A Note on the Innovation-Competition Relationship under Endogenous Horizontal Product Differentiation*

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8 October 2009

Abstract

We show how a computational model of dynamic duopoly with multidimensional investment can reconcile apparently contradictory findings in the empirical literature on competition and innovation. Two such findings are that industry-level innovation rates are inverse-U shaped in average profit margins, but U-shaped in a concentration index. To explain these findings our model adds endogenous product locations to a standard quality-ladder model of dynamic competition in investment. We show that the above findings, and others, can be explained as outcomes of Markov-perfect equilibria at different distributions of the parameters representing consumer tastes and the costs of investment.

*Part of this research previously appeared in an earlier draft of a companion paper, Narajabad and Watson (2009).

1 Introduction

In this note we show that a model of dynamic oligopoly with endogenous qualities and product locations can explain a range of reported industry-level correlations between innovation and measures of competition. Innovation refers to the rate at which an industry improves the frontier level of quality or production costs, commonly measured with indexes such as patent counts. A large empirical literature considers the question of whether this rate of increase is helped or hindered by an increase in the degree of competitive rivalry in an industry. This literature is notable for the variety of results achieved, and general agreement on its conclusions seems lacking.[1] Our purpose here is to show that seemingly contradictory findings from this literature may be reconciled within the same oligopoly model if product locations are considered to be endogenous.

The model is a computational dynamic duopoly framework explained in detail in Narajabad and Watson (2009). Two firms in an industry play an infinite-horizon game of repeated price competition. Each firm produces a single product indexed by a quality level and a degree of horizontal differentiation from its rival's product. Time is discrete and in each period a firm may invest in two dimensions – to improve product quality and to change the location of its product in a Hotelling space of horizontally differentiated consumer tastes. For simplicity quality improvement takes the form of steps up a quality ladder (with the restriction that no firm can ever get more than one step ahead of its rival), while product locations are restricted to either end of the Hotelling line. We use numerical techniques to study how the Markov-perfect equilibria of this model depend on the primitives: consumer transport costs, the marginal utility of quality, and the cost of product relocations.

Allowing multidimensional investment, with endogenous product locations, distinguishes our framework from most other models in the dynamic oligopoly literature. To illustrate the role of horizontal differentiation we pick out two results from the empirical literature. The first is the well-known inverse-U finding of Aghion, Bloom, Blundell, Griffith and Howitt (2005). Those authors compared patent rates in a panel of UK industries with a measure of average industry profit margins and found innovation peaking at the intermediate profit levels. They explain this finding by exogenously vary-

[1] See Cohen and Levin (1989) and Symeonidis (1996) for surveys.

ing the degree of collusion in a theoretical dynamic duopoly with investment in cost reduction.

Seemingly in contradiction with this result is a finding of Flath (2009). Estimating Solow residuals for a panel of Japanese manufacturing industries, Flath finds the rate of increase in these residuals to be U-shaped in concentration, measured by the average industry Herfindahl index. Our model can explain both observed patterns, albeit at different parameter values. Part of the explanation is that, in a model with endogenous product locations, profit margins and concentration indices are not necessarily monotonically related. In equilibrium medium levels of concentration may be associated with the lowest average profit margins, a fact that allows us to reconcile apparently opposite responses of innovation to related measures of competition.

To show how our model can explain other patterns in the data, we consider also the findings of Geroski (1990). Working with UK panel data, he fitted a measure of innovation to a model linear in concentration and other explanatory variables. Concentration is found to have a significantly negative effect. We show a parameter distribution which reproduces this negative linear relationship in our model.

We focus on explaining innovation at the industry level, rather than the firm level. For recent studies of the latter type, see, e.g., Blundell, Griffith, and Van Reenen (1999), and Okada (2005). The stylized two-firm nature of our model, with no entry or exit, makes it difficult to compare its firm-level predictions with real-world industries where individual firms compete with varying numbers of rivals. While our analysis is not an econometric approach, it suggests that in any such approaches endogenous product locations may have a role to play in explaining disparate patterns of innovation.

2 Model

Our framework, explained in more detail in Narajabad and Watson (2009), aims at incorporating multidimensional investment decisions into a dynamic duopoly. We abstract away from entry and exit and model a discrete-time infinite-horizon game of repeated price competition between two firms who invest in both quality improvement and horizontal differentiation. Firms each produce a single perishable good, defined in any period by a quality level and a location at either end of the Hotelling unit interval. Consumers with unit demands are uniformly distributed on this interval. They have purchase utilities that are linear in price and a quality index q, and linear or quadratic in the distance to the point of purchase, with transport-cost parameter α. We assume that q is high enough relative to α to ensure that the market is always covered.

Firms play repeated stage games of price competition. In each period they may also make simultaneous investments to improve quality and relocate products. These investments have stochastic outcomes. Improvements in quality q take the form of step-by-step advances up a ladder. As in, e.g., Aghion and Howitt (1997), we assume that the quality leader never gets more than one step ahead on this ladder, reflecting, for example, spillovers through which a laggard is able to copy some of the leader's technological advances. Parameter $k > 0$ measures the size of a single step up in q, i.e., the marginal utility of quality improvements.[2] Product relocations refer to switches from one end of the Hotelling line to the other. Depending on the present location of the rival's product this could imply more or less horizontal differentiation between goods. The cost of investing in such switches is indexed by a parameter $\gamma > 0$.[3] [4]

Let an industry's current state be a pair (s, δ), where $s = 0$ or 1 as the firms' products are, or are not, currently located at the same end of the Hotelling line, and where $\delta \in \{-k, 0, +k\}$ is the current gap between the firms' qualities. There are thus six possible states. Define a firm's strategy as a state-conditional two-dimensional choice of investment in product relocation and quality improvement. We focus on symmetric Markov-perfect equilibria in investment strategies. Despite the small state space the model is not very tractable analytically and numerical analysis is required. With the algorithm of Pakes and McGuire (1994) we solve for equilibrium over a range of parameter values. Multiple equilibria are frequently encountered – reasons for this are discussed in our companion paper, where we show that most such cases can be reduced to uniqueness by assuming that: (a) k is not too small, and (b) Pareto-dominated equilibria are not played.[5]

[2] Note that quality improvement can equivalently be redefined as investment in cost reduction.

