Information Technology and the Rise of Household Bankruptcy*

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Abstract

Several studies have attributed the rise of household bankruptcy in the past two decades to the decline of social stigma associated with default. Stigma explanations, however, cannot account for the increase in the supply of unsecured credit during this period. I try to explain both of these facts as a result of a more informative credit rating technology.

I study an adverse selection environment where borrowers are heterogeneous with respect to their default costs. Creditors have access to a rating technology which provides an exogenous signal about borrowers' default costs. Equilibrium contracts subject each borrower to a credit limit such that creditors' expected profit, conditional on the signal about borrower's default cost, is zero. As the exogenous signal becomes more informative, credit market supplies larger credit limits for borrowers with high default costs, and reduces credit limits for borrowers with low costs of default. Hence a more informative signal allows those with high default costs to borrow more, making them more likely to default, while decreasing borrowing and default by those with low default costs. The net effect could be a rise of average unsecured debt and default.

Using the Simulated Method of Moments, I estimate the model's parameters to match the increases in the average consumer credit card limit, the average unsecured consumer debt level and the spread of the credit limit distribution from 1992 to 1998, using the Survey of Consumer Finance's data. The quantitative example of the stylized model can generate an increase half the size of the observed rise of bankruptcy filings from 1992 to 1998, which indicates the important role of creditors' information about their borrowers' riskiness in the trends of the unsecured consumer credit market.

Keywords: Consumer Bankruptcy, Information and Market Efficiency, Rating Agencies.

JEL Classification: G14, E44, K35, E21.

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1 Introduction

The paper provides an *informational* explanation for three main trends observed in the US consumer credit market since 1980s: the rise of *household bankruptcy*, the increase in *unsecured consumer debt*, and the rise of *supply of unsecured consumer credit*. The main focus of this paper is the Improvement in creditors' information for assessing borrowers' *default cost* and the resulting change in *allocation of credit*. Heterogeneity in borrowers' default costs not only generates heterogeneous propensity to default, but also heterogeneous utilization of available credit card limits. Therefore, when creditors offer more informed *credit contracts*, supply of credit rises and hence borrowing increases. More interestingly, better allocation of credit allows more borrowers to accumulate large enough unsecured debt such that default becomes optimal. In addition to higher supply of unsecured credit and hence indebtedness, better informed credit contracts could result in higher default frequencies and larger amounts of debt discharg.

Household bankruptcy filings have been increasing in the US for the past quarter-century. In 1984, 0.33% of American households filed for bankruptcy. The number of filers rose to 0.93% of households in 1991 and continued to increase up to 1.46% in 2003.[1] Moreover, debt discharge rate also increased. That is, households were defaulting on larger unsecured debts.[2] This trend can also be spotted in the Canadian data (Livshits, MacGee and Tertilt [21]), suggesting that the increase should not be solely attributed to legal changes in the US.

During this period, supply of unsecured credit for households flourished, mainly through credit cards. While in 1989, 56% of American households had credit card and 29% of them carried positive balances on their credit card accounts, fifteen years later, 71% of them had credit card and 40% were carrying debt on their accounts. Those with positive balance are called *revolvers* in the literature. The average credit card debt of revolvers almost doubled from about $\$1,800$ in 1989 to about $\$3,300$ in 2004.[3] But households' credit card debts were not subject to the same credit limits. During this period the average credit card limit available for an American household more than doubled; they rose from about $\$7,100$ in 1989 to about $\$15,200$ in 2004.[4]

The importance of credit card debt in a household's decision to file for bankruptcy has been well documented (for example by Domowitz and Sartain [11] and Sullivan, Warren and Westbrook [28].) Therefore, understanding the dynamics behind the expansion in the supply of credit cards and their usage is critical for the study of the rise of household bankruptcy.

Barron and Staten [5] document that expansion of the credit card industry would not be possible without rapid improvements in information technology and credit rating technologies. In 1997, credit bureaus issued some 600 million reports about credit seekers, (Padilla and Pagano [26]), and in the following decade credit scores produced by the Fair Isaac and Company, known as FICO scores, became the industry's standard tool for assessing borrowers' credit worthiness. Edelberg [13] shows that creditors increasingly used risk-based pricing of interest rates in consumer loan markets during the mid-1990s, and Berger [6] reports that the improvement in the lending capacity was due to improvements in information technology used by the banks. Moreover, Musto [24] documents the importance of creditors' information on borrowers' riskiness for the supply of unsecured consumer credit.

This paper tries to explain the rise in the number of bankruptcy filings as a result of an improvement in the credit market's assessment of borrowers' riskiness. This might sound counter intuitive at first. When creditors separate borrowers according to their riskiness, they will tighten their supply of credit for riskier borrowers, which will make them less likely to default. However, at the same time, safer borrowers will receive larger borrowing limits, which allows them to borrow more and become more likely to default. This happens because even safer borrowers, ceteris paribus, are more prone to default once they accumulate large enough unsecured debt.

[1] Just before the sweeping changes to America's bankruptcy code took effect at the end of 2005, the number of bankruptcy filers jumped to 1.8% of American households. Unsurprisingly, the number of filers plummeted after the change went into effect. The number of filers has picked up again and returned to its earlier levels.

[2] See Sullivan, Warren and Westbrook [28]

[3] All dollar amounts are in 1989 constant prices.

[4] A household's credit card limit is defined as the sum of the limits on all of the household's credit cards.

The change of credit rating technologies can have significant implications for the supply of unsecured consumer credit as well as household bankruptcy. If rating technologies do not work well and the credit market lacks information on borrowers riskiness, then it will be difficult to separate different types of borrowers. I call this case the *pooling case*. The equilibrium supply of credit will be so tight that safer borrowers borrow very little, and therefore will not pay much to cover losses made from lending to riskier borrowers.[5] Now, If rating technologies improve, then credit market obtains information on borrowers' riskiness, which could be used to separate different types of borrowers. I call this case the *separating case*. Differentiating between borrowers, creditors will cut back their supply of credit for riskier borrowers. But compared with the pooling case, the reduction will be small and their default rate will not fall significantly. On the other hand, creditors will significantly increase credit supply for safer borrowers, who are now separated from riskier ones. Safer borrowers will be able to borrow much more than the pooling case and hence will default more frequently.[6] I call this mechanism, the *informational* explanation for the rise of household bankruptcy.

There are different sources of heterogeneity amongst borrowers, but this paper focuses on the heterogeneity in default costs. Default on unsecured debt and the resulting bad credit history, not only limits future access to unsecured credit, but it also has negative effects on cost of receiving secured debts like mortgage and car loan, as well as negative consequences for job opportunities, insurance costs and rental costs (See Chatterjee, Corbae and Rios-Rull [9].) Moreover, as documented by White [29], even after controlling for all different current and future pecuniary costs of filing for bankruptcy, borrowers show large unexplained heterogeneous propensities to default.[7] Obviously other heterogeneities, like differences in borrowers' income processes, have also clear implications for heterogeneity in borrowing and default decisions. However, this paper abstracts from the other heterogeneities and assumes heterogeneity in a reduced form non-pecuniary default cost.[8] That is, even with the same debt level, current income and expected future income, it is more costly for some borrowers to default than the others. This could be due to their higher dependency on using other markets like job, insurance or rental markets, which makes the negative consequences of default more severe for them, or it could be simply due to a higher stigma of default for them. A higher cost of default would apparently make borrowers safer for lending.

Default costs not only affect borrowers' default decisions, but also influence their borrowing patterns. More specifically, when a borrower has accumulated enough debt to find default a possible future outcome, different default costs, and hence default probabilities, translate into different marginal costs of borrowing. Therefore, in addition to having heterogeneous probability of default, borrowers with different default costs show heterogenous propensity to borrow out of an available credit limit.

The heterogeneity in propensity to borrow is essential for the increase in the average supply of unsecured credit after switching from the pooling case to the separating case. That is, although riskier borrowers' debt levels and default probabilities are very elastic with respect to credit supply, safer borrowers could have a low and relatively inelastic debt level and default probabilities in the pooling case. Hence, after switching from the pooling case to the separating case, there will be a small decline in credit supply for riskier borrowers, while credit supply increases much more for safer borrowers, generating a net increase in the average credit supply.

More specifically, I study a model in which borrowers start with a temporary low income, but expect their income to switch permanently to higher levels in future. However, there is uncertainty about the time of switching to the permanent income, as well as its level. While waiting for the realization of their permanent income, borrowers can borrow up to a credit limit, which is determined by a competitive credit market. Depending on how long they wait until the realization of permanent income, borrowers can accumulate different levels of unsecured debt. Once a borrower realizes her permanent income, depending on her debt and realized permanent income level, as well as her default cost, she chooses to repay her debt, or default.

[5]For example, if riskier borrowers are much more riskier than safer ones and there are a lot of them in the pool of borrowers, borrowing and default will be mostly done by riskier borrowers and safer ones will be effectively inactive.

[6]Borrowers' responsiveness to terms of credit contracts, and specifically credit limits, is well documented by Gross and Souleles [16].

[7]See also Gross and Souleles [15] and Fay, Hurst and White [14].

[8]Note that with constant relative risk aversion, all pecuniary default costs could be represented by a non-pecuniary cost, as long as they are proportional to the defaulter's consumption. Most increases in defaulters' cost of living, like higher renting cost, car insurance rate etc., are proportional to living costs.

Despite being quite stylized, the model does a good job in showing the importance of information on borrowers' riskiness for allocation of credit. In a simple quantitative example, the model generates an increase in the supply of unsecured credit, unsecured debt and default rate, as the result of switching from the pooling case to the separating case. It also generates dispersion in credit supply and unsecured debt, with more debt accumulated by safer borrowers. Since most of the increase in default is due to *safer* borrowers, and they only default if their debt amount is relatively large, the model generates an increase in the debt discharge rate. These implied changes in the allocation of credit card limits, debt distribution and debt discharge are qualitatively consistent with the trends observed in the US data. Some of the changes generated by this stylized model are even quantitative consistent with the data, showing the importance of the underlying mechanism addressed by this paper. In particular, a simple calibrated stylized version of the model can generate an increase half the size of the observed rise of bankruptcy filings from 1992 to 1998, while accounting for the increase in the average unsecured consumer debt, and the rise of the average and dispersion of the supply of unsecured credit limits.

The literature provides other explanations for the rise of household bankruptcies. The common explanation attributes it to the fall of stigma attached to bankruptcy. Gross and Souleles [15] report that ceteris paribus, a credit card holder in 1997 was almost one percentage point more likely to declare bankruptcy than a card holder with the identical risk characteristics in 1995. Fay, Hurst and White [14] report that even after controlling for state and time fixed effects, households are more likely to file for bankruptcy if they live in districts with higher aggregate bankruptcy filings rates. The stigma explanation, however, has counterfactual implications for the equilibrium supply of unsecured credit and debt. If borrowers become less reluctant to default on their debt, then shouldn't creditors restrain their supply of credit? and wouldn't this result in less borrowing rather than more? Athreya [1] and Livshits et al. [21] note this counterfactual effect and suggest a reduction in the transaction costs of lending to account for the rise of unsecured consumer debt.[9,10]

While able to account for the rise of unsecured credit supply, a fall in credit transaction costs cannot explain the observed spread in the distribution of consumer credit card limits.[11] That is, the rise of credit limit has been larger for some households than the others. Gross and Souleles [15] report that creditors extended larger unsecured credit lines to less riskier borrowers, suggesting that the spread of credit supply is mostly associated with the improvement in creditors' assessment of borrowers' riskiness. Using a combination of the rise of stigma and fall of transaction costs to address different trends of the consumer credit industry ignores an important innovation of this industry: the consumer credit risk rating. This paper studies the implication of this innovation for the trends of the unsecured consumer credit market.

Recently, Sanchez [27], Drozd and Nosal [12], Livshits, MacGee and Tertilt [22], and Athreya, Tam and Young [2] address the increase in household bankruptcy from informational prospectives in environments with ex-ante heterogeneous borrowers. As I explain in more detail in the last section, despite using elegant modeling and more rigorous quantitative setups compared to this paper's stylized model, most of them cannot generate either the observed increase in the dispersion of unsecured consumer debt, or the observed rise of the consumer debt discharge rate.

Section (2) provides some facts from the US data on the number of household bankruptcy filings, the distributions of credit card limits and debts, as well as the changes in these distributions across time. Section (3) describes a model of borrowers' demand for credit, the responsiveness of their demand to credit contracts, in particular credit limits, and the change in propensity to default when credit supply increases. Then the model is used to show how a more informed credit market supplies more credit, while increasing the dispersion of credit supply. Section (4) provides a simple quantitative example and shows the model is relevant for explaining the rise of credit supply, unsecured consumer debt and the number of household bankruptcies. Section (5) concludes.

[9] Athreya [1] also uses the same reduction to generate the rise in filings, which leads to a significantly higher debt to income ratio than the level observed in the data. Livshits et al. [21] use a combination of decline in stigma and fall in transaction costs to explain the changes in filings and the ratio of unsecured debt to income.

[10] Another possible explanation for more bankruptcy filings could be the rise of "uncertainty" in households' income and emergency expenses. This explanation implies a similar counterfactual decline in the supply of unsecured credit. Moreover, Livshits et al. [21] find its effect on the rise of bankruptcy filing numbers to be insignificant.

[11] The next section on Data and Motivation reports the observed changes in the distribution of consumer credit card limits. In particular, the distribution of credit card limits did not just shift rightward; it also spread out.

