Inflation and Price Competition

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PRELIMINARY

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Motivation

- The price elasticity of demand for a given brand is primarily determined by changes in the number of customers purchasing that product, *extensive margin*, rather than changes in the quantity purchased by each customer, *intensive margin*.
  - Levin and Yun (08)
- Inflation can affect firms' price competition.

- What can we learn from micro foundation models of money about this?

Model

- Based on Rocheteau & Wright (05) and Burdett & Judd (83)
- Sellers with heterogenous production costs
- Sellers announce linear prices and commit to meet the demand at their posted prices
- Buyers get price quotes, randomly drawn from the pool of sellers, and visit the seller with the lowest price in their sampled set. (i.e. *nonsequential search*)
- A seller's price affects two margins:
  - *Extensive margin*: number of buyers visiting a seller
  - *Intensive margin*: quantity purchased by each visiting buyer

Environment

- Discrete time, infinite horizon, $t = 1, 2, ..$
- Unit one continuum of agents: half permanently *seller* & the other half *buyer*
- **Decentralized Market**
  - Sellers realize their linear production cost, α, from a distribution $\Lambda(\cdot)$ with support $[\underline{\alpha}, \overline{\alpha}]$.
  - Sellers post their price $p(\alpha)$
  - Buyers choose their search intensity $s \geq 1$, incurring utility cost $c(s)$, and sampling s sellers' prices randomly, $c(1) = c'(0) = 0$ and $c''(\cdot) > 0$
  - Buyers visit the sellers with the lowest price in their samples, and choose their demand $d(p, m)$, receiving utility $u(d)$ and spending $p \cdot d$ of their money holdings
  - Sellers incur utility cost $-D(p) \cdot d(p) \cdot \alpha$ and receive $D(p) \cdot d(p) \cdot p$ units of money
- **Centralized Market**
  - Agents receive transfer τ, readjust their money holdings and receive $U(x) - h$

Seller's Problem

- Taking buyers' search intensity, $\overline{s}$, and the price distribution of other sellers, $F(\cdot)$, as given, a seller with productivity α solves:

$$\begin{aligned} V_S(\alpha, \overline{s}) &= \max_p E\left\{D(p)d(p)(p-\alpha)\right\} \\ &= \max_p \left\{\overline{s}\cdot(1-F(p))^{\overline{s}-1}d(p)(p-\alpha)\right\}. \end{aligned}$$

with FOC:

$$\underbrace{(\overline{s}-1)\cdot\frac{f(p)}{1-F(p)}}_{\textit{extensive margin loss}} = \underbrace{\frac{1}{p-\alpha}+\frac{d'(p)}{d(p)}}_{\textit{intensive margin gain}}$$

DM Price Function

Using equilibrium condition $F(p(\alpha)) = \Lambda(\alpha)$ we have:

$$p'(\alpha) = (s-1) \cdot \frac{\lambda(\alpha)}{1-\Lambda(\alpha)} \cdot \left(\frac{1}{p(\alpha)-\alpha} + \frac{d'(p)}{d(p)} \right)^{-1}$$

Note that at the monopolistic price, p^M, we have

$$\frac{1}{p^M - \alpha} + \frac{d'(p^M)}{d(p^M)} = 0$$

Buyer's Problem

- Taking the DM price function, $p(\cdot)$, as given, a buyer with real balance, m, solves:

$$V_B(m) = \max_{s \geq 1} -c(s) + Ev_p(m),$$

where $p = \min\{p_1, ..., p_s\}$ is the lowest price offered among the s sampled sellers, and

$$v_p(m) = \max_{d(p,m) \leq \frac{m}{p}} u\left(d(p,m)\right) + EW(m - p \cdot d(p,m) + \tau),$$

which would imply:

$$d(p,m) = \min\left\{m/p, u'^{-1}(p)\right\}.$$

Buyer's Money Holding

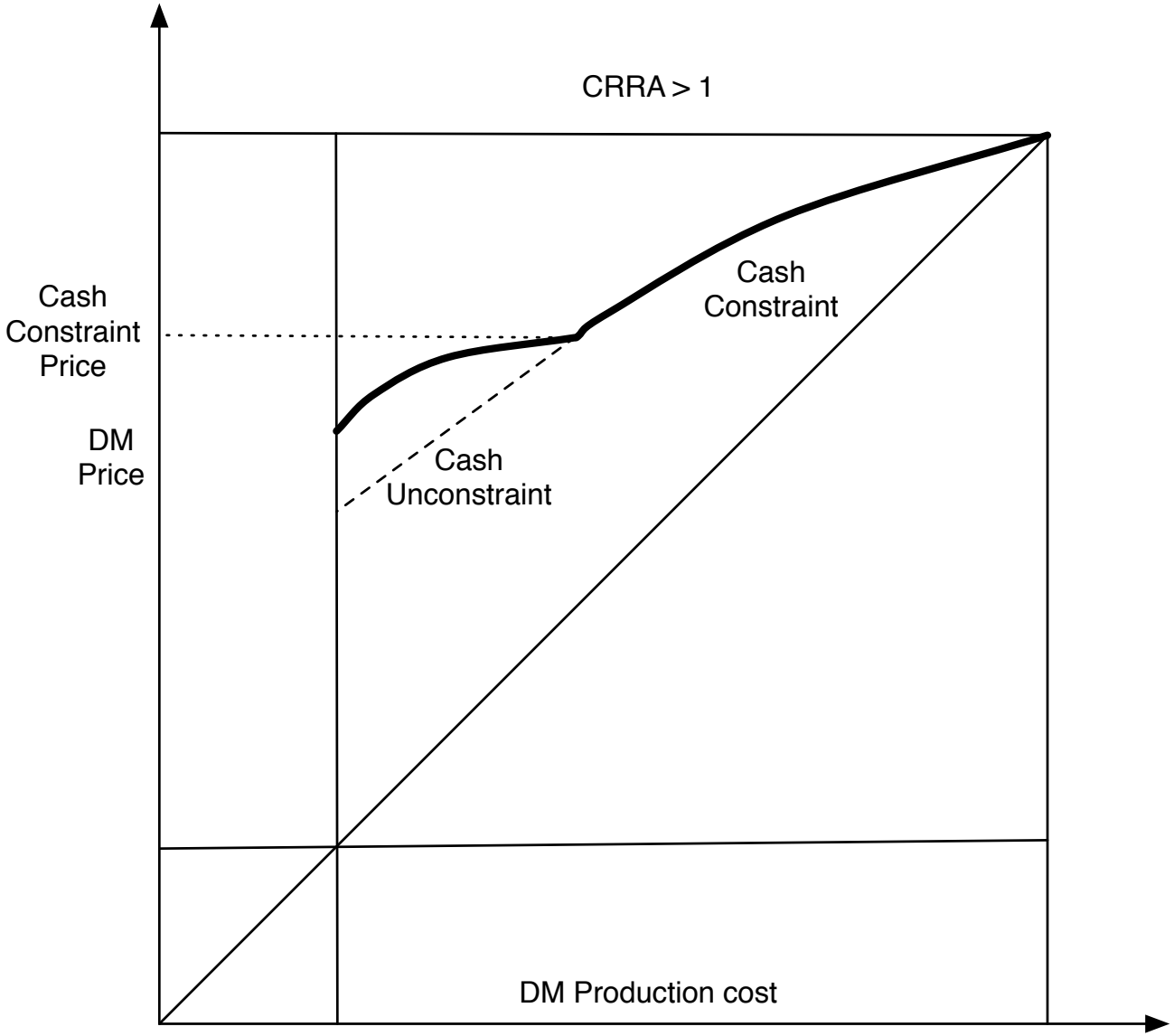
- For inflation rate, π, a buyer's optimal money holding, m^*, is determined by:

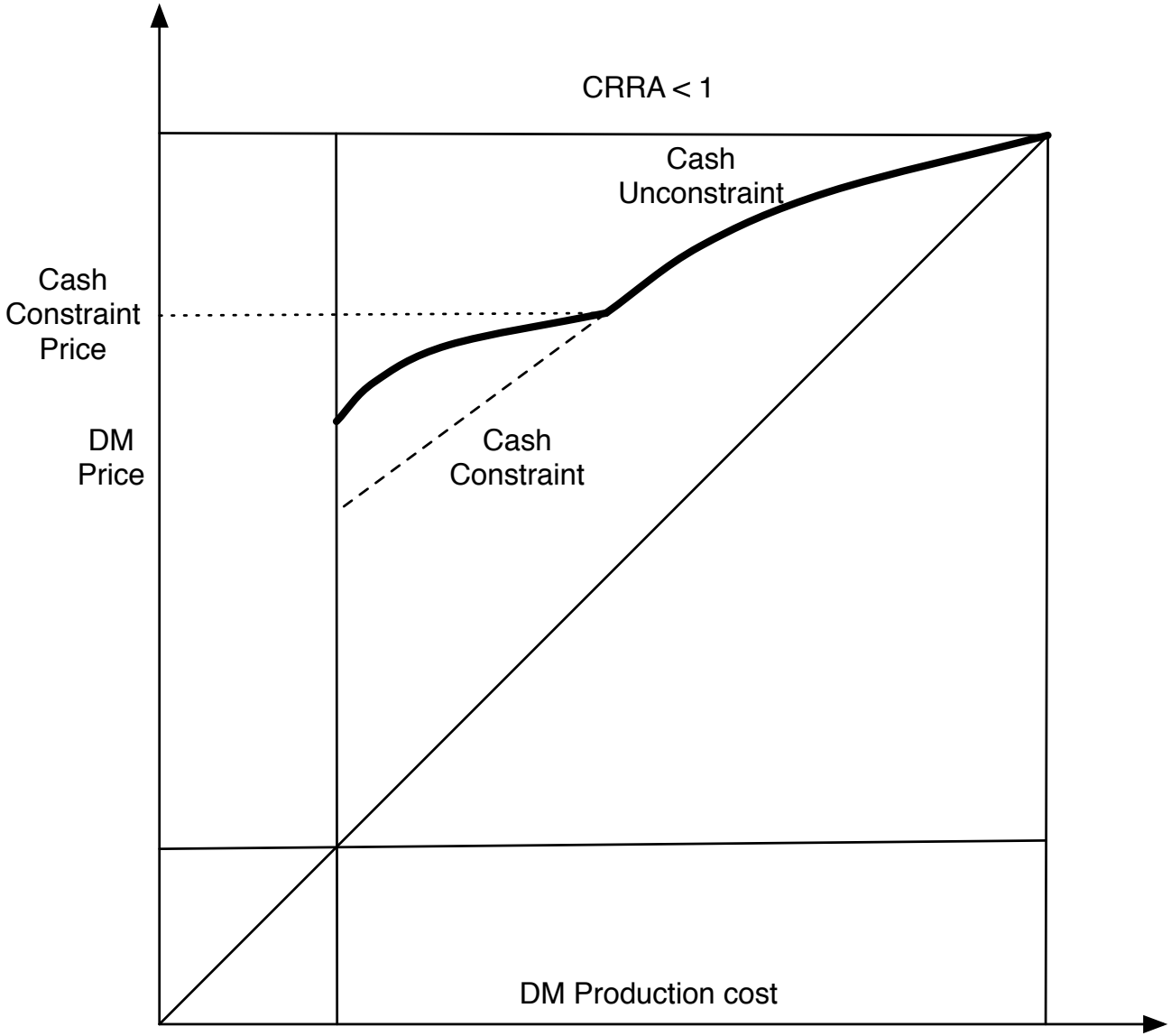
$$\begin{aligned}\frac{1}{\beta(1-\pi)} &= EV'_B(m^*) \\ &= \int_{\alpha} \frac{\{1+[u'(d(p,m^*)-p(\alpha)]\cdot d_2(p,m^*)\}}{\mathbf{d}(1-(1-\Lambda(\alpha))^s)}.\end{aligned}$$

Note that if the cash constraint does not bind, that is $d(p,m) = u'^{-1}(p)$, then bringing more money to the next period only delivers utility in the centralized market.