[3] Like quality investment, successful implementation of a product relocation depends in part on a random component. This stochastic element helps ensure existence of equilibrium.

[4] In addition to k, α, and γ, a fourth parameter ϕ indexes the cost of quality improvement. This parameter can be normalized to zero.

[5] Our companion paper looks for cases of multiplicity by starting the Pakes-McGuire

In keeping with other literature we define innovation as the average probability of an improvement in the frontier technology (i.e., in the maximum quality) under the invariant distribution of Markov-perfect equilibrium strategies.[6] To study the relationship between this measure and competition indicators used in empirical work, we first summarize the effects on innovation of the primitives in our model. Two in particular relate to traditional notions of 'competitiveness': the transport cost α, representing the heterogeneity in consumer tastes, and γ, the cost of product relocations.[7] Both parameters operate through their effect on the industry's long-run 'horizontal composition', i.e., on the proportion of time it spends with products co-located or separated. Numerical analysis analysis indicates that if k is not too low then innovation is maximized when products are permanently co-located on the Hotelling line.[8] Concomitantly, when locations are continually changing, innovation tends to increase as the long-run frequency of co-location rises. Large enough falls in α will thus tend to raise innovation, because as α falls the dominant incentive is for the quality leader to co-locate with the laggard so as to eliminate the latter's local market power.[9] Effects of γ will depend on α. If α is high then increases in γ lead the industry to spend more time in separated location states, where innovation is lower. If α is low then the opposite applies, as the increased costs of relocation dull the laggard's incentives to horizontally differentiate its product.

algorithm at many different initial values. Recently Besanko, Doraszelski, Kryukov and Satterthwaite (2008) have shown that in cases of multiplicity this algorithm will likely be unstable at a substantial portion of the equilibria, and therefore will not be able to locate all equilibria. Our other paper therefore only claims uniqueness within the set of equilibria at which the Pakes-McGuire algorithm is stable.

[6]Reference will be made below to some cases where an equilibrium implies multiple ergodic sets, in which case we arbitrarily choose one of the invariant distributions to use in plotting our innovation rates.

[7]The effects of increases in the third parameter k are straightforward: innovation rises, since the returns to moving ahead in the quality race have increased.

[8]In Narajabad and Watson (2008) we prove this analytically in a continuous-time model with fixed product locations. When firms are colocated their joint surplus is larger, because less of this surplus is lost in competition for the marginal consumer. This raises the incremental returns to successful innovation.

[9]The effect of lower α on innovation is not necessarily monotonic. For intermediate values of k there may be an initial dip as α falls, followed eventually by a larger rise, leading to an overall U shape.

3 The inverse U

Consider then Aghion et al. (2005, hereafter 'ABBGH'), who study the empirical relationship between competition and innovation by matching patent data to measures of industry competitiveness for firms listed on the London Stock Exchange over the period 1973-94. Industry competitiveness is represented by average price-cost margins, which are proxied by variable profits, taken from accounting reports. An industry's innovation rate is measured by its average patent activity. They find a clear inverse-U relationship between variable profits and patenting, with the innovation measure being highest at intermediate profit levels.[10] The authors show that this pattern can be explained by a simple continuous-time dynamic duopoly model, in which competition is parameterized by an exogenously set degree of collusion. Varying the collusion parameter from zero to 100% produces the competition-innovation relationship observed in the data.

For completeness we reproduce the relevant figure from ABBGH as figure 1 below. Note that the competition indicator is constructed as one minus the Lerner index, so that competition increases from the left. It may seem difficult for our model to reproduce this empirical relationship, since we never find an inverse-U shape of innovation with respect to changes in either α or γ. However this is not the correct reference point, because the 'explanatory' variable in ABBGH is actually variable profit, not product substitutability or relocation costs, and in our model the relationship between the parameters (in particular, α) and long-run average variable profit is not necessarily monotone.

The upper panel of figure 2 shows long-run average industry stage-game profits in equilibrium , as a function of $\log(\alpha)$, at different values of γ. Industry profits in the present model are equivalent to sales-weighted price-cost margins. If the switching cost is high then there are two invariant distributions. In that case the figure plots industry profits for both ergodic sets: $s = 0$ and $s = 1$. These cases are shown as dotted curves – the curve that is constant in α represents $s = 0$. For comparison the lower panel of figure 2 shows the corresponding long-run innovation rates.[11]

[10] Instrumental-variable techniques are used to deal with the potential endogeneity of profits. See also Scherer (1967) for earlier empirical work which finds a similar pattern.