2 Data and Motivation

This section documents three major trends in the US unsecured consumer credit market: (1) increase in household bankruptcy filings, (2) increase of the average and the dispersion of households' credit card debt, and (3) increase of the average and the dispersion of households' credit card limits. This paper tries to explain these three trends simultaneously in a unified framework. I also report two important trends documented by other studies, which motivate the mechanism developed in this paper: (1) relatively high elasticity of households' credit card debt to (exogenous) increase in their credit card limits, even for those with debt levels well below their credit limits, and (2) the rise of defaulting households' discharged unsecured debt.

Households can file for bankruptcy under chapter 7 or 13. Under chapter 7, their unsecured debt such as credit card debt, installment loans, medical bills and damage claims are discharged; and filers lose all of their assets above their state specific exemption levels.[12] Under chapter 13, filers do not lose their assets, but must propose a plan to repay a portion of their debts from their future income. Since households have the right to choose between the chapters, they are only obliged to use future earnings to repay debt to the extent they would repay under chapter 7. Those who file under chapter 13, are allowed to file again under chapter 7, but chapter 7 filers cannot refile for another 6 years. The bankruptcy flag remains in a filer's credit history for 10 years (see Musto [24].)

Approximately 70% of those who seek bankruptcy protection, file under chapter 7 and two thirds of those who file under chapter 13 end up filing again under chapter 7.[13] This paper, however, does not distinguish between filing under the two chapters and studies a notion of bankruptcy similar to filing under chapter 7.

Figure (1) shows the number of bankruptcy filings by American households over the past quarter-century.[14] Except three short periods of 1992-94, 1997-2000 and 2003-04, bankruptcy filings has been on the rise from 1984 to 2005. The drastic change of the US bankruptcy code caused a sudden jump in the number of bankruptcy filers before the new code took effect at the end of 2005, and then reduced the number of bankruptcy filings significantly. Despite a much tighter bankruptcy code, the number of filings has increased sharply and returned to its levels before the change of the bankruptcy code, most likely due to the recent economic downturn. This paper will focus on the rise of household bankruptcy filings prior to the change of the bankruptcy code in 2005. It is worth noting that even during 1994 to 1997, a period of robust economic expansion, bankruptcy filings rose by 63%. Using the Survey of Consumer Finance (SCF), I find that 11% of American households had at least once filed for bankruptcy in their lives by 2004, and from those who had filed, 69.4% of them had filed in the past 10 years. That is, more than 7% of American households had a bankruptcy flag on their credit history in 2004.

Households' credit card debt increased dramatically during this period. Table (1) reports the average credit card debt, both for all households as well as those who have *revolving* debt on their credit cards, from the SCF 1989-2007 in constant prices. The revolving credit card debt are households' total balances on their credit card accounts after their last payments. The average household credit card debt more than doubled, increasing from $\$954$ in 1989 to $\$1,851$ in 2004 and $\$2,507$ in 2007. This was not only due to the increase in the fraction of revolver households, but it was mostly due to the increase in the average debt level of the revolvers. More precisely, the fraction of revolver households rose from about 30% in early 1990s to about 40% of households in early 2000s. Meanwhile, the average debt level of revolvers doubled and soared from $\$1,828$ in 1989 to $\$3,295$ in 2004 and $\$4,299$ in 2007. Just from 1992 to 1998, during an economic boom, revolvers' debt increased by a factor of 59%.

But the increase in the average amount of credit card debt does not fully summarize the change of the cross sectional distribution of households' credit card debt. As reported in Table (1), the standard deviation of this distribution also doubled from 1989 to 2004, and continued to grow in 2007. From 1992 to 1998 alone, the standard deviation of revolvers' debt rose by a factor of 72%, resulting in 8.25% increase in the coefficient of variation (i.e. std./mean) of this distribution. In short, not only the average household's credit card debt increased,

[12] Exemption levels differ across the US states.

[13] See Li and Sarte [20] for an elaborate study of bankruptcy filers' choice of chapter.

[14] The percentage of filers for the 1984-95 period are reported from Fay et al. [14]. The number of filings for the 1995-2009 period are from www.uscourts.gov and the number of households for this period are from www.census.gov.

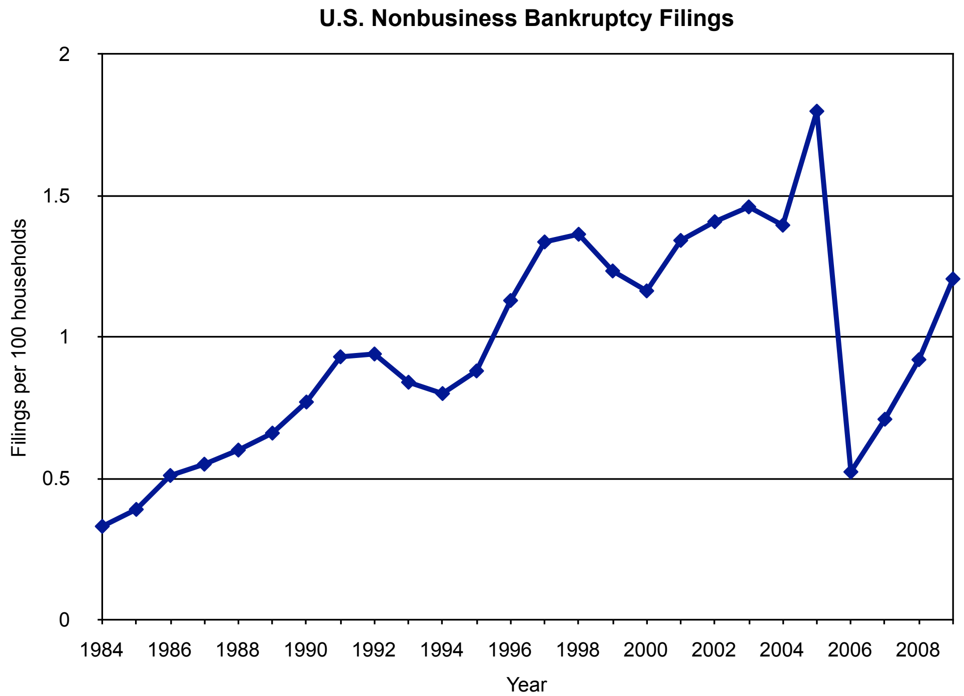

Figure 1: The Rise of Household Bankruptcy

but it increased more dramatically for some households, causing the distribution of households' credit card debt to spread out.

Credit card contracts usually consist of a credit limit and an interest rate. Table (1) reports the average and standard deviation of households' total credit card limit. Households were not simply borrowing more out of their existing credit limits. The average credit card limit, both for revolvers and non-revolvers, more than doubled during this period. In only six years, from 1992 to 1998, the average credit limit of revolvers increased by a factor of 75%. It is important to realize that the increase in the average credit limit, was not restricted to those who were borrowing on their credit cards. The average credit limit for all credit card holders, including revolvers and non-revolvers, rose by 79% during these six years. That is, those who were not carrying any debt on their credit cards received even a slightly higher raise in their credit limits.

The standard deviation of the cross section of credit card limits almost tripled from 1992 to 2001. More specifically, the coefficient of variation (i.e. std./mean) for the credit card holders' limits rose by more than 40% during these twelve years. For revolvers, the rise in the standard deviation of credit limits was even more pronounced. The coefficient of variation rose by 79% from 1.011 to 1.809 for revolvers' credit card limits during these nine years. This observation is critical for this paper's main mechanism.

Not being a panel data, I cannot use the SCF for measuring the elasticity of households' credit card debt to increase in their credit card limits.[15] However, Gross and Souleles [16] study borrowers' response to exogenous change in credit supply, using a proprietary dataset on credit card accounts' activites during mid 1990s. They report an average "marginal propensity to consume (MPC) out of liquidity" (i.e. dDebt$/d$Limit) in the range of $10 - 14\%$. Importantly, their study finds that MPC is significant even for borrowers with debt levels well below their credit card limits. Average MPC of 14% implies a \$790 increase in the average credit debt for the observed

[15] Another commonly used dataset on household finance, The Panel Study of Income Dynamics (PSID), does not report credit limits in exact amounts.

		1989	1992	1995	1998	2001	2004	2007
Credit Card Debt*	Mean	954	1,025	1,346	1,696	1,453	1,851	2,507
(Card Holders)	Debt> 0	1,828	1,947	2,404	3,098	2,707	3,295	4,299
	Std.	2,120	2,303	3,076	3,979	4,172	4,246	5,932
	Debt> 0	2,648	2,878	3,788	4,958	5,390	5,228	7,255
Credit Limit*	Mean	7,092	7,157	10,390	12,802	13,548	15,223	17,231
(Card Holders)	Debt > 0	7,125	6,579	9,832	11,505	11,964	13,643	15,867
	Std.	11,296	8,223	13,151	17,861	22,055	20,911	22,551
	Debt > 0	9,624	7,204	11,233	15,696	21,645	18,066	20,468
Interest Rate	Mean	–	–	14.51	14.45	14.36	11.49	12.89
(Card Holders)	Debt > 0	–	–	14.14	14.48	14.20	11.81	12.61
	Std.	–	–	4.29	4.63	5.24	6.42	6.58
	Debt > 0	–	–	4.46	5.04	5.62	6.63	7.01
Card Holders**		55.91%	62.32%	66.45%	67.54%	72.72%	71.46%	70.15%
Revolvers ** (Credit Card Debt> 0)		29.18%	32.83%	37.21%	36.97%	39.01%	40.14%	40.92%

**Reported in 1989 dollars using CPI index.*

***Percentage of the entire sample.*

Table 1: Summary of the US Households' Credit Card Usage from the *Survey of Consumer Finance*

$5,645 increase of the average credit limit from 1992 to 1998. The actual average increase of debt level is $671 for this period, suggesting that the rise of credit card debt could be mostly attributed to the rise of credit card limits.

So far, I have reported the credit card limit and debt for an average household. But how about the credit card limit and debt level of those who file for bankruptcy? The households who are reported as bankrupt in the SCF, have usually completed their legal processes and their debt levels are discharged. Moreover, after filing for bankruptcy, credit cards are cancelled so no information is available from the SCF on the filers' credit card limits. Not being a panel dataset, the SCF does not allow me to observe the credit card limit and debt of defaulters just before filing for bankruptcy. Therefore, this paper uses the financial description of bankruptcy filers from Sullivan et al. [28].

Credit Card Debt (Ratio to Income)	1991		1997	
Mean	$10,193	(.531)	$12,608	(.767)
s.d.	$13,751	(.755)	$15,380	(1.154)
25th percentile	$2,702	(.122)	$3,864	(.167)
median	$6,112	(.310)	$8,262	(.469)
75th percentile	$12,807	(.645)	$14,188	(.874)

Table 2: Credit Card Debt listed in bankruptcy filings from Sullivan et al. (1989 dollars)

Table (2) summarizes the distribution of credit card debt listed in bankruptcy filings in 1991 and 1997 reported by Sullivan et al. [28]. Since the SCF data collection and report takes about one year, the data on filers' credit card debt corresponds to 1992 and 1998 data from the SCF. The table also reports the ratio of credit card debt to income for bankruptcy filers. The average credit card debt of filers rose by a factor of 24%, and the median increased by a factor of 35%, suggesting that the distribution not only shifted rightward but also spread out. The increase in the ratio of credit card debt to income is even higher. The average of the ratio of credit card debt to income for filers rose 44% while the median increased by 51%. In short, those who filed for bankruptcy in 1997 were defaulting on much higher credit card debts than those who filed in 1991.

Next I address two aspects of the unsecured credit market, which has been used by other papers to explain the rise of household bankruptcy filings in the US: (1) fraction of households who have credit card and those who have revolving credit card debt, and (2) the long-term elasticity of credit card debt to (exogenous) change in credit card interest.

The fraction of households who had credit card, rose from around 55% of the US households in the late 1980s, to about 70% during 2000s (Table (1).) During the same period, the fraction of households with positive debt on their credit cards (i.e. revolvers) increased from about 30% to 40% of all households. These upward trends happened mostly during 1990s and both of these fractions did not change very much from 2001 to 2007. These trends, which are called *democratization* of credit cards, have played an important role in the increase of the indebtedness of households and hence their higher tendency to default. However, as noted before, the increase in the average credit card holder household's unsecured credit card debt is mostly due to the increase in the debt level of those who use their credit cards to borrow (i.e. *intensive margin*) rather than the increase in the fraction of those who borrow on their credit cards (i.e. *extensive margin*.)

More specifically, if the intensive margin of unsecured credit card debt would have remained in its 1989 level, $\$1,828$ for revolvers, and only the extensive margin would have changed from its 1989 level to its 2001 level, the average card holders' credit card debt, would have increased from $\$954$ in 1989 to $\$980$, instead of $\$1,453$ level of 2001. This observation suggest the active margin of change in households' credit card debt during 1990s was the intensive margin. Therefore, this paper will focus on how the change in lenders' information about borrowers could have resulted in higher indebtedness of borrowers, rather than adding more to the pool of borrowers. I should emphasize that the main mechanism of the paper is also capable of generating the change along the extensive margin, however, it seems the data suggests this margin was less important during the past twenty years.

The change in the interest rate on credit card debt plays an important role in some papers' explanation for the rise of household bankruptcy.[16] Table (1) reports the average and standard deviation of credit card interest rates from the SCF 1995-2007. (Other than a general consumer credit interest rate reported for 1983, the SCF did not collect the credit card interest rates prior to 1995.) From 1995 to 2001, when the Bank Prime Loan Rate, *MPRIME*, fluctuated between 8.00% and 9.50%, the average credit card interest rate remained around 14.5%, and its standard deviation rose almost one percentage point from 4.29% in 1995 to 5.24% in 2001.[17] The average credit card interest rates in 2004 decreased to 11.49% while MPRIME dropped to $4.00\% - 5.00\%$. The variation of credit card interest rates across households also rose. Specifically, the standard deviation increased to 6.42% in 2004 and 6.58% in 2007.

