \sum_{i=1}^{I}\int_{\widehat{Y}} D^i(p, z)\, d\nu(z) = \sum_{i=1}^{I}\omega^i + \sum_{j=1}^{J}\int_{\widehat{Y}} z_j d\nu(z)\right\} \neq \emptyset$$

and thus $P(\nu) \neq \emptyset$.

Let now $y^A \in \widehat{Y}^A$ be an arbitrary production plan for the replica $\mathcal{E}_r^A$ and ν_{y^A} the distribution it induces on $\widehat{Y}$. Again, with a change of variable, equation (1) can be written as

$$\sum_{i=1}^{I}\int_{\widehat{Y}} D^i(p,z)\,d\nu_y(z) = \sum_{i=1}^{I}\omega^i + \sum_{j=1}^{J}\int_{\widehat{Y}} z_j d\nu_y(z).$$

which shows that $P^A(y^A) = P(\nu_{y^A})$.

Assume now that $\widehat{Y}$ is closed and convex. This implies that $\widehat{Y}$ is compact (see the appendix) and thus $\mathcal{M}\left(\widehat{Y}\right)$ is a compact metric space with the Prokhorov metric. Then P is compact-valued and upper hemi-continuous (see [4] for a proof). ■

3 Convergence to Walras Equilibrium

In this section we analyze the limit behavior of a sequence of oligopolistic equilibria of the replica economies $\mathcal{E}_r^N$, as $N \to \infty$, when firms follow objectives that are compatible with their shareholders' interests. We call these equilibria Cournot S-equilibria (where S stands for "shareholders"). Our aim is to show that such a sequence of Cournot S-equilibria converges, in a sense to be defined later, to a Walras equilibrium of $\mathcal{E}_r^\infty$. First, we define Cournot S-equilibria of the finite replicas and Walras equilibria of $\mathcal{E}_r^\infty$.

If, for $j \in J$, S_j is the set of firm j's shareholders in the basic economy $\mathcal{E}$, then $S_j \times \{a\}$ is the set of firm (j,a)'s shareholders in the replica $\mathcal{E}_r^A$. Let $\mathbf{p} : \mathcal{M}\left(\widehat{Y}\right) \to \Delta^{L-1}$ be a measurable selection from P and define $\mathbf{p}^A : \hat{Y} \to \Delta^{L-1}$ by $\mathbf{p}^A(y^A) \stackrel{def}{=} \mathbf{p}(\nu_{y^A})$. Let also $\overline{y}^A$ be a feasible production plan in $\mathcal{E}_r^A$ and denote by $\overline{y}^A_{-(j,a)}$ the production profile (as prescribed by $\overline{y}^A$) of all the firms except (j,a).

Definition 3.1 *The production plan $\overline{y}^A_j(a) \in \widehat{Y}_j$ is called S-efficient for firm (j,a) given $\overline{y}^A_{-(j,a)}$ and the price selection $\mathbf{p}^A$ if and only if there exists no production plan $y^j \in \widehat{Y}_j$ such that:*

$$u^i\left(D^i\left(\mathbf{p}^A(\widetilde{y}^A), \widetilde{y}^A(a)\right)\right) \geq u^i\left(D^i\left(\mathbf{p}^A(\overline{y}^A), \overline{y}^A(a)\right)\right),$$

for all $i \in S_j$, with strict inequality for at least one $i' \in S_j$ and a μ-positive measure of $a \in A$, where $\widetilde{y}^A(b) \stackrel{def}{=} \begin{cases} \overline{y}^A(b) & \text{if } b \neq a \\ \left(\overline{y}^A_{-j}(a), y^j\right) & \text{if } b = a \end{cases}$.

Thus, a production plan is S-efficient for a firm if, given the production plans chosen by the other firms and the equilibrium price selection, there does not exist another production plan such that every shareholder is better off in the new market equilibrium. It is important to note here that S-efficiency is a very weak condition, since different production choices made by a firm may generate equilibrium allocations for that firm's shareholders which are not Pareto comparable. It is therefore likely that the set of S-efficient production plans is large.

Definition 3.2 *A feasible production plan* $\overline{y}^A$ *is called Cournot* S*-efficient (for a given equilibrium price selection* $\mathbf{p}^A$*) if and only if, for all* j *and* μ*-almost every* a*,* $\overline{y}_j^A(a)$ *is* S*-efficient for firm* (j,a) *given* $\overline{y}_{-(j,a)}^A$ *and* $\mathbf{p}^A$*.*

Definition 3.3 *A production plan* $y^\infty \in Y^\infty$ *for the replica* $\mathcal{E}_r^\infty$ *is called a Walrasian equilibrium production plan if and only if there exists a price vector* $p \in P(\lambda \circ (y^\infty)^{-1})$ *such that, for every* $j \in J$ *and* μ*-almost every* $a \in [0,1]$*,* $py_{(j,a)} \geq py_j$*, for every* $y_j \in Y_j$*.*

Assume that $\left(y^N\right)_{N=1}^\infty$ is a sequence of Cournot S-efficient production plans of the replicas $\mathcal{E}_r^N$ (corresponding to some, possibly different, price selections) and let $\left(\nu^N\right)_{N=1}^\infty$ be the associated sequence of distributions on $\widehat{Y}$, i.e., $\nu^N \overset{def}{=} \nu_{y^N}$.

Definition 3.4 *We say that the sequence of production plans* $\left(y^N\right)_{N=1}^\infty$ *converges to the production plan* y^∞ *of* $\mathcal{E}_r^\infty$ *if and only if the associated sequence of distributions is weakly convergent to the distribution of* y^∞*, i.e.,* $\nu^N \Longrightarrow \nu \overset{def}{=} \lambda \circ (y^\infty)^{-1}$*. In this case we write* $y^N \to y^\infty$*.*

Theorem 3.5 *Let* $(y^N)_{N=1}^\infty$ *be a sequence of Cournot* S*-efficient production plans of the replicas* $\left(\mathcal{E}_r^N\right)_N$ *such that* $y^N \to y^\infty$*. If* y^∞ *belongs to the relative interior of* $\widehat{Y}^\infty$ *in* Y^∞ *and* $P\left(\lambda \circ (y^\infty)^{-1}\right)$ *is a singleton, then* y^∞ *is a Walrasian equilibrium production plan for* $\mathcal{E}_r^\infty$*.*

Proof. Let $\mathbf{p}^N$ be the price selection associated with y^N and $p^N = \mathbf{p}^N\left(y^N\right)$. Then $p^N \in P\left(\nu^N\right)$. Since $\mathcal{M}\left(\widehat{Y}\right)$ is compact and P is upper hemi-continuous with compact values, $\left(p^N\right)_N$ belongs to a compact set and thus has a convergent subsequence. With an abuse of notation we assume that $p^N \to p \in \Delta^{L-1}$. Upper hemi-continuity of P implies then that $p \in P(\nu)$ and since $P(\nu)$ is a singleton, $P(\nu) = \{p\}$.