[11] Note that in this figure and the next we measure time-averaged stage-game profits, not the discounted stream of future profits. This variable-profit measure excludes the costs of switching locations and of improving quality. This is consistent with the use of variable

The figure shows that long-run average stage-game profits are maximized at high values of α, where firms choose separate locations and have a high degree of local market power. Profits tend to be minimized at intermediate values of α, where there is more intense competition but where a laggard still has an incentive to maintain horizontal differentiation from the quality leader (which differentiation erodes the leader's profit). At low values of α equilibrium will in the long run see firms permanently colocated in the space of horizontal tastes. Average profits then converge to an intermediate level, reflecting a weighted average of high profits to the quality leader and zero profits to any other firm (because Bertrand competition pushes the profits of all but the quality leader to zero).

A key observation from the two panels of figure 2 is that the highest long-run innovation rates are associated with intermediate average profit levels. This gives rise to the profit-innovation relationship shown in figure 3. To construct this figure we simulated a dataset by drawing 500 random triples $(k, \log(\alpha), \gamma)$ from a joint uniform distribution on $[8.0, 9.2] \times [-1.7, +2] \times [-2.5, +3]$. For each triple we solved for the equilibrium and plotted long-run innovation against long-run average stage-game profit. We then ran a kernel regression (with Gaussian kernel) of innovation on profits over this 'dataset', which is the curve shown.[12] Comparing figures 1 and 3 we see that over the parameter values shown our model produces the inverse-U relationship reported by ABBGH. The inverse U arises here because: (a) high and low stage-game profits are both associated with low or medium innovation levels; (b) intermediate profit levels can be associated with low or medium innovation rates (e.g., at $\log(\alpha) = 1.5$ in figure 2), or with the highest innovation rates (e.g., $\log(\alpha) \leq -0.5$), but if $\log(\alpha)$ is uniformly distributed then the latter are more likely.[13]

accounting profits by ABBGH.

[12]When a given pair $(\log(\alpha), \gamma)$ implies a 'no-movement' equilibrium with two ergodic sets we arbitrarily chose to plot the innovation rate for $s = 1$. If we instead randomized between $s = 0$ and $s = 1$ the figure would look essentially the same, because we would just get more observations at the 'peak' of the hill shape, where innovation is maximized.

[13]Note that in figure 3 'competition' increases from the right, as stage-game profits fall, whereas in figure 1 competition increases from the left.

4 The U shape

Figure 4 reproduces figure 2 from Flath (2009), who studies innovation in a panel of 74 Japanese manufacturing industries over the period 1961-90. After estimating Cobb-Douglas production functions for each industry, Flath calculates industry-specific average annual rates of increase in the Solow residual. In figure 4 these rates of increase are plotted against the industry's average Herfindahl index over the sample period, to show how innovation varies with concentration across industries. The figure shows a flattish but overall U-shaped relationship. Also shown are the fitted values from a regression of the innovation measure on concentration, including a quadratic term in concentration which is reported to be significantly positive.

Our model is able to reconcile this U-shaped relationship with seemingly contradictory finding of ABBGH, albeit at different parameter values to those shown in figure 3. Part of the reason is that when product locations are endogenous the relationship between concentration ratios and price-cost margins is not necessarily monotone. For example in our model medium-to-high price-cost margins can arise when firms are horizontally differentiated and tastes are quite heterogeneous, in which case firms share the market and concentration will be low, or when the firms produce identical products, in which case the market is frequently monopolized by a quality leader and concentration will be high.

Figure 5 shows equilibrium long-run averages of the innovation rate and the Herfindahl index, as functions of $\log(\alpha)$, for various values of γ. These are plotted for a lower value of k, the marginal utility of quality, than in figure 2, and thus the implied innovation rates are lower. In the top panel, as in figure 2, the general trend of innovation is upward as tastes become more homogeneous. Here however there is a pronounced initial dip in innovation as α falls, which is not present at the higher value for k in figure 2. This is because of the reduced incentives to invest by a quality laggard, which leads the industry, still with permanently separated products, to spend more time in states of asymmetric quality, where aggregate innovation is lower. As α continues to decline the industry eventually leaves the region of permanent horizontal differentiation, colocation becomes more frequent, and average incentives for quality improvement increase.

The bottom panel of figure 5 shows the Herfindahl index to be monotonically increasing as α falls – compare with the non-monotonic relationship between α and profits in figure 2. As α falls the quality leader's market share increases, not just because tastes are more homogeneous but also because the industry spends more time with undifferentiated product locations. Figure 6 shows the implications for the innovation-concentration relationship. To draw this figure we computed equilibria for 300 randomly sampled triples $(k, \log(\alpha), \gamma)$ from a uniform distribution on $[1.7, 1.9] \times [-0.75, 2] \times [-1.3, 3]$. When an equilibrium implied two ergodic sets we plotted outcomes for the case of $s = 1$ – if we were to plot outcomes for $s = 0$ instead the figure would look much the same but there would be more observations with high innovation and high concentration at the top right of the figure. Also shown is the fitted curve from a regression of innovation on concentration with quadratic term.

The overall impression is of a U-shaped relationship, as in the results of Flath. Low levels of concentration are consistent with horizontally differentiated firms, which in turn are associated with medium-to-low innovation levels. As tastes become more homogeneous and the industry becomes more concentrated the innovation rate initially falls, because of the reduced incentive for investment by a quality laggard. Eventually the leader develops an incentive to colocate its product with that of its rival, which nullifies the rival's market power and raises overall incentives for quality improvement. In consequence the industry becomes more concentrated and innovation turns up, reaching its highest level when products are permanently colocated.

5 Innovation declining in concentration

Geroski (1990) studies the relationship between innovation and competition in a panel of 73 British industries over the period 1970-79. Innovation is in each industry a count of the number of innovations introduced over this period, derived from a survey of industry experts. This count is regressed on industry characteristics, including a number of competition measures such as concentration, entry and exit propensities, import penetration, and the prevalence of small firms. Concentration enters through a linear term only, and Geroski finds that, all else equal, its effect is significantly negative.