Fixed Search Intensity

- Assume that the monopoly markup is bounded, then, $\alpha \to \overline{\alpha}$ implies $p(\alpha) \to \alpha$. (The seller with the highest price charges zero markup.)
- $p'(\overline{\alpha}) = 1 - \frac{1}{s} > 0$.
- If $-u''(d)d/u'(d)$ is not too much smaller than one, then
  - the price function is increasing in α
  - the markup, $p(\alpha) - \alpha$ is decreasing in α.
- Depending on the spending on DM consumption being increasing or decreasing in DM price (i.e. $-u''(d)d/u'(d)) > 1$ or < 1)
  - *CRRA*> 1: Buyers are cash-constrained at high cost sellers
  - *CRRA*< 1: Buyers are cash-constrained at low cost sellers
- Elasticity of DM demand with respect to DM price differs for cash constrained and unconstrained buyers.
  - Equilibrium price function has a kink at the *cash-constraint price*: $d(p, m) \cdot p = m$.


CRRA > 1
Cash
Constraint
Price
DM
Price
Cash
Constraint
Cash
Unconstraint
DM Production cost


CRRA < 1
Cash
Unconstraint
Cash
Constraint
Price
DM
Price
Cash
Constraint
DM Production cost

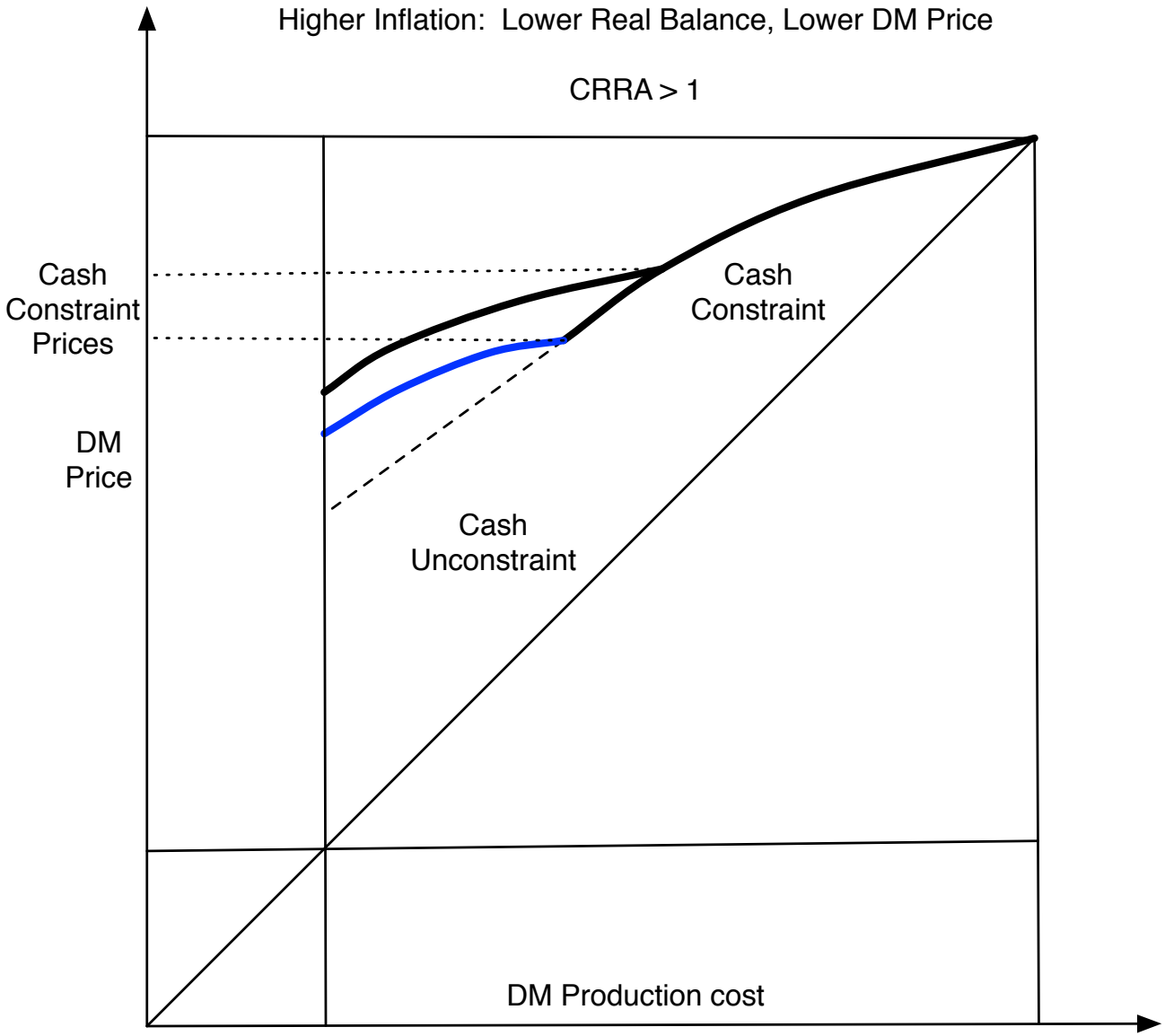
Inflation with Fixed Search Intensity

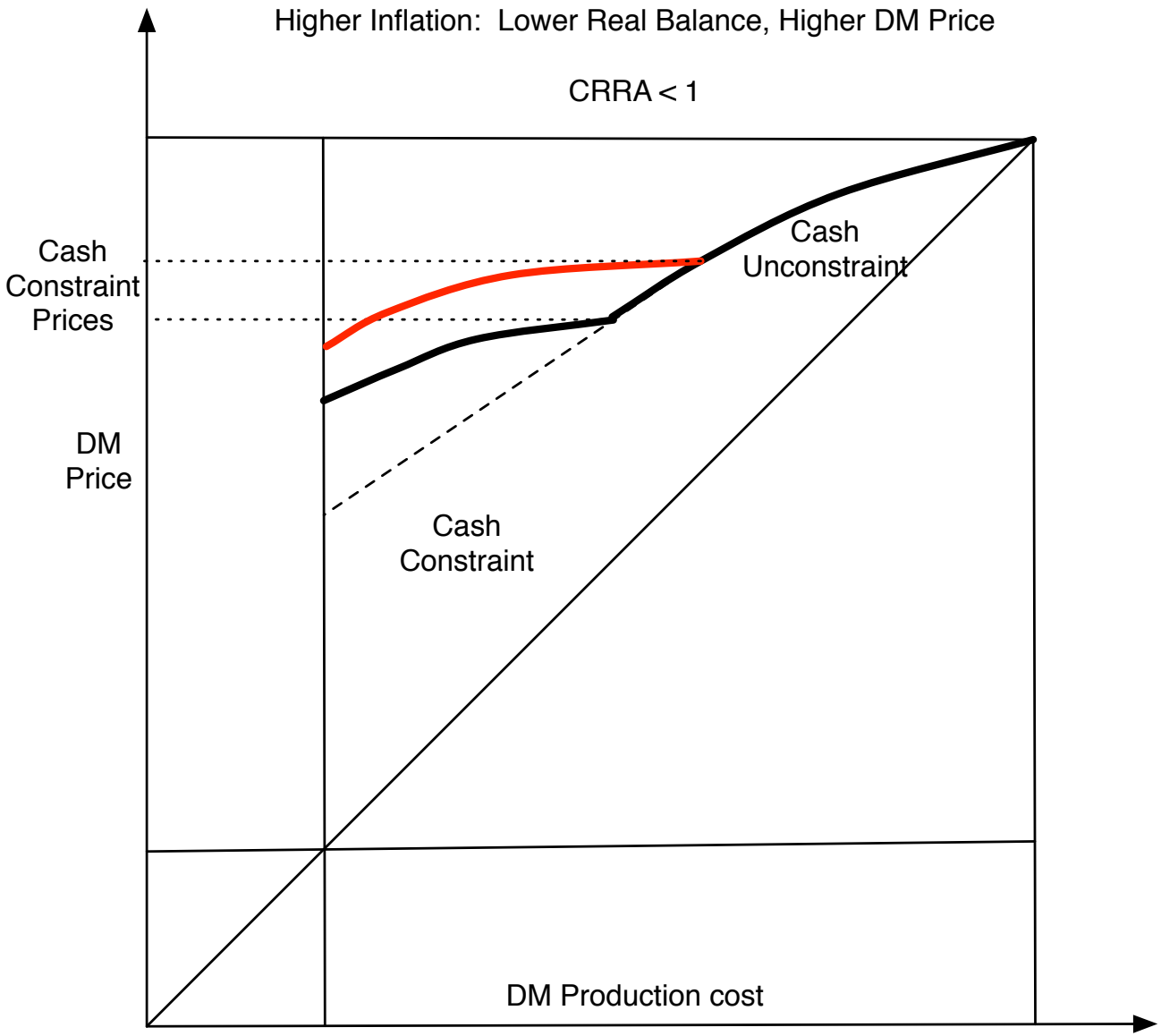
- Optimal money holding, m^*, is decreasing in inflation.
  - Lower real balance increases the range of sellers at which the buyer would be cash constrained .
- Depending on the spending on DM consumption being increasing or decreasing in DM price (i.e. $-u''(d)d/u'(d)) > 1$ or < 1)
  - *CRRA*> 1: Price function is decreasing in inflation
  - *CRRA*< 1: Price function is increasing in inflation
- Note:

$$p'(\alpha) = (s-1)\cdot\frac{\lambda(\alpha)}{1-\Lambda(\alpha)}\cdot\left(\frac{1}{p(\alpha)-\alpha}+\frac{d_1(p,m)}{d(p,m)}\right)^{-1}$$

