STABILIZED FINITE ELEMENT METHODS FOR INCOMPRESSIBLE FLOWS WITH EMPHASIS ON MOVING BOUNDARIES AND INTERFACES

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Abstract

Two finite element formulations for incompressible fluid dynamics are presented. The space-time velocity-pressure formulation is used for flow problems involving moving boundaries and interfaces. The formulation is based on a time-discontinuous Galerkin method, supplemented with least-squares-type stabilization terms. The discussion centers on the application of the method to various problems with deforming domains. In particular, the formulation is employed to simulate three-dimensional sloshing in a storage tank subjected to external vibrations.

The stress-velocity-pressure formulation is developed, essentially for viscoelastic flows. Treatment of stress as a separate unknown allows for inclusion of complex constitutive relations. Least-squares-type stabilization terms provide, once more, robustness to otherwise potentially unstable formulation. The method is first tested on Newtonian fluid flows past a circular cylinder in two dimensions. Then, by using simple viscoelastic constitutive model, the formulation is applied to a standard test problem.

The strategies for the solution of large systems of equations arising from the finite element discretization of the above formulations are also discussed. Particular emphasis is placed on iterative methods and the implementations on massively parallel supercomputers, paving the way for solving very large-scale practical problems, including those in three dimensions.

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List of Figures

List of Tables

List of Boxes

Contents