The simultaneous rise of the spreads of credit limits and interest rates, indicate that creditors offered more differentiated credit terms to their borrowers. Variation in credit limits, however, increased far more than that of interest rates, especially prior to 2004. While the standard deviation of limits rose by a factor of 68% from 1995 to 2001, the standard deviation of interest rates increased by a factor of 22%.[18] Moreover, Gross and Souleles [16] report the long-term elasticity of debt to the interest rate to be approximately -1.3. Hence the change in standard deviation of the interest rates from 4.46% in 1995 to 5.04% in 1998, while the average credit card interest rate for revolvers remained $14 - 14.5\%$, would imply at most $5 - 6\%$ increase in the average revolver's credit card debt. But the average revolver's credit card debt rose by a factor of 29% from 1995 to 1998.

The relatively smaller cross sectional variation in revolvers' credit card interest rate, compared to variation of their credit limits, hints at the importance of credit limits for the rise of revolvers' credit card debt during 1990s. Therefore, this paper focuses on the quantity of unsecured consumer credit supply, namely credit card limits,

[16] For example see Athreya et al. [2].
[17] MPRIME is reported from the Board of Governors of the Federal Reserve System.
[18] Stickiness of credit card interest rates have been studied by Ausubel [3] and Calem and Mester [8].

rather than the price of unsecured credit, namely credit card interest rates.[19] It is worth mentioning that since 2001, the cross sectional variation in credit card interest rates rose significantly. Thus, a comprehensive study of the change in credit card supply during 2000s should study both aspects. But this paper restricts its study to credit limits and hence uses data from 1990s for its quantitative example.

To summarize, the data on the US unsecured consumer credit market suggests that households' credit card limits rose significantly during the past quarter-century. This increase was more pronounced for some households than the others, resulting in the rise of the cross-sectional dispersion of credit limits. Households used their credit cards to accumulate more unsecured credit card debt. Larger number of households found it optimal to file for bankruptcy, and when they did, they tended to default on larger unsecured debt levels. The next section provides a framework to study how those defaulters could get access to more unsecured credit and accumulate larger debts before defaulting on them.

3 Model

Households borrow for different reasons, using different forms of credit contracts, and decide to repay or default on their debt considering different factors. In order to model households' borrowing and default decision, I make three major simplifying assumptions about: (1) households' bankruptcy decision, (2) their demand for borrowing and accumulation of unsecured debt, and (3) the form of unsecured credit contracts between borrowers and creditors. Using a stylized model based on these assumptions, I study the implication of an improvement in creditors' information about borrowers' *riskiness*, for the equilibrium supply of credit, unsecured debt, default frequency and debt discharge.

Abstracting from the legal details of the process, after filing for bankruptcy, unsecured debt of defaulters are discharged, they lose their assets above the exemption level and the bankruptcy flag remains on their credit history for a certain period of time. Having a bad credit history not only limits supply of unsecured credit, but also has negative effect on cost of receiving secured debts like mortgage and car loan. It even has negative consequences for job opportunities, insurance cost and rental costs, since the potential employers, landlords and insurance companies have access to the applicant's credit history. (See Chatterjee et al. [9].) Moreover, even after controlling for all different current and future pecuniary costs of filing for bankruptcy, borrowers show large unexplained heterogeneous propensities to default.[20] This observation is used as an indication for a *stigma* associated with filing for bankruptcy.

More specifically, I abstract from all different sources for costs of bankruptcy, and assume that upon filing for bankruptcy, debt is discharged and borrowers are excluded from credit market. Moreover, they have to incur a non-pecuniary default cost, which is equivalent to losing a constant fraction of future income under constant relative risk aversion utility functions. I assume borrowers are endowed with heterogeneous default costs. That is, some borrowers incur higher costs upon defaulting, which makes them less likely to default and hence *safer* borrowers compared to those with lower cost of default, who are, ceteris paribus, *riskier* for lending. Finally, as I will discuss below, this default cost is the main attribute of borrowers' creditworthiness in creditors' evaluation.

Unsecured debt is an important determinant of bankruptcy filings.[21] Households might end up with high unsecured debt, due to some exogenous expense shocks, or they could simply accumulate their unsecured debt over time. Although expense shocks play an important role in household bankruptcy decision, their effect is mostly through tipping households with large unsecured debt into bankruptcy.[22] This paper abstracts from expense shocks and focuses on accumulation of unsecured debt and in particular credit card debt.

[19] Another challenge for studying the effect of interest rate on debt level lies in the fact that credit card prices contain other dimensions like cash back rates, flyer miles and other point programs on which no data is available from the SCF.

[20] See for example Gross and Souleles [15], White [29] and Fay et al. [14].

[21] Secured debts like mortgage and car loan, can also affect a household default decision, specially if the interest rate charged on those loans depend on the household's credit score. However, by definition secured loans are collateralized and unlike unsecured debts cannot be discharged upon bankruptcy.

[22] See Chatterjee et. al [10] and Livshits et al. [21].

As noted by Calem, Gorday and Mester [7], credit card balances show high persistence with annual autocorrelation of 0.90. That is, households use credit cards mostly for medium- or long-term financing rather than for short-term or unanticipated liquidity shocks. Hence, this type of unsecured debt is accumulated over time. Notice that a high persistence Markov income process with transitory shock and no income profile is unlikely to generate steady and gradual accumulation of unsecured debt. With those income processes, borrowing is mostly used to smooth consumption whenever a short-term income or liquidity shock is received.[23] I exploit this point in modeling of the household demand for unsecured borrowing.

In particular, I use an extremely parsimonious income process. I assume households start with an initial income and then at a random time their income switches permanently to a random level, which is likely to be higher. With this income process, households borrow for medium- or long-term financing of the difference between their consumption and initial income, until realization of their permanent level of income. One can think about this paper's income process as a simplified version of the *Heterogeneous Income Profile*, studied by Guvenen [17] and [18], under which households are subject to large heterogeneity in their income process and they learn about their profile when the change occurs.

Households use different types of credit contracts for borrowing. This paper focuses on unsecured debt contracts and in particular credit card contracts. These contracts consist of a credit limit and an interest rate. Average American household sampled in the SCF, has about four credit cards, usually with different terms. However, they tend to carry their revolving unsecured debt on a credit card which offers the lowest interest rate and highest credit limit. Moreover, although the creditors have the right to change the terms of contract whenever they want, the interest rate and credit limit do not change frequently. That is, unless there is a delinquency in minimum payment or a run-up of debt over a short period of time, creditors tend to keep the terms of credit contract unchanged.[24]

For simplicity, I assume creditors are committed to their offered terms of contract (i.e. interest rate and credit limit) and borrowers can only choose a single contract.[25] Moreover, in order to focus on the role of credit limits, I assume all of the offered credit contracts have the identical interest rate, which is above the risk free rate.[26] Despite being restrictive, this assumption is consistent with the US data during 1990s, since most of the heterogeneities in the credit card contracts during 1990s were along the credit limit dimension and variations in the interest rates were much more limited. It also allows to study the response of unsecured consumer debt to changes in credit card limit.

Musto [24] documents that creditors change their supply of credit as they lose information on the creditworthiness of borrowers due to the removal of the bankruptcy flag from their credit histories ten years after the filing date. This observation shows the importance of creditors' information about the characteristics of their borrowers for the supply of unsecured credit. Many factors can be used to predict a borrower's propensity to default. In addition to demographic and occupational information, credit rating agencies use borrowers' credit history to assess their creditworthiness. Hence borrowers should be potentially cautious about the effect of their borrowing/repayment decisions on their future terms of credit contracts. Dynamic signaling models seem to be useful for the study of borrowing/repayment decisions. However, these models are usually very difficult to analyze.[27]

[23]For an elaborate model of credit card usage with Markov income process, see Chatterjee et al. [10]

[24]Although this is not explicitly stated in credit card contracts, credit card companies are implicitly committed to maintain the original line of credit with the specified interest rate, as long as the borrowing pattern does not deviate significantly from the expectation they had at the time of extending credit to the borrower. A reputation mechanism can implement such behavior in the absence of an explicit commitment.

[25]Note that without this assumption, borrowers could be offered a continuum of contracts with incremental credit limits and increasing interest rates. In that case, they would start borrowing from the contract with the lowest interest rate and as debt level rose, contracts with higher interest rates would be used. Since higher debt level would increase borrowers' default probability, creditors would make zero expected profit on all of these contracts. This would be identical to having a menu of interest rates for different debt levels.

[26]The framework is general enough to allow for contracts to vary in both of credit limit and interest rate dimensions. In an earlier version of the paper, I had assumed variable credit limits and interest rates. However, in order to highlight the role of credit scores and prevent the creditors to sort borrowers based on their choice of contract, the timing of the environment was different. In particular, borrowers would choose their contracts before realization of their types. In the current version, however, borrowers know their types when they choose their contracts. Nevertheless, since there is only one instrument available for creditors, namely credit limit, they cannot use differentiate contracts to sort borrowers.

[27]For an example of a model with dynamic updating of creditors' beliefs about borrowers' creditworthiness see Chatterjee et al. [9]. Extension of credit over time in their model depends on the evolution of household credit scores or Bayesian posteriors of household type.

This paper assumes a parsimonious credit rating technology, and uses the improvement in informativeness of this rating technology to study what happens when creditors become more informed about the creditworthiness of borrowers. I assume creditors receive a public signal about borrowers' types, i.e. default cost, when credit contracts are offered. I use more informative signals as a proxy for the improvement of credit rating technology. Therefore, I abstracts from the details of how credit scores are developed and focus on the informational content of credit scores when households start to use their credit cards for borrowing.

In the following subsections, first I describe the environment, then borrowers' problem will be studied. In particular, I use the model to study how change of credit limit affects borrowing patterns and default decisions of borrowers with different riskiness. I show, under certain conditions, increasing credit limit would raise borrowing and default. Moreover, ceteris paribus, riskier agents borrow more, accumulate more debt and default more frequently. Then I characterize creditors' problem, which is basically a static problem of choosing credit limit offers, based on signals about borrowers' riskiness. Existence of equilibrium will conclude this section. I assume the informativeness of signals about borrowers' riskiness to be constant in this section. In the next section, Discussion and Quantitative Example, I use the stylized model to study how credit supply, borrowing and debt amounts, default frequency and level of debt discharge might change as a result of a more informative signal about borrowers' riskiness.

3.1 Environment

Consider a continuous time economy populated by a unit measure of agents denoted by $i \in (0, 1)$. Agents live forever, discount future at rate r and their instantaneous utility from consumption is given by a strictly increasing and strictly concave function $u(\cdot)$. There is also a competitive market of risk neutral creditors with access to funds at rate r, who can only lend at rate $\rho \geq r$.

At time zero, each agent i realizes her risk type $\theta(i) \in [0, 1]$, which is private information. There is a *Rating Technology* which sends a public signal $\tilde{\theta}(i)$ about each agent's type $\theta(i)$. The conditional distribution of types given signals, denoted by $\psi(\theta|\tilde{\theta})$, is public information.

Agents are endowed with a stream of income, which is subject to idiosyncratic shocks. In particular, they all start with an *initial* income, $y^I > 0$, until a random switching time, governed by a Poisson process with the arrival rate δ. Once switching time arrives, agents draw their publicly observable *permanent* income, y^P, from a distribution $F(\cdot)$. After the permanent switch, they receive y^P for the rest of their lives. I assume $F(\cdot)$ has a support over $[\underline{y}^P, \infty)$ with $\underline{y}^P > 0$, does not have any mass point, and $E[y^p] \gg y^I$.

Agents can borrow from creditors while waiting for realization of their permanent incomes. Lending contracts are exclusive and can only be made after receiving public signals about agents' types. Specifically, I assume contracts to have a fixed common interest rate $\rho \geq r$ and a credit limit L, determined competitively by credit market, based on public signals on agents' types. Moreover, creditors are committed to their contract until uncertainty about agents permanent income is resolved. In the meantime, agents can accumulate debt, D, at the interest rate, ρ, up to their credit limits, L.

At any point in time agents can exercise their *bankruptcy* option and default on their debt. If an agent defaults all of her debt will be discharged but she cannot borrow from the credit market anymore. After bankruptcy, agent i can only consume $\theta(i)$ fraction of her income for the rest of her life. Note that losing $(1 - \theta(i))$ fraction of income, is supposed to capture all the costs associated with the bankruptcy that are recognized in the literature, regardless of their source and nature. Finally, once agent i with debt D realizes her permanent income $y^P(i)$, she chooses between filing for bankruptcy and consuming $(\theta(i) \cdot y^P(i))$, or committing to repay her debt and hence consuming $(y^P(i) - r \cdot D)$ for the rest of her life. Note that I assume once uncertainty about agents' permanent income is resolved they can commit to repay their debt using the risk-free rate, r.

To summarize, at time zero competitive creditors receive signals about agents' risk types, and offer credit contracts that consist of a fixed interest rate and signal specific credit limits.[28] Then, agents use their credit lines until realization of their permanent incomes. At any time, agents can default on their debt, which will cause them to lose a type-dependent fraction of their income forever.

3.2 Agent's Problem

After realization of types, θ, and choosing the exclusive credit contract, denoted by a pair of credit limit and interest rate, (L, ρ), agent decides on how much to borrow/repay, and whether to file for bankruptcy or not. In particular, agent chooses $b(t)$, the amount of borrowing/repayment on her credit line if she does not realize her permanent income by time t. Also, she decides whether to default on her debt at time t, if she does not realiz y^P by that time. Since agents can only default once, we denote the time of filing for bankruptcy by T^*. That is, if agent's permanent income is not realized by time T^*, she defaults at that time. Obviously agent can choose to not default on her debt before realization of her permanent income, in which case we have $T^* = \infty$.