One way to explain this observation in terms of our model would be to choose parameters so that the curve of figure 6 is right-truncated at the base of the U, thereby giving us a declining innovation-concentration relationship. For example this could be arranged by assuming $\log(\alpha) \geq -0.5$, setting γ high (so that there is no product relocation), and plotting innovation for the ergodic set where $s = 1$. Then the innovation curve would just be a monotone transformation of the rightmost part of the lower dotted curve in the top panel of figure 5.

In the present context this explanation seems a little unsatisfactory, in that it uses a value for k, the marginal utility of quality, that is markedly different to that used to derive the inverse U in figure 3 – compare $k \in [8.0, 9.2]$ in the latter figure with $k = 1.8$ in figure 5. It might be argued that this is potentially contradictory, since Geroski and ABBGH both use UK data, and since there is some overlap in their sample periods. As an alternative way of explaining Geroski's finding, consider the innovation-concentration relationship shown in figure 7. This figure is constructed in the same way as figure 6, but for a different distribution of parameters. We now allow k and $\log(\alpha)$ to be correlated, by generating k as a linear function of $\log(\alpha)$ plus random noise, assuming that $(\log(\alpha), \gamma)$ is distributed joint uniform on $[-1.5, 2] \times [1, 3]$. The dependence of k on $\log(\alpha)$ is set to yield k values in the range $[2.5, 6.5]$, and in figure 7 yields a sample correlation of 0.96 between k and $\log(\alpha)$.

Along with the 300 'data points', generated by random sampling from this new parameter distribution, the figure shows the line fitted in a regression of innovation on concentration with linear term only. In keeping with the results of Geroski, the curve is downward sloping. This trend reflects the positive correlation between taste heterogeneity $\log(\alpha)$ and the marginal utility of quality k. When the former is high firms tend to split the market by operating at separate product locations, and concentration is then low. If k is simultaneously high then these will be the cases with the highest innovation rates. High concentration is observed when α is low, which induces colocation of products and frequent monopolization of the market; if k is simultaneously low then these cases will see relatively less innovation.

Figure 7 uses values for k which are closer to those in the inverse U of figure 3 than the low numbers used in figure 6. The values in figures 3 and 7 still do not overlap, but note that the data underlying the inverse U of ABBGH extend 15 years beyond those in Geroski (1990). This might justify a difference in parameters between the two figures. For our purposes the main point of figure 7 is to illustrate the role that correlated parameter values might play in explaining certain empirical patterns. A positive correlation between transport costs and tastes for quality might not be implausible if, for example, both variables are thought of as functions of consumer's income.

6 Discussion

For simplicity, and in keeping with other studies, we have defined innovation as the rate of increase in the industry's frontier quality level. This definition ignores quality improvements by a laggard in state $s = 1$, i.e., when products are horizontally differentiated. Such improvements make a contribution to gross surplus, because they are appreciated by consumers located near the laggard, and hence it might be argued that they should be counted as innovations. (Certainly it is conceivable that such improvements could show up in patent data, depending on how far patent protection is extended in the horizontal direction.) We have reproduced figures 3, 6 and 7 using the rate of increase in gross surplus as an alternative definition of the innovation rate. In each case the general shape of the relevant innovation-competition relationship is the same as previously, and our analysis of these figures would be unaltered under this definition of innovation.

In principle our framework makes some predictions which might be tested with data from the industries in the studies cited above. For example the model predicts that, all else equal, industries with fairly homogeneous consumer tastes will be relatively over-represented among the high-innovation medium-profit industries at the top of the inverse U of ABBGH. Such industries should also be relatively numerous at the upper right-hand end of Flath's inverse U. In practice the implementation of such tests would not be straightforward because of the need to control for the marginal utility of quality and the costs of product relocation. These variables may be difficult to measure in a consistent way across industries.

Instead our framework is mainly intended to emphasize the useful role that endogenous product locations might play in the study of the determinants of innovation. We have shown that a model which incorporates this ingredient is capable of explaining a range of observed relationships between innovation and competition measures. By virtue of its stylized nature our model is silent on other relationships of interest, such as that between innovation and entry. Moreover it comes with some problems of multiplicity (discussed in Narajabad and Watson 2009) which would complicate econometric applications. We leave for future work the task of developing econometric frameworks which combine endogenous horizontal differentiation with tractable treatments of these related important issues.

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Figure 1:

From Aghion, Bloom, Blundell, Griffith & Howitt (2005):