We show first that, for every $j \in J$ and μ-almost every $a \in A$, $y_j^\infty(a) \in \arg\max\left\{pz_j \mid z_j \in \widehat{Y}_j\right\}$. This proves that y^∞ is a Walras equilibrium of the truncated economy in which firms' production sets are $\left(\widehat{Y}_j\right)_{j=1}^J$. Then we prove that every equilibrium of the truncated economy is an equilibrium of $\mathcal{E}_r^\infty$.

Consider an arbitrary $j \in J$ and suppose that $y_j^\infty(a) \in \arg\max\left\{pz_j \mid z_j \in \widehat{Y}_j\right\}$ fails on a positive-measure set. Let $\widetilde{y}_j \in \arg\max\left\{pz_j \mid z_j \in \widehat{Y}_j\right\}$. Then, $\exists X \subset [0,1]$ with $\lambda(X) > 0$ such that, for every $a \in X$, $p\widetilde{y}_j > py_j^\infty(a)$. Since utilities u^i are strictly increasing in each argument, this implies that

$$u^i\left(D^i\left(p,\widetilde{y}_j,y_{-j}^{\infty}(a)\right)\right)>u^i\left(D^i\left(p,y_j^{\infty}(a),y_{-j}^{\infty}(a)\right)\right)\ \forall i\in S_j,\ \forall a\in X.$$

Let $\epsilon^i(a)\overset{def}{=}u^i\left(D^i\left(p,\widetilde{y}_j,y_{-j}^{\infty}(a)\right)\right)-u^i\left(D^i\left(p,y_j^{\infty}(a),y_{-j}^{\infty}(a)\right)\right)>0.$

We will show that this leads to a contradiction with $\left(y^N\right)_N$ being Cournot S-efficient. The proof is done in 3 steps.

<u>*Step 1*</u>: We prove first that there exists a non-empty subset $X'\subseteq X$ such that $\min_{i\in S_j}\inf_{a\in X'}\epsilon^i(a)>0$.

Clearly, if $\min_{i\in S_j}\inf_{a\in X}\epsilon^i(a)>0$ we can take $X'=X$. Suppose now that $\min_{i\in S_j}\inf_{a\in X}\epsilon^i(a)=0$. Let $\{i_1,...,i_k\}=\left\{i\in S_j\mid \inf_{a\in X}\epsilon^i(a)=0\right\}$, where $1\leq k\leq \#(S_j)$. Then for any $N\in\mathbb{N}$, the sets $X_N^{i_0}\overset{def}{=}\left\{a\in X\mid \epsilon^i(a)<\frac{1}{N}\right\}$ are non-empty. Moreover, $X_{N+1}^{i_0}\subseteq X_N^{i_0}\ \forall N\in\mathbb{N}$. For every $N\in\mathbb{N}\setminus\{0\}$ define $Z_N^{i_0}=X_N^{i_0}\setminus X_{N+1}^{i_0}$ and $Z_0^{i_0}=X\setminus X_1^{i_0}$. Clearly, the sets $\left(Z_N^{i_0}\right)_{N\in\mathbb{N}}$ are mutually disjoint and $\bigcup\limits_{N\in\mathbb{N}}Z_N^{i_0}=X$. Since X has a positive measure, at least one of the sets $\left(Z_N^{i_0}\right)_{N\in\mathbb{N}}$, say $Z_{N_0}^{i_0}$, must have a positive measure and $\inf_{a\in Z_{N_0}^{i_0}}\epsilon^{i_0}(a)\geq\frac{1}{N_0+1}>0$. If for all $i\in S_j\setminus\{i_0\}$ $\inf_{a\in X}\epsilon^i(a)>0$ (i.e., if $k=1$) then by taking $X'=Z_{N_0}^{i_0}$ we obtain $\min_{i\in S_j}\inf_{a\in X'}\epsilon^i(a)>0$. If $k>1$ then repeat the construction taking the set $Z_{N_0}^{i_0}$ as X and then iterate for all the remaining i-s for which $\inf_{a\in X}\epsilon^i(a)=0$. The $(k-1)$-th iteration delivers a positive measure set, $Z_{N_k}^{i_k}$ for which $\min_{i\in S_j}\inf_{a\in Z_{N_k}^{i_k}}\epsilon^i(a)>0$. Take then $X'=Z_{N_k}^{i_k}$ and define $\epsilon\overset{def}{=}\min_{i\in S_j}\inf_{a\in X'}\epsilon^i(a)$.

<u>*Step 2*</u>: Since $X'\subseteq\bigcap\limits_{i\in I}\left\{a\in[0,1]\mid \frac{\epsilon}{2}<\epsilon^i(a)\right\}$ and $\lambda(X')>0$, it follows that

$$\lambda\left(\bigcap_{i\in I}\left\{a\in[0,1]\mid \frac{\epsilon}{2}<\epsilon^i(a)\right\}\right)>0. \tag{2}$$

Let

$$G\overset{def}{=}\bigcap_{i\in I}\left\{z=\left(z^1,z^2,...z^J\right)\in\widehat{Y}\mid u^i\left(D^i\left(p,z_j,z_{-j}\right)\right)<u^i\left(D^i\left(p,\widetilde{y}_j,z_{-j}\right)\right)-\frac{\epsilon}{2}\right\}.$$

G is an open set in $\widehat{Y}$ and, since $\nu^N\Longrightarrow\nu$, $\liminf\limits_{N\to\infty}\nu^N(G)\geq\nu(G)$. Clearly, $y^{\infty}(a)\in G\ \ \forall a\in X'$. Hence, $\nu(G)=\lambda\left(\{a\in[0,1]\mid y^{\infty}(a)\in G\}\right)\geq\lambda(X')>0$ and thus $\liminf\limits_{N\to\infty}\nu^N(G)>0$.