The sequential problem for a type θ agent with the initial income y^I, who is offered contract (L, ρ), is given by:

$$V(\theta; L) \quad = \quad \max_{b(t), T^*} \left\{ \begin{array}{c} \int_0^{T^*} e^{-(\delta+r)t} \left[u(y^I + b(t)) + \delta W(D(t); \theta) \right] dt \\ + \int_{T^*}^{\infty} e^{-(\delta+r)t} \left[u(\theta y^I) + \delta W^D(\theta) \right] dt \end{array} \right\} \tag{1}$$

where the debt level at time $t < T^* \leq \infty$, denoted by $D(t)$, must satisfy the credit limit constraint:

$$D(t) = \int_0^t e^{\rho(t-\tau)} b(\tau) d\tau \quad \leq \quad L. \tag{2}$$

$W(D; \theta)$ is the expected value of switching to permanent income for a type θ agent with D units of debt. $W^D(\theta)$ is the expected value of switching to permanent income for a type θ agent who has defaulted before realization of her permanent income. In particular,

$$W^D(\theta) = \frac{1}{r} \int u(\theta y^P) dF(y^P). \tag{3}$$

When a type θ agent with D units of debt realizes her publicly observable permanent income y^P, since $r \geq r$, she has no incentive to borrow except when she intends to default on her debt. Hence creditors will not allow a borrower to use the remaining part of her credit limit after switching to her permanent income. That is, upon realization of her permanent income agent has to decide either to pay back her debt or default. If agent defaults, the present value of consuming θ fraction of her income stream is:

$$\frac{1}{r} u(\theta y^P).$$

If agent decides to pay back her debt, since there is no uncertainty about her future income, she will be charged the risk-free interest rate, r, and

the present value of her consumption stream is given by:

$$\frac{1}{r} u(y^P - rD).$$

[28] Note that the contracting phase does not involve any dynamic decision making.

Therefore, if $y^P - rD \geq \theta y^P$, agent will choose to consolidate her debt at the interest rate r and repay it back. Otherwise, she will default on her debt and consume $(\theta \cdot y^P)$ for the rest of her life. That is:

$$W(D;\theta) = \frac{1}{r}\left[\int_0^{\frac{rD}{1-\theta}} u(\theta \cdot y^P)dF(y^P) + \int_{\frac{rD}{1-\theta}}^{\infty} u(y^P - rD)dF(y^P)\right]. \tag{4}$$

Overall, agents have three decisions to make: (i) whether to default or pay back their debt, after realization of permanent income, (ii) whether to default or not, before realization of permanent income, that is to set $T^* < \infty$ or $T^* = \infty$, and (iii) the sequence of borrowing/payment, $b(t)$.

If agent decides to default before realizing her permanent income, she does not do so before using all of her available credit limit, since she can continue to borrow and default later.[29] Let T denotes the time when the debt level, D, reaches the credit limit L. If agent defaults before realizing her permanent income, that is $T^* < \infty$, then $T = T^*$. Later we will show, under certain conditions, T is finite. That is, even if $T^* = \infty$ and default does not occur before realization of permanent income, debt level will reach credit limit in finite time, if realization of permanent income takes long enough.

Let's denote agent's borrowing/payment at the time of reaching credit limit by $b^* = b(T)$. If agent does not default at the limit, she has to pay the interest charge of debt to satisfy the credit limit. That is, $b^* = -\rho L$. Therefore, if the agent does not default after reaching her credit limit, the continuation value equals:

$$\frac{1}{\delta + r}[u(y^I - \rho L) + \delta W(L;\theta)].$$

But if she defaults, the continuation value will be

$$\frac{1}{\delta + r}[u(\theta \cdot y^I) + \delta W^D(\theta)].$$

So the agent defaults at the credit limit only if:

$$[u(\theta \cdot y^I) + \delta W^D(\theta)] \quad > \quad [u(y^I - \rho L) + \delta W(L;\theta)]. \tag{5}$$

Now if (5) holds and the agent defaults at the limit, then $b^* = b(T) = b(T^*) \geq -\rho L$. In general:

Lemma 1 *Borrowing/payment at the credit limit, $b^* = b(T) \geq -\rho L$, satisfies:*

$$u'(y^I + b^*) \quad \leq \quad \frac{[u(y^I + b^*) + \delta W(L;\theta)] - [u(\theta y^I) + \delta W^D(\theta)]}{\rho L + b^*}, \tag{6}$$

with equality if $b^* > -\rho L$. Moreover, if (5) holds then (6) uniquely determines b^*.

Proof. See the Appendix. ■

The left hand side of (6) is the marginal utility of consumption just before reaching the credit limit. The right hand side is the marginal cost of reaching the default time at a faster pace, due to the increase in consumption. More specifically, the numerator of the right hand side is the difference between the streams of utility before and after default. The denominator is the rate of debt increase or equivalently the rate of approaching the credit limit and hence default.

[29] Notice that the option of bankruptcy is still available even if permanent income is realized.

Given agent's borrowing/payment and default decisions at credit limit, let's study borrowing decision before reaching the credit limit. The Hamiltonian for agent's problem (1) before reaching the limit is given by:

$$\mathcal{L} = e^{-(\delta+r)t}[u(y^I + b) + \delta W(D;\theta)] + \lambda[\rho D + b], \tag{7}$$

and the optimal solution must satisfy:

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \rho D + b = \dot{D} \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial D} = e^{-(\delta+r)t}\delta W(D;\theta) + \rho\lambda = -\dot{\lambda} \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial b} = e^{-(\delta+r)t}u'(y^I + b) + \lambda = 0. \tag{10}$$

We have the solution for $b(T)$ from (6), which implies the $\lambda(T)$ from (10). Now solving for λ backward from (9) and then substituting it in (10) for $t < T$ we have:

$$\begin{aligned} u'(y^I + b(t)) &= e^{-(\delta+r-\rho)(T-t)}u'(y^I + b(T)) \\ &\quad - \int_t^T \delta e^{-(\delta+r-\rho)(\tau-t)} W_D(D(\tau);\theta)d\tau. \end{aligned} \tag{11}$$

The left hand side of (11) is the marginal utility from increasing consumption and hence debt at time t. The right hand side shows the two marginal costs of increasing debt. The first expression is the marginal cost of getting closer to the credit limit, and hence becoming credit constrained. The second expression is the marginal cost of the extra debt if agent realizes permanent income before reaching the credit limit.

To clarify these two marginal costs, note that If agent increases borrowing for a small period of time but otherwise leaves her consumption stream unchanged, then she will reach her credit limit sooner. As a result, agent's consumption has to decline for a period of time just before the time she would be reaching her credit limit without altering her current consumption stream. On the other hand, if agent realizes her permanent income before reaching her credit limit, she will be carrying more debt, which means lower expected value of switching to her permanent income.[30]

We can also consider borrowing/payment, b, as a function of debt, D, and credit limit L. That is $b(t) = b(D(t);L)$. Then by taking the derivative of (10) with respect to time and then substituting for $\dot{\lambda}$ from (9) we have:

$$\frac{db(D;(L,\rho))}{dD} = \frac{\dot{b}}{\dot{D}} = \frac{(\delta + r - \rho)u'(y^I + b) + \delta W_D(D;\theta)}{u''(y^I + b)(\rho D + b)}. \tag{12}$$

Using $b(L;L) = b^*$ from (6) as the boundary condition, we can solve for $b(D;L)$ for $D < L$, using the differential equation (12).[31] Moreover, we can use this approach to state some properties of the borrowing/payment function.

Lemma 2 *$b(D;L)$ is continuous in debt level, D, and credit limit, L.*

Proof. By construction $b(L;L) = b^*$ is continuous in L , from (6). The continuity of $b(D;L)$ for $D < L$ follows from continuity of the solution of the differential equation (12). ■

Lemma 3 *For $L_1 < L_2$, if in the solution for (1) it is optimal to increase debt up to credit limit, then:*

$$b(L_1;L_1) \quad < \quad b(L_1;L_2).$$

[30]Notice that $W_D(D(\tau);\theta) \leq 0$.

[31]I use this method to compute the borrowing/payment function in the quantitative examples of the next section.

Proof. See the Appendix. ■

Theorem 4 *If in the solution for (1) it is optimal to increase debt up to credit limit, then $b(D; L)$ is strictly increasing in L.*

Proof. Suppose not. Then $\exists D^* > 0$ and $L_1 < L_2$, such that,

$$b(D^*; L_1) \geq b(D^*; L_2).$$

Since $b(L_1; L_1) < b(L_1; L_2)$ then by continuity $\exists\ D^{**} > 0$ such that,

$$b(D^{**}; L_1) = b(D^{**}; L_2).$$

But in that case, (12) implies $\forall D \geq D^{**}$

$$b(D; L_1) = b(D; L_2),$$

which contradicts $b(L_1; L_1) < b(L_1; L_2)$. ■

The theorem states that when agents receive higher credit limits they will accumulate more debt. According to (11), there are two factors contributing to the curbing of their borrowing. First, as debt rises and agent gets closer to her credit limit, she becomes more borrowing constrained. Second, the marginal cost of repayment after realization of permanent income is increasing in debt level. These two effects can make borrowing a decreasing function of debt. The first factor is always in place, but the second one may not be. Specially, once the agent chooses to default on her debt, despite the increasing marginal cost of repayment, the increasing propensity to default can actually decrease the marginal cost of debt.

Figure (2) shows the borrowing streams and the resulting debt levels while agent is waiting for the realization of her permanent income, subject to two different credit limits. With the lower limit, agent does not default after reaching her credit limit. Hence, once the debt level reaches the credit limit, she has to maintain the negative "borrowing" level of $b = -\rho \cdot L$, until she realizes her permanent income. But with the higher credit limit, agent defaults once her debt reaches this higher credit limit.[32] The Figure also depicts another important response of borrowing streams to a change of credit limit. As agent approaches her credit limit, she curbs her borrowing due to the first factor described above. However, at the beginning of her borrowing period, as time passes and her debt rises, the agent might actually increase her borrowing. This happens because the second factor explained above is not in place. That is, the marginal cost of debt after realization of permanent income is actually declining, due to the rise of propensity to default.

For low enough debt levels, that agent does not default after realization of permanent income, marginal cost of debt after realization of permanent income is an increasing function of debt. But for high enough debt levels, that the probability of default after realization of permanent income is positive, as debt rises, the probability of repayment falls. The latter effect could in fact decrease the marginal cost of debt after realization of permanent income. The following lemma summarizes these effects.

Lemma 5 *If $F(\cdot)$ is concave, then $W_D(D; \theta)$ is decreasing in D for $D \in [0, \frac{(1-\theta)\underline{y}^P}{r}]$ and increasing for $D > \frac{(1-\theta)\underline{y}^P}{r}$.*

Proof. See the Appendix. ■

Notice that so far we have simply assumed agent continues to accumulate debt until reaching credit limit. But what if the revolving interest rate, ρ, is too high, or the expected rise in the permanent income is too small? Would agents ever reach their credit limits then? In particular if the solution of (12) is such that $\exists \tilde{D} < L$ where $b(\tilde{D}; L) = -\rho\tilde{D}$, then agent stops increasing her debt after reaching to the debt level $\tilde{D}$. The following lemma provides a sufficient condition for optimality of increasing debt level up to the credit limit.

[32] The figure shows that there are times when agent borrows more with the lower credit limit. This is because by that time, more debt have been accumulated under the higher credit limit. More specifically, $b(D; L_1) < b(D; L_2)$ still holds for $L_1 < L_2$, although for some t, $b_{L_1}(t) > b_{L_2}(t)$.

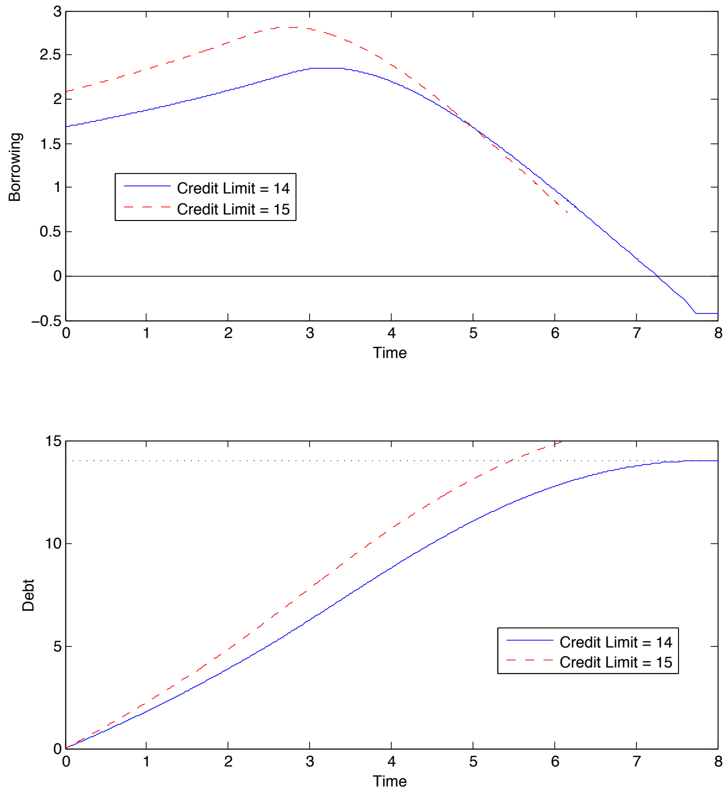

Figure 2: Borrowing and Debt Subject to Different Credit Limits

Lemma 6 *For a concave $F(\cdot)$, if $(\delta+r-\rho)u'(y^I)+\delta W_D(\frac{(1-\theta)\underline{y}^P}{r};\theta) > 0$, then for any credit limit, it is optimal for agent to increase her debt up to any credit limit.*

Proof. See the Appendix. ■

Although this condition is independent of credit limit, L, it depends on the contract's interest rate, ρ. Obviously it also depends on the initial income, y^I, the inverse of the expected length of the borrowing period, δ, the permanent income distribution, $F(\cdot)$, and the cost of default $(1-\theta)$. Despite providing a sufficient condition for borrowing to continue until debt level reaches to credit limit, the lemma is silent about whether agent would actually reach credit limit in finite time or not. The next theorem answers this question.