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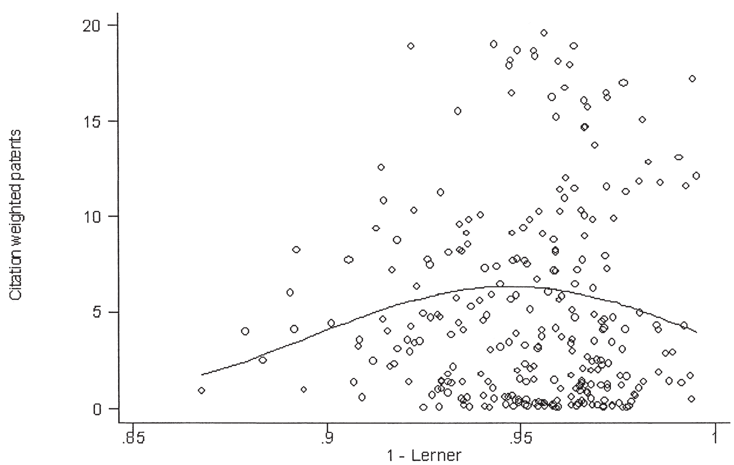
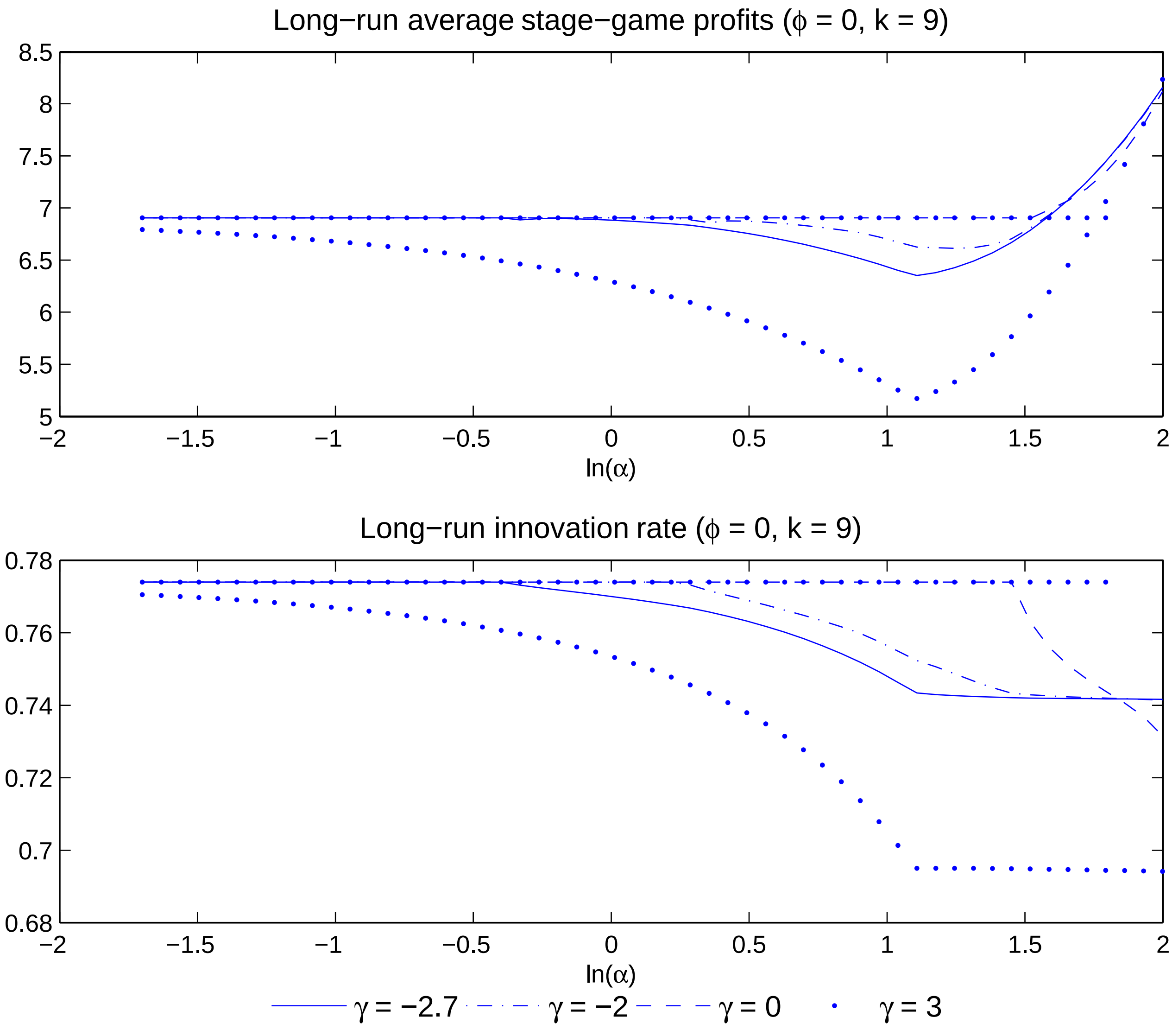