This implies the existence of some $\delta>0$ and $N_\delta\in\mathbb{N}$ such that $\nu^N(G)>\delta$, $\forall N\geq N_\delta$ or, equivalently,

$$\#\left\{n\mid u^i\left(D^i\left(p,y_j^N(n),y_{-j}^N(n)\right)\right)<u^i\left(D^i\left(p,\widetilde{y}_j,y_{-j}^N(n)\right)\right)-\frac{\epsilon}{2},\ \forall i\in S_j\right\}>N\delta,\ \ \forall N\geq N_\delta.$$

Hence, for every $N \geq \max\left\{N_\delta, \frac{1}{\delta}\right\} \overset{def}{=} \widehat{N}_\delta$ there exists n_N such that

$$u^i\left(D^i\left(p, y_j^N(n_N), y_{-j}^N(n_N)\right)\right) < u^i\left(D^i\left(p, \widetilde{y}_j, y_{-j}^N(n_N)\right)\right) - \frac{\epsilon}{2},\ \forall i \in S_j$$

and thus

$$u^{(i,n_N)}\left(D^{(i,n_N)}\left(p, y^N\right)\right) < u^{(i,n_N)}\left(D^{(i,n_N)}\left(p, \widehat{y}^N\right)\right) - \frac{\epsilon}{2}, \forall (i, n_N) \in S_{(j,n_N)} \quad (3)$$

where $\widehat{y}^N(n) = y^N(n)$ for any $n \neq n_N$ and $\widehat{y}_k^N(n_N) = \begin{cases} y_k^N(n_N) & \text{if } k \neq j \\ \widetilde{y}_j & \text{if } k = j \end{cases}$.

Denote by $\widehat{\nu}^N$ the distribution of $\widehat{y}^N$. We show next that $\widehat{\nu}^N \Longrightarrow \nu$. Since P is upper hemi-continuous and $P(\nu)$ is a singleton this implies that $p^N\left(\widehat{y}^N\right) \to p$ (passing to a subsequence, if necessary).

Let B be a ν-continuity set, i.e., $\nu(\partial B) = 0$. It is enough to prove that $\underset{N\to\infty}{lim}\ \widehat{\nu}^N(B) = \nu(B)$. We know that $\underset{N\to\infty}{lim}\ \nu^N(B) = \nu(B)$, because $\nu^N \Longrightarrow \nu$. Since $\widehat{y}^N(n) = y^N(n)$ for any $n \neq n_N$ and $\widehat{\nu}^N(B) = \frac{1}{N}\#\left\{n \mid \widehat{y}^N(n) \in B\right\}$, it follows that $\widehat{\nu}^N(B)$ and $\nu^N(B)$ differ by at most $\frac{1}{N}$, and thus $\left|\widehat{\nu}^N(B) - \nu^N(B)\right| \leq \frac{1}{N}$. This implies that $\underset{N\to\infty}{lim}\ \widehat{\nu}^N(B) = \underset{N\to\infty}{lim}\ \nu^N(B)$ and thus $\underset{N\to\infty}{lim}\ \widehat{\nu}^N(B) = \nu(B)$ for every ν-continuity set B. Thus, $\widehat{y}^N \to y$.

Step 3: Consider now the functions $V^i : P\left(\widehat{Y}\right) \times \widehat{Y} \to \mathbb{R}$ defined by

$$V^i(p, y) \overset{def}{=} u^i\left(D^i(p, y)\right),\ i \in S_j.$$

These functions are uniformly continuous on their domains, because $u^i(\cdot)$ and $D^i(\cdot,\cdot)$ are continuous and $P\left(\widehat{Y}\right) \times \widehat{Y}$ is a compact set. Then, since $p^N \to p$ and $\mathbf{p}^N\left(\widehat{y}^N\right) \to p$, $\exists N_\epsilon$ such that

$$V^i\left(p^N, y^N(n_N)\right) < V^i(p, y^N(n_N)) + \frac{\epsilon}{4}, \quad (4)$$

$$V^i\left(p, \widehat{y}^N(n_N)\right) < V^i(\mathbf{p}^N\left(\widehat{y}^N\right), \widehat{y}^N(n_N)) + \frac{\epsilon}{4}, \quad (5)$$

for every $i \in S_j$ and any $N \geq N_\epsilon$. Inequalities (3), (4) and (5) imply then that for N sufficiently large,

$$V^i\left(p^N, y^N(n_N)\right) < V^i(\mathbf{p}^N\left(\widehat{y}^N\right), \widehat{y}^N(n_N)),\ \forall i \in S_j,$$

or, equivalently,

$$u^{(i,n_N)}\left(D^{(i,n_N)}\left(p^N, y^N\right)\right) < u^{(i,n_N)}\left(D^{(i,n_N)}\left(\mathbf{p}^N\left(\widehat{y}^N\right), \widehat{y}^N\right)\right),\ \forall i \in S_j.$$

This is a contradiction with the assumption that y^N is a Cournot S-efficient production plan, which proves that y must be a Walras equilibrium of the truncated economy in which firms' production sets are $\left(\widehat{Y}_j\right)_{j=1}^J$.

We show now that y is a Walras equilibrium of $\mathcal{E}_r^\infty$. Suppose that it is not. Then for some (j,a), $py_j^\infty(a) < \max\{pz_j \mid z_j \in Y_j\}$ and thus there exists some $y_j \in Y_j \setminus \widehat{Y}_j$ such that $py_j^\infty(a) < py_j = \max\{pz_j \mid z_j \in Y_j\}$. Convexity of Y_j implies that $\alpha y_j^\infty + (1-\alpha) y_j \in Y_j$ for every $\alpha \in [0,1]$. Since $y_j^\infty(a)$ belongs to the relative interior of $\widehat{Y}_j$ in Y_j, for α sufficiently close to 0, $\alpha y_j^\infty + (1-\alpha) y_j \in \widehat{Y}_j$. But $p\left(\alpha y_j^\infty + (1-\alpha) y_j\right) > py_j^\infty(a)$, which contradicts that $y_j^\infty(a)$ is profit maximizing in $\widehat{Y}_j$. ■

The theorem relies heavily on $P(\nu)$ being a singleton set. This condition is needed to insure continuity of the price selection at the limit point. While we cannot dispense with that completely, the requirement can be relaxed, with a construction like in Roberts [18]. That allows for multiplicity of equilibria at the limit point, but requires regularity of the limit equilibrium and thus its *local* uniqueness. Even so, it remains a strong condition since, as pointed out by Roberts himself, existence of critical equilibria is non-pathological. Allen [3] pointed out that this negative result is considerably alleviated if, instead of simple price selections, one uses *randomized* price selections (i.e., selections from the correspondence *coP* instead of P; this amounts to saying that firms hold non-trivial beliefs over the possible market clearing prices). As opposed to the case of simple price selections, the existence of continuous randomized price selections is a generic result (see also Mas-Colell and Nachbar [15]). Allen proves therefore that, if firms maximize their *expected* profits with respect to some non-trivial beliefs over prices, convergence of Cournot equilibria (in which firms maximize profits) to competitive equilibria does obtain generically. However, the problem is more complex here and Allen's approach cannot be directly applied. The reason is that, as opposed to the standard Cournot model in which the firms maximize profits, in our model S-efficiency dictates firms to make pairwise comparisons between a status quo and an alternative. For that, a firm has to use its beliefs over two different equilibrium sets. To make this comparison meaningful, some global beliefs need to be defined. This was done in [4]. Whether allowing for firms' non-trivial, global beliefs over prices (as defined in [4]) does indeed improve the convergence result is an interesting question which remains open for now and will be subject of future research.