Theorem 7 *For concave $F(\cdot)$, if $(\delta+r-\rho)u'(y^I)+\delta W_D(\frac{(1-\theta)\underline{y}^P}{r};\theta) > 0$, then $\exists T > 0$ such that $\forall t \geq T$ we have $D(t) = L$. That is, if realization of permanent income takes long enough, then debt reaches the credit limit.*

Proof. Lemma 6 guarantees that for any debt level $D < L$, agent increases debt level above it. Suppose debt level does not reach the credit limit in finite time, that is $T = \infty$, then for a large enough t such that $D(t) \approx L$ and $b(t) \approx -\rho L$, following (11) we have:

$$
\begin{aligned}
u'(y^I + b(t)) &= -\int_t^{\infty} \delta e^{-(\delta+r-\rho)(\tau-t)} W_D(D(\tau);\theta) d\tau \\
&\approx \frac{-\delta}{\delta+r-\rho} W_D(L;\theta),
\end{aligned}
$$

which contradicts $(\delta + r - \rho)u'(y^I - \rho L) + \delta W_D(L;\theta) > 0$. ■

Next we study the effect of agents' riskiness on borrowing. In particular, we show that due to lower cost of default, riskier agents face lower marginal cost of borrowing, which in turn induces them to borrow more. First we need to study the effect of agents' riskiness on borrowing/repayment at the credit limit.

Lemma 8 *Borrowing at the limit, $b^\theta(L;L)$, is increasing in the agent's riskiness, θ, and strictly increasing if default is optimal at the limit.*

Proof. Following lemma 1, if default is not optimal at the limit, then $b^\theta(L;L) = -\rho L$. If default is optimal, then following (6), $b^\theta(L;L)$ is governed by:

$$u'(y^I + b) \cdot (\rho L + b) - u(y^I + b) + u(\theta y^I) - \delta \left\{ W(L;\theta) - W^D(\theta) \right\} \quad = \quad 0.$$

Therefore:

$$\frac{db}{d\theta} \quad = \quad -\frac{y^I u'(\theta y^I) + \delta \int_{\frac{rL}{1-\theta}}^{\infty} y^P u'(\theta y^P) dF(y^P)}{u''(y^I + b) \cdot (\rho L + b)} > 0,$$

where the inequality follows from $u''(\cdot) < 0 < u'(\cdot)$ and $b^\theta(L;L) > -\rho L$. ■

The proof relies on the observation that the benefit of delaying default until realization of permanent income is a decreasing function of agent's riskiness. That is, $\frac{d}{d\theta} \left\{ W(L;\theta) - W^D(\theta) \right\} < 0$. Next we use (12) to study the effect of the agents' riskiness on their borrowing before reaching their credit limit.

Theorem 9 *Borrowing, $b^\theta(D;L)$, is increasing in agent's riskiness, θ.*

Proof. See the appendix. ■

The proof follows because the marginal cost of debt after realization of permanent income is lower for riskier agents. A lower marginal cost, induces riskier agents to borrow more and accumulate debt at a faster pace.

Using the effect of change in credit limit on borrowing and debt, next we study the effect of credit limit on default rate. From (5), the probability of defaulting on D units of debt after realization of permanent income, is given by:

$$F\left(\frac{rD}{1-\theta}\right). \tag{13}$$

When agent is offered a larger credit limit, she will accumulated more debt. Since the time of switching to permanent income is exogenous and independent of the debt level, the probability of having more debt at the time of switching, and therefore the probability of default, is higher with a larger credit limit.

Agent might also default before realization of permanent incomes, once they reach to credit limit. This is shown in the example of Figure (2). Hence, a higher credit limit not only induces agents to accumulate more debt by the time of realization of permanent income, it might also induce agents to default before realization of permanent income, once they reach to their credit limit.

In summary, a higher credit limit induces agents to borrow more, and hence makes them more likely to default. Moreover, riskier agents, who have lower cost of default, not only are more likely to default after realization of permanent income, but also accumulate more debt before realization of permanent income.

3.3 Creditors' Problem

In the previous subsection we studied decision rules of a type θ agent with the initial income y^I and a credit contract (L,ρ). Creditors take these decision rules as given, hence, their expected profit from offering a contract (L,ρ) to a type θ agent with the initial income y^I is given by:

$$\Pi(\theta;L) \quad = \quad \int_0^{T^*} e^{-(\delta+r)t}\{-b(t)+\delta D(t)[1-F(\frac{rD(t)}{1-\theta})]\}dt,$$

where $b(t)$ and T^* are the borrowing stream and default time decisions which solve (1) for a type θ agent with the initial income y^I who is offered a credit contract (L,ρ). Moreover $D(t)$ is the implied debt amount from (2). Notice that when a type θ agent realized her permanent income while having debt level $D(t)$, the probability of repaying her debt is $[1-F(\frac{rD(t)}{1-\theta})]$ from (13).

When a creditor in the competitive credit market offers a credit contract, the only available information is the public signal $\tilde{\theta}$ from the Rating Technology. Recalling that the conditional probability of drawing type θ given the signal $\tilde{\theta}$ is given $\psi(\theta|\tilde{\theta})$, a creditor's expected profit from offering credit contract (L,ρ) to an agent with a rating signal $\tilde{\theta}$ is:

$$\Pi(\tilde{\theta};L) \quad = \quad \int \Pi(\theta;L)d\psi(\theta|\tilde{\theta}). \tag{14}$$

Recall that when agents receive their credit contracts, they have realized their private information about their true risk types, θ. However, the offered credit contracts can only vary in their credit limits, L, and they all offer the same fixed interest rate, ρ.[33] It is easy to show that all agents (weakly) prefer contracts with higher credit limits. Therefore, the equilibrium credit limit for an agent with public rating signal $\tilde{\theta}$ is given by:

$$L_{\tilde{\theta}} \quad = \quad \max\left\{L|\Pi(\tilde{\theta};L)=0\right\}. \tag{15}$$

Notice that when the signals are not fully informative and hence agents with different types are offered the same contract, then some types generate positive profit for creditors which is used to compensate for losses made on the other types. Figure (3) shows an example for how creditor's profit changes as a function of offered credit limit, L, for a fix interest rate $\rho > r$. The Figure depicts profit from two types with different default costs. As credit limit increases creditor's profit rises, since agents can borrow more but yet do not find it optimal to default. Hence for $\rho > r$ creditor's expected profit rises. For a large enough credit limit though, riskier agents accumulate enough debt to find it optimal to default, therefore creditor's profit from them declines as credit limit rises. However, safer agents still generate more profit. As credit limit increases further, eventually safer agents also accumulate enough debt to find the default option optimal. As more safe agents default, the expected profit from them falls as well and eventually becomes negative. Next section provides sufficient conditions under which the solution for the creditor's problem (15) exists.

3.4 Equilibrium Existence

In this subsection we show the existence of equilibrium, which is characterized by the solution of the creditors' problem (15) for offering credit contract (L,ρ) to an agent with the rating signal $\tilde{\theta}$.

Lemma 10 $\Pi(\theta;L)$ *is continuous in L.*

[33] In an earlier version of the paper, creditors could choose the interest rate as well. However, in order to avoid market failure due to asymmetric information, I had assumed that agents do not learn their types until after choosing their credit contracts.

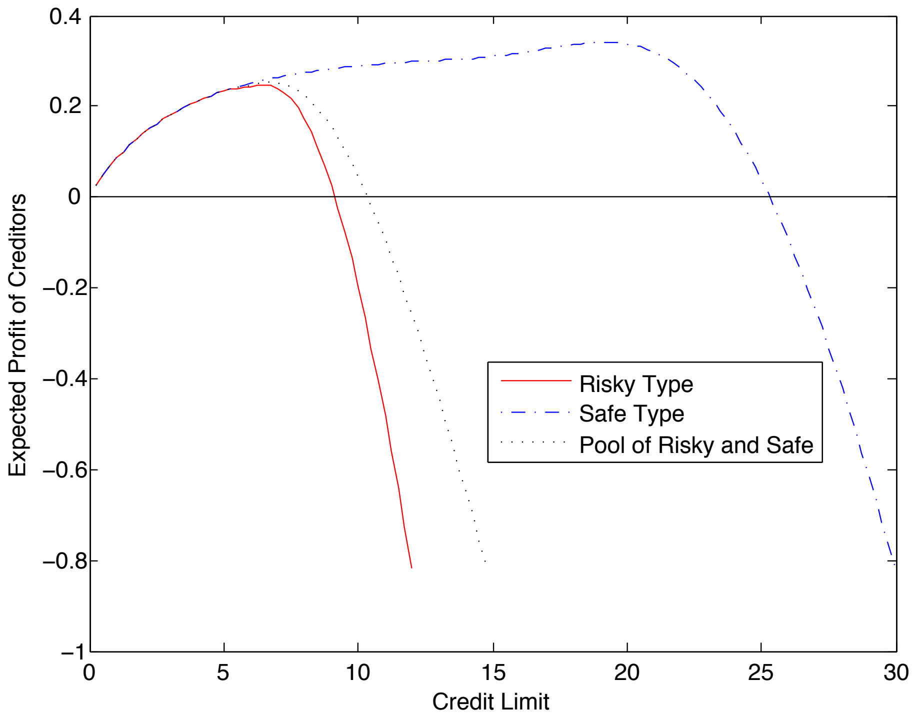

Figure 3: Creditors' Expected Profit From Extending Credit Limit to a pool of Risky and Safe Borrowers

Proof follows from the continuity of the decision rule b and lack of mass points in $F(\cdot)$.[34,35]

Lemma 11 *If the support of $\psi(\cdot|\tilde{\theta})$ is bounded away from zero, the utility function $u(\cdot)$ is unbounded from above and the expected present value of all future income for an agent with signal $\tilde{\theta}$ is bounded, then:*

$$\lim_{L\to\infty} \Pi(\tilde{\theta}; L) < 0 \ \forall \rho.$$

Proof. See the Appendix. ■

The proof shows for $\rho \geq r$, as credit limit increases, so does the expected value of the contracts for agents with signal $\tilde{\theta}$. That is, $\lim_{L\to\infty} V(\theta; L) \to \infty$. Then uses the fact that with $\Pi(\tilde{\theta}; L) \geq 0$ the expected utility $E_{\theta|\tilde{\theta}} V(\theta; L)$ is bounded from above.

Theorem 12 *If the support of $\psi(\cdot|\tilde{\theta})$ is bounded away from zero, the utility function $u(\cdot)$ is unbounded from above and the expected present value of all future income for an agent with signal $\tilde{\theta}$ is bounded, then the creditor's problem (15) has a unique solution. In particular, if $E(y^p) \gg y^I$, then this solution is positive.*

Proof. For a given interest rate $\rho > r$, agent's expected value $V(\theta; L)$ is increasing in L. This is because agent can always opt out and does not use all of her credit limit. Let's $L^*(\tilde{\theta}; \rho)$ denotes the largest L such that $\Pi(\tilde{\theta}; L) = 0$. Since $\Pi(\tilde{\theta}; 0) = 0$ and $\Pi(\tilde{\theta}; L)$ is continuous in L, the previous lemma guarantees the existence and uniqueness of $L^*(\tilde{\theta}; \rho)$, which solves (15). Now since $\rho > r$, if $E(y^p) \gg y^I$, then for a small enough $\epsilon > 0$, $E_{\theta|\tilde{\theta}} V(\theta; \epsilon) > 0$ and $\Pi(\tilde{\theta}; \epsilon) > 0$, and therefore $L^*(\tilde{\theta}, \rho) > 0$. ■

[34] Note that $\Pi(\theta; L)$ is also continuous in ρ.

[35] When the credit limit is such that it makes borrowers indifferent between default or staying at the credit limit after reaching the limit, then creditor's profit depends on the fraction of agents who default after reaching the limit. Therefore, $\Pi(\theta; L)$ is a correspondence of L, however, a continuous one.

4 Discussion and Quantitative Examples

In the previous section I introduced a stylized model of unsecured borrowing and default. In this section, I use a quantitative example of this model to show how the change in the allocation of credit, which is the result of tailoring credit contracts to borrowers' true risk types, not only increases the average and dispersion of credit card limit and debt distributions, but it can also increase the frequency of default as well as the amount of discharged debt. In the setup of this model, higher correlation between the Rating Technology's signals about borrowers' risk types and their actual risk types allows creditors to sort borrowers according to their risk types and offer differentiated contracts.

Since in the stylized model of this paper, the only available information, at the time when creditors offer credit contracts to borrowers, is assumed to be the exogenous signals of the Rating Technology, credit contracts are only conditional on these signals. That is, agents with identical signals will receive contracts with identical terms. Therefore, in equilibrium creditors commit to provide the same credit limit to borrowers with the identical signals.[36] However, agents with different risk types but the same contract may choose different borrowing patterns.[37] Depending on the informativeness of the Rating Technology's signals about borrowers' true risk types, borrowers with different default costs might be pooled together and offered identical credit limits, or might be separated and receive differentiated credit limits. Therefore, allocation of credit limits, equilibrium distribution of unsecured debt, default frequency and debt discharge would change as the correlation between the Rating Technology's signals about borrowers' risk types and their actual risk types changes.