and

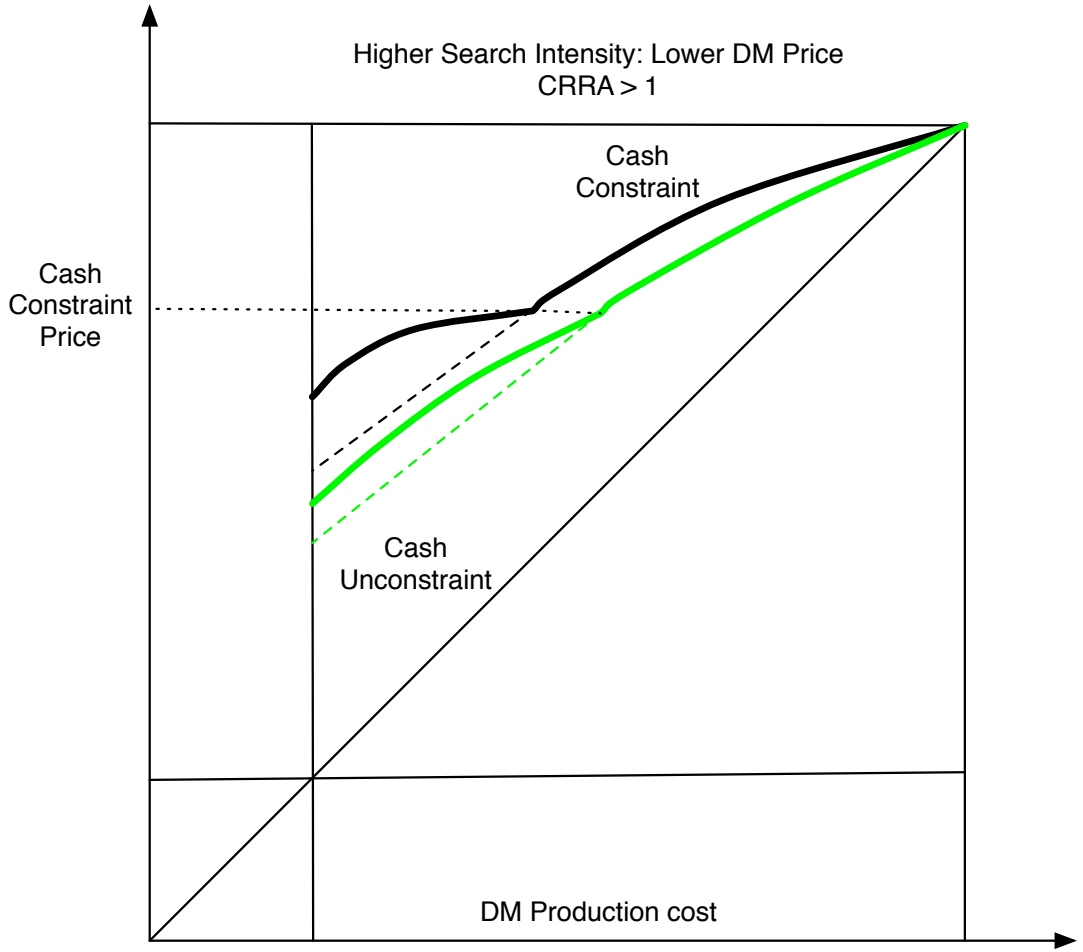
$$\frac{d_1(p,m)}{d(p,m)} = (\frac{-1}{p})\cdot\begin{cases} 1 & u'^{-1}(p) > m/p \\ \frac{u'(d)}{-u''(d)\cdot d} & u'^{-1}(p) \leq m/p \end{cases}$$

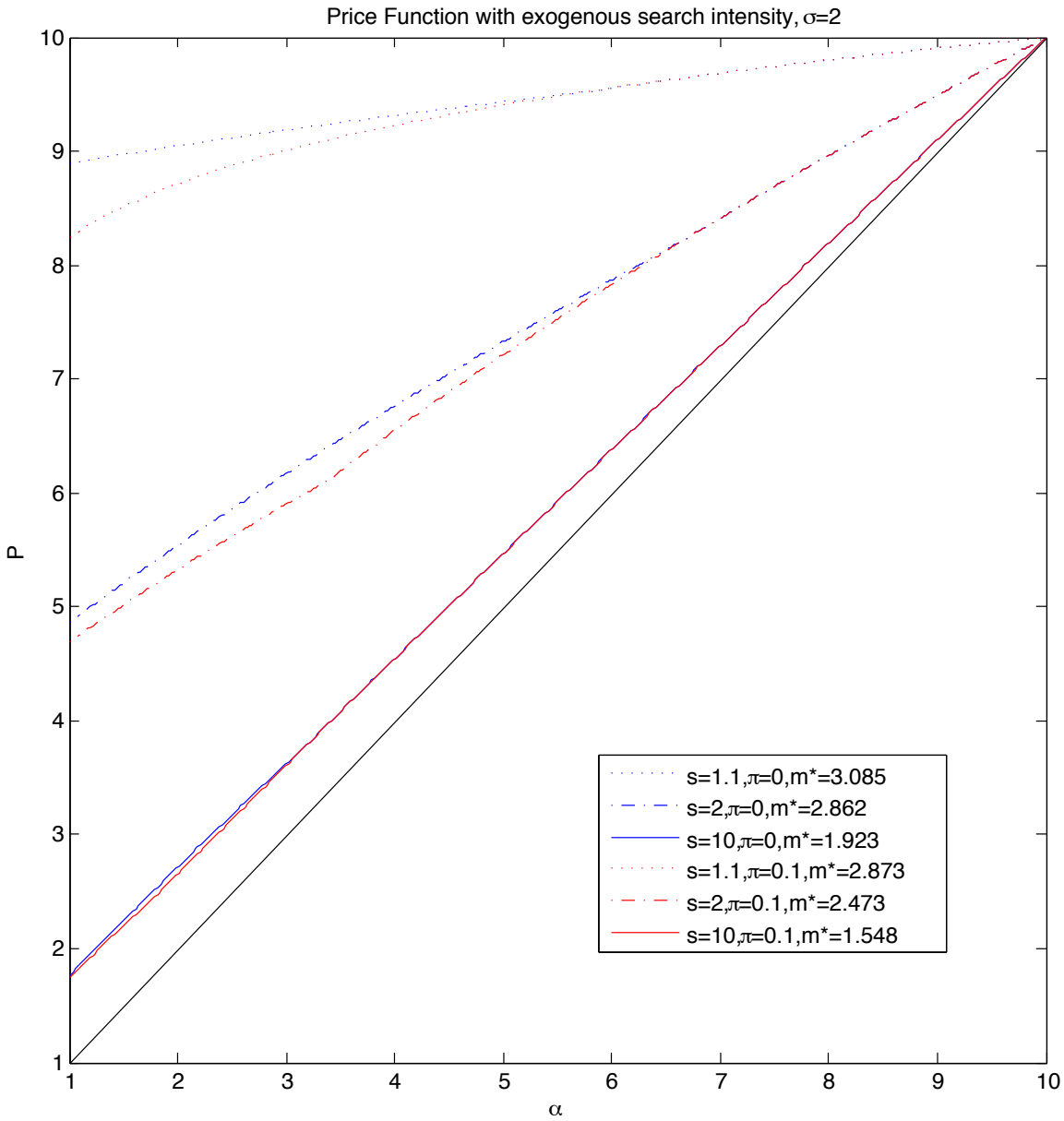

Higher Inflation: Lower Real Balance, Lower DM Price
CRRA > 1
Cash
Constraint
Prices
DM
Price
Cash
Constraint
Cash
Unconstraint
DM Production cost