4 The Role of the Ownership Structure

This section shows that the problem of convergence to competitive equilibria is aggravated even more if we allow different ownership structures for the firms. Our previous results apply to a particular sequence of finite economies, that were replicas of a basic economy. Using sequences of replica economies to draw inferences about the equilibrium behavior in large economies is a technique widely used in the literature. Starting with the well-known construction of Debreu and Scarf for pure exchange economies, replica economies have proved to be a useful tool in studying the limit properties of different strategic equilibrium outcomes. The replication technique we used for our production economy is known in the literature. It was first introduced by Nikaido and later used, among others, by Aliprantis, Brown and Burkinshaw, [2], and Florenzano and Laureana del Mercato, [10]. However this is not the only way one can construct replicas of a particular economy, even when similarity of the clones is a concern. Alternatively, each replica may be constructed such that each clone of a certain type holds the same number of shares in firms of the same industry. This is a technique used, for example, by Roberts, [18], Mas-Colell, [14], and Allen, [3], among others. A natural question to ask is whether the result of the previous section is robust to changes in the replication technique. We provide here an example to show that it is not.

Consider the same basic economy as in section 2, but change the construction of the ownership structure of the replicas as follows. In the N-fold replica every consumer (i,n) owns $\frac{1}{N}\theta_j^i$ shares in *each* firm (j,k), where $n,k \in \{1,2,...,N\}$. Denote the new N-fold replica economy by $\widetilde{\mathcal{E}}_r^N$. For the continuum replica, denoted by $\widetilde{\mathcal{E}}_r^\infty$, consider the ownership structure given by

$$\Theta_j^i : [0,1] \times \mathcal{B}([0,1]) \to [0,1], \quad \Theta_j^i(s,T) \overset{def}{=} \theta_j^i \lambda(T).$$

Here $\Theta_j^i(s,T)$ represents the ownership of consumer (i,s) in firms with names belonging to $\{j\} \times T$. Thus, in both cases, the ownership structures of the new replicas $\widetilde{\mathcal{E}}^A$ are given by mappings

$${}^A\Theta_j^i : A \times \mathcal{A} \to [0,1], \quad {}^A\Theta_j^i(a,T) = \theta_j^i \mu(T).$$

Given a feasible production plan y^A, consumer (i,a)'s intermediate endowment in the replica $\widetilde{\mathcal{E}}^A$ is:

$$\widetilde{\omega}^{(i,a)} = \omega^i + \sum_{j=1}^{J} \theta_j^i \int_A y_j^A(a)\, d\mu(t).$$

Thus, if we denote by $\overline{y}_j^A = \int_A y_j^A(a)\, d\mu(t)$ the average production in the j-th industry, then $\widetilde{\omega}^{(i,a)} = \omega^i + \sum_{j=1}^{J} \theta_j^i \overline{y}_j^A$. Since the intermediate endowments depend on the production plans chosen by the firms only through the averages of each industry, so do the demand functions and thus the equilibrium prices. Thus if we denote by $\widetilde{P}^A : Y^A \rightrightarrows \Delta^{L-1}$ the exchange equilibrium price correspondence of the replica $\widetilde{\mathcal{E}}^A$, we have $\widetilde{P}^A(y^A) = P^1(\overline{y}^A)$.

Let now $y^* \in Y$ be a Cournot S-efficient production plan of the basic economy, which is not a Walras equilibrium production plan. For every $N \in \mathbb{N}$ construct $y^{*N} \in Y^N$ such that $y^{*N}(n) = y^*$ for every $n \in \{1,...,N\}$. Then y^{*N} is a Cournot S-efficient production plan of the N-fold replica. Clearly $y^{*N} \to y^{*\infty}$, where $y^{*\infty}(t) \overset{def}{=} y^*$, $\forall t \in [0,1]$. $y^{*\infty}$ is a Cournot S-efficient production plan of the continuum replica $\widetilde{\mathcal{E}}_r^\infty$, but not a Walras equilibrium. Hence we found a sequence of Cournot S-efficient production plans of the finite replicas, which does not converge to a Walras equilibrium as $N \to \infty$. Therefore, an equivalent of theorem 3.4 does not hold for the sequence of replicas $\widetilde{\mathcal{E}}_r^N$.

The reason for this failure is that not all Cournot S-efficient production plans of the continuum replica $\widetilde{\mathcal{E}}_r^\infty$ are Walras equilibria. Note that for both sequences of finite replicas, $\mathcal{E}_r^N$ and $\widetilde{\mathcal{E}}_r^N$, it is true that their sequences of Cournot S-efficient production plans converge to a Cournot S-efficient production plan of the corresponding limit economy. However, in $\mathcal{E}_r^\infty$ every Cournot S-efficient production plan is a Walras equilibrium production plan, while the same is not true for $\widetilde{\mathcal{E}}_r^\infty$. This dissimilarity between the two continuum replicas is generated by their only different feature: the ownership structure. Thus, apart from the inherent discontinuities of the exchange equilibrium price selections, the firms' ownership structures may be an additional cause of the failure of strategic equilibria to converge to a Walras equilibrium of the limit economy.

5 Topological Structure of the Space of Economies

This section defines a convergence topology on the space of private ownership economies with a finite number of types of consumers and a finite number of types of firms.

Let $\mathcal{B}([0,1])$ denote the Borel σ-algebra of $[0,1]$. The interval $[0,1]$ is interpreted as the space of consumers' and firms' names. Let also $u^i : \mathbb{R}_+^L \to \mathbb{R}$, $i = 1, ..., I$ be continuous, strictly increasing (in each argument) and strictly quasi-concave utilities and $\omega^i \in \mathbb{R}_{++}^L$, $i = 1, ..., I$, the set of all possible endowments of goods. Then $\mathcal{U} = \left\{ \left(u^i, \omega^i\right) \mid i = 0, ..., I \right\} \cup \{(\mathbf{0}, 0_L)\}$ denotes the space of possible types for consumers, where $\mathbf{0}$ denotes the constant function 0. The space of all possible technologies (firms' types) is $\mathcal{Y} = \left\{ Y_j \subseteq \mathbb{R}^L \mid j = 0, ..., J \right\}$, where $Y_0 = \{0\}$ and, for every $j = 1, ..., J$, the set Y_j is assumed to be non-empty, closed and convex, and exclude "free lunches" (i.e., $Y_j \cap \mathbb{R}_+^L = \{0\}$).