In the previous section we showed that, since agents with high default costs (safe borrowers) are more likely to pay back their debt, their marginal cost of borrowing is higher compared to agents with low default costs (risky borrowers). Hence facing the same credit limits safer agents accumulate less debt, and the responsiveness of their debt level to an increase in credit limit is also smaller. That is, safer borrowers have lower propensity to utilize their credit limits. If the credit limit is increased above the equilibrium level, then debt level of low default cost agents, which will make negative expected profit, will increase more than debt level of high default cost agents, which will make positive expected profit for creditors. Therefore with uninformative Rating signals, and the resulting pooling credit contracts, supply of unsecured credit will be relatively tight.

If the Rating Technology's signals are informative about borrowers' true types, however, borrowers will be separated and credit contracts will be offered according to their riskiness. While in the pooling case there is cross-subsidization across types, in the separating case the expected profit from high default cost borrowers cannot be used to subsidize the expected losses made on low default cost borrowers. Hence, once separated from riskier borrowers, the equilibrium supply of credit will increase significantly for safer borrowers. Apparently, higher credit limits will allow safer borrowers to accumulate more debt and default more often and on larger debt levels compared with the pooling case. The opposite will happen for riskier borrowers.

If we switch from the pooling case to the separating case, the analytical results of the previous sections are useful to explain how credit supply, unsecured debt, default rate and debt discharge rate for each risk type would change. The net results of switching from the pooling to the separating case, however, are ambiguous. In particular it is not clear whether the rise of safer borrowers' credit limit, unsecured debt, default frequency and discharge rate, would offset the corresponding declines for riskier borrowers. I use a quantitatively relevant numerical example of the model, to study the overall implications of the separation of different risk types.

In the following subsections, I use quantitative examples of the stylized model to study the relevance of informativeness of credit ratings about borrowers' riskiness for the trends of the US unsecured consumer credit market, which were reported in the Data and Motivation section. In particular, the first quantitative example uses

[36] As it was discussed in the previous section, since credit contracts are assumed to have a fixed interest rate and all agents (weakly) prefer contracts with higher limits, within the pool of borrowers who have the same signal, creditors cannot separate different types by offering them different contracts.

[37] Moreover, ex-post realized heterogeneity in the arrival time and level of permanent income, causes agents with identical credit contracts and even identical risk types to accumulate different amounts of debt and make different default decisions.

a version of the model with only two risk types under two extreme cases of completely uninformative and fully-informative signals of the Rating Technology, and sets the parameter values to match the relevant data moments. The second set of examples, use the stylized model to study the relevance of the stigma explanation of the rise of bankruptcy (i.e. fall of the average default cost of borrowers) by keeping the informativeness of the Rating signals constant and changing either the riskiness of types or the composition of risk types.[38]

4.1 Informativeness of Credit Ratings

To provide a quantitative example for the effect of the increase in the informativeness of credit ratings on unsecured consumer credit market, I use the stylized model with two risk types. I compare equilibrium credit supply, unsecured debt, default rate and debt discharge under two extreme cases: (1) fully uninformative signals from the Rating Technology, which forces creditors to pool borrowers with high and low default costs and offer them the same credit limit, (2) fully informative Rating signals, which allows creditors to separate safer and riskier borrowers and offer them different credit limits. I calibrated some of the stylized model's parameters to commonly used values and set some of them to match the targeted moments of interests from the data. To shed some light on the importance of the differentiation of borrowers according to their riskiness, I then compare some moments of interest, which are not targeted, with the corresponding moments generated by the calibrated model. In the rest of this subsection, first I describe the moments of interest from data, then explain the calibration/estimation of the parameters and finally report the results.

Moments of interest are chosen to summarize the trends of unsecured consumer credit market, reported in the Data and Motivation section. Specifically, changes in the average and the standard deviation of the ratio of credit card limit to income, the average and the standard deviation of the ratio of credit card debt to income, default rate, and the average discharged credit card debt for defaulters, are useful to capture the hitherto mentioned trends.

Values of the moments of interest from SCF 1992 and 1998 are used to set the stylized model's parameters and study the quantitative relevance of the model. These two years are chosen for three reasons. First, as it was mentioned in the Data and Motivation section, I rely on Sullivan et al. [28] for the data of discharged credit card debt of defaulters. They report these values using a sample of bankruptcy filers in 1991 and 1997. As noted before, since the SCF data collection and report take about one year, the data on discharged credit card debt corresponds to 1992 and 1998 data from the SCF. Second, the stylized model focuses on the change of the unsecured credit limit, while assuming a constant interest rate. As it was reported in the Data and Motivation section, this focus is more consistent with the changes of the consumer credit card contracts during 1990s. Third, bankruptcy filings rose significantly during this period, despite being a period of robust economic growth. Hence, the usual causes of the rise of bankruptcy, such as high unemployment rate, are unlikely to have played an important role during this period.

Instead of using the average and the dispersion of credit card limits and debts for the entire sample of households in the SCF, I use the moments of interest for a particular set of households, who correspond to the stylized model's borrowers. Most households do not borrow on their credit cards (see the bottom row of Table (1)). Therefore I restrict my attention to revolvers, who have positive balance on their credit card accounts after their last payments. The model predicts that households increase their credit card debt until realization of their permanent income, then either default on their debt or pay it back. The SCF is not a panel dataset and cannot be used for direct study of dynamics of household credit card debt accumulation and de-accumulation. The SCF, however, reports households' answer to the following question:

> Thinking only about Visa, Mastercard, Discover, Optima and store cards, do you almost always, sometimes, or hardly ever pay off the total balance owed on the account each month?

[38] In an earlier version, the stylized model was also used to show the rise of unsecured credit supply along the extensive margin, i.e. the increase in the fraction of households who had credit cards. This was done by having a third risk type with no default cost, that is $\theta = 1$, in addition to the high and low default cost types. Borrowers who were pooled by this third type would receive zero credit limit in the equilibrium with uninformative signals. Then, switching to informative signals allows for the rise of credit supply along the extensive margin.

Roughly speaking, half of the revolvers answer "they hardly ever pay off the total balance." This group of households, who I call *A-revolvers*, are accumulating credit card debt. I use the moments of interest for A-revolvers, assuming them to be the ones who correspond to the debt accumulating borrowers of the stylized model. Moreover, I assume that all bankruptcy filers are A-revolvers, that is households do not pay off their total credit card balance before filing for bankruptcy.

Table (3) reports the moments of interest. Fraction of A-revolvers from all households did not change significantly from 1992 to 1998 in the SCF. The average ratios of credit card limit and credit card debt to annual income for A-revolvers, however, rose significantly.[39] The increase in the standard deviation of the distribution of credit limit to income ratio for A-revolvers, was even more pronounced. Therefore, the coefficient of variation for this distribution, defined as standard deviation divided by mean, increased from 1992 to 1998. Assuming all bankruptcy filers are also A-revolvers, the reported ratio of bankruptcy filers to A-revolvers would account for the change in the bankruptcy filing of A-revolvers from 1992 to 1992.[40] Finally, the last row reports the increase in the average credit card debt discharged upon bankruptcy.

		1992	1998
Fraction of A-revolvers* from Total Sample**		16.05%	17.31%
Credit Card Limit/Income** for A-revolvers*	Mean	25.97 %	42.48%
	Std.	41.38 %	77.50%
	Coef. of Variation	159.34%	182.44%
Cred Card Debt/Income** for A-revolvers*	Mean	10.66%	16.95%
Ratio of Bankruptcy Filers to A-revolvers		5.86%	7.39%
Discharged Credit Card Debt/Income for Bankruptcy Filers***	Mean	53.10%	76.70%

* *A-revolvers* are households who are accumulating credit card debt and "hardly ever pay off" their total credit card balance.
** Data from the SCF 1992 and 1998.
*** Data from Sullivan et al. [28]

Table 3: Moments of Interest

I make two functional form assumptions about the the utility function and the distribution of the permanent income of the stylized model for this quantitative exercise. In particular, I use log utility function, and assume the permanent income, y^P, is drawn from a truncated exponential distribution:

$$F(y^P) = 1 - e^{-\eta(y^P - \underline{y^P})}.$$

There are eight parameters to be calibrated/estimated for the quantitative example of the stylized model with two types using the assumed functional forms for the utility function and permanent income distribution. In addition to the time discount factor, r, the credit card interest rate, ρ, the arrival rate parameter for the Poisson process of the realization of the permanent income, δ, the two parameters for the two types' riskiness, $0 < \theta_L < \theta_h < 1$, and the distribution parameter for types, $\mu(\theta_h)$ (which would also determine $\mu(\theta_l) = 1 - \mu(\theta_h)$), two parameters of the permanent income distribution, η and $\underline{y^P}$, should be set for the quantitative exercise.[41] Taking

[39] These figures are reported after dropping less than half a percentage of the subsample who report zero or negative income.

[40] Since the fraction of A-revolvers in the population increased, looking at the ratio of defaulters to A-revolvers underestimates the rise of bankruptcy filings. However, the results of this quantitative example are not sensitive to using the ratio of defaulters to the average number of A-revolvers across two years.

[41] Since I use log utility and the targeted moments are the ratios relative to income, the initial income, y^I, could take any value without loss of generality.

the time unit to be 3 months, I calibrate $r = \ln(1.01)$ to be consistent with the 4% average annual growth rate. I set $\rho = \ln(1.03)$ to be consistent with a 12% interest charge on credit lines.[42] The other six parameters are estimated to match the moments of interest.

Accounting for the spread of credit card limits by the switch from the pooling case to the separating case of the stylized model with two risk types, is worth noting. The estimated parameters are set to match the moments generated by the model in the pooling case of the stylized model with two risk types to the data moments from 1992, and the moments generated by the model in the separating case to the data moments from 1998. Clearly creditors had some information on borrowing households' risk types in 1992, and did not have full information in 1998. In particular, households were not offered a single credit card contract in 1992, as the pooling case of the stylized model suggests. Instead, a distribution of credit card limits were supplied by creditors in 1992. However, the spread of the distribution of credit limits increased from 1992 to 1998.[43] In the pooling case of the stylized model with two types, both types are offered a single credit limit, L_P, while in the separating case two credit limits, $L_S^{\theta_l}$ and $L_S^{\theta_h}$, are offered depending on the agent's type $\theta \in \{\theta_h, \theta_l\}$. The 1992 data provides a distribution of credit limits. To study the increase of the spread of the credit limit distribution from 1992 to 1998, suppose any credit limit L from the distribution of credit limits in 1992 was replaced by two credit limits, $(L_S^{\theta_l}/L_P) \cdot L$ and $(L_S^{\theta_h}/L_P) \cdot L$, with weights $\mu(\theta_l)$ and $\mu(\theta_h)$, in the distribution of credit limits for 1998. In this case, the following would hold for the coefficients of variation for these three distributions:

$$CV(L_{1998})^2 + 1 = (CV(L_{1992})^2 + 1)(CV(L_S)^2 + 1) \quad (16)$$

where L_{1992} and L_{1998} are the distribution of credit limits in 1992 and 1998, L_S is the distribution of credit limits in the separating case, and $CV(\cdot) = std.(\cdot)/mean(\cdot)$, is the coefficient of variation of these distributions.

Using the Simulated Method of Moments, I estimate the three parameters related to the permanent income, plus the three parameters related to the default cost and its distribution, to match six targeted moments. Four of the moments are the average ratios of credit limits to income and credit card debt to income for 1992 and 1998. The fifth targeted moment is the coefficient of variation of the credit limits in the separating case implied from 1992 and 1998 data using (16). This moment captures the increase in the spread of the credit limit distribution from 1992 to 1998. The last targeted moment is the default rate of the A-revolvers in 1998. Notice that the exercise does not target the default rate of the A-revolvers in 1992.[44]

The computational process of the parameter estimation is straightforward. To generate the targeted moments by the model, I compute the borrowing/payment paths backward, using the 4th order Runge-Kutta method for the ordinary differential equation (12). Given the simplicity of the model, the targeted moments are computed pretty fast for each set of parameters, despite having three zero profit conditions. An identity weight matrix is then used to minimize the percentage deviation of the six moments generated by the model, from the six targeted data moments. This provides us consistent estimates for the parameters. The estimated values for the parameters are reported in Table (4) and the moments generated from the model are reported in Table (5) (the values from data are inside the parentheses.)

The six parameters are estimated to match six moments from the data. The default rate of 1992, however, is not amongst the targeted moments and hence can be used to check for consistency of the model. According to the

[42] Notice that the average credit card interest rate in this period is around $14.50 - 15.00\%$. However, since the model generates the equilibrium credit limits by equalizing the creditors' profit to zero, we should consider the creditors' operational costs. I approximate this cost to be 3% from the difference between the Bank Prime Loan Rate and the Federal Fund Rate.

[43] In this quantitative exercise I account for the increase of the spread of the distribution of credit limits as a result of a more informative Rating Technology. If the Rating Technology sent signals about certain risk characteristics of borrowers in 1992, by 1998 the signals still contained the information about those characteristics. However, as the Rating Technology became more informative, it could provide some additional information about the risk characteristics of borrowers. We can interpret the switch from the pooling case (uninformative signal) to the separating case (fully informative signal) as the provision of additional information by the Rating Technology on borrowers' characteristics.

[44] In an earlier version of the paper, an alternative exercise of targeting the default rate of the A-revolvers in 1992, but not 1998, was also reported. The results are similar and available upon request.