FIGURE I

Scatter Plot of Innovation on Competition

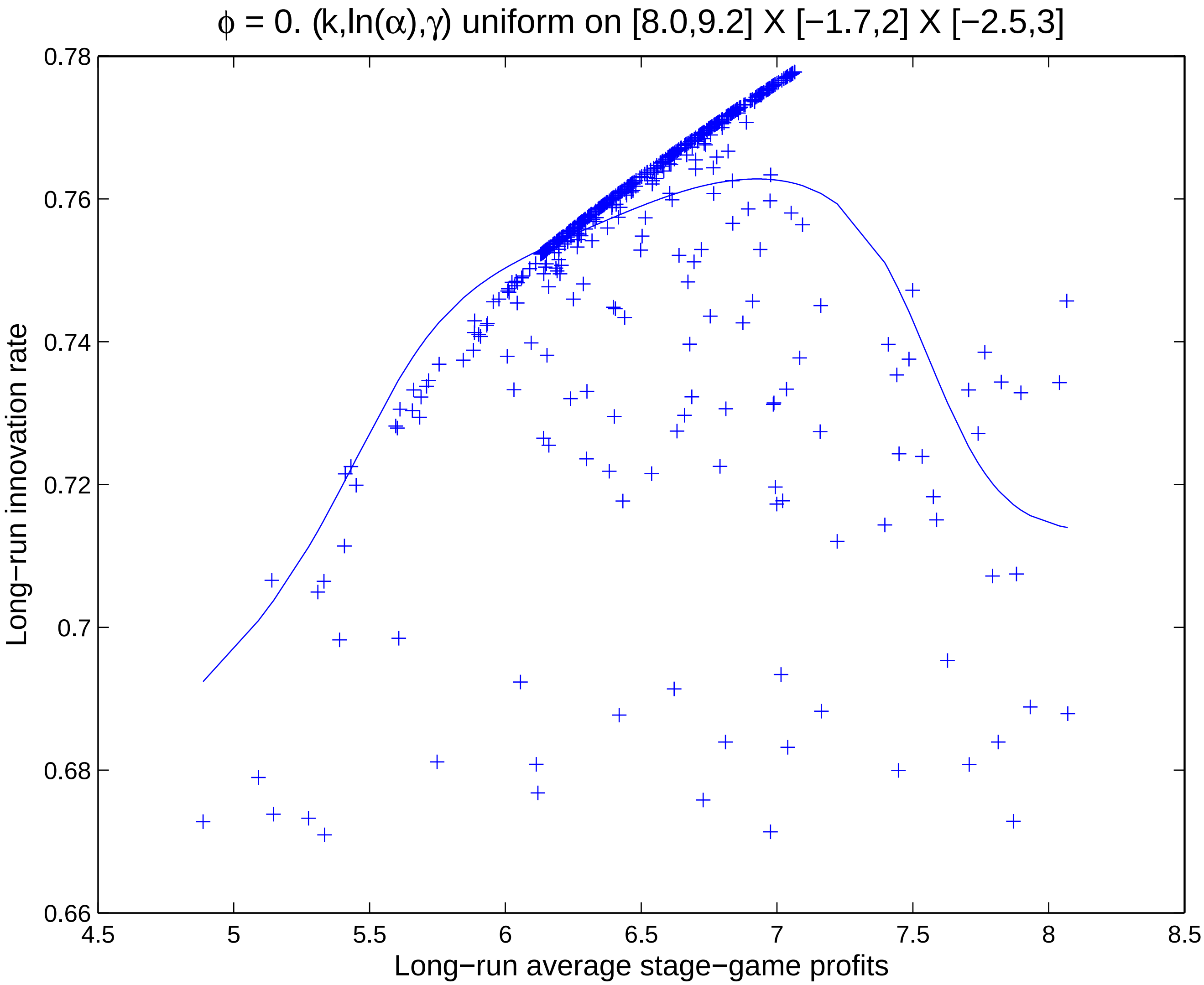
The figure plots a measure of competition on the x-axis against citation-weighted patents on the y-axis. Each point represents an industry-year. The scatter shows all data points that lie in between the tenth and ninetieth deciles in the citation-weighted patents distribution. The exponential quadratic curve that is overlaid is reported in column (2) of Table I.

Figure 2: Long-run average stage-game profits, and long-run innovation, as functions of α



Note: When gamma = 3 there are two invariant distributions.

Figure 3: Innovation as a function of long-run average stage-game profits



Note: Curve shows kernel regression over 500 random "data points". (Gaussian kernel, bandwidth = 0.44.) When there are two invariant distributions, the case of s=1 is plotted.

Figure 4:

From David Flath (2009), "Industrial concentration, price-cost margins, and innovation":

Figure 2. Plot of regression estimate in Table 6, Model 4.