Definition 5.1 *An economy with finite types can be represented by a* 5*-tuple* $(\nu, \mu, \mathcal{C}, \mathcal{F}, \Theta)$, *where:*

1. ν *and* μ *are probability measures on* $([0,1], \mathcal{B}([0,1]))$. *They describe the distributions of consumers and, respectively, firms over the space* $([0,1], \mathcal{B}([0,1]))$,
2. $\mathcal{C} : ([0,1], \mathcal{B}([0,1])) \to (\mathcal{U}, \mathcal{P}(U))$ *is measurable and its projection on* $\mathbb{R}_+^L$ *is* ν*-integrable. For every* $s \in [0,1]$, $\mathcal{C}(s) \in \mathcal{U}$ *gives the characteristics of consumer* s*, i.e., his/her utility and endowment of goods. The mapping* $\mathcal{C}$ *describes the* ***consumption sector****,*
3. $\mathcal{F} : ([0,1], \mathcal{B}([0,1])) \to (\mathcal{Y}, \mathcal{P}(\mathcal{Y}))$ *is measurable and describes the* ***production sector****,*
4. $\Theta : \mathcal{B}([0,1]) \otimes \mathcal{B}([0,1]) \to [0,1]$ *is a probability measure on the product space* $\mathcal{B}([0,1]) \otimes \mathcal{B}([0,1])$ *with the following properties:*
   (a) *for every* $S \in \mathcal{B}([0,1])$, $\Theta(S, \cdot)$ *is absolutely continuous with respect to* μ,
   (b) *for every* $T \in \mathcal{B}([0,1])$, $\Theta([0,1], T) = \mu(T)$ *and* $\Theta(\cdot, T)$ *is absolutely continuous with respect to* ν.

Θ *describes the* ***ownership structure*** *of the economy..*

Since for a given $S \in \mathcal{B}([0,1])$, $\Theta(S,\cdot)$ is absolutely continuous with respect to μ, there exists a Radon-Nikodym derivative, $\gamma(S,\cdot) : [0,1] \to \mathbb{R}_+$ which is μ-integrable and $\Theta(S \times T) = \int_T \gamma(S,t)\, d\mu(t)$ for every $T \in \mathcal{B}([0,1])$. It can be proved that $\gamma(\cdot,t) : \mathcal{B}([0,1]) \to \mathbb{R}_+$ is a probability measure on $([0,1], \mathcal{B}([0,1]))$. $\gamma(\cdot,t)$ describes the ownership distribution of firm t. Similarly, there exists a mapping $\theta : [0,1] \times \mathcal{B}([0,1]) \to \mathbb{R}_+$ such that: *i)* for every $s \in [0,1]$, $\theta(s,\cdot)$ is a measure on $([0,1], \mathcal{B}([0,1]))$, *ii)* $\theta(\cdot,T)$ is ν-integrable for every $T \in \mathcal{B}([0,1])$, and *iii)* $\int_S \theta(s,T)\, d\nu(s) = \Theta(S \times T)$. Therefore,

$$\int_S \theta(s,T)\, d\nu(s) = \int_T \gamma(S,t)\, d\mu(t) = \Theta(S \times T) \tag{6}$$

An economy $\mathcal{E}$ is called *finite* if ν and μ are atomic measures with finite numbers of atoms. An economy $\mathcal{E}$ for which both ν and μ are atomless is called a *continuum* (or atomless) economy.

EXAMPLE:

If λ is the Lebesgue measure and $\delta_{\{x\}}$ is the Dirac measure on $[0,1]$ (defined as $\delta_{\{x\}}(X) = \begin{cases} 1, & x \in X \\ 0, & \text{otherwise} \end{cases}$, then the replica economies $\mathcal{E}_r^N$, $\widetilde{\mathcal{E}}_r^N$, $\mathcal{E}_r^\infty$ and $\widetilde{\mathcal{E}}_r^\infty$ can be represented as follows: $\mathcal{E}_r^N = \left(\nu^N, \mu^N, \mathcal{C}^N, \mathcal{F}^N, \Theta^N\right)$, $\widetilde{\mathcal{E}}_r^N = \left(\widetilde{\nu}^N, \widetilde{\mu}^N, \widetilde{\mathcal{C}}^N, \widetilde{\mathcal{F}}^N, \widetilde{\Theta}^N\right)$, $\mathcal{E}_r^\infty = (\nu, \mu, \mathcal{C}, \mathcal{F}, \Theta)$, $\mathcal{E}_r^\infty = \left(\widetilde{\nu}, \widetilde{\mu}, \widetilde{\mathcal{C}}, \widetilde{\mathcal{F}}, \widetilde{\Theta}\right)$,

1. $\nu^N = \widetilde{\nu}^N = \frac{1}{NI} \sum_{k=1}^{NI} \delta_{\left\{\frac{k}{NI}\right\}}$, $\nu = \widetilde{\nu} = \lambda$,

2. $\mu^N = \widetilde{\mu}^N = \frac{1}{NJ} \sum_{k=1}^{NJ} \delta_{\left\{\frac{k}{NJ}\right\}}$, $\mu = \widetilde{\mu} = \lambda$,

3. $\mathcal{C}^N(s) = \widetilde{\mathcal{C}}^N(s) = \begin{cases} \left(u^i, \omega^i\right), & \text{if } s = \frac{(i-1)N+n}{NI} \text{ for some } n \in \{1, ..., N\} \\ (\mathbf{0}, 0_L), & \text{if } \nu^N(\{s\}) = 0, \end{cases}$,

   $$\mathcal{C}(s) = \widetilde{\mathcal{C}}(s) = \sum_{i=1}^{I} \left(u^i, \omega^i\right) \delta_{\left[\frac{(i-1)}{I}, \frac{i}{I}\right)}(\{s\}),$$

4. $\mathcal{F}^N(t) = \widetilde{\mathcal{F}}^N(t) = \begin{cases} Y_j, & \text{if } t = \frac{(j-1)N+n}{NJ} \text{ for some } n \in \{1, ..., N\} \\ (\mathbf{0}, 0_L), & \text{if } \mu^N(\{t\}) = 0, \end{cases}$

   $$\mathcal{F}(t) = \widetilde{\mathcal{F}}(t) = \sum_{j=1}^{J} Y_j \delta_{\left[\frac{(j-1)}{J}, \frac{j}{J}\right)}(\{t\}),$$

5. $\Theta^N(S \times T) = \frac{1}{NJ} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{n=1}^{N} \theta_j^i \delta_{\left\{\frac{(i-1)N+n}{NI}\right\}}(S)\, \delta_{\left\{\frac{(j-1)N+n}{NJ}\right\}}(T)$.