		Parameters	Estimated Values
Permanent Income*	Arrival Rate	δ	0.1326
	Lower Bound**	$\underline{y}^P$	0.51576
	Std.**	$1/\eta$	1.2579
Default Cost***	High Risk Type	$1-\theta_h$	0.011
	Low Risk Type	$1-\theta_l$	0.026
	Fraction of High Risk	$\mu(\theta_h)$	0.5293

* Permanent income's arrival is assumed to be a Poisson process with parameter δ, and is drawn from a truncated exponential distribution with c.d.f. $F(y^P) = 1 - e^{-\eta(y^P - \underline{y}^P)}$.
** Values are normalized for the initial income, $y^I = 1$.
*** Type θ borrowers are assumed to keep θ fraction of their income after default.

Table 4: Estimated Parameters

data, the default rate by A-revolvers rose from 5.86% in 1992 to 7.39% in 1998. Switching from the separating case to the pooling case, the model generates a decrease of default rate from 7.39% to 6.61%, which is 51% of the change in bankruptcy filings observed in the data.[45] Given the simple structure of the model and the fact that the exercise did not target the increase in the number of bankruptcy filings, the results are appealing.

The ratio of credit card debt to income generated by the model and reported from data are also provided in Table (5) under the Additional Moments section. The data moments are from Sullivan et al. [28]. The average credit card debt of a defaulter, generated by the model in the pooling case is close to the reported data. However, the debt to income ratio of defaulters in the separating case generated by the model is much higher than the reported data. The model predicts that when switching from the pooling case to the separating case, defaulters' debt to income ratio almost doubles, while Sullivan et al. [28] reports only 44% increase in this ratio. It should be noted that while these two moments were not targeted in the estimation process, the stylized model's ability to come close to matching them is remarkable, particularly in the pooling case. Similarly, the average propensity to consume out of liquidity (dDebt$/d$Limit) implied by the stylized's model's switch from the pooling to the separating case, 38%, is larger than the 10-14$\%$ range reported by Gross and Souleles [16] for the marginal propensity to consume (MPC) out of liquidity. Nevertheless, it still demonstrates the relevance of the stylized model.

Comparing the allocation of credit limits across the two types under the pooling and the separating cases, and the resulting effects on each type's debt, default rate and discharge rate, highlights the stylized model's main mechanism. Table (5) reports credit limit, debt, default rate and average debt of a defaulter for both types in both cases generated by the model. In the pooling case, agents with the high default cost (i.e. the safe type) borrow very little and do not default at all. Due to their small debt, they do not generate enough profit for the creditors to compensate for the losses made on the riskier agents. That is, under the pooling case, the cross-subsidization across different risk types is not significant. Hence, when types are separated, borrowing and default by the riskier agents do not change significantly. In particular, the average utilization rate of credit limit (Debt/Limit) shows a very small change and remains around 55%. However, in the separating case, the safer types are offered a much larger credit limit. Therefore, they can accumulate larger debts which results in more defaults by the "safe" agents. Optimality of default, encourages the safe types to utilize their credit limits. Hence, by switching from the pooling to the separating case, safe borrowers' utilization of credit limits rises from 22% to 34%. The model generates the rise in the average credit card debt discharge of bankruptcy filers, because the safe type agents only default when their debt level is really high. Therefore, the rise of default in the separating case, goes hand in hand with the rise of the debt discharge rate.

[45] In an alternative exercise of targeting default rate of 1992 but not 1998, going from the pooling case to the separating case, the model generates an increase of default rate from 5.86% to 6.42%, which is more than 37% of the increase in bankruptcy filings in the data.

			Pooling	(Data 1992)*	Separating	(Data 1998)*
Targeted Moments		Av. (Cred Lim/Income)	25.96%	(25.97 %)	42.51%	(42.48%)
		Av. (Cred Card Debt/Income)	10.66%	(10.66%)	16.95%	(16.95%)
		Default Rate			7.39%	(7.39%)
		Relative Rise in Coefficient of Variation of Credit Lim			47.22%	(47.25%)
Additional Moments		Default Rate	6.61%	(5.86%)		
		Av. (Cred Card Debt/Income) for Defaulters	53.51%	(53.10%)	102.04%	(76.70%)
	Type θ_h (Low Cost of Default)	Cred Lim/Income	25.96%		23.58%	
		Av. (Cred Card Debt/Income)	15.15%		12.74%	
		Default Rate	12.48%		8.77%	
		Av. (Cred Card Debt/Income) for Defaulters	53.51%		53.76%	
	Type θ_l (High Cost of Default)	Cred Lim/Income	25.96%		63.80%	
		Av. (Cred Card Debt/Income)	5.61%		21.68%	
		Default Rate	0.00%		5.85%	
		Av.(Cred Card Debt/Income) for Defaulters	$---$		159.70%	

* Data moment are for A-revolvers and are computed using credit card limit, debt and income from the SCF. Credit card debt to income ratio of defaulters are from Sullivan et al. [28].

Table 5: Summary of Generated Moments and Corresponding Data Moments

Finally, the estimated values for the six parameters (Table (4)) are reasonable. The estimated permanent income parameters, imply that on average households' permanent income is about 75% larger than their initial income, and it takes about two years before switching to permanent income. Generally speaking, these characteristics seem to be consistent with the income shocks that the US households experience during periods of financial distress such as unemployment spells.[46] The risky type is estimated to lose about one percentage of her income after default and the cost of default is about two and half times larger for the safe type. About half of the population is estimated to be of each type. That is, with only two types, we do not need a small fraction of agents with a significantly different default costs from the rest of the population to explain the observed trends in data as a result of separation of risk types.

4.2 Stigma

The next quantitative examples provide a comparison of the informational explanation of the rise of bankruptcy with the stigma literature. That literature holds the view that a decrease in the stigma, hence the costs associated with the bankruptcy, is a major driver of the increase in bankruptcy filings. I show that by focusing on the agents' costs alone, one may be able to generate the level of bankruptcy filings observed in data, however, several other interesting moments will behave counterfactually.

If all risk types had become riskier from 1992 to 1998, that is their cost of default had decreased, then the equilibrium credit limits would have decreased for all types. Hence, the stigma mechanism cannot explain the rise of unsecured credit supply. In particular, keeping the separating informational structure unchanged and using the estimated values for the permanent income process and the fraction of each risk type, I increase the default cost of both types at the same rate in order to reduce bankruptcy filings from 7.39% down to 6.61%. This would mimic

[46] Although the income process is a simplified version of the *Heterogeneous Income Profile*, studied by Guvenen [18], the parameters are not calibrated to capture the long-term income process. Instead, they are determined by borrowers' medium-term demand for unsecured debt.

the change in default rate generated by switching from the separating case to the pooling case, discussed in the previous subsection. This is done by increasing $(1-\theta_h) = 0.011$ to 0.011×1.255 and increasing $(1-\theta_l) = 0.026$ to 0.026×1.255. Having higher default costs, increases the average credit limit to 55.98% of income, raises the average debt level to 19.14% of income and increases the average debt to income ratio of defaulters from 102.04% to 140.95%. All of these changes contradict even the direction of the trends observed in data by going from 1998 back to 1992.

But what if there was a change in the composition of risk types and a higher fraction of borrowers became risky from 1992 to 1998? Using the estimated values for the parameters of the permanent income process and default costs, while keeping the separating informational structure, I reduce the fraction of high risk agents, to generate a decrease in bankruptcy filings similar to the decrease in bankruptcy filings generated by the switch from the separating to the pooling case in the the previous subsection's exercise. This is attained by decreasing $\mu(\theta_h) = 52.93\%$ to 26.0%, which would reduce bankruptcy filings from 7.39% down to 6.61%. Note that I keep the estimated values of the other parameters unchanged. The lower fraction of risky agents, increases the average credit limit to 53.34% of income, raises the average debt level to 19.35% of income and increases the average debt to income ratio of defaulters from 102.04% to 131.38%. All of these changes contradict the trends observed in data from 1992 to 1998.[47]

5 Last Thoughts

This paper was the first attempt in providing an informational explanation for the rise of household bankruptcy. I suggest a mechanism to simultaneously account for the increase in the unsecured credit supply and the corresponding increase in the average credit card debt. The rise of credit supply follows from separating borrowers with different degrees of riskiness. The rise of bankruptcy is explained by the increase in the supply of unsecured credit to borrowers with high default costs, which allows them to accumulate more credit card debt. While these borrowers are less likely to default on small debts, they are more likely to default when they accumulate large amounts of unsecured debt. Hence, the debt discharge rate rises as well as bankruptcy filings.

Using the simulated method of moments I provide a simple quantitative example of matching the average credit limit and debt levels, as well as the increase in the spread of the credit limit distribution. The model can generate an increase in the default rate which is about half of the increase in the default rate observed in the data. Given the simple stylized structure of the model, this is quite appealing. The result shows the importance of the heterogeneity of borrowers' default costs over other sources of heterogeneities like variation in income processes.

An important prediction of the stylized model of this paper is the increase in the supply of credit for safer borrowers, which results in higher indebtedness of these borrowers and therefore more default by them on larger unsecured debt amounts. Empirical evidence from Gross and Souleles [15] supports that creditors extended larger unsecured credit lines to less riskier borrowers, and hence safer borrowers accumulated more debt. Sullivan et al. [28] report the rise of unsecured debt discharged after bankruptcy. However, rigorous confirmation of this prediction requires more detailed models and careful empirical studies for future research.

Going forward, it is pertinent for this literature to carefully identify and address key features of the data. Recently four papers have addressed the increase in household bankruptcy from informational prospectives in environments with ex-ante heterogeneous borrowers. Despite their elegant modeling and rigorous quantitative setups, they still miss some key features highlighted by this paper's stylized model, requiring attention in the future studies. In particular, Sanchez [27] models the improvement in information about borrowers' heterogeneous

[47] To generate a decrease in the default rate similar to 1992 level, 5.86%, the fraction of risky agents should be reduced to 0.05%. This change would increase the average credit limit to 63.60% of income, raise the average debt level to 21.63% of income and increase the average debt to income ratio of defaulters from to 159.16%.

In a similar exercise, using the parameter values estimated to match 1992 default rate under pooling case, I keep the pooling structure but increase the fraction of high risk types to generate an increase of bankruptcy filings equal to the separating case. Again higher fraction of risky agents, decreases the equilibrium credit limit, increases the average credit card debt, and the debt to income ratio of defaulters slightly decreases. All of these changes contradict the significant increasing trends observed in the data.

income processes, by a decline in the cost of *monitoring*. Although the model generates an increase in the supply of unsecured credit and debt, it neither generates the observed dispersion of these two variables in the US data, nor the the rise of debt discharge rate. This happens because the distinction between safer and riskier borrowers is only in their income processes, hence, even after separation from riskier borrowers, safer borrowers would not borrow more than riskier ones and would not default on larger debts. The model developed by Drozd and Nosal [12] for the decline of credit solicitation cost in a search environment with varying borrower characteristics, is an informational explanation at heart. That is, if credit contracts are updated more frequently, they will reflect borrowers' characteristics more accurately. Although they also assume credit card contracts to be combination of credit limits and interest rates,[48] borrowers default if and only if they reach their credit limits. This is a counterfactual, since a significant fraction of borrowers are at their limits but do not default and a lot of borrowers default before reaching their credit limits.[49] Moreover, they also fail to generate enough dispersion in credit card limits and cannot account for the rise of debt discharge rate. Livshits et al. [22] provide a stylized model to show how more informed signals about borrowers' income process allow creditors to offer more and better specified credit contracts, when offering contract is costly. As noted before, since the main source of heterogeneity is in borrowers' income processes, the informational improvement in their model fails to generate increase in the debt discharge rate.

Athreya et al. [2] is the closest one to this paper. They provide a rigorous quantitative accounting for contribution of different sources of borrowers' heterogeneity to the rise of bankruptcy. Their paper, confirms the importance of heterogeneity in default costs for explaining the observed trends in the US data over the other sources of borrower heterogeneities. Moreover, unlike the other two quantitative models they use a life cycle model and show the importance of *unfulfilled expectation of permanent income* for most of defaulters, a feature captured by my model's simple income process. However, they model credit contracts as one period maturing bonds. Beside computational complications, their setup forces borrowers to default in response to short lived bad shocks, which increases incentive to default artificially. That is, "credit tightens exactly during the period which it is most needed." Therefore, despite their success in generating the rise of unsecured debt and bankruptcy filings, their model generates counterfactually time-invariant debt discharge, since the setup prevents borrowers from accumulating large amounts of debt with intention to default. My paper's simple credit arrangement, however, avoids this counterfactual prediction and generates an increase in the debt discharge rate.

Understanding the causes and nature of the rise of household bankruptcy has important policy implications. If this rise was due to the decline of bankruptcy stigma, the policy response should have been a tighter bankruptcy code to increase the cost of bankruptcy. The 2005 change of bankruptcy code, tried to accomplish this goal. But if the rise of bankruptcy filings resulted from a more informed credit market, then tightening of the bankruptcy code was not necessarily required. While the model is too stylized to be used for determining the optimal bankruptcy code, it suggests that any model used for such policy studies needs to address the characteristics of the supply side of the unsecured consumer credit market, such as changes in the creditors' technology for rating and sorting of their borrowers.

Last but not the least, is the implications of the consumer credit market trends for the aggregate variables. Huggett [19] and Aiyagari [4] pointed out the importance of credit limits for households' precautionary saving motives and therefore the aggregate price of capital. This paper does not address the saving decision of households, but tackles the question of how credit limits are allocated, which has an important role for saving decisions. It is true that households who borrow on their credit cards usually do not belong to the top quartile of the wealth distribution and therefore their contribution to the aggregate productive capital stock is not significant. However, access to unsecured credit could have an important implication for the entrepreneurial activities at the household level which should not be omitted in a thorough welfare analysis of the consumer credit market.