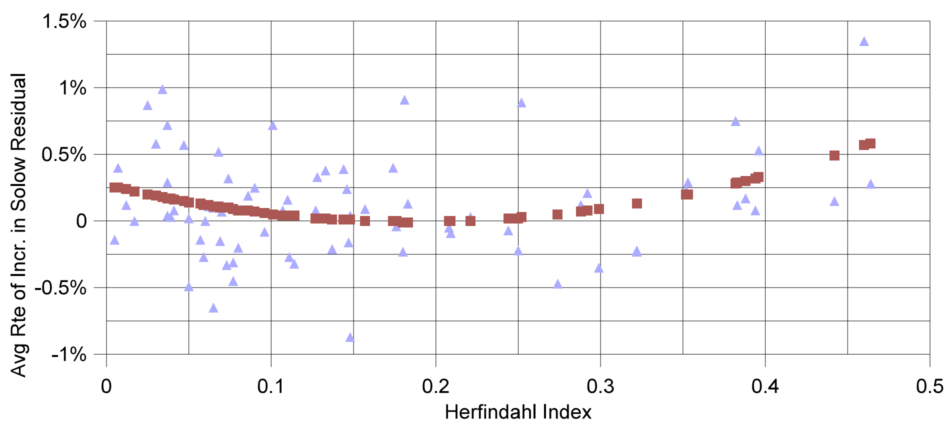
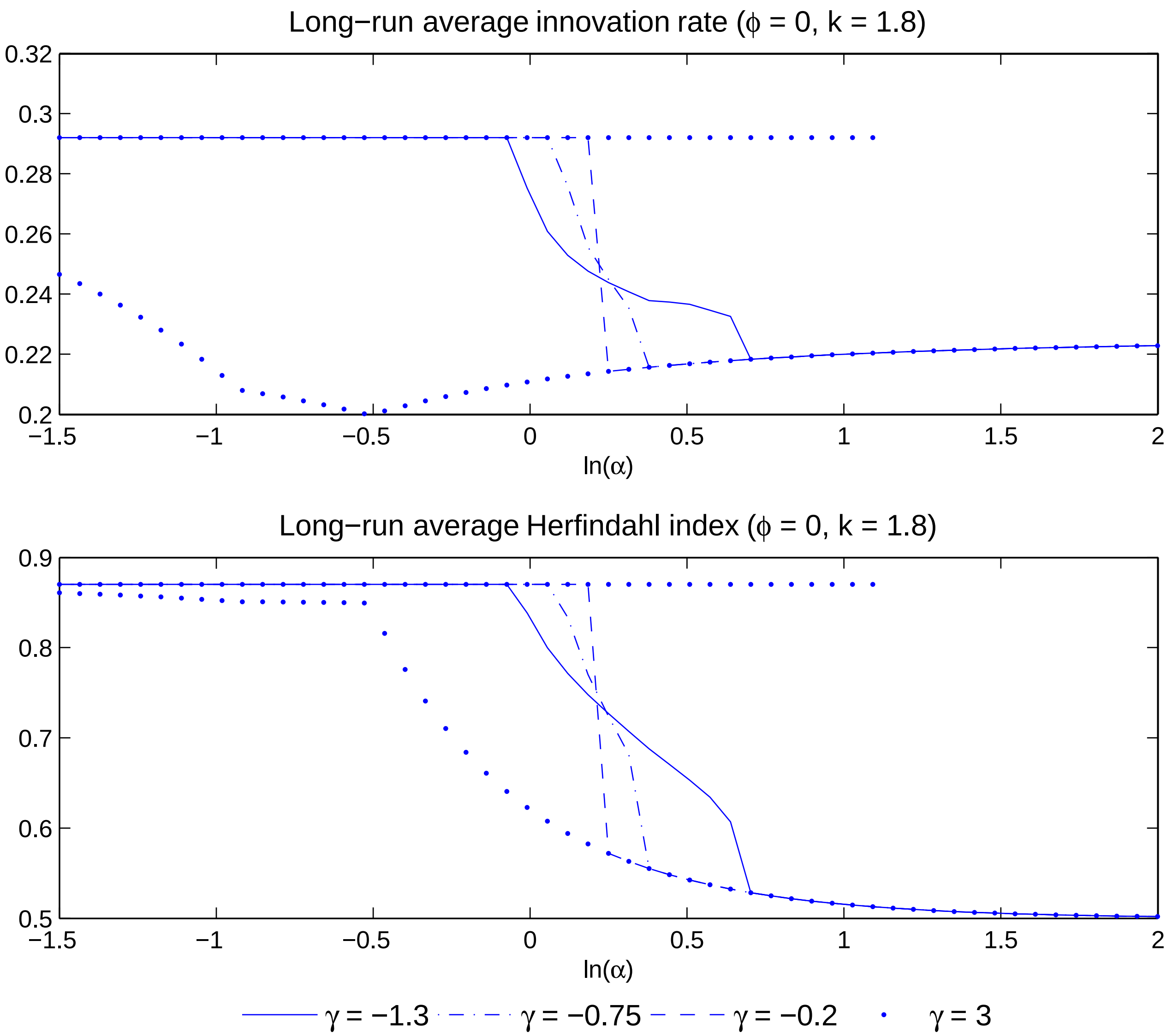
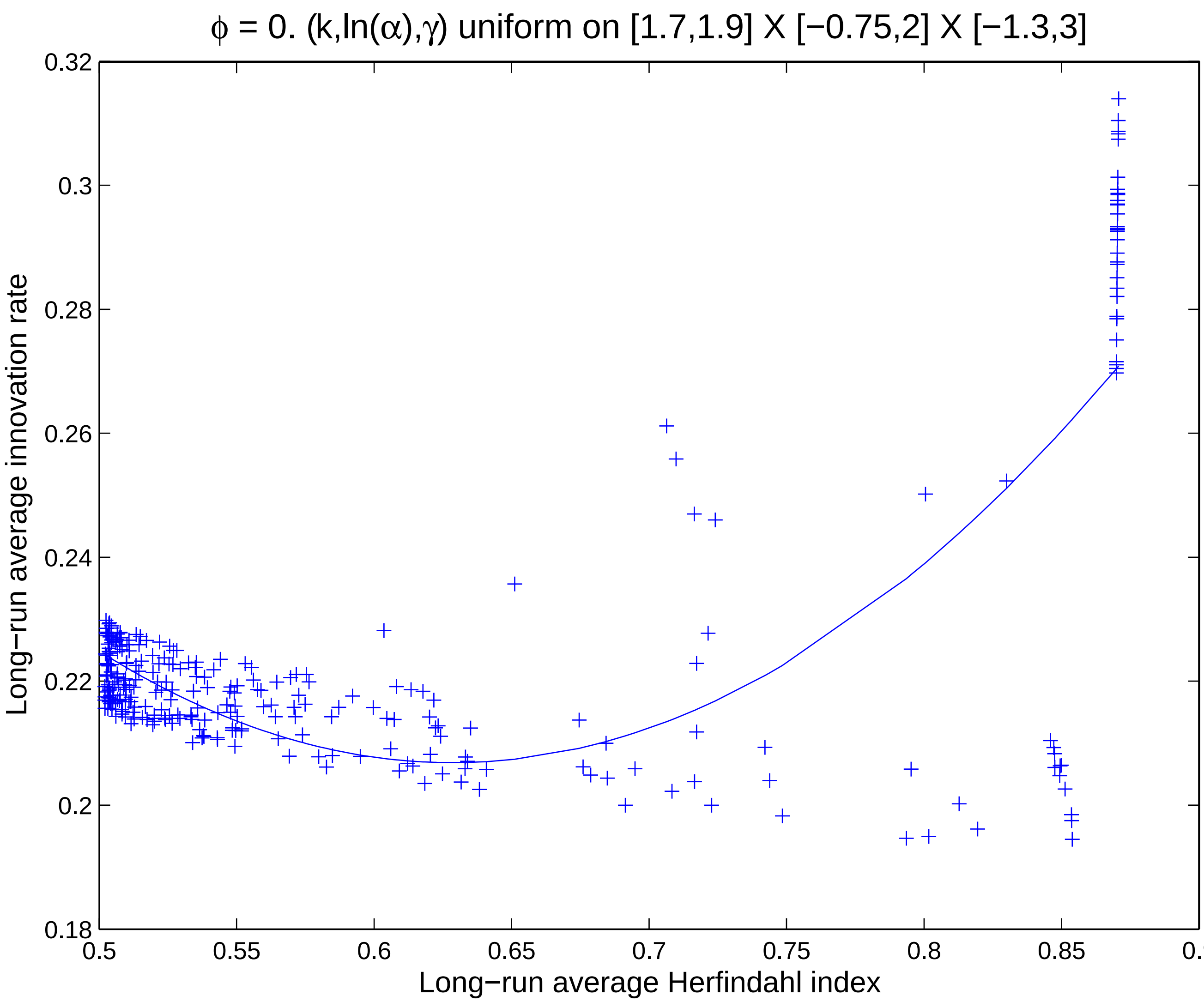