Definition 5.2 *An allocation for the private ownership economy $\mathcal{E}$ is a pair (x, y) such that:*

1. $x : ([0,1], \mathcal{B}([0,1])) \to \mathbb{R}^L$ *is ν-integrable, with $x(s) \in \mathbb{R}^L_+$ ν-a.e.*
2. $y : ([0,1], \mathcal{B}([0,1])) \to \mathbb{R}^L$ *is $\theta(s, \cdot)$-integrable for ν-a.e. $s \in [0,1]$ and $y(t) \in \mathcal{F}(t)$ μ-a.e.*

The allocation (x, y) is called feasible if and only if:

$$\int_0^1 x(s) d\nu(s) = \int_0^1 \omega(s) d\nu(s) + \int_0^1 y(t) d\mu(t)$$

Given a production plan, $y : ([0,1], \mathcal{B}([0,1])) \to \mathbb{R}^L$, consumer s's intermediate endowment is given by,

$$w^y(s) \overset{def}{=} \omega(s) + \int_0^1 y(t)\theta(s, dt). \tag{7}$$

Note that $w^y : ([0,1], \mathcal{B}([0,1])) \to \mathbb{R}^L$ is ν-integrable and

$$\int_0^1 w^y(s) = \int_0^1 \omega(s) d\nu(s) + \int_0^1 \int_0^1 y(t)\theta(s, dt)\, d\nu(s) =$$
$$= \int_0^1 \omega(s) d\nu(s) + \int_0^1 y(t) \int_0^1 \theta(s, dt)\, d\nu(s) = \int_0^1 \omega(s) d\nu(s) + \int_0^1 y(t) d\mu(t).$$

Denote by $\mathcal{E}(y)$ the associated pure-exchange economy and let $P(y) \subseteq \Delta^{L-1}$ be the set of Walrasian equilibrium price vectors of $\mathcal{E}(y)$. Let also $x^y : [0,1] \times \Delta^{L-1} \to \mathbb{R}^L$ be the consumers' demand, i.e.,

$$x^y(s, p) \in \arg\max \left\{ u^s(x) \,\middle|\, x \in \mathbb{R}^L_+,\ px \le w^y(s) \right\} \quad \nu - a.e.,$$

and $\mathbf{p}$ a measurable selection from P with the property that $\mathbf{p}(y) = \mathbf{p}(y')$ whenever $P(y) = P(y')$. Define then

$$D^s(\mathbf{p}(y), y) \overset{def}{=} x^y(s, \mathbf{p}(y))$$

as being consumer s's demand in the exchange equilibrium corresponding to y. Let also $S_t = \text{supp}\, \gamma(\cdot, t) \in \mathcal{B}([0,1])$ be the set of firm t's shareholders.

Definition 5.3 *A feasible production plan y^* is called Cournot S-efficient if and only if, for μ-almost every $\bar{t} \in [0,1]$ the following holds: there does not exist $y \in \mathcal{F}(\bar{t})$ such that*

$$\begin{aligned} \gamma\left(\{s \mid u^s(D^s(\mathbf{p}(\widetilde{y}), \widetilde{y})) \ge u^s(D^s(\mathbf{p}(y^*), y^*))\}, \bar{t}\right) &= 1, \\ \gamma\left(\{s \mid u^s(D^s(\mathbf{p}(\widetilde{y}), \widetilde{y})) > u^s(D^s(\mathbf{p}(y^*), y^*))\}, \bar{t}\right) &> 0, \end{aligned} \tag{8}$$

where $\widetilde{y} : [0,1] \to \mathbb{R}^L$ is defined as $\widetilde{y}(t) = \begin{cases} y^*(t) & \text{if } t \ne \bar{t} \\ y & \text{if } t = \bar{t}. \end{cases}$

Given a continuum economy $\mathcal{E}$, we want to attach a meaning to $\mathcal{E}^N \to \mathcal{E}$ where $\mathcal{E}^N$ are finite economies. This is done in the following definition:

Definition 5.4 *If* $\mathcal{E}^N = \left(\mu^N, \nu^N, \mathcal{C}^N, \mathcal{F}^N, \Theta^N\right)$ *is a sequence of private ownership economies we say that* $\mathcal{E}^N \to \mathcal{E} = (\mu, \nu, \mathcal{C}, \mathcal{F}, \Theta)$ *if and only if:*

1. $\nu^N \Rightarrow \nu$, $\mu^N \Rightarrow \mu$, $\Theta^N \Rightarrow \Theta$ *(weak convergence of measures),*
2. $\nu^N \circ \left(\mathcal{C}^N\right)^{-1} \Rightarrow \nu \circ \mathcal{C}^{-1}$, $\mu^N \circ \left(\mathcal{F}^N\right)^{-1} \Rightarrow \mu \circ \mathcal{F}^{-1}$ *(weak convergence of measures).*

If $\left(\mathcal{E}^N\right)_N$ is a convergent sequence of private ownership economies, $\mathcal{E}^N \to \mathcal{E}$, and y^N is a Cournot S-efficient production plan for economy $\mathcal{E}^N$, we are interested in the properties of the limit of the sequence $\left(y^N\right)_N$. In particular, we would like to know if $\left(y^N\right)_N$ converges, in a sense to be defined later, to a Walras equilibrium production plan of the limit economy $\mathcal{E}$. The following definition introduces our notion of convergence for sequences of feasible production plans.

Definition 5.5 *If* y^N *is a feasible production plan for the economy* $\mathcal{E}^N$ *and* y *is a feasible plan for* $\mathcal{E}$*, we say that the sequence* $\left(y^N\right)_N$ *converges to* y *if and only if*

$$\nu^N \circ \left(w^{y^N}\right)^{-1} \Rightarrow \nu \circ \left(w^y\right)^{-1}. \tag{9}$$

Definition (5.5) is a generalization of other notions of convergence defined in the literature. For example, in replica economies $\left(\widetilde{\mathcal{E}}_r^N\right)_N$ condition (9) is equivalent to the convergence of the average production plans of each industry. In $\left(\mathcal{E}_r^N\right)_N$ the condition is equivalent to $\mu^N \circ \left(y^N\right)^{-1} \Rightarrow \mu \circ y^{-1}$.