[48] Mateos-Planas and Rios-Rull [23] show how the *credit line* contract (combination of an interest rate and a credit limit) arises endogenously in the equilibrium.

[49] At least partly to indicate their "*good faith*" to their bankruptcy judge.

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Appendix

Lemma 1 *Borrowing/payment at the credit limit, $b^* = b(T) \geq -\rho L$, satisfies:*

$$u'(y^I + b^*) \quad \leq \quad \frac{[u(y^I + b^*) + \delta W(L;\theta)] - [u(\theta y^I) + \delta W^D(\theta)]}{\rho L + b^*},$$

with equality if $b^* > -\rho L$. Moreover, if (5) holds then (6) uniquely determines b^*.

Proof. If (5) does not hold, i.e. agent does not find it optimal to default at the credit limit, then $b^* = -\rho L$, which satisfies (6). If (5) holds and agent defaults once reaches her credit limit, then (6) uniquely determines b^*. To see this point note that (6) can be rearranged as:

$$u'(y^I + b^*)(\rho L + b^*) - [u(y^I + b^*) + \delta W(L;\theta)] + [u(\theta y^I) + \delta W^D(\theta)] \quad = \quad 0. \tag{17}$$

If (6) holds, then left hand side of (17), which is decreasing in b^* due to the concavity of $u(\cdot)$, is positive for $b^* \geq -\rho L$. Derivative of the left hand side of (17) with respect to b^* is $u''(y^I + b^*)(\rho L + b^*)$, which is negative for $b^* \geq -\rho L$. Moreover, since $u(\cdot)$ is strictly concave as $b^* \to \infty$, the left hand side approaches zero. Then by continuity, (6) has a unique solution.

The optimality of the unique solution for (17) follows from using calculus of variation for borrowing amount $\mathbf{b}$, when $D = L - \epsilon$, for a very small $\epsilon > 0$. Suppose agent wants to maximize her utility for the next $\overline{\triangle t}$ period of time, where $\overline{\triangle t}$ is small enough. Also, suppose she will definitely default afterward. If agent borrows a constant stream of $\mathbf{b}$ before reaching her credit limit, when she is going to default, then it takes approximately $\triangle t = \frac{\epsilon}{\rho D + \mathbf{b}}$ units of time to reach to the credit limit. So agent will approximately receive a value equal to:

$$(\frac{\epsilon}{\rho D + \mathbf{b}})[u(y^I + \mathbf{b}) + \delta W(L;\theta)] + (\overline{\triangle t} - \frac{\epsilon}{\rho D + \mathbf{b}})[u(\theta y^I) + \delta W^D(\theta)].$$

Taking the derivative of this expression with respect to $\mathbf{b}$ and setting it equal to zero yields (17). ■

Lemma 3 *For $L_1 < L_2$, if in the solution for (1) it is optimal to increase debt up to credit limit, then:*

$$b(L_1; L_1) \quad < \quad b(L_1; L_2).$$

Proof. If $b(L_1; L_1) = -\rho L_1$, that is agent does not default at her credit limit, then since she increases debt level up to her limit, we should have $b(L_1; L_2) > -\rho L_1$.

If $b(L_1; L_1) < -\rho L_1$, that is agent defaults at credit limit L_1, then from (5) it follows that she also defaults at credit limit L_2, therefore $b(L_1; L_1)$ and $b(L_2; L_2)$ are both governed by (17). From (17) it follows that:

$$\frac{db^*}{dL} \quad = \quad \frac{-\rho u'(y^I + b^*) + \delta_\theta W_D(L;\theta)}{u''(y^I + b^*)(\rho L + b^*)}. \tag{18}$$

Now comparing (18) and (12), it follows that

$$\frac{db^*}{dL} \quad > \quad \left.\frac{db(D;L)}{dD}\right|_{D=L},$$

hence by continuity of b^* and $b(D; L)$, it follows that $b(L_1; L_1) < b(L_1; L_2)$. ■

Lemma 5 *If $F(\cdot)$ is concave, then $W_D(D;\theta)$ is decreasing in D for $D \in [0, \frac{(1-\theta)\underline{y}^P}{r}]$ and increasing for $D > \frac{(1-\theta)\underline{y}^P}{r}$.*

Proof. For $D \in [0, \frac{(1-\theta)\underline{y}^P}{r}]$, concavity of $W(D;\theta)$ follows from concavity of $u(\cdot)$, since agent does not default after realization of permanent income. For $D > \frac{(1-\theta)\underline{y}^P}{r}$ we have:

$$
\begin{aligned}
W_{DD}(D;\theta) &= \frac{d}{dD}\left\{-\int_{\frac{rD}{1-\theta}}^{\infty} u'(y^P - rD)dF(y^P)\right\} \\
&= \frac{r}{1-\theta}u'(\frac{\theta rD}{1-\theta})F'(\frac{rD}{1-\theta}) + r\int_{\frac{rD}{1-\theta}}^{\infty} u''(y^P - rD)dF(y^P) \\
&= \frac{r}{1-\theta}\cdot u'(\frac{\theta rD}{1-\theta})F'(\frac{rD}{1-\theta}) + ru'(y^p - rD)F'(y^p)|_{y^p=\frac{rD}{1-\theta}}^{y^p=\infty} - r\int_{\frac{rD}{1-\theta}}^{\infty} u'(y^P - rD)d^2F(y^P) \\
&= \frac{\theta r}{1-\theta}\cdot u'(\frac{\theta rD}{1-\theta})F'(\frac{rD}{1-\theta}) - r\int_{\frac{rD}{1-\theta}}^{\infty} u'(y^P - rD)d^2F(y^P) \\
&> 0
\end{aligned}
$$

where the inequality follows from the concavity of $F(\cdot)$ and $u'(\cdot) > 0$. ■

Lemma 6 *For a concave $F(\cdot)$, if $(\delta + r - \rho)u'(y^I) + \delta W_D(\frac{(1-\theta)\underline{y}^P}{r};\theta) > 0$, then for any credit limit, it is optimal for agent to increase her debt up to any credit limit.*

Proof. Using the previous lemma, if $(\delta + r - \rho)u'(y^I) + \delta W_D(\frac{\underline{y}^P(1-\theta)}{r};\theta) > 0$, then it is guaranteed that $\frac{\delta + r - \rho}{\delta}\cdot u'(y^I) + W_D(D;\theta) > 0$ for $\forall D > 0$.

Suppose agent stops increasing her debt level above $\overline{D}$, that is $b(\overline{D}) = -\rho\overline{D}$, while $\overline{D} < L$. In this case the marginal benefit from increasing the borrowing amount is $(\delta + r)u'(y^I + b(\overline{D}))$ and the marginal cost is $\rho u'(y^I + b(\overline{D})) - \delta W_D(\overline{D};\theta)$. Concavity of $u(\cdot)$ guarantees $(\delta + r - \rho)u'(y^I + b(\overline{D})) + \delta W_D(\overline{D};\theta) > 0$, hence it is optimal to increase $b(\overline{D})$ from $-\rho\overline{D}$ and therefore raise debt level above $\overline{D}$. ■

Theorem 9 *Borrowing, $b^\theta(D;L)$, is increasing in agent's riskiness, θ.*

Proof. First, notice that the marginal cost of debt after the realization of permanent income, $-W_D(D;\theta)$, is decreasing in riskiness, θ. That is:

$$
W_{D,\theta}(D;\theta) = \frac{r^2 D}{(1-\theta)^2}\cdot u(\frac{\theta rD}{1-\theta})\cdot F'(\frac{rD}{1-\theta}) \geq 0.
$$

Lemma (8) guarantees that $b^\theta(L;L)$ is increasing in θ. Next we prove that $b^\theta(0,L)$ is also increasing in θ. Define $V(\theta;L;D)$ as the value of having debt level $D < L$ with a contract (L,ρ), while waiting for the realization of the permanent income by a type θ agent. By definition, we have:

$$
\begin{aligned}
V(\theta;L;D) &= \frac{1}{r+\delta}\left\{u(y^I + b^\theta(D;L)) + \delta W(D;\theta) + (b^\theta(D;L) + \rho\cdot D)V_D(\rho;L;D),\theta)\right\} \\
&= \frac{1}{r+\delta}\left\{u(y^I + b^\theta(D;L)) + \delta W(D;\theta) - (b^\theta(D;L) + \rho\cdot D)u'(y^I + b^\theta(D;L)\right\}, \quad (19)
\end{aligned}
$$

where the second equality follows from the optimality of the choice of $b^\theta(D; L)$. That is, $V_D(\theta; L; D) + u'(y^I + b^\theta(D; L)) = 0$, for $D < L$. Using (19) for $D = 0$, we have:

$$V(\theta; L; 0) \quad = \quad \frac{1}{r+\delta}\left\{u(y^I + b^\theta(0; L)) + \delta W(0, \theta) - b^\theta(0; L)u'(y^I + b^\theta(0; L))\right\}. \qquad (20)$$

Notice that, since the support of the permanent income distribution, $F(\cdot)$, is bounded above zero, agent does not default on zero debt level. Therefore $W(0, \theta)$ is independent of θ, hence we have:

$$\frac{d}{d\theta}\{V(\theta; L; 0)\} \quad = \quad \frac{1}{r+\delta}\left\{-b^\theta(0; L)u''(y^I + b^\theta(0; L))\right\} \cdot \frac{d}{d\theta}\left\{b^\theta(0; L)\right\}. \qquad (21)$$

Since default cost is decreasing in θ, and therefore agents with higher θ can derive higher utility by following the borrowing stream of those agents with lower θ, by construction, $V(\theta; L; 0)$ is increasing in θ. Following (21), and by the concavity of $u(\cdot)$, $b^\theta(0; L)$ will be also increasing in θ.

Now suppose $\exists \tilde{D} > 0$ and $\theta_1 < \theta_2$ such that $b^{\theta_1}(\tilde{D}; L) > b^{\theta_2}(\tilde{D}; L)$. Since $b^{\theta_1}(0; L) \leq b^{\theta_2}(0; L)$, there exist $0 \leq D^* \leq \tilde{D}$ such that $b^{\theta_1}(D^*; L) = b^{\theta_2}(D^*; L)$. Without loss of generality, we can assume D^* is the supremum of such D^*s, and by continuity of $b^{\theta_1}(\cdot; L)$ and $b^{\theta_2}(\cdot, L)$ in debt level, we still have $b^{\theta_1}(D^*; L) = b^{\theta_2}(D^*; L)$. But then using (12) and the fact that $-W_D(D; \theta)$ is decreasing in θ, we have:

$$\frac{db^{\theta_1}(D; L)}{dD}|_{D=D^*} \quad < \quad \frac{db^{\theta_2}(D; L)}{dD}|_{D=D^*}$$

which contradicts $\exists \tilde{D} > 0$ and $\theta_1 < \theta_2$. ■

Lemma 11 *If the support of $\psi(\cdot|\tilde{\theta})$ is bounded away from zero, the utility function $u(\cdot)$ is unbounded from above and the expected present value of all future income for an agent with signal $\tilde{\theta}$ is bounded, then:*

$$\lim_{L\to\infty} \Pi(\tilde{\theta}; L) < 0 \ \ \forall \rho.$$

Proof. Let's note

$$E_{\theta|\tilde{\theta}} V(\theta; L) = \int V(\theta; L) d\psi(\theta|\tilde{\theta}).$$

First we show for any $\rho \geq r$ we have $\lim_{L\to\infty} V(\theta; L) \to \infty$. Next we show if $\Pi(\tilde{\theta}; (L, \rho)) \geq 0$ then $E_{\theta|\tilde{\theta}} V(\theta; L)$ is bounded from above.

For any credit limit L, consider the plan of borrowing $b = \frac{\rho L}{e^\rho - 1}$ during $t \in [0, 1]$, and then defaulting. Also always defaulting after realization of permanent income. The expected present value of lifetime utility of this plan is equal to

$$\int \frac{1}{\delta + r}\left[(1 - e^{-(\delta+r)})u(y^I + \frac{\rho L}{e^\rho - 1}) + e^{-(\delta+r)}u(\theta y^I) + \delta V^D(\theta)\right] d\psi(\theta|\tilde{\theta}).$$

This is a lower bound for $E_{\theta|\tilde{\theta}} V(\theta; L)$, and if the support of $\psi(\cdot|\tilde{\theta})$ is bounded away from zero, this lower bound goes to infinity as $L \to \infty$, due to the unboundedness of $u(\cdot)$. Hence $E_{\theta|\tilde{\theta}} V(\theta; L)$ is unbounded as $L \to \infty$.

Let $\overline{y_{\tilde{\theta}}}$ denotes the stream of incomes which has the same present value as the expected present value of all future income for an agent with the rating signal $\tilde{\theta}$ at interest rate r. That is

$$\overline{y_{\tilde{\theta}}} = \int\int \frac{1}{r+\delta}(ry^I + \delta y^P) dF(y^P) d\psi(\theta|\tilde{\theta}).$$

Due to the concavity of $u(\cdot)$, if $\Pi(\tilde{\theta}; L) \geq 0$ then $E_{\theta|\tilde{\theta}} V(\theta; L) \leq \frac{1}{r} u(\overline{y_{\tilde{\theta}}})$. Since $E_{\theta|\tilde{\theta}} V(\theta; L)$ is bounded from above when $\Pi(\tilde{\theta}; L) \geq 0$ and is unbounded when $L \to \infty$, we conclude $\lim_{L\to\infty} \Pi(\tilde{\theta}; L) < 0$ ■