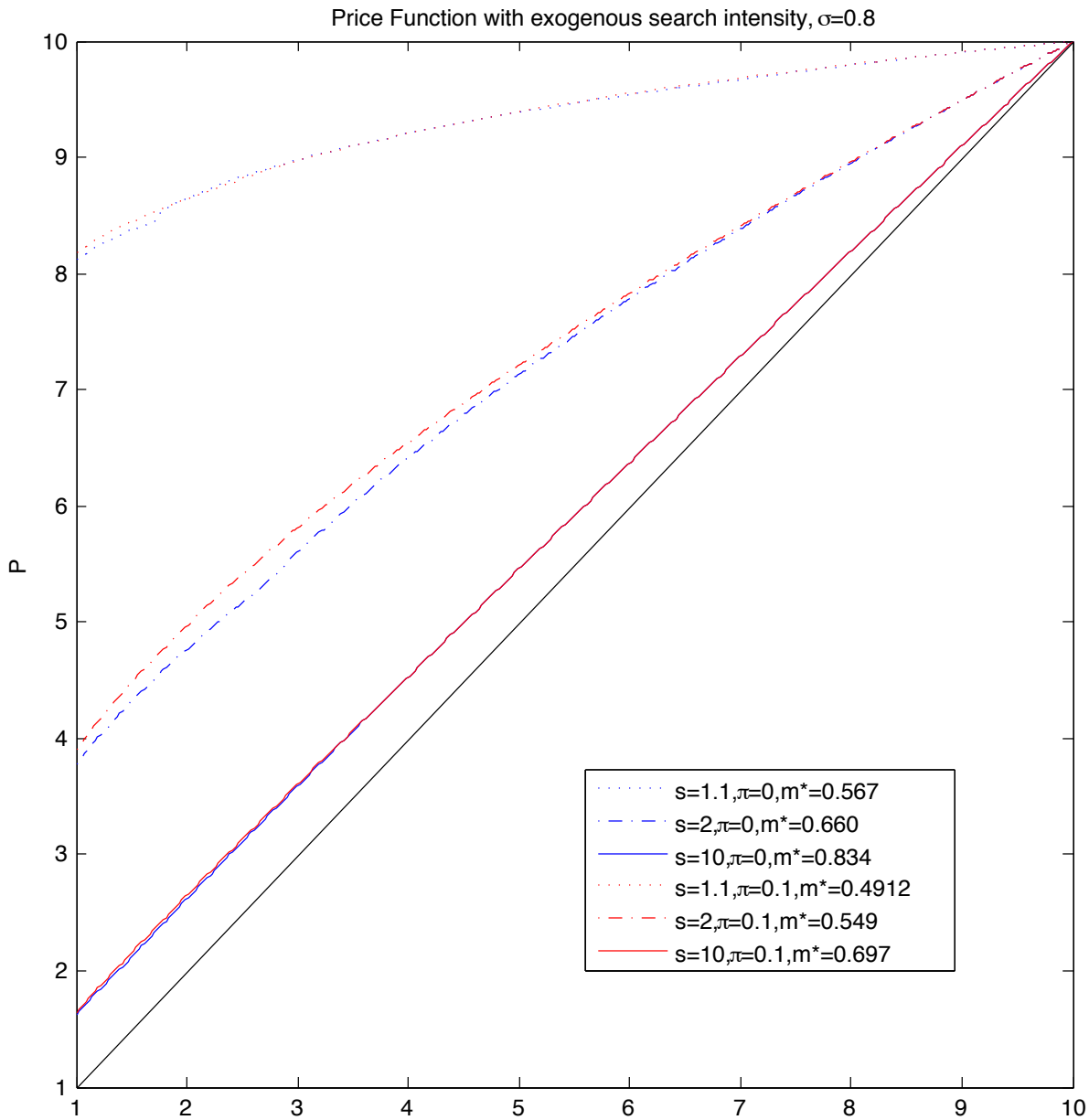

Higher Inflation: Lower Real Balance, Higher DM Price
CRRA < 1
Cash
Constraint
Prices
DM
Price
Cash
Unconstraint
Cash
Constraint
DM Production cost

Search Intensity and Price

Price function is decreasing in the search intensity, s.