6 Characterization of Ownership Structures

This section investigates the role of the ownership structure in the convergence to competitive equilibrium and states the main results of the paper. As shown before, the failure to achieve convergence to competitive equilibrium was not due to the lack of continuity of the strategic equilibrium correspondence, but to the possibility that strategic equilibria may not coincide with the Walras equilibria even when firms are infinitesimal, if certain ownership structures prevail. This section gives a characterization of the ownership structures that guarantee convergence to Walras equilibria of a limit economy for every sequence of Cournot S-efficient production plans of a convergent sequence of economies.

To obtain the Walras equilibrium outcome in the limit, each firm should be unable to affect prices and should maximize profits. We show in the sequel that a single firm's choice has indeed no effect on prices in an atomless economy, but the choice of an S-efficient production plan may not lead to profit maximization for *some* ownership structures.

Consider first the effect on prices. Let $\bar{t} \in [0,1]$ be arbitrary. As proved bellow, the exchange equilibrium price sets are not affected by firm $\bar{t}$'s choice of the production plan.

Proposition 6.1 *If y and y' are two feasible production plans such that $y(t) = y'(t)$ for every $t \neq \bar{t}$, then $P(y) = P(y')$.*

Proof. The firm $\bar{t}$ can influence the exchange equilibrium price sets if and only if that firm's production plan choice affects the intermediate endowments of a positive measure of consumers. From formula (7) one can infer that firm $\bar{t}$ affects the intermediate endowment of consumer s if and only if $\bar{t}$ is an atom of $\theta(s,\cdot)$, and thus $\theta\left(s,\left\{\bar{t}\right\}\right) > 0$. Let $S = \left\{s \in [0,1] \mid \theta\left(s,\left\{\bar{t}\right\}\right) > 0\right\}$ be the set of consumers whose wealth is affected by the firm $\bar{t}$'s choices. If we assume that $\nu(S) > 0$, then $\int_S \theta\left(s,\left\{\bar{t}\right\}\right) d\nu(s) > 0$. On the other hand, according to formula (6), $\int_S \theta\left(s,\left\{\bar{t}\right\}\right) d\nu(s) = \int_{\left\{\bar{t}\right\}} \gamma(S,t)\, d\mu(t) = 0$. This proves that $\nu(S) = 0$, and thus a single firm cannot influence the equilibrium prices. ■

We turn now to the issue of profit maximization and determine those ownership structures under which any S-efficient production plan is a profit maximizer.

Theorem 6.2 *Cournot S-efficient production plans of an atomless economy coincide with its Walras equilibrium production plans if and only if:*

$$\gamma\left(\left\{s \in [0,1] \mid \theta\left(s,\left\{\bar{t}\right\}\right) > 0\right\},\bar{t}\right) > 0 \text{ for } \mu\text{-almost every } \bar{t} \in [0,1]. \tag{10}$$

Condition (10) imposes two requirements. First is that almost all firms have some shareholders with non fully-diversified portfolios of shares. The second requirement is that the set of those undiversified shareholders is non-negligible relative to the set of all shareholders. These two conditions insure that firm $\bar{t}$'s choices have a non-negligible effect on the wealth of a significant subset of shareholders. Because the price effect is absent in an atomless economy, those shareholders unanimously approve profit maximization and thus any Cournot S-efficient production plan for $\bar{t}$ has to be profit maximizing.

Proof. Let y^* be a Cournot S-efficient production plan of the atomless economy. According to proposition (6.1), $P(y^*) = P(\widetilde{y})$ and thus $\mathbf{p}(y^*) = \mathbf{p}(\widetilde{y})$ for every $\widetilde{y}$ defined as above. Then the production plan y^* is Cournot S-efficient if and only if, for μ-almost every $\bar{t} \in [0,1]$, there does not exist $y \in \mathcal{F}\left(\bar{t}\right)$ such that

$$\begin{aligned} \gamma\left(\left\{s \mid u^s\left(D^s\left(\mathbf{p}(\widetilde{y}),\widetilde{y}\right)\right) \geq u^s\left(D^s\left(\mathbf{p}(y^*),y^*\right)\right)\right\},\bar{t}\right) &= 1, \\ \gamma\left(\left\{s \mid u^s\left(D^s\left(\mathbf{p}(\widetilde{y}),\widetilde{y}\right)\right) > u^s\left(D^s\left(\mathbf{p}(y^*),y^*\right)\right)\right\},\bar{t}\right) &> 0. \end{aligned}$$

Since $\mathbf{p}(y^*) = \mathbf{p}(\widetilde{y})$,

$$\begin{aligned} &\left\{s \mid u^s\left(D^s\left(\mathbf{p}(\widetilde{y}),\widetilde{y}\right)\right) \geq u^s\left(D^s\left(\mathbf{p}(y^*),y^*\right)\right)\right\} = \\ &\qquad = \left\{s \mid \mathbf{p}(y^*)\left(\int_{[0,1]} \left(\widetilde{y}(t) - y^*(t)\right)\theta(s,dt)\right) \geq 0\right\} \end{aligned}$$

and therefore S-efficiency of y^* implies that for μ-almost every $\bar{t} \in [0,1]$ and every $y \in \mathcal{F}(\bar{t})$,

$$\begin{aligned}\gamma\left(\left\{s \mid \mathbf{p}(y^*)\left(\int_{[0,1]} y^*(t)\theta(s,dt) - \int_{[0,1]} \widetilde{y}(t)\theta(s,dt)\right) > 0\right\}, \bar{t}\right) &> 0 \text{ or} \\ \gamma\left(\left\{s \mid \mathbf{p}(y^*)\left(\int_{[0,1]} y^*(t)\theta(s,dt) - \int_{[0,1]} \widetilde{y}(t)\theta(s,dt)\right) = 0\right\}, \bar{t}\right) &= 1.\end{aligned}$$

Since $y^*(t) - \widetilde{y}(t) = 0$ for every t except $\bar{t}$, the integral $\int_{[0,1]} (y^*(t) - \widetilde{y}(t))\,\theta(s,dt)$ is different than zero if and only if $\bar{t}$ is an atom of $\theta(s,\cdot)$. In that case,

$$\int_{[0,1]} (y^*(t) - \widetilde{y}(t))\,\theta(s,dt) = \left(y^*(\bar{t}) - y\right)\theta\left(s,\{\bar{t}\}\right).$$

Therefore y^* is a Cournot S-efficient production plan if and only if for μ-almost every $\bar{t} \in [0,1]$ and every $y \in \mathcal{F}(\bar{t})$, either

$$\begin{aligned}\gamma\left(\left\{s \mid \mathbf{p}(y^*)\left(y^*(\bar{t}) - y\right)\theta\left(s,\{\bar{t}\}\right) > 0\right\}, \bar{t}\right) &> 0 \text{ or} \\ \gamma\left(\left\{s \mid \theta\left(s,\{\bar{t}\}\right) = 0\right\}, \bar{t}\right) &= 1.\end{aligned}$$

If $\gamma\left(\left\{s \mid \theta\left(s,\{\bar{t}\}\right) = 0\right\}, \bar{t}\right) = 1$ for μ-almost every $\bar{t} \in [0,1]$ then every feasible production plan is in fact a Cournot S-equilibrium of the continuum economy. Clearly then, not every strategic equilibrium is a Walras equilibrium.