Figure 5: Long-run innovation, and the long-run average Herfindahl index, as functions of α



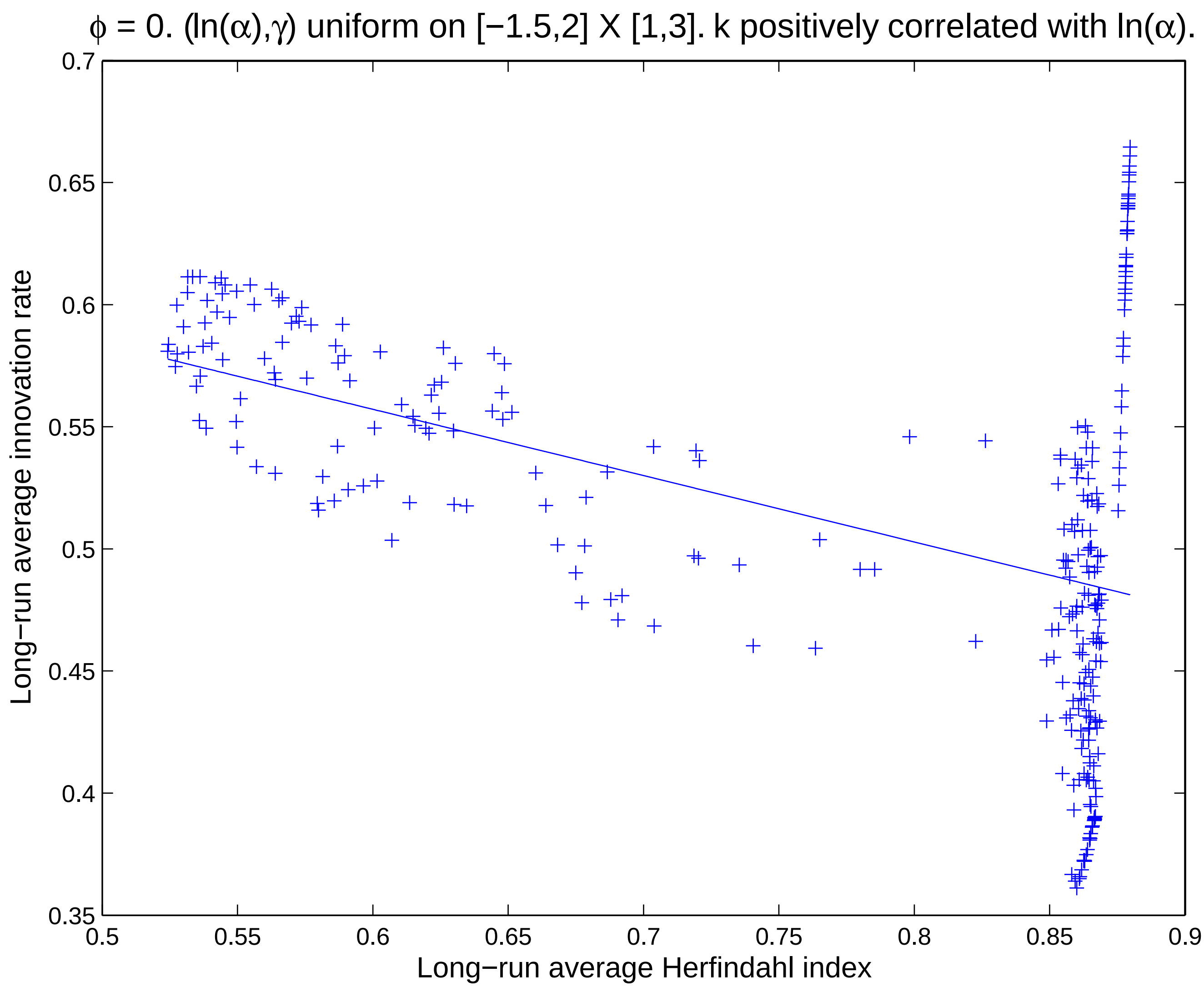
Note: When gamma = 3 there are two invariant distributions.

Figure 6: Innovation as a function of long-run average concentration



Note: Curve shows OLS regression with quadratic term over 300 random "data points".

Figure 7: Innovation as a function of concentration – linear specification



Note: Linear regression line shown, 300 "data points". Values for k lie in [2.5,6.5].
When there are two invariant distributions, the case of s=1 is plotted.