Price Function with exogenous search intensity, σ=2
P
α
s=1.1,π=0,m*=3.085
s=2,π=0,m*=2.862
s=10,π=0,m*=1.923
s=1.1,π=0.1,m*=2.873
s=2,π=0.1,m*=2.473
s=10,π=0.1,m*=1.548


Price Function with exogenous search intensity, σ=0.8
P
10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
6
7
8
9
10
s=1.1,π=0,m*=0.567
s=2,π=0,m*=0.660
s=10,π=0,m*=0.834
s=1.1,π=0.1,m*=0.4912
s=2,π=0.1,m*=0.549
s=10,π=0.1,m*=0.697

Welfare and Inefficiency

$$\Psi = -c(s) + \int_{\underline{\alpha}}^{\overline{\alpha}} \{u(d(p, m^*)) - \alpha \cdot d(p, m^*)\} \mathbf{d}(1 - (1 - \Lambda(\alpha))^s)$$

s.t.

$$d(p, m) = \min \left\{ m/p, u'^{-1}(p) \right\}$$

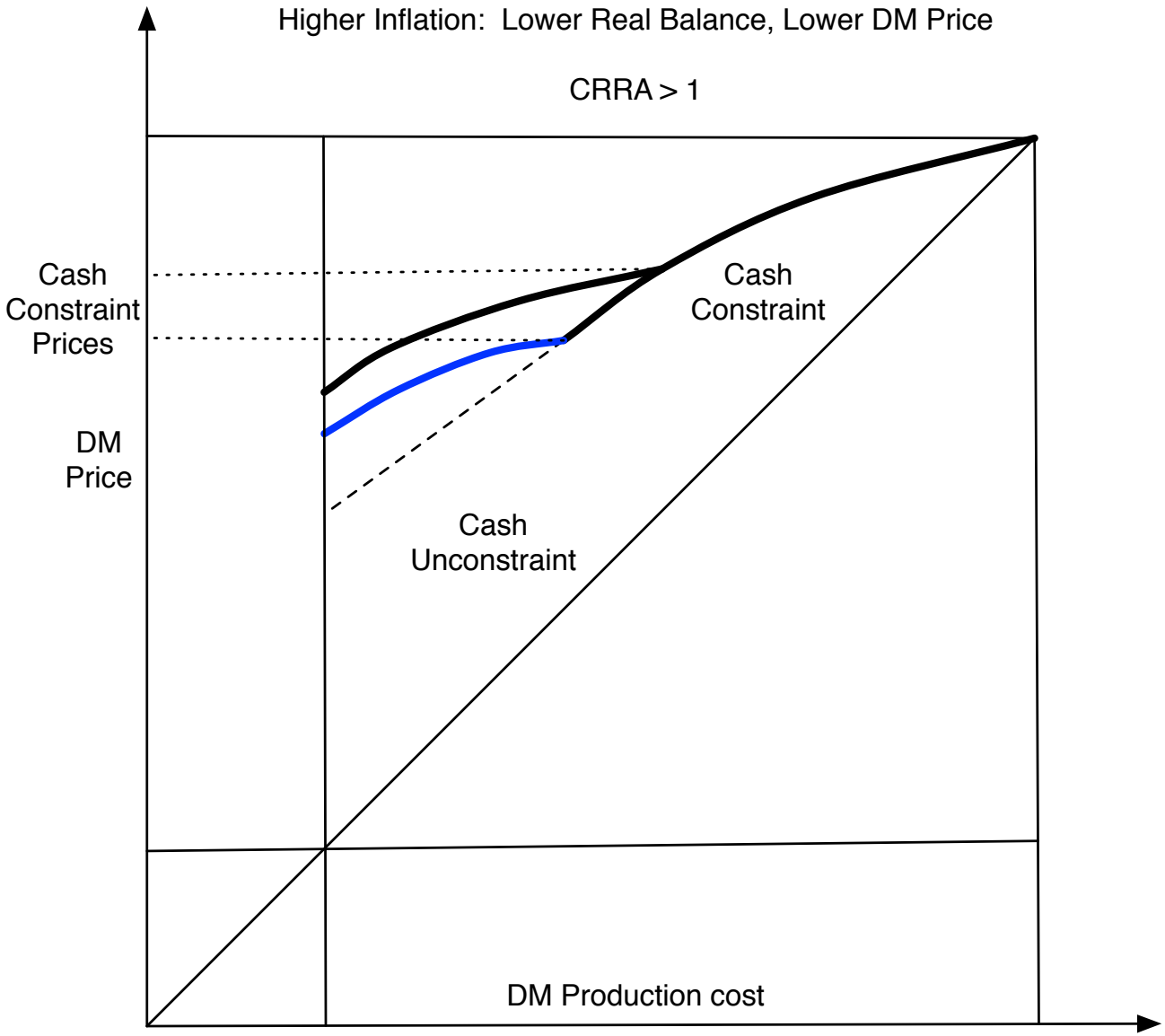
$$p'(\alpha) = (s-1) \cdot \frac{\lambda(\alpha)}{1 - \Lambda(\alpha)} \cdot \left(\frac{1}{p(\alpha) - \alpha} + \frac{d'(p)}{d(p)} \right)^{-1}$$

Three sources of inefficiency

- Buyers end up at high cost sellers
- Sellers charge higher than their marginal price
- Buyers are cash-constrained at some sellers

Inflation, Output and Welfare

- For $CRRA > 1$, increasing inflation from the Friedman Rule:
  - Increases DM output
  - Improves aggregate productivity of DM market
  - Has ambiguous effect on welfare
- Inflation makes buyers at high-cost sellers more cash-constrained
  - Reducing output by high-cost sellers, and making buyers at those sellers worse off
- Lowering real balance induces the marginal seller to reduce its price
  - Marginal seller's previous price would not make buyers cash-constrained but now it would
- Price competition lowers the price of low-cost sellers
  - Increasing output by low-cost sellers, and making buyers at those sellers better off
- Inflation redistributes from unlucky buyers, those at high-cost sellers, to the lucky ones, those at low-cost sellers.