However, if $\gamma\left(\left\{s \in [0,1] \mid \theta\left(s,\{\bar{t}\}\right) > 0\right\}, \bar{t}\right) > 0$ for μ-almost every $\bar{t} \in [0,1]$ then

$$\gamma\left(\left\{s \mid \left(y^*(\bar{t}) - y\right)\theta\left(s,\{\bar{t}\}\right) > 0\right\}, \bar{t}\right) > 0$$

if and only if $\mathbf{p}(y^*)\left(y^*(\bar{t}) - y\right) > 0$. Therefore, y^* is Cournot S-efficient if and only if $y^*(\bar{t})$ is profit maximizing for μ-almost every $\bar{t} \in [0,1]$ and thus y^* is a Walras equilibrium production plan. ■

Note that condition (10) is always violated if the measures $\theta(s,\cdot)$ are atomless for every $s \in [0,1]$. This happens, for instance, in the example of section 4, since $\widetilde{\theta}((i,t),\{j\} \times \{t\}) = \theta_j^i \lambda(\{t\}) = 0\ \forall t \in [0,1]$. On the other hand, if the measures $\gamma(\cdot,t)$ and $\theta(s,\cdot)$ have finite supports for every $t \in [0,1]$ and $s \in [0,1]$ – i.e., if every firm has only a finite number of shareholders and every consumer owns only a finite number of firms – then $\theta(s,\{t\}) = \gamma(\{s\},t)$ for every $s \in [0,1]$ and $t \in [0,1]$. In this case, condition (10) is trivially satisfied and thus every Cournot S-equilibrium is profit maximizing. This is the case of the atomless economy defined in section 3.

Theorem 6.3 *Let* $\left(\mathcal{E}^N\right)_N$ *be a sequence of finite private ownership economies that converges to the atomless economy* $\mathcal{E}$*. Let also* y^N *be a Cournot* S*-efficient production plan for* $\mathcal{E}^N$ *such that* $y^N \overset{N\to\infty}{\longrightarrow} y$*. If the ownership structure of* $\mathcal{E}$ *satisfies condition (10) and* $P\left(\nu \circ (w^y)^{-1}\right)$ *is a singleton, then* y *is a Walrasian equilibrium production plan for* $\mathcal{E}$*.*

Proof. Let $\mathbf{p}^N$ be the price selection associated with y^N and $p^N = \mathbf{p}^N\left(y^N\right)$. Then $\left(p^N\right)_N$ belongs to a compact set and thus has a convergent subsequence. With an abuse of notation we assume that $p^N \to p \in \Delta^{L-1}$. Upper hemi-continuity of P implies then that $p \in P\left(\nu \circ (w^y)^{-1}\right)$ and since $P\left(\nu \circ (w^y)^{-1}\right)$ is a singleton, $P\left(\nu \circ (w^y)^{-1}\right) = \{p\}$.

Assuming that y is a Walras equilibrium production plan for $\mathcal{E}$ we obtain existence of a set $A \subseteq [0,1]$ such that $\mu(A) > 0$ and $y(t)$ does not maximizes profits at prices p in $\mathcal{F}(t)$. Then $\exists\, j \in \{1, ...J\}$ and $\overline{y}_j \in Y_j$ such that $\mu\left(A \cap \mathcal{F}^{-1}(Y_j)\right) > 0$ and $py(t) < p\overline{y}_j = \max\{pz \mid z \in Y_j\}$.

Since the ownership structure of $\mathcal{E}$ satisfies condition (10), there exists a subset A' of A such that $\mu\left(A' \cap \mathcal{F}^{-1}(Y_j)\right) > 0$ and $\gamma(\{s \in [0,1] \mid \theta(s,\{t\}) > 0\}, t) > 0$ for every $t \in A' \cap \mathcal{F}^{-1}(Y_j)$. Then

$$\gamma\left(\left\{s \in [0,1] \mid u^s\left(D^s\left(p, \widetilde{y}^t\right)\right) > u^s\left(D^s(p,y)\right)\right\}, t\right) > 0,$$

where $\widetilde{y}^t(t) = \overline{y}_j$ and $\widetilde{y}^t(t') = y(t')$ for $t' \neq t$.

Moreover, it can be proved that there exists $\varepsilon > 0$ and $A'' \subseteq A$ such that $\mu\left(A'' \cap \mathcal{F}^{-1}(Y_j)\right) > 0$ and

$$\gamma\left(\left\{s \in [0,1] \mid u^s\left(D^s\left(p, \widetilde{y}^t\right)\right) > u^s\left(D^s(p,y)\right)\right\}, t\right) > \varepsilon,\ \forall t \in A'' \cap \mathcal{F}^{-1}(Y_j),$$
$$u^s\left(D^s\left(p, \widetilde{y}^t\right)\right) > u^s\left(D^s(p,y)\right) + \varepsilon,\ \forall t \in A'' \cap \mathcal{F}^{-1}(Y_j),\ \forall s \in S_t.$$

....To be continued..... ■

A similar condition (to (10)) on the firms' ownership structures is derived, in a different context, by Hart, ([11]). Using a sequence of replicas of a stochastic economy, he proves that value/profit maximization is *approximately* unanimously supported by a firm's shareholders. This means that, as the number of firms in the economy increases, the maximum increment in utility that any shareholder of a given firm can obtain out of replacing that firm's value maximizing plan by his/her most preferred production plan tends to zero. A necessary condition for Hart's result is the existence of at least one consumer whose shareholdings in the firm's profits stay bounded away from zero as the economy is replicated. Hart's condition is the "sequential version" of our condition (10) i.e., condition (10) applied to replica economies.

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