Higher Inflation: Lower Real Balance, Lower DM Price
CRRA > 1
Cash
Constraint
Prices
DM
Price
Cash
Constraint
Cash
Unconstraint
DM Production cost

Endogenous Search Intensity

- Taking the other buyers' real balance, $\overline{m}$, and the search intensity, $\overline{s}$, and therefore the DM price function, $p_{\overline{m},\overline{s}}(\cdot)$, as given, a buyer chooses
  - real balance, m^*
  - search intensity, s^*

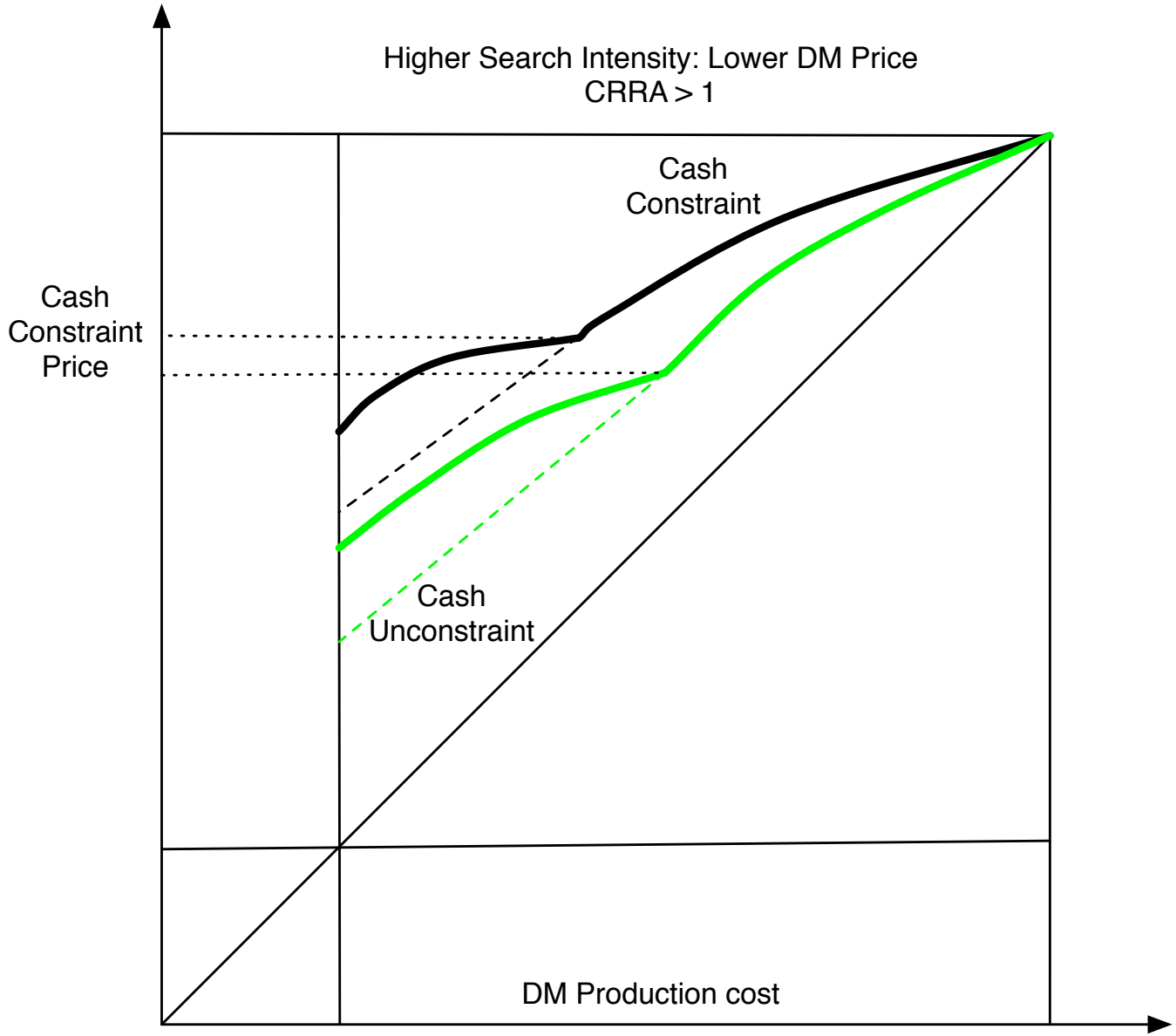
$$c'(s^*) = \int \left\{ d(p_{\overline{m},\overline{s}}, m^*) \cdot u'(d(p_{\overline{m},\overline{s}}, m^*)) \cdot \frac{p'_{\overline{m},\overline{s}}(\alpha)}{p_{\overline{m},\overline{s}}(\alpha)} \right\} \cdot(-\ln(1-\Lambda(\alpha)))(1-\Lambda(\alpha))^{s^*} \cdot \mathbf{d}\alpha$$

$$\frac{1}{\beta(1-\pi)} = \int_{\alpha} \{1 + [u'(d(p_{\overline{m},\overline{s}}, m^*) - p_{\overline{m},\overline{s}}(\alpha)] \cdot d_2(p_{\overline{m},\overline{s}}, m^*)\} \cdot \mathbf{d}(1-(1-\Lambda(\alpha))^{s^*})$$

- In the symmetric equilibrium: $\overline{m} = m^*$ and $\overline{s} = s^*$.

Suboptimal Search Intensity?

- Higher $\bar{s}$ could lower the DM prices, $p_{\bar{s}}(\cdot)$, which in turn increases the payoff to increase s^*.
- This type of complementarity results in suboptimal equilibrium.
- But could the monetary authority increase search intensity?


Higher Search Intensity: Lower DM Price
CRRA > 1
Cash
Constraint
Cash
Constraint
Price
Cash
Unconstraint
DM Production cost

Inflation and Search Intensity

- Facing inflation, and a fixed search intensity, buyers cannot lower their real balance very much, as it would raise the welfare lose at high price (i.e. high cost) sellers increasingly.
- But with *endogenous* search intensity, buyers can increase their search intensity to lower the chance of ending up at high-price (high-cost) sellers.
  - This effect can lower DM prices in the face of inflation even futher, lowering the inefficiency due to price distortion
  - DM output and efficiency could increase even further
  - The effect on welfare is still ambiguous, as we have to consider the welfare loss due to search cost as well.